Natural Gas Supply in Denmark
- A Model of Natural Gas Transmission and the Liberalized Gas Market

A Masters Thesis submitted to the department of Informatics and Mathematical Modeling at the Technical University of Denmark

Author: **Lars Bregnbæk**

Supervisors: **Thomas K. Stidsen, Assistant Professor**
Hans Ravn, Dr. Techn.

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Abstract

In the wake of the liberalization of European energy markets a large area of research has spawned. This area includes the development of mathematical models to analyze the impact of liberalization with respect to efficiency, supply security and environment, to name but a few subjects. This project describes the development of such a model.

In Denmark the parallel liberalization of the markets of natural gas and electricity and the existence of an abundance of de-centralized combined heat and power generators of which most are natural gas fired, leads to the natural assumption that the future holds a greater deal of interdependency for these markets.

A model is developed describing network flows in the natural gas transmission system, the main arteries of natural gas supply, from a technical viewpoint. This yields a technical bounding on the supply available in different parts of the country. Additionally the economic structure of the Danish natural gas market is formulated mathematically giving a description of the transmission, distribution and storage options available to the market. The supply and demand of natural gas is put into a partial equilibrium context by integrating the developed model with the Balmorel model, which describes the markets for electricity and district heat. Specifically on the demand side the consumption of natural gas for heat and power generation is emphasized.

General results and three demonstration cases are presented to illustrate how the developed model can be used to analyze various energy policy issues, and to disclose the strengths and weaknesses in the formulation.

Contents

Nomenclature

α	constant ≈ 1 for pipe flow equations
β^e	cost factor for electricity distribution
β^h	cost factor for heat distribution
β^x	cost of transmission
χ_i	export rate at node i
ΔE	energy gain
$\Delta^{s,t}$	duration of time period s, t
$\Delta^{s,t}$	duration of time segment s, t
ϵ_a^h	distribution loss factor for heat
ϵ_r^e	distribution loss factor for electricity
Γ_s	fraction of purchased storage capacity, which must be in the storage facility by season s
ι_i	injection rate into facility
κ_{EN}^s	monthly entry capacity booked in month s
κ_{EN}^Y	annual entry capacity booked
κ_{EX}^s	monthly exit capacity booked in month s
κ_{EX}^Y	annual exit capacity booked
Λ_i	stock of a facility
$\mathbf{A}$	node-pipe incidence matrix
$\mathbf{G}$	graph describing the transmission network
$\mathbf{s}$	vector of source strengths in the network

$\mathcal{G}$ set of generation technologies

$\mathcal{M}$ set of emission types

$\mathcal{A}$ set of areas

$\mathcal{C}$ set of countries

$\mathcal{D}$ set of distribution areas

$\mathcal{E}$ set of edges

$\mathcal{I}$ subset of areas where a storage facility is located.

$\mathcal{L}$ point of linearization

$\mathcal{P}$ subset of areas where production of gas occurs or gas can be exported.

$\mathcal{R}$ set of regions

$\mathcal{S}$ set of seasons

$\mathcal{T}$ set of time segments

$\mathcal{Y}$ set of years

ν_i supply rate at node i

Ω added heat

$\bar{\iota}_\sigma$ injection allowance for storage product σ

$\overline{\psi}_e^l$ slope of linearization plane in direction of pressure

$\overline{\upsilon}_e^l$ slope of linearization plane in direction of flow rate

$\bar{\varepsilon}_\sigma$ extraction allowance for storage product σ

$\overline{m}_{c,m}$ limit on emission type m in country c

$\overline{p}_e^l$ intersect of linearization plane

$\overline{V}_\sigma$ volume allowance for storage product σ

$\Phi(\cdot)$ function representing the emissions resulting from the generation profile

$\pi_i^{s,EXP}$ export price at i in season s

$\pi_i^{s,IMP}$ import price at i in season s

ρ density

ρ^s cost of monthly capacity contract as fraction of annual contract

ρ_n density under normal conditions

σ index of storage products

$\sqrt{\frac{1}{f}}$ transmission factor

τ^{ι}	injection tariff
τ_{δ}^{ξ}	distribution tariff in distribution area δ for price step ξ
τ_{EN}^{Y}	tariff for annual entry capacity
τ_{EX}^{Y}	tariff for annual exit capacity
τ_V	volumetric transmission tariff
ε_i	extraction rate from facility
Ξ	set of price steps for the distribution system operators
ζ_{σ}	product units purchased of storage contract σ
A	area of pipe cross-section
c_B	back-pressure ratio between electricity and heat production
c_v	specific heat capacity at constant volume
D	is the pipe diameter
dh_f	head loss due to friction
E	efficiency factor of a pipe
e_d	electricity made available to the consumer after loses in transmission, distribution etc.
e_s	generated electricity
f	friction factor dependent on the pipe roughness and Reynolds number
f_t	theoretical friction factor
F_x	net forces acting in direction x
$F_{i,g}^{s,t}(\cdot)$	Function describing the natural gas consumption of technology g in area i in time segment s, t
g	acceleration of gravity
g_g	generation constraints of technology g
h_s	generated heat
K_a	cost of heat and power generation within a certain area
p	pressure
Q	volumetric flow rate
q	added heat
Q_n	volumetric flow rate under normal conditions
R	the universal gas constant

R_i	residual demand for natural gas
T	temperature
t^e	energy tax rate on electricity
t^h	energy tax rate on heat
u	mass flow rate
U^e	utility function of electricity
U^h	utility function of heat
V	volume
W	performed work
w	average flow velocity across over a cross section of pipe
w_g^{ξ}	weight associating distribution price steps with generation types
$x^{r,\rho}$	transmission of electricity between regions
$X^{x(r,\rho)}$	investment in electricity transmission capacity
Z	compressibility factor of natural gas

CHAPTER 1

Introduction

This project encompasses the development of a technical supply model of the Danish natural gas transmission system, and a mathematical description of the economic structures relevant for the supply of natural gas. The two models interact to shed light on the interplay between the three markets of namely natural gas, electricity, and district heat.

1.1 Motivation

The liberalization of the Danish energy markets, particularly for electricity and natural gas, has been accompanied by a wide range of interesting challenges. The degree of integration and market interaction between network bound energy supply forms is particularly interesting in light of the central energy policy issues of efficiency, supply security, and environmental impact.

The interplay between energy markets is perhaps most evident when considering a gas fired combined heat and power plant (CHP). This is a meeting point between the electricity, district heating markets and the natural gas market. Some de-centralized CHPs have been producing power on market-like terms since January 1st 2005, while still under the obligation to supply district heating in accordance with demand. This is a complex economical production node, where the decisions taken by the plant operator are nested in the development of three markets.

It is an open question how de-centralized CHP plants will react to the new market structure on a long term basis, yet their potential for impact on the above mentioned policy issues is considerable. In 2002, the total electricity output from de-centralized CHPs was 22 PJ out of a total 127 PJ of generated electricity [12]. Thus around 17% of the electricity production, which is mostly gas fired, has an uncertain future.

The second motivational factor is that the development of a model for analyzing energy policy requires a combined look at technical as well as economic aspects of the energy supply system. This combination of analyzing technical systems (i.e. the natural gas transmission network) with respect to their capability for energy supply using an economically based market optimization, by means of mathematical modeling, basically sums up the scientific interest of the author of this thesis.

1.2 Objective

The objective of the project is to develop a decision support tool for addressing challenging energy policy issues and issues pertaining to energy systems analysis. The emphasis is upon the development of a mathematical description of the natural gas sector from an integrated technical and economical viewpoint. This description is integrated with the Balmorel model of electricity and district heating (www.Balmorel.com), hoping thereby to achieve a comprehensive representation of the three network bound energy supply forms. Hence developing a tool for analyzing the interplay between these markets.

