Complete Rerouting Protection

T. Stidsen and P. Kjærulff

Informatics and Mathematical Modeling, Technical University of Denmark, DK-2800 Kongens Lyngby, Denmark

tks@imm.dtu.dk

Protection of communication against network failures is becoming increasingly important and in this paper we present the most capacity efficient protection method possible, the complete rerouting protection method, when requiring that all communication should be restored in case of a single link network failure. We present a linear programming model of the protection method and a column generation algorithm. For 6 real world networks, the minimal restoration overbuild network capacity is between 13% and 78%. We further study the importance of the density of the network, derive analytical bounds and study methods to speed up the column generation algorithm.

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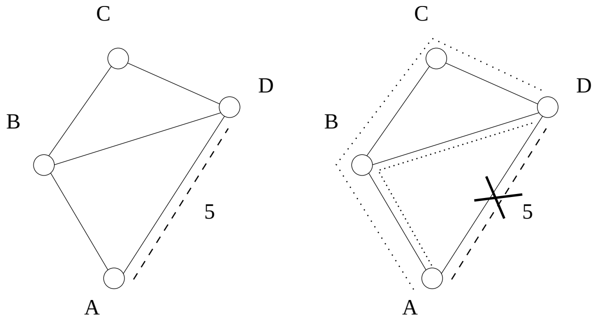
1. Introduction

Reliability of communication networks has become of major importance in the last decades. This has lead to a significant research in different technologies which can make the communication networks more reliable. A communication network may fail in a number of ways: Power outages on switches, switch software failure, switch hardware failure, cable cuts etc. Cable cuts by e.g. entrepreneurs is one of the most frequent types of network failures and they are difficult to prevent. This type of network failures, called link failures, is the focus in this paper.

Whenever a link in a communication network fails, communication either has to wait for the link to be repaired or the communication has to be rerouted through unharmed parts of the network. Rerouting in case of link failure can usually be performed much faster than physical recovery of the failed link, but it requires additional capacity on the network. This extra capacity we will term the Restoration Over Build (ROB) network capacity.

The communication networks considered in this paper are the circuit switched high capacity backbone networks, which transports a wide variety of communication traffic: Telephone calls, internet, etc. We consider the static case where a fixed connection, called a circuit, of a certain bandwidth is setup between two switches (nodes) in the network. In Figure 1 (a) a bidirectional circuit is established between node A and node D on the link AD of communication volume 5, marked as the dashed line. Hence there needs to be a communication capacity of 5 on link AD. If link AD fails, see Figure 1 (b), two alternative routes marked with dotted lines, can be used to reestablish the circuit: One using the links AB, BD and one using the links AB, BC, CD. Both routes may also be used to collectively cover the loss of communication. In order to be able to recover from the AD link failure, we need at least 5 units of unused capacity on the alternative routes. If we assume that the circuit AD is the only communication circuit, the non-failure (NF) network capacity is 5. If the circuit is rerouted along the AB, BD path we need 5 units capacity on both links and hence the ROB network capacity is 10. The Relative Restoration



(a) No failure (b) Single link failure

Over Build (RROB) network capacity is the ROB network capacity divided with the NF network capacity, i.e. RROB = 2.0.

The routing and protection model we describe here is chosen to be as simple as possible. The communication circuits we consider are bidirectional. We only consider single link failures. We assume that there are no limits on the capacity of each link and that the cost of a routing and protection plan is the sum of the cost of the required link capacities. We assume no limits on the paths which may be used, except that failed links cannot be used at all. We further assume that 100% protection is required, i.e. that the full communication flow is re-established. We assume that we can split the communication in any way necessary.

A number of different protection methods have been suggested, see Section 2 for a brief overview of some of these. They differ in (at least) two important aspects: The speed of the recovery and network capacity required. The speed of the recovery depends on a number of intrinsic details in the protection method which is not the focus of this paper, see [1] for a detailed discussion of the required recovery time.

It is important to acknowledge that a tradeoff exists: The faster the recovery, the more capacity is required. Hence it is impossible to find *one* overall best rerouting method. The best choice depends on the particular situation faced by the telecommunication network operator depending on e.g. the maximally allowed rerouting time, the technology available for the switches etc.

