A Partial Load Model for a Local Combined Heat and Power Plant

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Proceedings of the 1st Nordic Optimization Symposium, Copenhagen 2006

Abstract

Small distributed combined heat and power (CHP) plants constitute a not insignificant share of the power production in Denmark, particularly the western part of the country. Since January 1st, 2005 these plants have been required to act on market terms, i.e. sell the power produced on the spot market, rather than at the fixed feed-in tariff previously used.

Many versions of the unit commitment problem have been discussed in the literature, from the so-called economic dispatch (the task of dispatching the entire system at least cost given a certain demand) to optimal bidding to an electricity spot market in various shades. However, CHP is rarely touched upon and then mainly from the system point of view. This paper considers a local CHP plant faced with bidding into the spot market while taking into account assorted physical limitations.

As prices thus are unknown at the time of production planning, a simple stochastic model is utilised to construct an optimal plan under uncertainty. In the model, the CHP unit is segmented and with it all the parameters (cost, electricity to heat ratio, etc.) connected to it. This enables taking into account varying efficiency and production costs as well as enabling the handling of pollutant emissions, the extent of which depend in various ways on the level at which production takes place.

A case study is conducted using data from a typical local CHP plant and the years 2003 through 2006 are simulated to assess the accuracy of the stochastic model compared to the deterministic case.

1 Introduction

Local combined heat and power (CHP) production in Denmark has, in recent years, undergone several changes. Perhaps most significant was the transition from unloading production at a feed-in tariff to selling it at NordPool[1] [1]. In connection herewith, the plants must plan their production and their bids to the power market at a time when the sales prices for electricity are yet unknown.

Most Danish local CHP plants originated as heat plants whose raison d'être was servicing a local district heating network. For this reason the sale of electricity is primarily seen by the plant operators as a way of financially compensating for the expenses of heat production which the plants are often contractually bound to supply to the local network. The local CHP plants are all thermal, with fuel types including (but not limited to) waste, bio gas, and straw but a large majority are fuelled by natural gas [10]. They often also possess a heat storage facility (typically a large hot water tank) and/or a purely heat producing unit. The local CHP plants range in size from less than one hundred kW to nearly 100 MW and the gas-fired units are particularly characterised by their rapid commitment ability. In this paper a typical representative of local Danish CHP plants is considered, i.e. a gas-fired unit of roughly 10 MW, and thus disregarding ramp rates in the model is not unreasonable.

The unit commitment problem in energy production planning typically involves scheduling generators in order to meet system demand while taking into account the physical constraints connected herewith and minimising operating costs (see e.g. [4] and [28]). For more detailed overviews of work performed on the problem, see [15], [18], and [24].

In line with the deregulation of various electricity markets in recent years, e.g. in California, Spain, and the Nordic market, another type of unit commitment problems have arisen, namely those that consider the problem from the point of view of the individual power producers acting on the market. This has resulted in several papers on how generating companies

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[1]The Nordic power exchange [17].

(GENCOs) with a portfolio of several large production units should bid to the market. The approach varies: considering imperfect information ([2], [29]), taking risk aversity into account [22], the response of a thermal unit to an electricity spot market [3] (this is similar to the approach in the present paper, except that prices are known at the time of planning and the unit is not a CHP), considering an oligopolistic GENCO [7], and, in contrast, veiwing the GENCO as a price-taker ([8], [11] (the latter also considers the Nordic spot market).

A second approach is to consider the problem of unknown prices at the time of planning by utilising e.g. a time series model fitted to historical electricity prices to forecast the prices of the bidding day, [9], [16]. Applying this to the Nordic power system would probably require a transfer function model (see e.g. [5]), as the Nordic spot prices depend heavily upon the level of the Norwegian and Swedish water reserviors and thus on precipitation.

Combined heat and power is not often taken into consideration but is of particular interest in the Danish system due to the large penetration of said technology, especially in the western part of the country. Typically, when CHP is considered, the economic dispatch problem is still prevalent [6], [13], [25], [27], though probabilistic production simulation of power systems with CHP may also be found [14], [26].