Requirements for the model include that it should describe the capacity and incentives of relevant market players with a suitable level of accuracy, and thus be able to predict market development. The model should take into account the technical capabilities of existing systems and the organisational/economic structures under which they are operated.

1.3 Structure

This thesis is structured as follows. Chapter 2 contains a presentation of relevant energy supply systems and markets. A historic review is combined with a discussion of current and future challenges in the sector. The liberalization process which is ongoing in Europe is discussed with special emphasis on the Danish model. Chapter 3 outlines the overall structure of the developed model and how it is intended to be implemented. Chapter 4 is a quick review of the Balmorel model. A comprehensive look at theory of, and models for, compressible flow leads to the development of a transmissions model in chapter 5. The structure of the Danish market for natural gas is outlined in chapter 6. This leads to the model formulation which encompasses the economics of natural gas supply. Chapter 7 describes various configuration options and the data set which is implemented. Sample results are presented in chapter 8 and in chapter 9 some inspirational demonstration cases are presented. Finally chapter 10 sums up the project contents and presents concluding remarks. As the thesis contains an abundance of symbolic terms, attention is drawn to the nomenclature in the beginning of the thesis, for reference.

1.4 Reading the Thesis

An understanding of operations research on an introductory level is necessary, as well as familiarity with basic mechanics and thermodynamics. As such, the use of linear, mixed-integer and binary programming will not be addressed (see for example [20] and [21]). A brief insight into conic programming is provided as this is not commonly applied even in OR circles. More emphasis is placed on the field of fluid dynamics. Without giving a comprehensive review of the entire field, the theory necessary for gas flow calculations and modeling is presented extensively.

1.5 Acknowledgements

The author is grateful to those who have provided assistance in connection with this project, and would especially acknowledge and express appreciation for the assistance provided by Gastra (Energinet Danmark), specifically to Jess Bernt Jensen and Torben Brabo for taking the time to provide the necessary insight into the business of gas transmission.

CHAPTER 2

Network Bound Energy Supply

This chapter is a comprehensive description of the Danish network bound energy supply. The basics of energy supply and markets are initially discussed. Four areas are described in the context of infrastructure, organization, and liberalization. These areas are:

1. Electricity supply
2. Natural gas supply
3. District heating
4. Combined heat and power (CHP)

These areas are naturally interdependent and in a post-liberalized energy market the developments in one area have an increasing impact on the other areas.

Figure 2.1 provides an overview of the system of energy supply as a whole, showing the distribution of electricity and district heating capacity, connectivity to the natural gas networks and availability of public heating supply.

2.1 Energy Markets and Supply Systems

The basic objective of an energy supply system is naturally supplying consumers with demanded energy commodities. The objective, when establishing a market structure for energy commodities, is to ensure that production and delivery is performed efficiently, to make the consumer able to obtain the lowest possible price, while maintaining a focus on issues such as supply security and any environmental implications.

The existence of a market for various energy commodities, relies on the presence of infrastructure to enable their delivery from producer to consumer. The technical systems enabling supply (e.g. the electricity grid and natural gas transmission and distribution networks) are often considered natural monopolies, since the investment costs which would be

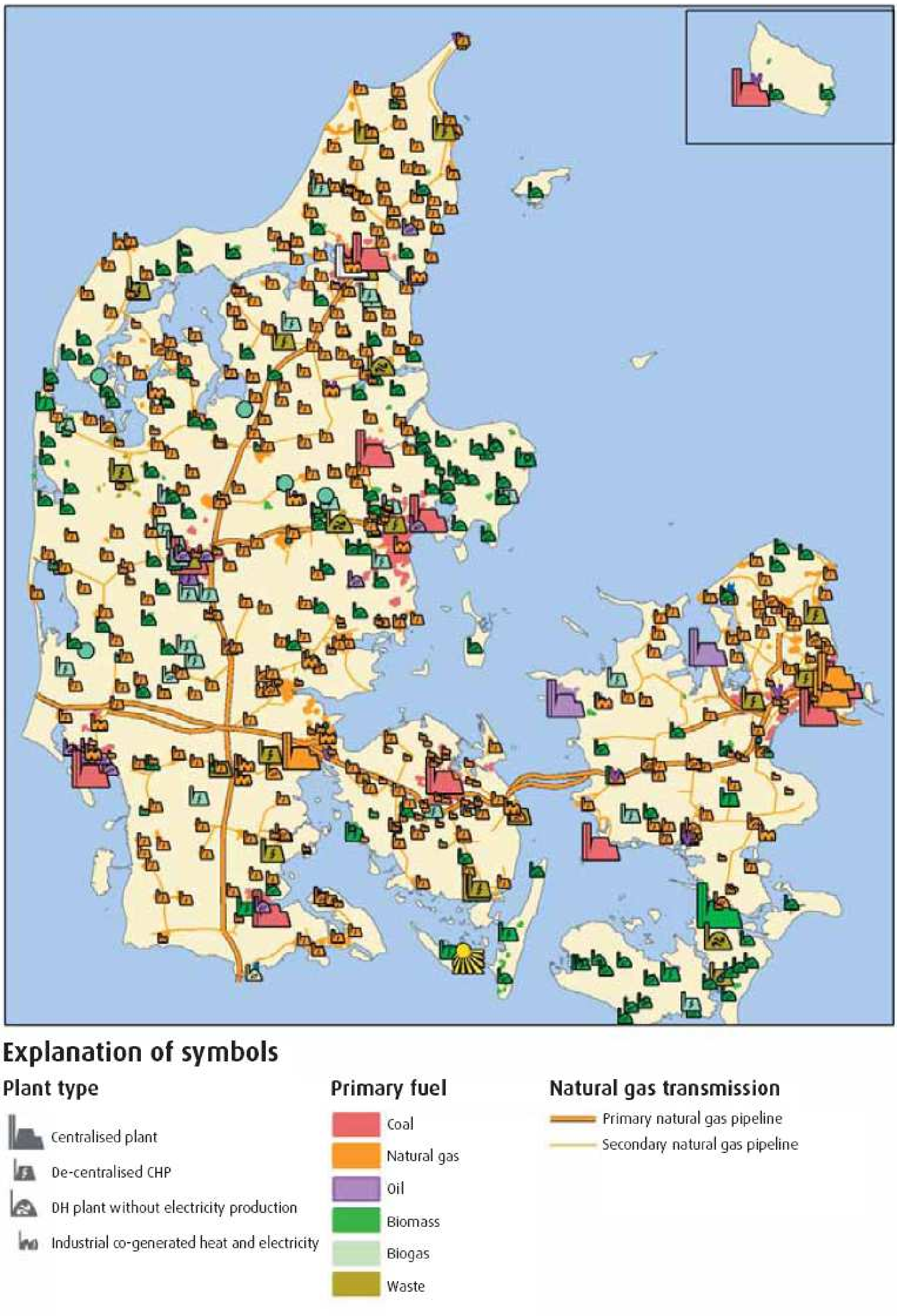

Figure 2.1: Danish energy production and supply. Heat and power generation facilities are distributed throughout the country. Where available most are connected to the natural gas network. The penetration of bio-fuels especially in the public heating sector is also a noticeable trait for the Danish energy supply. (SOURCE: Danish Energy Authority)

inflicted on each market player to develop and maintain his own technical system, supersedes the potential for efficiency gain by having a perfectly competitive market.

The availability of energy and the security of supply is a public commodity as a matter of policy. The consumption of energy units (e.g. molecules of natural gas or MWh of electricity) is a private commodity. For this reason, and the natural monopoly consideration, energy and energy supply is sold and purchased through two organizational structures; a public service structure and a market oriented structure. The key to liberalizing energy markets is to separate the public and the private commodities in order to enable transparency for consumers with respect to energy prices and to ensure that super-visional structures are able to asses the performance of companies in charge of supplying public commodities.