The fastest protection method is *1+1* (APS) protection [2], where the signal is sent over two physically disjoint paths from start to end of the network. This means that any single link failure can be recovered in the destination node, because if the signal disappears, the destination node can simply use the signal from the other path. Unfortunately 1+1 protection requires RROB network capacity to be significantly more than 1.0. 1+1 protection is one of the two most widely used protection methods for circuit switched networks, the other is ring protection.

In this paper we present a new protection method: Complete Rerouting (CR). This is the most capacity efficient protection method for circuit switched networks and it is, to the best of our knowledge, the first time it has been described, though we have cited it in [3]. The required capacity depends both on the protection method *and* on the efficiency of the planning method used when planning routing and protection. In this article we will both present a Linear Programming (LP) model and a column generation optimization algorithm to solve the developed model.

In the remainder of this paper we will in Section 2 give a brief description of some of the developed protection methods for circuit switched network protection.

In Section 3 the LP model and the column generation algorithm is described. The results when applying the column generation algorithm to 6 real world network is given in Section 4. Finally we will attempt a conclusion in Section 5

2. Previous Work

Because of the importance of reliable communication networks, a lot of research has been performed. It is beyond the scope of this paper to thoroughly present all the different protection methods and we refer to [2] for a comprehensive and recent survey of the field.

In general the protection methods can be classified according to the structure they protect:

- Span protection, i.e. all circuits using the failed link are rerouted between the end nodes of the failed link [2, 4]
- Path restoration, i.e. the paths which fail are individually protected through re-routing. Examples are: Global [5], 1+1 (APS) protection [2], Shared Backup Path Protection [2].
- Ring protection and p-cycle protection, all the links are part of an overlaid structure, a ring or a p-cycle. Ring protection [2, 3, 6]. p-cycle protection [2, 3, 7]

The above classification is quite broad and many variations exists for each protection method.

3. The Complete Rerouting Protection Model

All of the protection methods described in Section 2 make one crucial assumption: Only circuits which fails are rerouted. While this is a natural and logical assumption, it limits the possibilities when planning the protection. When performing complete rerouting protection, all the circuits can be rerouted in case of a link failure. This means that for each possible link failure, a complete routing plan is established. These plans are optimized in order to minimize the cost of the required link capacities in the network.

3.A. The Complete Rerouting Protection Linear Program

Consider a network consisting of a set of nodes V indexed by i, j, k, l, q, r and a set of links L. The links are indexed by unordered end-node pairs, $ij \in L : i, j \in V$. The cost per capacity unit of each link is given by the constants $c_{ij} \in R^+$. Furthermore, a set of circuits demands D, indexed by unordered node pairs, $kl \in D : k, l \in V$ are defined. The constant $d_{kl} \in N_0$ is the number of circuit demanded between nodes k and l. The failure situations, one for each link in the network, are indexed by unordered node pairs, $qr \in L : q, r \in V$. For each demand kl and each failure situation qr a set of paths $P^{kl,qr}$ is defined and the constant $PATH^{kl,qr}_{p,ij} \in \{0, 1\}$ has the value 1 if path p for demand kl in failure situation qr use link ij and 0 otherwise. The flow on each path p for each demand kl in each failure situation qr is defined by a variable $x^{kl,qr}_p \in R^+$. The necessary capacity for each link is defined by the variable $y_{ij} \in R^+$. We can now formulate the CompleteRerouting LP model, given a set of allowable paths $P^{kl,qr}$ to use:

Complete-Rerouting($P^{kl,qr}$)

minimize:

$$\sum_{ij} c_{ij} \cdot y_{ij} \tag{1}$$

subject to:

$$\sum_{p \in P^{kl,qr}} x_p^{kl,qr} \geq d_{kl} \quad \forall\, kl \in D, qr \in L \tag{2}$$

$$y_{ij} - \sum_{kl} \sum_{p \in P^{kl,qr}} PATH_{p,ij}^{kl,qr} \cdot x_p^{kl,qr} \geq 0 \quad \forall\, ij, qr \in L, ij \neq qr \tag{3}$$

$$x_p^{kl,qr}, y_{ij} \in R^+ \tag{4}$$

Equation (1) calculates the cost of the required capacities in the network. Equation (2) ensures that each demand is satisfied in each failure situation. Equation (3) ensures that for each failure situation there is enough capacity on each link. Equation (4) defines the domains of the variables $x_p^{kl,qr}$ and y_{ij}.