The present paper employs a different perspective than the abovementioned works: it examines CHP but concentrates on a single small plant - in this respect a GENCO is considered. However, as it is a local CHP that is considered and not a large GENCO which produces only electricity, the perspective is somewhat skewed in comparison to the mentioned papers. Here, the production of the CHP plant is examined in terms of heat production: the primary concern of the local CHP plant is meeting the heat demand, as mentioned previously. Simultaneously, the plant manager must take into account that at the time of bidding (more than 12 hours prior to time of operation), electricity spot prices are unknown. Note, that the bid is made for the entire 24 hours of the coming day of operation at the same time. If prices are expected to be low (i.e. less than the cost of production), the CHP plant may satisfy heat demand in that period by extracting heat from the storage facility or utilising the heat only production unit. When high prices are expected, clearly it is advantageous to bid electricity from the CHP production unit, provided there is room in the storage facility for the heat produced which is not fed directly into the local network. Preliminary work on this subject may be found in [20], [21]. Further, this paper takes into account the possibility of segmenting the load on the CHP unit, as several factors vary according to the load level, e.g. the production efficiency of the unit and amount of pollutants emitted during production.

The paper is organised as follows. The unit commitment problem with uncertainties is formulated in Section 2; in Section 3 a case study is conducted using data from [19]; and finally Section 4 concludes upon the work and suggests avenues for further research.

2 Problem formulation

In the present paper a model is presented, elaborating on previous work ([20]), which alleviates planning difficulties for a single local CHP plant, consisting of a combined heat and power generating unit, a heat boiler, and a heat storage facility. The model creates a day-ahead production schedule for the plant, taking into account restrictions regarding production capacity of both the CHP unit and the heat boiler; start-up constraints for the CHP unit and the heat boiler; capacity limitations of the heat storage facility; meeting heat demand; while minimising production expenses, including starting costs for both production units. The planning is done under uncertainty, in the respect that a number of price scenarios each weighted with a probability are taken into consideration in the objective function. Each scenario consists of a set of 24 hourly electricity spot prices.

There are, however, other factors that may influence planning besides the expected sales price of the electricity generated, such as emission of environmental pollutants that are generated during power production, or the fact that production may be more efficient, and thus more economical, at certain load levels. These concerns are also taken into account in the present model. In concurrance with previous arguments, ramp rates are disregarded. Also, the plant is considered price taker in the market, as it is but one of many such units which participate.

2.1 Constraints

By segmenting the load variable (m_{ot}) and the electricity-to-heat ratio (c_m) as well as the production cost (c_{kv}) of the CHP unit using the index $g \in \mathcal{G} = \{1, ..., G\}$, it is possible to take into account that power production is not necessarily linear within the capacity limits of the CHP generator. Total production, or 'load', is assumed continuously varying between the minimum of segment 1 and the maximum of segment G (which equals the installed production capacity of the unit). Thus maximum for one segment equals minimum for the next and segment g needs to be fully loaded before any production can take place in segment $g+1$. The total production on the CHP unit is the sum of the production on the individual segments.

When thus segmenting the production cost on the CHP unit it is important to bear in mind the hazards involved in doing so. In the case study (Section 3) it is assumed that the marginal cost of production (MCP) is constant on each segment and, so to say, decreasing as a function of the load, as illustrated in Fig. 1, left. This allows for convexity of the total cost as a function of the load (cf. Fig. 1, right).

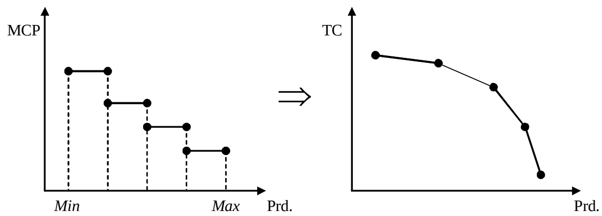

Figure 1: Structure of the marginal cost of production (MCP) and the consequences for the total cost.