2.2 Three Interconnected Markets

In Denmark three energy systems form a very interesting and interconnected structure. The electricity and district heating systems meet in combined heat and power (CHP) generation facilities, of which most are natural gas fired, and spread widely over the country. As such the three networks interface in the technology of co-generation.

The structure of today's Danish energy markets is a product of the European single market project, the purpose of which is to increase competition and efficiency with respect to national and European level concerns for supply security and environmental conservation. As a result, the electricity and natural gas sectors have been reformed in parallel.

The purpose of the liberalization is as quoted from the Treaty *to secure the free movement of goods, persons, services and capital within the internal market* in this case with regard to the markets of electricity and natural gas. The emphasis is placed on increased transparency and access for market players to ensure an integrated, competitive and efficient market. This encompasses the establishment of general principles for a framework at community level, while leaving the implementation of this framework to the member-states, in recognition of the differences in the structures of the national energy systems.

There is a certain degree of freedom for member-states to subsidize or otherwise prioritize electricity generated from renewable and co-generation methods.

Recently, in an effort to further integrate the energy sector, the three system companies bearing system responsibility for electricity and natural gas supply (Eltra, Elkraft System and Gastra) have been merged into one company bearing the full weight of system responsibility pertaining to electricity and natural gas supply, Energynet.dk. This, along with a large number of mergers between market players, some planned and others already performed, is a very obvious example of why the kind of research undertaken in this project is highly relevant, in light of current developments.

2.3 Electricity Supply

Electricity supply in Denmark is mainly secured by three types of generation.

1. Central plants
2. De-centralized combined heat and power plants
3. Wind power

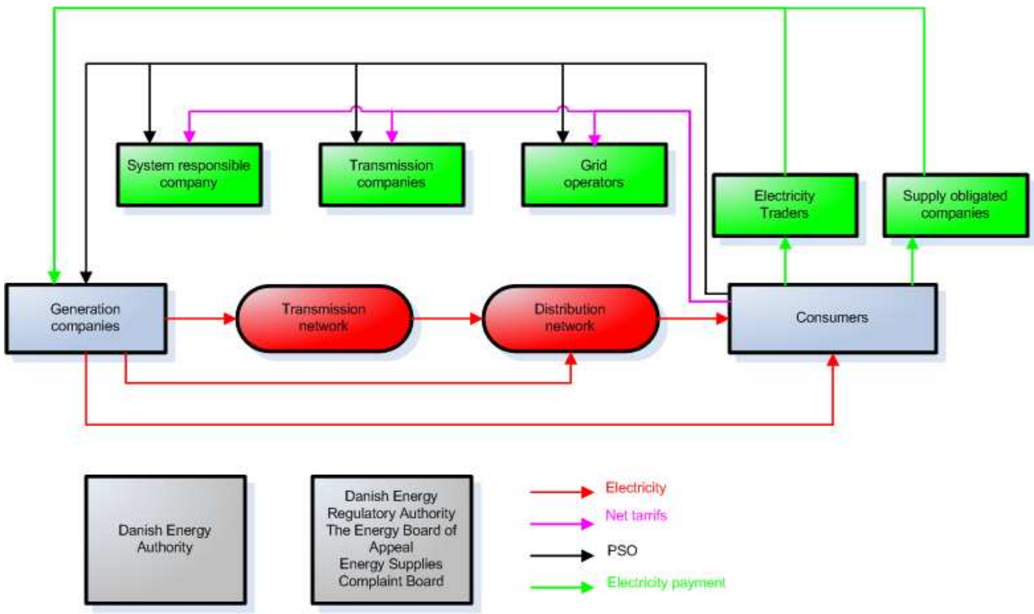

Figure 2.2: Organization of the electricity sector (SOURCE: Danish Energy Authority[28])

The central plants were originally large powerplants, mainly oil fired, until the energy crisis in the 1970s. Almost all have since been converted to combined production of heat and electricity, and they now supply Denmark's largest cities with district heating while retaining a large share of the total electricity production. Most of them are today fired by either coal, biomass or natural gas, in part to decrease dependency on insecure oil supplies. Each plant is located on one of the 15 central power plant locations in the largest Danish cities.

De-centralized combined heat and power plants were originally local suppliers of district heating, organized at municipal level or as consumer owned private companies. Many of these heat producers have through the 1990s been converted to co-generation due to the introduction of a favorable subsidy on combined production enabling decentralized CHPs to sell electricity at a feed-in tariff on prioritized terms. There are approximately 600 de-centralized plants currently in operation.[28]

Wind power is still prioritized on the electricity market. There are around 5,400 wind turbines spread out around the country. The total wind generation capacity accounts for 3118 MW by January 1st 2005 and in 2004 18.5% of electricity generation was performed by wind power.[28]

2.3.1 Organization of the Electricity Market

Today, the Danish electricity supply structure is organized as displayed on figure 2.2, as a result of the market liberalization.

The figure (2.2) demonstrates how the roles of the different actors are connected. The red arrows show the actual electricity flows. Green arrows show how payment for delivered electricity occurs. The black arrows show payments for the public service obligations, which the different companies serve the consumer. Finally, the purple arrows demonstrate the flow of network tariffs from the consumer to the network companies.

A further description of what the payments mean follows below:

1. Electricity trading takes place on market terms. Either by bilateral agreements or on one of the power exchanges (either the Nordic exchange, Nord Pool, or the German exchange, EEX). The supply obligated companies supply customers who do not wish to take advantage of the free choice of electricity supplier. These companies supply customers at regulated prices.

2. Network tariffs are payments to cover the expenses of the delivery of electricity from producer to consumer. This covers expenses of the system responsible company, transmission operators, and grid operators.

3. PSO payments cover the common interests of the electricity market. This includes supply security, subsidies for environmentally friendly production, energy related research etc.

Electricity trading on the Nord Pool power exchange is a bid-ask process between generators and electricity traders. The system price (spot-price) is formed 24 times in the day-ahead market. All traders can take bids at the spot-price assuming there is sufficient capacity for transmission. If this is not the case, the exchange forms area prices which reflect the supply situation. Aside from the day-ahead market, Nord Pool also deals with futures in electricity.[31]

2.3.2 Legal Foundation for Liberalized Electricity Markets

On a European level directive 96/92/EC provided general definitions for actor roles within the electricity systems of members states, and more importantly the un-bundling of accounts. This has later been replaced by directive 2003/54/EC and Regulation (EC No 1228/2003) governing conditions for cross-border trade in electricity. The rules enforce the principles of non-discriminant access to networks, as well as transparency. The rules dictate that efforts should be undertaken to ensure that system operators make available all necessary information for obtaining access to the network, with transparent and non-discriminant access prices. Also they dictate that system operators must preserve confidentiality of commercially sensitive information.[29]

The latest Danish implementation of these measures into national legislation are Law No 494 and 495 both of June 9th 2004.[28]

2.4 Natural Gas Supply

The Danish supply of natural gas originates in the off-shore oil and gas fields in the North Sea. Two high pressure pipelines extend along the sea bed and make landfall in Jutland. They meet at the Nybro gas treatment plant near the western coast of Denmark (see Figure 2.3), where up to 24 million cubic meters (energy content roughly equal to 1000 TJ) of gas can be treated daily. From Nybro two 30 inch transmission lines extend across Jutland towards the major junction at Egtved. From here one connection goes South to the Danish-German border at Ellund. Another goes North to the gas storage facility at Lille Torup and terminates in the city of Aalborg. Finally a transmission line runs all the way East across the country, passing Odense and crossing both "Belts" to arrive on the outskirts of Copenhagen near Karlslunde. From here one line proceeds to the Stenlille storage facility while others

Figure 2.3: The Danish natural gas transmission system (SOURCE: Gastra)

proceed to supply the area of Greater Copenhagen and the northern parts of Zealand. Ultimately a transmission line crosses Øresund to supply our Swedish neighbors.[30]

Most of these major transmission lines are 20-30 inches in diameter and perform at a maximum pressure of 80 bars. At no point in the transmission network is the pressure allowed to descend below 42 bars, in order to secure adequate pressure at the final delivery locations. Metering and regulation stations (M/R stations) are located along the transmission lines. From here, natural gas is extracted from the transmission system into the underlying distribution networks. Here the responsibility for network operation is also passed from the transmission system operator Gastra [30] to one of the four distribution system operators. These operators, along with the storage system operator, are public companies responsible for providing the basic services of natural gas supply. They develop products for capacity and volumetric throughput in the system, and provide balancing services. The model used in this article is a reflection of present and previous structures, of services available to the gas shipper.