3.B. Sub Problem

The LP model described in Section 3.A can be solved by any standard LP solver, but the number of paths can be huge, depending on the size and the density of the network. Instead a column generation algorithm is applied to solve the problem. For each iteration in the column generation algorithm, the LP model is solved using a *subset* $P_*^{kl,qr} \subseteq P^{kl,qr}$ of the paths. Using the dual variables $\alpha_{qr}^{kl} \geq 0$ (from equation (2)) and the dual variables $\beta_{ij}^{qr} \geq 0$ (from equation (3)), we can for each demand kl and each failed link qr calculate the reduced cost of the best backup path, see equation (5). The best path for demand kl when link qr has failed is given by the binary vector $a_{ij}^{kl,qr} \in \{0, 1\}$.

$$c_{reduced}^{kl,qr} = \sum_{ij} \beta_{ij}^{qr} a_{ij}^{kl,qr} - \alpha_{qr}^{kl} \tag{5}$$

Given that the dual variables β_{ij}^{qr} are positive, the shortest path can be found using the Floyd-Warshall algorithm [8] for each failure situation qr. The Floyd-Warshall algorithm is an $O(N^3)$ algorithm, which has to be applied, for each failure situation qr. This leads to a worst case solution time of each subproblem of $O(N^5)$ or, as most telecommunication networks are rather sparse, $O(N^3 \cdot L)$.

3.C. The Column Generation Algorithm

The column generation algorithm is given in Figure 1. First one dummy column is generated for each demand kl in each failure situation qr and included into the set of current paths $P_*^{kl,qr}$. The dummy columns are given artificially high costs to ensure that they will never be used in the final solution. Then the main loop is entered. The reduced master problem is solved given the current set of paths $P_*^{kl,qr}$. Then new paths are generated, by executing the Floyd-Warshall algorithm for each failure situation qr, using as link costs the dual β_{ij}^{qr} variables. The failed link can either be assigned an artificially high cost or be ignored by the Floyd-Warshall algorithm. All the paths with negative reduced costs, calculated by equation (5) are added to

Initialize $P_*^{kl,qr}$
repeat
    SOLVE CompleteRerouting($P_*^{kl,qr}$)
    repeat
        $x_{ij}^{kl,qr}$ =Floyd-Warshall(β_{ij}^{qr})
        Add path $x_{ij}^{kl,qr}$ to $P_*^{kl,qr}$ if $c_{reduced}^{kl,qr} < 0$
    until all failure situations qr covered
until No improving paths found

Fig. 1. Column Generation algorithm

the set $P_*^{kl,qr}$. At the end of the main loop it is checked if any new paths have been added to $P_*^{kl,qr}$, if not, the algorithm terminates.

The CR column generation algorithm will most often give fractional solutions. Most circuit switched connection methods will require that certain sizes of connections cannot be split, i.e. be bifurcated. Furthermore link capacities may be modular, i.e. link capacities can only be acquired in certain sizes. The same may be required of the protection methods. On the other hand these requirements are often posed by the technical equipment which may later be improved. Hence to evaluate the protection *methods*, it is beneficial to disregard current technical limitations. If we, on the other hand, we required some kind of integer solution on the CR protection method we loose the lower bound guarantee. Since we do *not* anticipate the CR will become an actual applied protection method, we are more interested in the lower bounding abilities and hence prefer the fractional solution.

A more important issue regarding the CR column generation algorithm is its runnning time. We want to be able to calculate protection lower bounds for networks of a realistic size. These issues are dealt with in Section 4.B.

4. Results

In this Section the complete rerouting protection method is tested by applying the column generation method to 6 real world networks. In order to enhance the analysis in Section 4.A we simplify optimization setup. The cost of the links is set to 1 ($c_{ij} = 1$) and the demand is set to 1 ($d_{kl} = 1$) for each un-ordered node pair, except for the France 2 [9] network.

In Table 4, a number of details for each of the networks are specified. The table contain seven columns. First the name of the network, then the number of nodes and the number of links in the network and then the average node degree. The fifth column contains the Non-Failure network capacity. The sixth column contains the ROB network capacity for the Complete Rerouting protection method. Finally column seven contains the RROB network capacity.