Nevertheless, as soon as the MCP changes character, there is the risk of non-convexity of the total cost as a function of the load. For instance if the MCP is linear on each segment with slopes of opposite signs or merely if one of the intermittent segments had higher MCP than the surrounding segments it would have dire consequences for the convexity of the total cost.

The maximum load capacity of a segment equals the difference between the upper and lower bounds of the segment. The minimum load capacity of segment 1 is given by $m^g_{min}, g \in \{1\}$ and is zero for the remaining segments. Because of the 'sequential additivity' of the segments and because the cost is not assumed convex, a constraint is needed to ensure that if the CHP unit is producing in the segment following the present, then the present segment must be fully loaded, i.e.

$$\sum_o m^g_{ot}\delta^s_{ot} \geq z^{s,g+1}_t \left(K^g_{kv} - m^g_{\mathsf{min}}\right), \qquad g \in \mathcal{G}, \forall s, t, \tag{1}$$

where $\sum_o m^g_{ot}\delta^s_{ot}$ is the heat production[2] on the CHP unit at time t; z^{sg}_t is a binary variable indicating whether or not the unit is producing in segment g and during period t, under scenario s; and K^g_{kv} is the heat production capacity of the CHP unit in segment g.

A similar constraint is introduced, linking the segments directly. If the CHP unit is running in the following segment, it must also be running in the present segment. This translates to

$$z^{sg}_t \geq z^{s,g+1}_t, \quad g \in \mathcal{G}\backslash\{G\}, \ \forall s, t. \tag{2}$$

Finally, a constraint that guarantees connectivity between start-up and production is needed. It ensures that a start-up is not planned in a given hour (i.e. $v^s_t = 1$) unless production is bid in the first segment of the hour in question (here, v^s_t indicates whether the CHP unit has been started during period t under scenario s). Conversely, if there is production, it should be at least minimum production. This results in the constraint

$$m^1_{\mathsf{min}} v^s_t \leq \sum_o m^1_{ot}\delta^s_{ot}, \ \forall s, t. \tag{3}$$

There are several additional constraints that concern the physical aspects of the problem. Firstly, heat demand must be met. Let V^s_t denote the volume of the heat storage at the beginning of period t under scenario s and d_t the heat demand in period t. Ensuring that the total amount of heat demanded equals the heat produced on both CHP unit and boiler plus any heat extracted from the storage facility during a given hour is accomplished by the constraints

$$V^s_{t+1} = V^s_t + \sum_{o,g} m^g_{ot}\delta^s_{ot} + m^s_{kt} - d_t, \quad \forall s, t = 1, \ldots, T-1, \tag{4}$$

$$V^s_1 = V^s_T + \sum_{o,g} m^g_{oT}\delta^s_{oT} + m^s_{kT} - d_T, \ \forall s. \tag{5}$$

Constraints (4) ensure that the amount of heat produced when producing electricity for sale on the spot market does not exceed the free space in the storage once heat demand has been met. Fixing the initial volume of the storage strictly between limits then guarantees (by constraints (5)) that the storage is neither completely empty nor full to capacity at the end of the time interval considered. The equality sign in both (4) and (5) signifies that excess heat may not be cooled off.

Then there are the production limits on the CHP unit, i.e. production must remain between capacity limits of each segment during any given period:

$$z^{sg}_t m^g_{\mathsf{min}} \leq \sum_o m^g_{ot}\delta^s_{ot} \leq z^{sg}_t K^g_{kv}, \qquad g \in \mathcal{G}, \forall s, t. \tag{6}$$

[2] For a thorough explanation of the use of the δ's, see Appendix A.

Further, the non-negative variable v_t^s, which indicates whether the CHP unit was started during period t under scenario s, is ensured binary by the optimisation and the constraint

$$v_t^s \geq z_t^{sg} - z_{t-1}^{sg},\ g \in \{1\}, \forall s, t = 2, \ldots, T. \tag{7}$$

The initial condition

$$v_1^s = z_1^{sg} - z_{init},\ g \in \{1\}, \forall s \tag{8}$$

handles the first period, where z_{init} is a parameter which indicates whether the CHP unit was running in the period immediately prior to the time considered.