2.4.1 Organization of the Market for Natural Gas

The breakdown of institutions in connection with the liberalization process has resulted in the new, and unbundled, structure of the Danish natural gas sector.

The overall systemic responsible company for the natural gas system is Gastra (EnergiNet Danmark), which is also responsible for the operation and development of the transmission system. There are five distribution networks, two of which are operated by DONG Distribution, and the remaining three are operated by Naturgas Fyn, Naturgas Midt-Nord and Hovedstadens Naturgas. The distribution companies are also corporately associated with

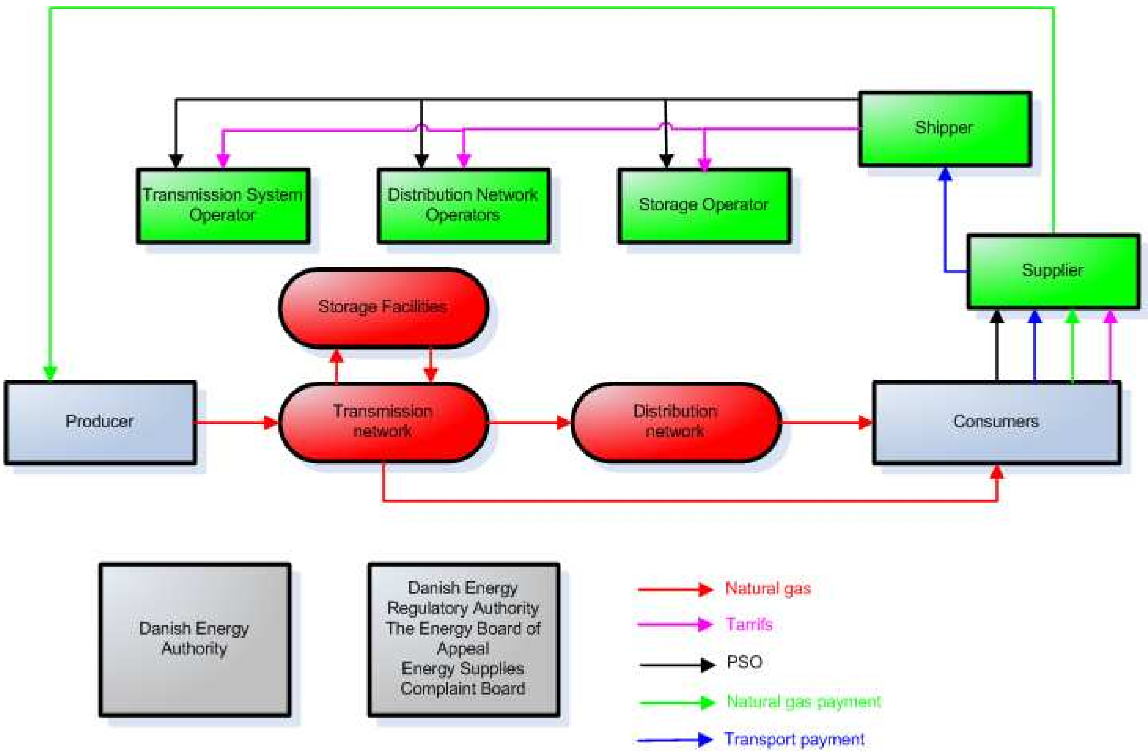

Figure 2.4: Danish natural gas market structure

the supply obliged companies and certain suppliers on market terms, however, each of these act a as an individual legal entity in accordance with the requirement for unbundling. The storage operator, DONG Lager, is also a separate entity and part of the DONG corporate structure.

There is still no exchange for natural gas. All gas is traded bilaterally. There is, however, a possibility for traders to swap gas amongst each other. Gastra has developed a virtual Gas Transfer Facility (GTF), which enables traders to swap gas in the network. Such a facility is also available for capacity within the transmission system, called the Capacity Transfer Facility (CTF). These virtual trading points are the first steps towards an actual trading point for natural gas. The next step, which is currently being investigated by Gastra in conference with Nord Pool, is to establish a virtual trading hub. Finally, an actual natural gas exchange might be developed in the not so distant future.[23]

2.4.2 Legal Foundation for Gas Market Liberalization

Directive 98/30/EC of the European Parliament and the Council of 22 June 1998 concerning common rules for the internal market in natural gas, provides the basis for the liberalization of the natural gas market. Proceeding this directive, two other directives (90/377/EEC and 91/296/EEC) had been adopted in 1990 and 1991 respectively. These directives called for transparency and reporting of prices for EU statistical purposes, and access rights to national high pressure transmission networks. The natural next step, in light of 96/92/EC, was the call for a non-discriminant and transparent access to natural gas supply services in the European Community, as had been done with regard to electricity as explained previously.

The contents of 98/30/EC is similar to that of 96/92/EC, but the differences in the technical/physical properties of electricity and natural gas and their connected systems set their mark on the specifics of the directive. There is the additional functional possibility of natural gas storage as opposed to electricity, and the possibility of dealing in liquefied natural gas (LNG). Basically the principles regarding market structure in 96/92/EC are echoed in 98/30/EC, calling for the separation (un-bundling) of different natural gas undertakings (production, transmission, distribution, supply, purchase and storage) as well as transparency and non-discrimination.

Directive 98/30/EC was replaced on June 26th 2003 by 2003/55/EC. All the mentioned documents can be found on the EUR-Lex website[29].

2.5 District Heating

Public heating supply is extensive in Denmark having connected 60% of all private homes to some form of public heating. Public heat planning was undertaken from 1979 and onwards and as a result a large number of municipality or private-consumer owned heat companies appeared.

The central planning of heating supply was a reaction to the energy crisis of the 1970s, and part of the larger project of national energy planning as a whole. The projected introduction of natural gas was undertaken in the same year, and naturally there was a political desire to utilize this investment efficiently.

As part of this planning, municipalities where given the authority to oblige private properties to be connected and supplied through the local public heating supply (district heating or individual natural gas heating), as this was developed. This obligation still stands today.

The supply of district heat is considered a natural monopoly since potential efficiency gains from having perfectly competitive markets, do not justify investments into parallel supply systems. This was also the case for electricity and natural gas supply as described in the previous sections. District heating, however, is not efficiently transported over great distances and as such heat generation is also most often considered to be a natural monopoly. Therefore, the generation of heat is a public obligation and the pricing of heat is regulated by the supervising authorities.

The basic guideline for regulation is that the business of producing and supplying heat must be self sustainable, which means that companies can only charge what is required to secure operating and investment costs.

The desire for energy efficiency and environmentally friendly production has spurred a reform of the district heating sector. Almost all district heating boilers have been replaced by either co-generation units, primarily fired by natural gas, or by bio-fuel heating units.

2.6 Combined Heat and Power (CHP)

Co-generation of heat and electricity became a matter of national priority with the combined heat and power agreement of 1986[25]. Combined generation of electricity and heat results in a higher total fuel efficiency than for separate generation. The process is basically to use the heat waste product from electricity generation, and use this in the local district heating network.