The most interesting results in this paper are contained in the last column in Table 4. The interesting fact is the RROB network capacity, i.e. the protection lower bound is significant and for three of the six networks 0.50 or larger. This bound enables us to give a better evaluation of different protection methods.

4.A. Protection bound and sparcity

The networks tested in Section 4 all have an average node degree between 3.21 to 4.73, i.e. they are rather sparse networks. This is very often the situation in

Table 1. The tested networks

	Nodes	Links	Avg. Node Degree	NF Capacity	CR ROB	CR RROB
Cost239 [10]	11	26	4.73	86	11.6	0.13
Europe	13	21	3.23	158	90.0	0.57
USA [9]	28	45	3.21	1273	641.2	0.50
Italy [11]	33	68	4.12	1718	581.4	0.34
France [9]	43	71	3.3	3473	1604	0.46
France 2 [9]	43	71	3.3	4043	3156.3	0.78

telecommunication networks, because of the high fixed charges required for creating new connections in the network. An interesting question is what the connection is between the sparseness of a network and the network protection capacity lower bound, i.e. sparcity, ROB, and RROB. To test this for a wider range of network densities, we choose the most dense network, Cost239. For this network we start with the smallest subset of links which guarantees protection, a ring. Then the links are randomly added until the original Cost239 network has been constructed. Afterwards, new links are randomly added until the fully connected network has been created. In this way we will test the importance of the network density with respect to the complete protection lower bound. Furthermore, we are able, to *analytically* calculate the optimal NF network capacity and CR network capacity for the two bounding cases the ring and the fully connected network, see Table 4.A. A brief description of these equations are given in Section 4.A.1 to Section 4.A.4 below. Notice that the equations for NF network capacity for the ring network and the CR network capacity for the ring network only holds for networks with an uneven number of nodes.

Table 2. Formulas for calculating network capacity for NF and CR for ring networks and fully connected networks

Network Type	NF	CR	ROBB
Ring	$\frac{(N+1)\cdot N\cdot(N-1)}{8}$	$\frac{(N+1)\cdot N\cdot(N-1)}{4}$	$2-1$
Full Network	$\frac{N\cdot(N-1)}{2}$	$\frac{N\cdot(N-1)}{2}\cdot\frac{N-1}{N-2}$	$\frac{N-1}{N-2}-1$

4.A.1. NF network capacity:Full Network

Shortest path routing in the fully connected network is simple: Use a capacity of 1 unit on each link. Each link corresponds to exactly a demand of 1 unit because we are considering unordered demand pairs. Hence the total capacity corresponds to the number of un-oriented links in the network, see equation (6).

$$NF_{full} = \frac{N\cdot(N-1)}{2} \tag{6}$$

4.A.2. NF network capacity:Ring Network

We will consider the case where we have a ring (network) with an un-even number of nodes N. Furthermore we will consider the case where there is a demand of 1 unit for each *ordered* pair of nodes. For each node in the ring, shortest path routing is performed to the $\frac{N-1}{2}$ nearest nodes on each side. We will have to use a capacity of 1 to the nearest, 2 to the next nearest and so forth. In order to adjust to the un-ordered case we finally divide by 2, see equation (7).

$$\begin{aligned} NF_{ring} &= 2 \cdot \left(\sum_{i}^{\frac{N-1}{2}} i \right) \cdot N \cdot \frac{1}{2} \\ &= \frac{\frac{N-1}{2} \cdot (\frac{N-1}{2} + 1)}{2} \cdot N \\ &= \frac{(N+1) \cdot N \cdot (N-1)}{8} \end{aligned} \tag{7}$$

4.A.3. CR network capacity:Full Network

In the case of a full network, we first route the traffic as in the NF case and hence need a capacity of one on each link. Only one demand use each link hence only this demand needs to be re-routed in case the links fails. the one unit is then split among all paths of length 2. There are $N-2$ of these. Hence an additional ROB on each link of $\frac{1}{N-2}$ is added. This leads to a CR_{full} as given in equation (8).

$$\begin{aligned} CR_{full} &= \frac{N \cdot (N-1)}{2} + \frac{N \cdot (N-1)}{2} \cdot \frac{1}{N-2} \\ &= \frac{N \cdot (N-1)}{2} \cdot \frac{N-1}{N-2} \end{aligned} \tag{8}$$