As for the boiler, the production limitations may be stated as

$$y_t^s m_{\min}^k \leq m_{kt}^s \leq y_t^s K_k,\ \forall s, t, \tag{9}$$

where, similar to the CHP unit, y_t^s is a binary variable indicating whether the boiler is in operation during period t under scenario s; m_{kt}^s is the heat produced on the boiler during period t, scenario s; $m_{\min}^k$ is the minimum production permitted on the boiler; and K_k is the production capacity of the boiler. The start-up variable w_t^s is controlled by the constraints

$$w_t^s \geq y_t^s - y_{t-1}^s,\ \forall s, t = 2, \ldots, T, \tag{10}$$

$$w_1^s = y_1^s - y_{init},\ \forall s, \tag{11}$$

where y_{init} indicates whether production was taking place on the boiler in the period prior to the time considered. Finally, the variable limits must be included:

$$0 \leq V_t^s \leq V_{\max},\ \forall s, t \tag{12}$$

$$z_t^{sg}, y_t^s \in \{0, 1\},\ \forall g, s, t \tag{13}$$

$$0 \leq m_{ot}^g, m_{kt}^s, v_t^s, w_t^s,\ \forall g, o, s, t. \tag{14}$$

2.2 Objective function

The costs considered in the model are the production costs of the CHP generator and the boiler, as well as the starting costs for both units.

$$\sum_{o,g} c_g^{kv} m_{ot}^g \delta_{ot}^s + m_{kt}^s c^k + c_{start}^{kv} v_t^s + c_{start}^k w_t^s \tag{15}$$

Emissions from a production unit naturally depends on the load. However, the efficiency of the unit often varies depending on the load as well, i.e. emissions may be relatively low at full load compared to half load. Including an emission cost in the objective function is relatively simple. Letting n_e^g indicate emission of type e and c_e^n the associated cost, emission expences may be considered in the model by including the term

$$\sum_e c_e^n \sum_{o,g} n_e^g m_{ot}^g \delta_{ot}^s \tag{16}$$

in the objective function. Thus, one may consider both environmental and economical consequences of whether production takes place under full or partial load. Income from sales of electricity produced on the CHP unit enters into the objective function as the term

$$\pi_t^s \sum_{o,g} c_m^g m_{ot}^g \delta_{ot}^s, \tag{17}$$

where π_t^s is the expected spot price for period t under scenario s.

Letting ϕ^s denote the probability for scenario s, the above terms may be incorporated into an optimisation model, the objective of which is to minimise

$$\sum_{s,t} \phi^s \{(15) + (16) - (17)\} \tag{18}$$

subject to constraints (1)-(14).

3 Case study

In the following section a case study is conducted using data from [19], where several factors are varied depending on the load. Simulations are carried out for the years 2003-2006 (the latter only partially, according to available data) in order to compare the stochastic model described in Section 2 with the deterministic case (in which prices are known in advance). The calculations were carried out using CPLEX 10.0.1 under GAMS [12].

3.1 Basic data

Several parameters are associated with a given segment: minimum and maximum production capacities, production costs, electricity to heat ratios, and emission of pollutants. This latter may of course be divided into several types of emission, which again may depend in various ways upon the load. The typical pollutants emitted from a gas-fired engine are CO, NO_x and UHC (Unburned Hydro Carbons), which therefore are the ones considered in the case study.

The local CHP plant considered in the case study has a CHP unit with a capacity of $K_{kv}^G = 10.981$ MW heat, the capacity of the heat boiler is $K_k =$

9.8 MW, and the capacity of the storage facility is $V_{max} = 150$ MW. The CHP unit has five segments ($\mathcal{G} = \{1, \ldots, 5\}$), to each of which the parameters listed above have a value associated. The heat demand is low in summer, high in winter, and gradually decreasing and increasing during spring and autumn, respectively. Additionally, there are typical diurnal variations (higher demand in the day than during the night).

Simulations were carried out for the years 2003-2005 as well as for the part of the year 2006 where data was available (from January 1st to September 29th) using the western Danish spot prices. The average daily prices of the mentioned years are depicted in for 2003-4 in Fig. 2 and for 2005-6 in Fig. 3.