Almost all central power plants have been converted to co-generation and the remaining central facilities serve only as back-up or peak load producers.

De-centralized CHP capacity has increased from roughly 200 MW in 1990 to nearly 2.5 GW in 2000. The incentive for co-generation has been the presence of a favorable feed-in tariff for unloading electricity into the network. The liberalization of the electricity market is now putting an end to this way of selling electricity. Now larger de-centralized producers must unload electricity in competition with other producers. The current direction of developments is that all de-centralized electricity producers will soon operate on market terms.

The subsidy of local co-generation has not completely disappeared. Were it so, it would be at the expense of the heat consumers, who are obligated to take part in the local heat supply. The subsidy is now put not on the electricity generation side, but on the heat supply side of the equation. This means that local CHPs now have to produce electricity to sell on the market so as to be able to reduce prices for its heat costumers.

2.7 Market Definition and Regulation

The two grey boxes in figures 2.2 and 2.4 describe defining and regulating national agencies. The first box containing the Danish Energy Authority is responsible for defining the rules of the energy markets in general. They interpret how legislation should be implemented in praxis. They also support research and development projects deemed in the public interest.

The other box contains the regulatory bodies of which the Energy Regulatory Board monitors the prices for PSOs and net-tariffs as well as the prices the supply obliged companies charge consumers. The Energy Board of Appeal handles civil complaints between consumers and electricity companies, whereas the Energy Supplies Complaints Board deals with complaints against the decisions of the Energy Authority and the Energy Regulatory Board.

2.8 Summary

Energy supply systems have been put into place all over Europe during the last century. In most cases the development of energy markets have been a matter of national concern, causing a spawning of publicly owned and managed energy supply companies, obliged to bring energy at a fair price to every corner of Europe. Many of these have since been privatized in recognition of the tendency that public monopolies are generally not economically efficient.

In line with the EU Directives concerning rules for the internal markets in electricity and natural gas, the structure of the sectors in Denmark have been developed to ensure open and transparent access to transmission and distribution systems. Also, an unbundling of accounts has occurred to ensure that the tariffs charged for transmission and distribution of energy commodities reflect the investment and operating costs of the transmission/distribution system.

The merging of the three system responsible companies (Eltra, Elkraft System and Gastra) to form the new company, Energinet.dk, is a development, which has made the research undertaken in this project more relevant than initially expected. It can be expected that the merger will serve to consolidate efforts between the electricity and gas sectors when addressing future challenges for energy supply, market development, efficient energy utilization, and relevant environmental concerns. One example of such activity is the research program entitled "A Model of and Analyses of an Integrated Gas and Electricity System." undertaken by EnergiNet Danmark in cooperation with relevant research institutions and supported by the Danish Energy Authority.

CHAPTER 3

Model Structure

The model developed in this project combines the technical and economic aspects of natural gas supply. The implementation is divided into two parts, which are described separately. However, in this chapter the overall structure of the implementation is presented to give a non-technical overview, which does not demand experience with mathematical modeling and operations research methods. The connection between the developed model and the Balmorel model is also described. The details and mathematical formulation of the technical model of natural gas supply is described later in chapter 5 and subsequently in chapter 6 the economics are formulated.

3.1 Overview

Figure 3 provides an overview of the integrated model. The important thing to note is the partial equilibrium model of the two commodities of electricity and natural gas. Each market takes input from and generates feed-back into the other market. The input taken by the electricity market from the market for natural gas affects the supply functions of electricity. Conversely output from the electricity market affects the demand function for natural gas. This is a reflection of the fact that natural gas is a primary energy source, whereas electricity is a secondary energy commodity.

The blue fields contain the fixed data and supply modeling of the natural gas model. The yellow fields contain fixed data and supply modeling of the electricity and district heating. Green fields contain the data that reflects the top-down elements of the model.

Finally the red fields indicate results of the model execution. Note that one result field, namely the field concerning investments gives feedback into the model. This reflects the fact that decisions regarding investments are transferred to the following years.

3.2 Flow Model

The flow model is basically a transportation model for natural gas. The flow model ensures the technical feasibility of the supply solution. These technical aspects are included directly

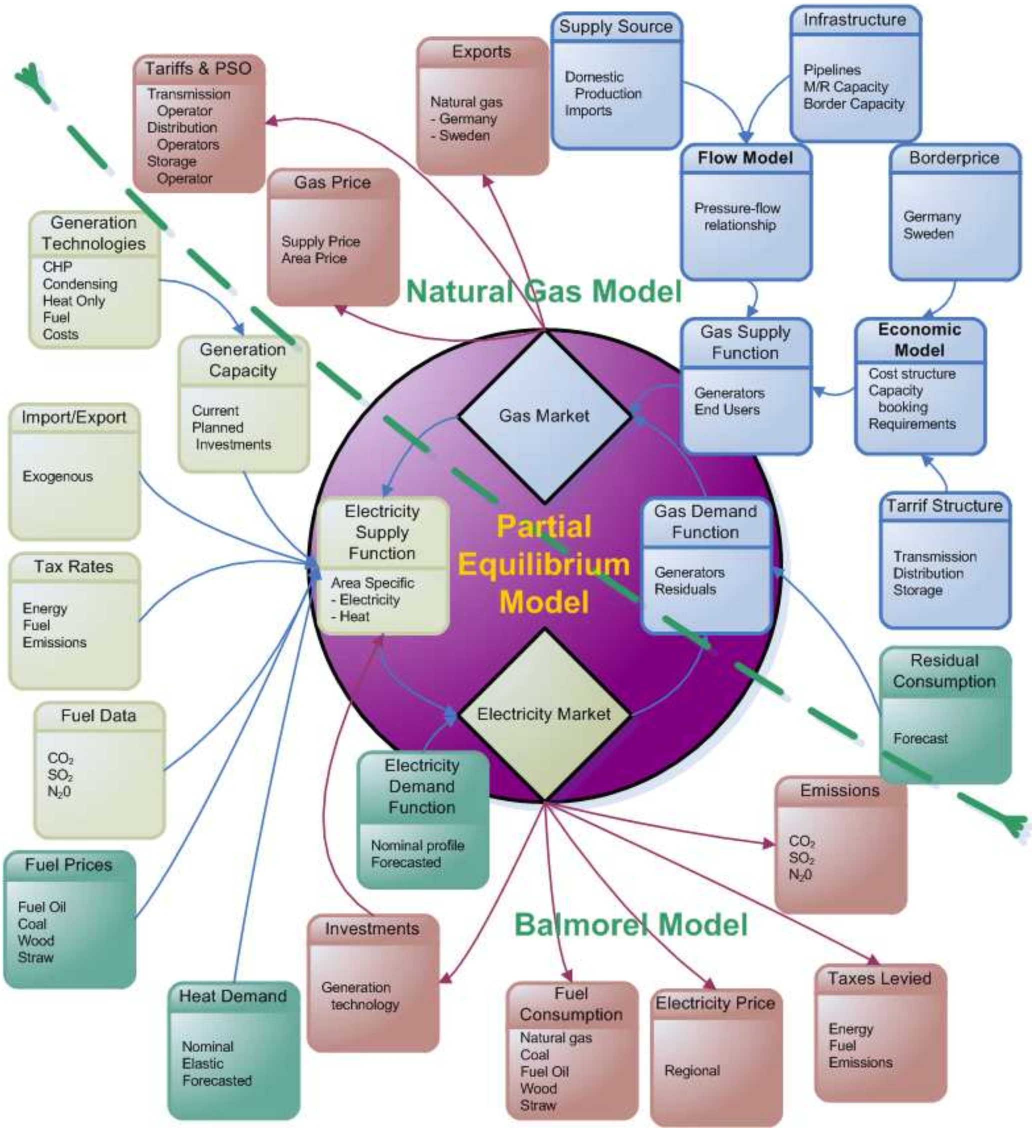

Figure 3.1: Overall model structure.

in the model in order to lure out the impact of for instance capacity shortfalls in economic terms i.e. the value of additional capacity.