4.A.4. CR network capacity:Ring Network

We will consider the case where we have a ring with an un-even number of nodes N and hence N links. Again we will consider the situation of ordered demands. Because we only consider single link failures, we hence have N failure situations, where all the communication should be routed on the $N-1$ remaining links which form a line. The maximal capacity of any of the remaining links are required on the two middle links. Since all links will become middle links of the line, in case of two specific failures, the maximal capacity is required for all links in the ring, see equation (9).

$$CR_{ring} = 2 \cdot \frac{(N-1)}{2} \cdot \frac{(N+1)}{2} \cdot \frac{N}{2} \tag{9}$$

4.A.5. Numerical simulations

Based on the Cost239 network we generated 30 different datasets. Each dataset contains 46 different networks with between 11 links, a ring, and 55 links, the fully connected network. The first 15 networks only consists of links from the original Cost239 network and the remaining networks consists of the Cost239 network with extra randomly chosen links. In Figure 2 the average NF network capacity and the average CR network Capacity for the networks is shown. The dotted lines are the lower bound (full network) and upper bound (ring network) for the NF network capacity and the dashed horizontal lines correspondingly are the lower and upper bounds for the CR protection method. The bounds are calculated according to equation (6), equation (7), equation (8) and equation (9). It can be seen that the analytic bounds holds for the ring network and the full network for both the NF network capacity and the CR network capacity. There are error bars corresponding to the standard deviation for both the NF network capacity and the CR network capacity. It can be seen that the error bars for the CR network capacity are quite significant for the more sparse networks, but they are reduced as the network density increases.

In Figure 3 the RROB network capacity for the CR protection method is shown together with error bars corresponding to the standard deviation for the ROBB network capacity. Furthermore, the ROBB network capacity for the 6 networks in Table 4 is shown in the graph and the RROB network capacity from Table 4.A is shown as the dashed lines. The predicted RROB network capacity holds for the ring network and for the full network, but as it can be seen the ROBB network capacity of the intermediate networks do *not* stay between the dashed lines. The graph for the RROB network capacity for the Cost239 network derived networks suggests that capacity savings can be achieved by expanding the networks with links up to an average node degree of 5. More experimentation is though needed to confirm this hypothesis. Furthermore, this may be different for other protection methods.

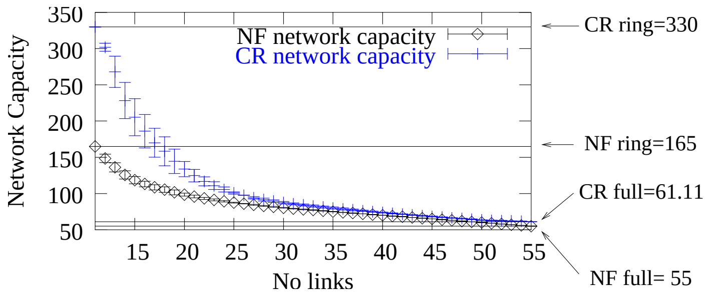


Fig. 2. Absolute network capacity for NF and CF

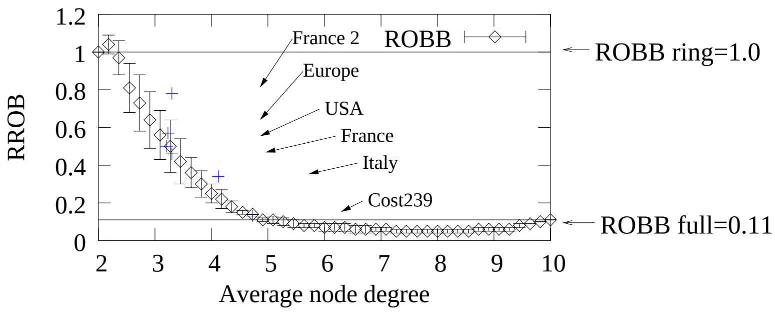


Fig. 3. RROB

4.B. *Computational Issues*

The experiments were performed on a 1000 MHz SUN Fire 3800 machine. Despite solving the Complete Rerouting problem using a column generation algorithm, the solution times for the larger and more dense networks is significant, see Table 4.B. The first column in Table 4.B contains the network name and the second column the number of rows in the LP program. The third column contains the initial number of rows, the fourth column the generated number of rows and the fifth column the total number of rows. Column six and seven contains the number of iterations and the number of seconds required respectively. Finally column eight contains the percentage of the computation time required for (re-)solving the master problem.