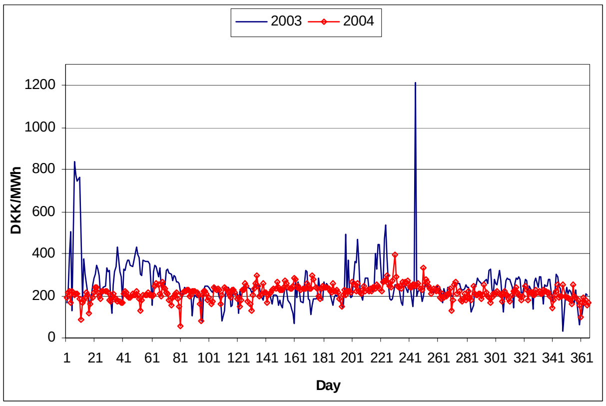

Figure 2: Daily average spot prices in the western Danish price area, 2003-2004.

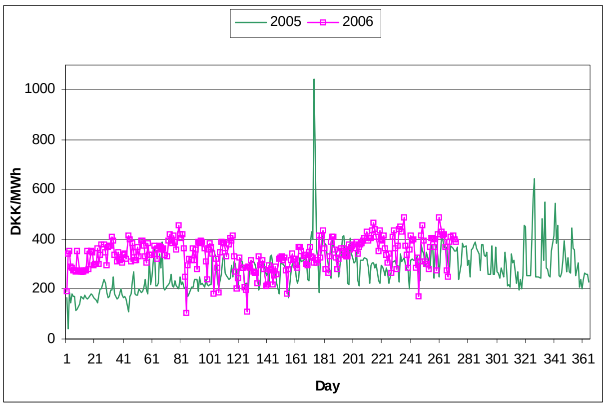

Figure 3: Daily average spot prices in the western Danish price area, 2005-2006. Note the difference in scale from Fig. 2.

Hydro power accounts for roughly half of the power production in the Nordic system and thus the available volume primarily in the Norwegian and Swedish reservoirs heavily influence the spot price. Years are categorised as 'dry', 'wet' and 'normal' as a consequence of the mentioned dependence. In dry years where precipitation has been sparse during the spring and summer months the spot price will typically increase (sometimes dramatically) towards the end of the year and the pattern may even be visible in the beginning of the following year. This was the case in the winter of 2002-2003 where aftereffects of the dry year 2002 may be seen in the beginning of 2003 (Fig. 2). In contrast, the year 2004 was very normal with only a slight downward trend at the very end of the year (Fig. 2).

In 2005 an increase in the level of the prices may be observed in the beginning of the year (Fig. 3). The level stabilises around 300 DKK/MWh which is an increase of around 100 DKK/MWh compared to the two previous years, cf. Fig. 2, and the level persists in 2006 so far with a few fluctuations and perhaps even a hint of increasing. This could be due to the fact that the market for CO_2 quotas initiated in 2005 and in Denmark it had been decided that the large power producers should bear the brunt of the CO_2 reduction which is part of the Kyoto protocol. As a consequence of decreasing coal prices and increasing oil prices, CO_2 quota prices have increased dramatically, which in turn influences the electricity spot price. Furthermore, the trend so far of the water levels in the reservoirs indicates that 2006 will also be a dry year and, as may be seen in Fig. 3, the spot prices have aleady begun an additional slightly increasing trend from the end of the summer. It would also seem that the volatility of the prices has increased in 2005 and 2006, perhaps in part due to the inclusion of the local CHP and other small distributed production units into the spot market from January 1st, 2005.

The spot price scenarios in the stochastic model described in Section 2 were constructed in the following manner. For any given day the spot prices of the previous five days of the same type[3] were given equal probability. To ensure bidding in cases where the prices of the previous days were low, an artificially high set of spot prices were included but given a comparatively small probability.

3.2 Simulation results

The calculations were carried out on an AMD Athlon Dual Core 2.01 GHz CPU with 1 GB of RAM

[3]The type distinguishes between weekdays and weekend days.

using CPLEX 10.0.1 under GAMS. From the results various observaitons may be made.