The aim is not to make an accurate operational model. This would be too complex to include in this model. Rather, the intention is to be able to extract the economic impact of technical restrictions. It is conceivable that later work could be able to include some elements of capacity investment etc., but this is beyond the scope of this project.

Capacity in the transmission system can be described in terms of pressure and flow. There is a limit to the strain one can subject pipeline components to, and therefore there are pressure defined operational limits in the transmission system. Pressure difference is the driving force, which causes flow in the transmission system. Therefore, the higher the pressure is at the source, the more flow can be pushed through the network. This is also the case on a distribution level, and therefore there are minimum pressure levels at transmission system outlets, in order to ensure that the distribution systems are able to push adequate flow through to their customers.

Natural gas transmission networks have the additional property of being able to store gas in the pipelines; a concept termed line-pack. This is done by raising the pressure in the transmission system by feeding in more natural gas than is taken out. This gives a buffer, which grants the operator a strong tool to react against outages, or can be used to compensate for short-term variations in demand or production.

3.3 Economic Model

Part of the model concerns the structure of the natural gas market. This module has two components. One is the determination of which contracts are purchased by industry from the system operators to gain the desired access to the system. This has regard for capacity and transmitted volumes in the transmission system and subordinately in the distribution networks. Also, it is a determination of which contracts are made with the storage system operator to ensure that storage capacity, injection and extraction capacity are all payed for as well as the variable costs of injection.

The second component is the construction of the optimization criteria, by the sum of costs inflicted upon the market.

3.4 Summary

The project is structured around the development of the Natural Gas Supply System Model. Model emphasis is placed on accurately describing the economic structure of the supply system, ensuring technical feasibility, and linking this with the Balmorel model.

CHAPTER 4

The Balmorel Model

The Balmorel model is a partial equilibrium model, which describes jointly an international electricity and heating system. The model was originally developed to shed light on international energy conditions in the Baltic Sea region and was in part financed by the Danish Energy Authority's Energy Research Programme around the year 2000.

4.1 Top-Down - Bottom-Up

The Balmorel model combines the approach of bottom-up modeling in a classic technical modeling tradition with top-down economic analysis, projections and forecasts. By describing mathematically the mechanisms, which define action and reaction to changes in the state of the system, the bottom-up part drives the model towards a stable state where, held up by boundary conditions describing the world outside the model dynamics, the model is able to produce results which are both realistic and comprehensive in terms of what they describe.

4.2 Market Equilibrium

The model is solved by optimizing the value of an objective function. The objective is an expression of difference between consumer utility and total cost of supply. As such the price of commodities is reflected in the cost that the final consumer is willing to pay, where a producer is able to supply at the bided price without generating a loss. Equilibrium is ensured by constraining the amount of energy commodities demanded by consumers at a point of consumption, to be equal to the amount supplied to that location.

Market equilibrium is the state of a market where supply and demand are equal for all considered commodities. The theory of general equilibrium applies to an entire economy, encompassing all goods traded in the economy.

Partial equilibrium theory states that developments in a described market, or a group of related markets, have negligible impact on other markets where prices are fixed. This is an attribute which makes it possible to add a great amount of detail to the description of the examined market, using only simple boundary conditions for describing the dependency on other markets. The development of partial equilibrium theory is attributed to Antoine Augustin Cournot [6] and Alfred Marshall [7].

4.3 Partial Equilibrium and Operations Research

Partial, and general, equilibrium theory relies on functions describing supply and demand. Operations research by tradition uses optimization models capable of simultaneous derivation of a massive number of variables according to some criterium, while subjected to constraints. The supply and demand curves of partial equilibrium theory are thus constructed and formulated mathematically, and the equilibria determined by imposing equality between supply and demand. This makes it possible to simultaneously take into account supply and demand conditions at all the market locations at the modeled time-steps, and determine equilibria for all these. By maintaining a linear model, where non-linear convex functions can be formulated by piecewise linear approximation, the size of the model, i.e. the number of constraints and variables, can be very large and yet maintain computational tractability.

In the Balmorel model, the considered commodities are heat and electricity. Fuel costs are exogenously fixed according to data and forecasts for developments in prices. This implies that the price of natural gas is exogenous in the Balmorel. In this project, an additional market is modeled in determining the correlation between electricity, district heat and natural gas, namely the market for natural gas.

4.4 Elements of the Balmorel Model

Equilibria are reflected in the solution of the model on a number of issues.

- Equilibria between consumer marginal utility and the marginal cost of supply by relevant geographical division and for ever modeled time-segment
- Equilibria between time-segments caused by presence of storage options.
- Equilibria between geographical divisions by transmission options.
- Equilibria of marginal utility between traded commodities.
- Equilibria between short-run and long run marginal costs implied by investment options.

In order to understand the implications of the above, consider the ideal system with infinite capacity for transmission, storage and distribution without loss, with no cost associated to these operations. One common price would appear for all geographical divisions and time-segments for which would reflect both the marginal cost of supply everywhere at any time as well as a global consumer utility.

The introduction of the aforementioned technical and economic elements impose limitations or costs on the transfer of resources between geographical divisions and times, and as such prices become geographically and temporally dependent.

4.4.1 Geography

Geographically, the Balmorel model is constructed on a three-level hierarchy of countries, regions and areas.

The country level features detail of national policy with regard to taxes and emission control as well as provides a logical geographical distinction for aggregation of results.

Regions are subdivisions of a country at a level where the geography described by the region can be assumed to feature a fully connected electricity distribution network. At an interregional level the process of electricity transmission is handled. Transmission bottlenecks appear at an interregional level. Electricity demand is also incurred on a regional level and hereby are electricity prices also determined at this level.

Areas are subdivisions of regions and can be assumed to feature a fully connected district heating network. Consumption of heat and production of both heat and power is associated with areas. Production capacity is naturally also installed at area level.

This has the positive side effect of giving more resolution with regarding to district heating. Since district heating systems are unable to transmit heat over great distances, increased resolution on the production and supply of district heating is also desirable. It was necessary to make new data for heating demand, the process of which is described in section 7.3.1.

This project concerns only Danish network bound energy supply. As such the set of countries $\mathcal{C}$ contains only Denmark. It would be fairly easy to include neighboring countries (at least with regard to electricity and heat), but this would impose additional requirements for computational power, and minor data adjustments.

The set of regions in Denmark contains two elements $\mathcal{R} = \{DK\_W, DK\_E\}$, since there are different electrical systems in Eastern and Western Denmark. As the focus is the natural gas transmission system, the logical choice of areas are areas supplied with natural gas from a specific metering and regulation station. There are about 50 of such areas in the set $\mathcal{A}$. This yields the positive side effect of greater resolution with regard to district heat, which is desirable since district heating systems are unable to transmit heat over great distances. This also makes it more likely that the model will use some of the smaller, and perhaps less efficient technologies whose main justification are their suitability for small scale heat supply. These are likely lost in overly aggregated models.

4.4.2 The Temporal Dimension

There are three temporal levels in Balmorel. The highest level is years, and each year is simulated without foresight regarding conditions in the subsequent years. There are two subdivisions of the year, generally called seasons and time periods. There is no restriction as to how these are to be interpreted, or to how many periods should be included. When simulating for one season and one time segment for example, this could correspond to simulation using annually averaged values. 12 seasons can correspond to months while 168 time-segments could indicate hourly averages for a week within the given month (season).