Table 3. Problem size and solution size

	Rows	Initial	Columns Generated	Total	Iterations	Sec.	resolve %
Cost239	2106	1456	2831	4287	24	17.1	93.5 %
Europe	2079	1659	2253	3912	44	3.8	56.1 %
USA	19035	17055	33145	50200	65	1062.5	90.1 %
Italy	40528	35972	105119	141091	86	104577.7	99.5 %
France	69154	64184	156831	221015	47	68968.0	98.6 %
France 2	69154	64184	76090	140274	130	22386.0	92.1 %

As can be seen from Table 4.B the running times are significant. The majority of the computation time is required for resolving the master problem, as can be seen from column eight in Table 4.B. This is quite surprising because we are using state of the art solvers, CPLEX 9.0 [12], and only *resolving* for each iteration, i.e. warmstarting from the previous solution. Since the main computational problem is solving the master problem, we have not considered faster algorithms for solving the subproblem, but concentrated on improving the solve time of the master problem.

We have tried three different approaches to reduce the computation time of the master problem: Reducing the number of columns by pricing out these, see Section 4.B.2, reducing the number of iterations performed by applying stabilization, see Section 4.B.2 and calculating non-optimal bounds, see Section 4.B.3.

4.B.1. Column Reduction

Reducing the number of columns in the master problem could possibly improve the solution time because only a fraction of the generated columns are actually used in the end. The columns accepted into the problem all have negative reduced costs according to equation (5), but the dual variables β_{ij}^{qr} and α_{qr}^{kl} changes during the iterations and hence columns which have at one iteration been included into the set $P_*^{kl,qr}$ may not have negative reduced costs in later iterations. Hence a test is added to the algorithm which test if already included columns have a reduced cost above a certain (positive) threshold, if yes they are excluded. This did however not improve solution time.

4.B.2. Stabilization

Stabilization of column generation algorithms is a well known technique [13, 14] which reduce the fluctuations in the dual variables (β_{ij}^{qr} and α_{qr}^{kl}) by either adding slack variables that are penalized or by adding bounds on the variables in the dual master problem. This was tested but it did not improve solution time. The reason is probably that the dual variables are *not* fluctuating wildly. To test this hypothesis, first the optimal values of the dual variables β_{ij}^{qr*} and α_{qr}^{kl*} were calculated. Then the column generation algorithm was re-run and the normalized Euclidean distance between the current dual variables (β_{ij}^{qr} and α_{qr}^{kl}) and the optimal dual variables (β_{ij}^{qr*} and α_{qr}^{kl*}) was calculated, see equation (10).

$$Euc = [\sum_{ij,qr}(\beta_{ij}^{qr} - \beta_{ij}^{qr*})^2 + \sum_{ij,kl}(\alpha_{qr}^{kl} - \alpha_{qr}^{kl*})^2]^{1/2}/[|L|^2 + |D| \cdot |L|] \tag{10}$$

For each iteration of the algorithm, the normalized Euclidean distance is calculated and plotted for the USA network in Figure 4.B.2. The curvature of the graphs which should indicate fluctuations in the dual variables is a slowly decaying Euclidean distance for most of the iterations followed by a steep decent in the end [15]. Judging from the graph in Figure 4.B.2, this could is not the case for the USA network, which is typical for the large networks where the running times are significant.

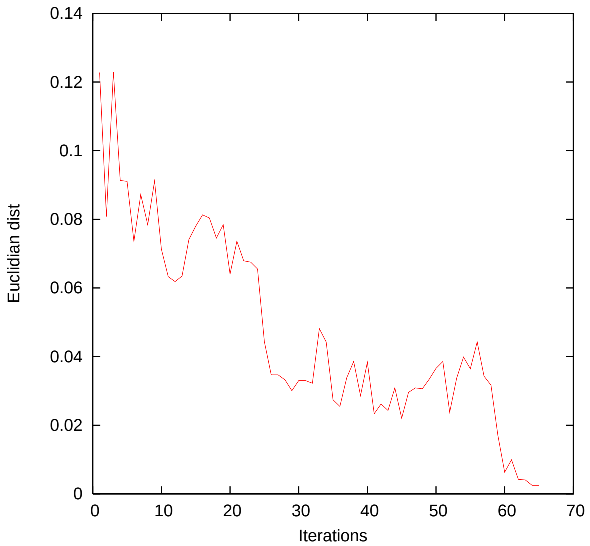


Fig. 4. Euclidean distance between optimal dual variables and current dual variables for the USA network