Consider the difference between the stochastic and the deterministic solutions, i.e. the 'value of full information'. The differences are depicted in Fig.s 4 and 5. As may be expected, the absolute error is least in the year 2004 where the spot prices were stable at the same level with no large oscillations the entire year (cf. Fig. 2). The deviations is somewhat larger in 2003, a year with several price spikes and a decreasing trend in the beginning of the year. From Fig. 4 it would seem that there is a delayed effect of these characteristica. The error relatively quickly establishes stability in the period of large oscillations in the spot prices (see Fig. 2). However this ability turns into a disadvantage once the spot prices stabilise wherefore the error level increases dramatically. The same effect is visible in the latter half of the year, as the error increases in the time following period of price spikes where the spot prices have stabilised somewhat.

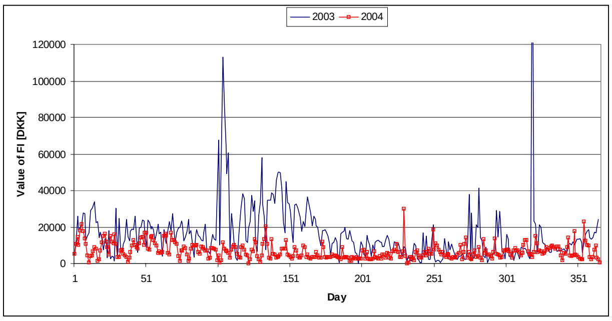

Figure 4: The value of full information for 2003 and 2004.

The tendency mentioned regarding the error in the 2003 simulations is somewhat repeated in the years 2005 and 2006. When there is a stable trend, the error adjusts after a small delay but abrupt changes in the spot price pattern usually affects larger errors. The effect of the single peak in 2005 is slightly different than the peaks in 2003. In 2006 the single peak affects a similarly singular peak in the error. This may be explained by the fact that the prices around the peak behave with relative stability (cf. Fig. 3) in comparison to the volatile behaviour of the spot prices surrounding the peaks in both the first and latter halves of 2003 (cf. Fig. 2).

The average daily error (ADE) and the total yearly deviation (TYD) from the full information case have also been calculated for each year (see Table 1).

The deviations are supported by the following reas-

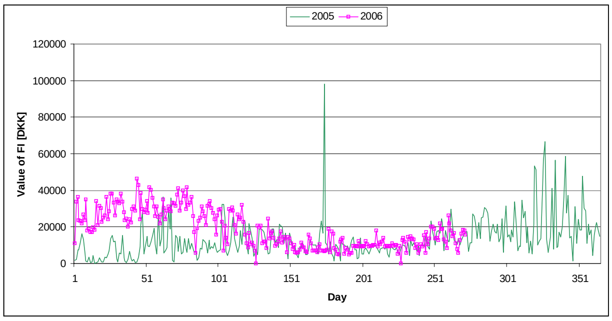

Figure 5: The value of full information for 2005 and 2006.

	2003	2004	2005	2006
ADE	9.04%	11.77%	3.30%	1.23%
TYD	1.72%	9.14%	1.46%	1.16%

Table 1: Percentwise average daily error and total yearly deviation.

oning. In years with large variations in the spot prices the potential for error may be large but the profit to be made when full information of the day-ahead prices is available is comparatively larger (as is the case in 2005, 2006, and to some extent also 2003). As the level of the prices has increased in 2005-6 compared to 2003, it is not suprising that the relative errors should be smaller in the first-mentioned years. In years with stable, but comparatively low, prices (such as 2004) the absolute deviations may be small, but as profits are also small even with full information available, relative deviations are large. If prices were stable, but high, it may be expected that the error was of a level with (or even less) than in 2005-6 and 2003. Considering the simplicity of the prognosis method in the stochastic model such errors as discussed above cannot be regarded as excessive.