The following sets describe the segmentation of the year into time periods:

$$\mathcal{S} = \{s_1, \ldots, s_{12}\} \tag{4.1}$$

$$\mathcal{T} = \{t_1, \ldots, t_{12}\} \tag{4.2}$$

The elements of the $\mathcal{S}$ set naturally represent the months of the year. The elements of $\mathcal{T}$ represent hours of a typical week in the appropriate month. The "hours" have varying weight (or duration) as some hours of the day and week are more interesting than others.

As mentioned, the set of years, $\mathcal{Y}$, controls the annual dynamics. A separate linear program is solved for each year and results are transferred to the succeeding years. This specifically concerns results regarding investments in capacity.

4.5 The Objective Function

The objective is to maximize the sum of consumer utility and the negative cost of production and supply. Consumer utility is formulated as follows:

Utility of electricity:	$\sum_{c\in\mathcal{C}}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}(c)} U^{e,r,s,t}(e_d^{r,s,t})$
Utility of district heat:	$\sum_{c\in\mathcal{C}}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\sum_{a\in\mathcal{A}(c)} U^{h,a,s,t}(e_d^{a,s,t})$

Here $e_d^{r,s,t}$ is the electricity made available to the consumer after loses in transmission, distribution etc. in the region r in the set of regions $\mathcal{R}(c)$ pertaining to the country c and the time segment (s,t). $U^{e,r,s,t}$ is the actual utility function of electricity dependent on consumer preferences. The utility of district heat is analogous to electricity, where $a \in \mathcal{A}(c)$ describes the and area a in the set of areas $\mathcal{A}(c)$ of country c.

In this project only non-elastic demands are employed, and as such the utility functions are constants for each time-segment and consumption location.

The associated costs contribution to the objective function are defined as follows.

	$-\sum_{c\in\mathcal{C}}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\Big\{$
Energy taxes	$\sum_{r\in\mathcal{R}(c)} t^e e_s^{r,s,t}(1-\epsilon_r^e) - \sum_{a\in\mathcal{A}(c)} t^h h_s^{a,s,t}(1-\epsilon_a^h)$
Generation costs	$\sum_{a\in\mathcal{A}(c)} K_a^{s,t}(e_s^{r,s,t}, h_s^{a,s,t})$
Transmission: operations and investments	$\sum_{(r,\rho)\in\mathcal{R}(c)^2, r\neq\rho}\left\{\beta^{x(r,\rho)}x^{r,\rho,s,t} + X^{x(r,\rho)}\right\}$
Distribution costs	$\sum_{r\in\mathcal{R}(c)} \beta_r^e \frac{e_d^{r,s,t}}{1-\epsilon_r^e} \sum_{a\in\mathcal{A}(c)} \beta_a^h \frac{h_d^{a,s,t}}{1-\epsilon_a^h}\Big\}$

Here e_s and h_s represent the generated amount of electricity and heat respectively. $\epsilon_r^e, \epsilon_a^h$ are the percentage loss in the distribution process. t^e and t^h are the energy tax rates associated with electricity and heat. $K_a^{s,t}(e_s^{r,s,t}, h_s^{a,s,t})$ is the cost function associated with a certain generation of heat and electricity in a given area. This function includes fuel costs, fuel taxes, emission taxes and operating costs. $\beta^{x(r,\rho)}$ is the cost of transmission between the regions r and ρ, $x^{r,rho}$ is the transmitted amount, while $X^{x(r,\rho)}$ represents an investment in transmission capacity. Finally β_r^e and β_a^h represent cost factors for distribution.

4.5.1 Investments

The model features the possibility to invest in both generation capacity and electricity transmission capacity between regions. These investments can either be endogenously performed at run-time, or, depending on the modeled scenario, be given as exogenous input data.

The investment option is limited by the shortsightedness of the temporal resolution. As annual planning is quasi-dynamic, the criteria for performing investments is a matter of the feasibility for the investment within the year in which it is undertaken. This means the comparison made is between the financial cost within the first year of operation with the efficiency gain in the overall system. In the following year the investment is treated as already existing capacity, and the investment costs are considered sunk costs.

4.5.2 Energy Transformation

In the Balmorel model various forms of energy transformation are supported. Technologies are described in terms of transformation potential, efficiency, cleanliness as well as economic parameters such as variable production costs, fixed annual costs and investment costs. In the following the basic forms of transformation supported by Balmorel are described. The technology types are exemplified with specific technologies, but may well be used to represent different technologies with similar technical characteristics. Generation constraints are formulated generally as a function of the produced amount of heat and electricity:

$$g_g^{s,t}(e_{s,g}^{s,t}, h_s^{s,t}) \leq 0, \forall g \in \mathcal{G} \forall s \in \mathcal{S}, \forall t \in \mathcal{T}$$

Single Energy Type Transformation

The two first technologies, illustrated on figure 4.1, represent technologies producing either only electricity or only heat. These can be exemplified by traditional condensing power plants, where the heat waist product is cooled by an intake of seawater, and traditional heat-only boilers, which produce only heat respectively.

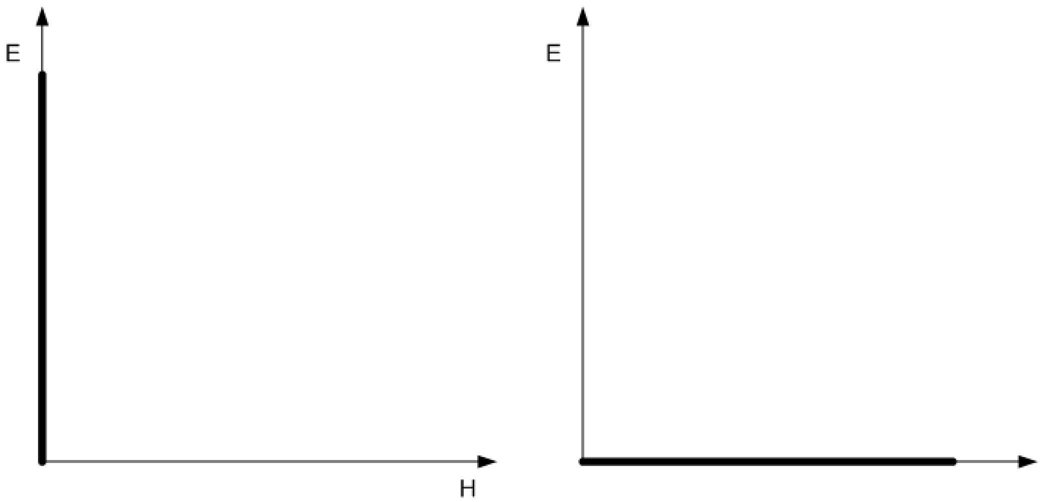

Figure 4.1: Electricity only and heat only production technologies.

CHP Technologies

Combined heat and power facilities come in many shapes and forms, but overall they can be divided into two types. These are fixed-ratio technologies, which produce heat and electricity at some near-constant ratio, and variable ratio technologies. Fixed ratio units are exemplified by gas engines or back-pressure gas turbines. Generally the ratio between electricity and heat is termed the c_B value of the technology. Variable ratio technologies are for example extraction steam turbines, where heat can be extracted at some point along the turbine to be used for district heating, or it can run along the full length of the turbine from where its temperature becomes too low to have practical use in the district heat network. Figure 4.2 illustrates the feasible region of these production technologies.