4.B.3. Non-optimal bounding

Because the objective in this paper is not finding actual protection schemes, but rather lower bounds, we can instead settle for getting the best possible lower bound within a given time limit. This is possible because we can, during the execution of the column generation algorithm, calculate guaranteed lower bounds according to equation (11). Because of the demand constraints (2), the greatest improvement which can be achieved for each iteration is to set all the new variables, one for each demand kl in each failure situation qr to 1. This (negative) value is now added to the current CR value achieving the bound. We illustrate the benefit of this bound for the USA network in Figure 5 where the ROBB network capacity and its lower bound is shown. It can be seen that whereas the ROBB network capacity is non-increasing, the lower bound is not non-decreasing.

$$CR_{bound} = CR_{current} + \sum_{qr,kl} min(0, c_{reduced}^{kl,qr}) \tag{11}$$

In Table 4.B.3 we have compare the precision, i.e. the number of digits of the RROB network capacity upon which the lower and the upper bound agree, with the computation time required. The first column in Table 4.B.3 is the name of the test network. The two next columns gives the time in seconds it takes for the bounds to agree on the first digit and the percentage of the total computation time it takes. Column four and five contain the same information for the second digit

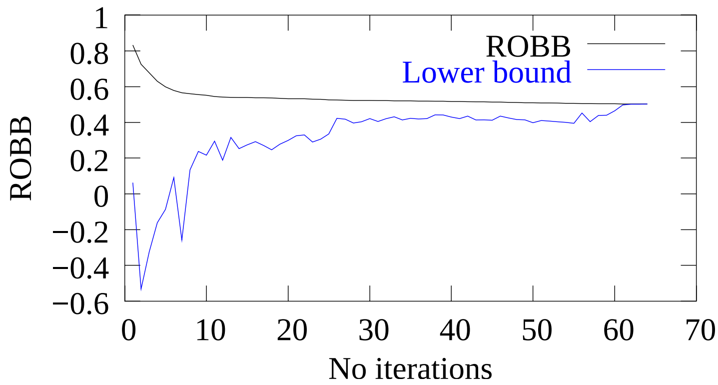


Fig. 5. ROBB and lower bound for USA network during column generation algorithm execution

and column six and seven the same information for full precision, which in this case corresponds to five digits. As it can be seen, just calculating the first digit of precision requires a significant time and only for the France 2 there are significant time savings to achieve. The missing data, e.g. 1 digit for USA simply means that the column generation algorithm found solutions agreeing on the first and second digit at the same time.

Table 4. Precision vs. time

	1		2		∞	
	Sec.	%	Sec.	%	Sec.	%
Cost239	13.1	75.7	17.3	100.0	17.3	100.0
Europe	2.6	66.7	-	-	3.9	100.0
USA	-	-	1253.3	99.1	1265.0	100.0
Italy	57845.2	54.7	101734.4	96.1	105825.2	100.0
France	70824.8	89.1	-	-	79482.5	100.0
France 2	1487.6	8.2	-	-	18154.4	100.0

5. Conclusion

In this paper we have presented Complete Rerouting protection method, which is the most capacity efficient protection method for 100% protection against link failures, i.e. it finds the lowest restoration over build network capacity.

We have presented an LP model for the complete rerouting protection method and also presented a column generation algorithm which can be used to find optimal solutions. This algorithm we then test on 6 real world networks with up to 43 nodes and 71 links. The found bounds can now be used as lower bounds when testing other protection methods. We have furthermore presented analytical bounds on the bounding cases of the ring network and the fully connected network. We have also demonstrated how lower bounds can be found based on the column generation algorithm, even though this algorithm is terminated before completion.

The complete rerouting method will most likely never be used for protection in networks because communication which is unharmed by a particular link failure will also have to be rerouted. For this to be acceptable, the capacity costs would have to be huge. The primary use of the Complete Rerouting method is for calculating lower bounds for protection of different networks. This is highly useful when evaluating different rerouting methods and we intend to demonstrate this in our future research.

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