4 Conclusions

A model was presented which considered the problem of constructing a production plan for a local CHP plant in order to bid into the day-ahead market while meeting heat demand. The model took into consideration the possibility of segmented production upon which several parameters (such as production cost, emissions, electricity to heat ratio) depended. Thus it was made possible to prioritise concerns regarding regulation of environmental pollutants. In the present model, emissions were penalised in the objective function. Another way of including environmental concerns would have been to include constraints which stated that emissions must be within certain limits. In that case one might still keep the penalty term in the objective function according to priorities.

Simulations were undertaken, running the model throughout the years 2003-2005 and the part of 2006 were data at present was available. The stochastic model was compared with a deterministic version, where prices were fully known in advance and the differences were examined. With the current prognosis approach in the model, the tendency was that while price trends were stable, errors were small. However, sudden changes in trends affected larger errors which would peter out as the new price trend became stable.

Further, the average daily errors in the years simulated ranged between 1.23% and 11.77%, largest in the year with the most stable but lowest prices (2004) and least in the year with a higher level of prices that had some fluctuations but in general lay around a.relatively stable mean value (2006). The total yearly deviations ranged from 1.16% to 1.72% with a single "outlier" of 9.14%. This outlier, again, was in 2004 which is not quite surprising when one considers that although the prices were very stable during that year, they were relatively low and thus profits would be small. When such is the case, even small absolute deviations easily become relatively large.

Taking into consideration the simplicity of the prognosis method utilised in the stochastic model, the performance must be deemed accaptable. However, it would be quite interesting, as a further avenue of research, to attempt to describe the Nordic spot price fluctuations using a time series model, use the predictions from such a model as the spot price input in the deterministic version of the presented stochastic model and see how it compares to the stochastic method of anticipating prices based on historic data presented in above.

Another issue is the deterministic nature of the heat demand in the present model. Whenever a model such as the present is to be utilised in reality issues of data arise. It would be useful, when optimising the production costs for a given local CHP plant, to have e.g. a fitted time series model for the local heat demand which the plant provides for (see also [23]).

A Appendix

In the simple linear model for a local CHP presented in a previous paper [21], a two-level price structure was introduced. The prices p_1 resp. p_2 indicated the surplus cost of producing electricity when the heat produced simultaneously could be utilised resp. when it could not (note, that $p_1 < p_2$). When spot prices were expected to be below p_1 nothing was bid to the spot market; were the spot prices expected to be above p_2 full capacity was bid; and were the spot prices expected to be between p_1 and p_2 the model would determine a suitable bid depending on the heat demand.

This simple approach may be less than expedient in certain cases. For instance, consider the case illustrated in Figure 6: here the two-price model enforces a shut-down in the intermediate period with low prices (below p_1) where it might be more desirable to continue production undeterred throughout the whole nine-hour period, e.g. if the start-up cost of the CHP unit is higher than the cost of producing during the low price period.

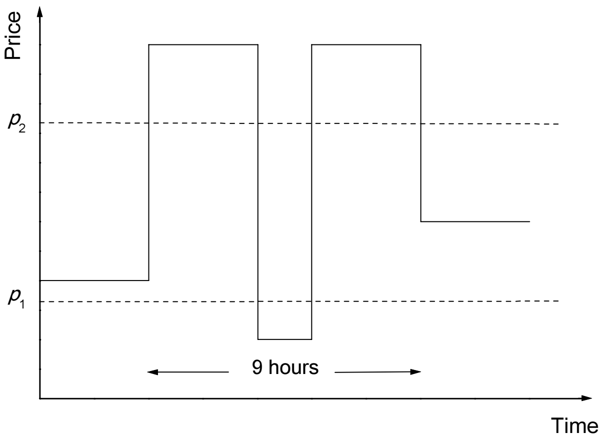

Figure 6: In the simple case, nothing is bid in the period with prices below p_1.

Alternatively, in order to handle situations as the example given above, define price levels for bids that adapt to the hourly spot prices. Consider a situation with e.g. five different spot price scenarios for a given hour, where $\pi^1 < \pi^5 < \pi^3 < \pi^2 < \pi^4$. In that case, six ordering levels are needed: one level below all spot prices (level $o = 1$), one level above all spot prices (level $o = 6$), as well as four intermidiate levels (levels $o = 2, \ldots, o = 5$). The parameter δ^s_{ot} is used to keep track of the order of the spot prices π^s_t for each hour, and is defined as

$$\delta^s_{ot} = \begin{cases} 1, & \text{if scenario } s \text{ has order } o \text{ in hour } t \\ 0, & \text{else} \end{cases},$$

where $t = 1, \ldots, T$, $s = 1, \ldots, S$, and $o = 1, \ldots, S + 1$.

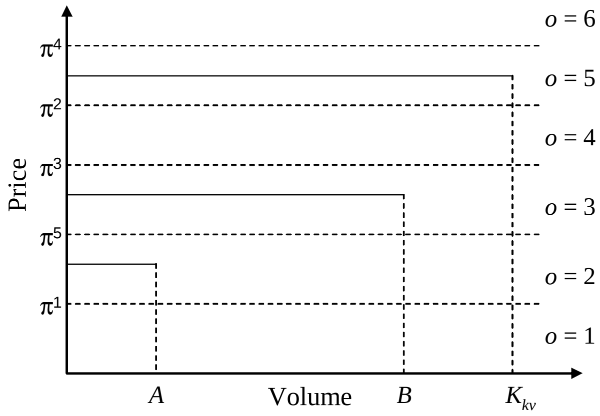

Figure 7: Ordering of spot prices for a single hour with associated total bid volumes.

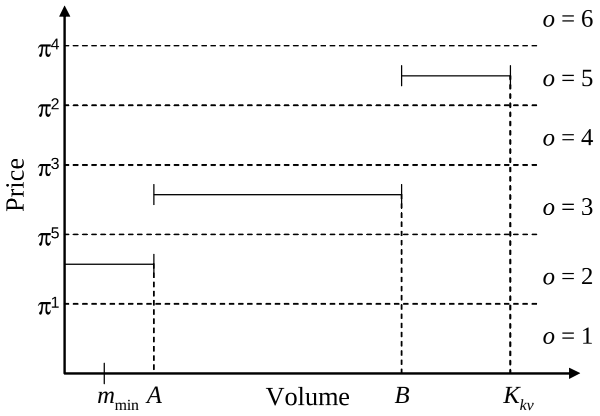

Figure 8: Ordering of spot prices for a single hour with associated bid volumes.

Each hourly price level has an associated bid volume, m_{ot}, measured in MWh_{heat}. Figure 7 illustrates the case with five ordered spot prices for a single hour.

The level below the lowest spot price π^1 (i.e. the first expected spot price for the hour in question) has order 1. This level is too low for the plant manager to willingly bid any volume (i.e. lower than the marginal production cost on the CHP unit). Once the price exceeds the spot price π^1 and enters level 2, the plant manager is willing to bid the volume A. When the price exceeds the fifth expected spot price, π^5, and reaches spot price level 3, the plant manager is willing to bid the total volume B. The full capacity, K_{kv}, of the CHP unit is not bid before the second expected spot price, π^2, is exceeded (i.e. level 5 is reached).

Note, that

- $\delta^s_{1t} = 1$ for all s and t, as all spot price scenarios have at least order 1;
- $\delta^s_{S+1,t} = 0$ for all s and t, as no spot price scenario will ever have order $S+1$.

Now, the constraints (6) may be reconsidered with the abovementioned explanation in mind. For the sake of simplicity, segmentation is disregarded, thus the constraints reduce to

$$z^s_t m_{\min} \le \sum_o m_{ot}\delta^s_{ot} \le z^s_t K_{kv}, \quad \forall s, t \tag{19}$$

The structure of the constraint is illustrated in Figure 8.

In this case, $m^1_{2t} = A$, $m^5_{3t} = B - A$, and $m^2_{5t} = K_{kv} - B$, where K_{kv} is the maximum capacity of the CHP unit. This is in accordance with constraint (19), as $A + (B - A) + (K_{kv} - B) = K_{kv}$ and $m_{\min} < A$.

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