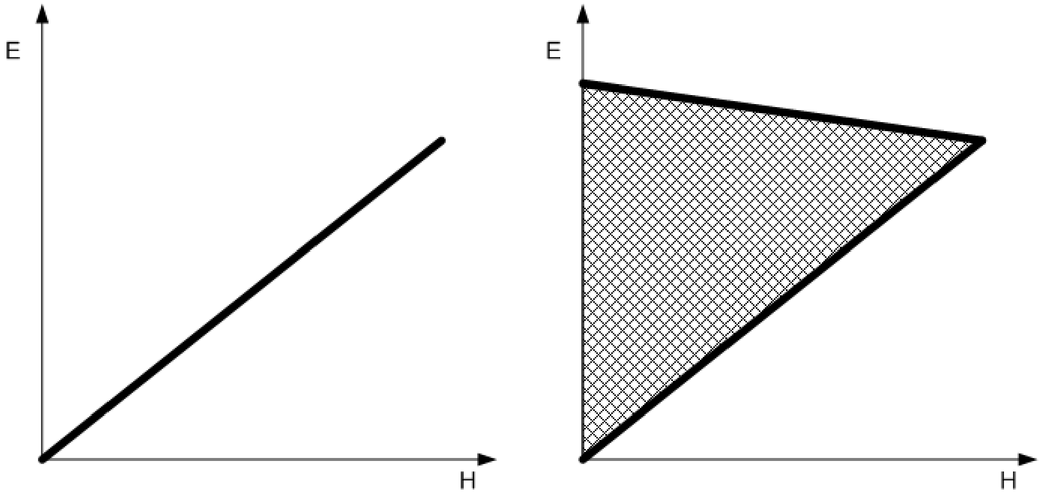

Figure 4.2: Combined heat and power technology types.

Storage Technologies

Heat and electricity storage facilities can also be described. Heat storage is generally a large insulated container with hot water. Electricity can be stored by hydrogen fuel cells or by pumping water into a reservoir, from where it at a later time can drop through a turbine releasing the energy potential. Figure 4.3 illustrates the production profiles of storage facilities.

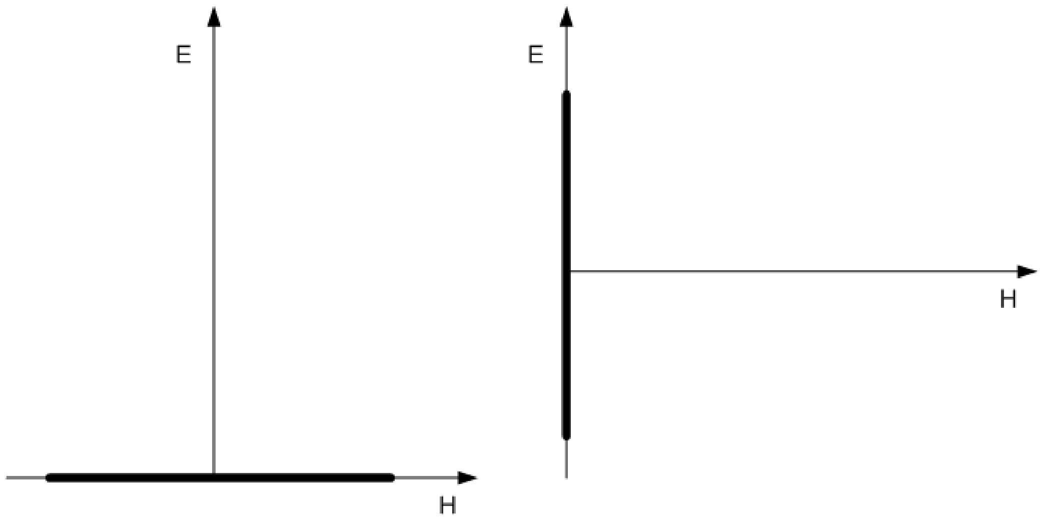

Figure 4.3: Heat and electricity storage technologies

Wind Power and Heat pumps

Finally a technology describes fixed electricity production units. These are wind or solar powered units who's production is fixed by the availability of wind or sunlight. Thus these appear as a point on the electricity-heat chart in figure 4.4. This technology option is sketched alongside heat pumps and similar technologies (such as electricity powered heat boilers), which use electricity to generate heat.

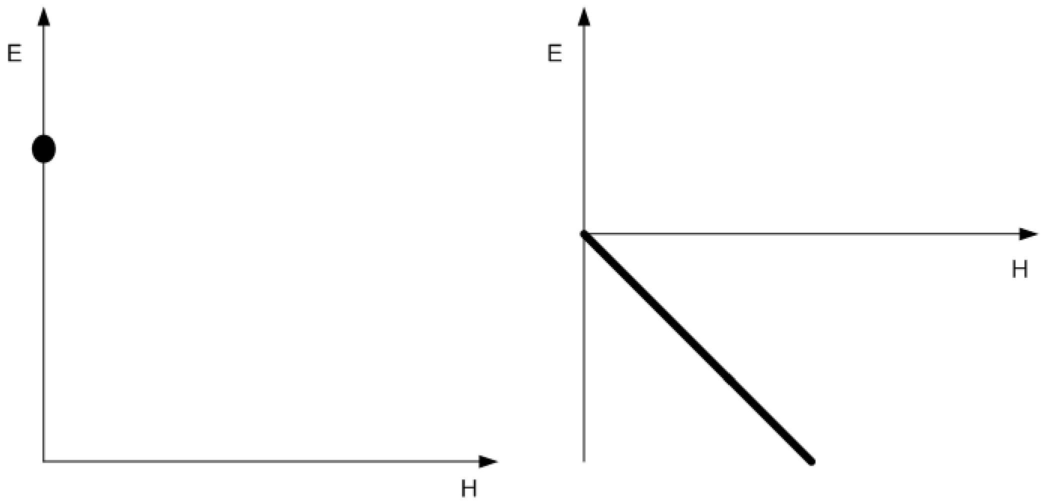

Figure 4.4: Heat and electricity storage technologies

4.5.3 Transmission and Distribution

Transmission of electricity is possible between regions, but transmission is limited by exogenous and endogenous transmission capacity. Transmission capacity is thus constrained by a simple linear flow model. Costs for transmission and loss in the network are also incurred. Regions serve as points of consumption for electricity. These are characterized by a loss factor, costs etc., in representation of a distribution network.

Transmission of district heat over great distances is infeasible, and thus heat demand must be supplied by generation from within each area. Areas are associated with distribution losses and costs with respect to heat, as with electricity consumption nodes above.

4.5.4 Energy Demands

Energy demands are represented by a nominal demand profile, which varies over time-segments. The built in data contains a representation of variations over the day, week and between seasons. The profile is applied to an annual demand by consumption node (region for electricity and area for heat). There is an option to introduce own-price elasticities, yet this is not applied in this project.

The demand satisfaction constraints can be stated for electricity as:

$$\sum_{g \in G(r)} e_{s,g}^{s,t} + \sum_{\rho \in \mathcal{R}(c), r \neq \rho} x^{(r,\rho),s,t}(1 - \epsilon^{x(\rho,r)}) = \frac{e_d^{r,s,t}}{1 - \epsilon_r^e}, \forall r \in \mathcal{R}, \forall s \in \mathcal{S}, \forall t \in \mathcal{T}$$

So demand for electricity is supplied by local production and net transmission into the region, subject to loses in distribution. For heat:

$$\sum_{g\in\mathcal{G}(a)} h_s^{s,t} = \frac{h_d^{a,s,t}}{1-\epsilon_a^h}, \forall a \in \mathcal{A}, \forall s \in \mathcal{S}, \forall t \in \mathcal{T}$$

4.5.5 Emission Quotas

Emissions can be limited by taxes or quotas. Where taxes appear in the objective function, quotas naturally take the form of constraints. This sort of emission policy is described by:

$$\sum_{g\in\mathcal{G}(c)} \sum_{s\in S} \sum_{t\in T} \Phi^m(e_{s,g}^{s,t}, h_s^{s,t}) \leq \overline{m}_{c,m}, \forall c \in C, \forall m \in \mathcal{M}$$

Where $m \in \mathcal{M}$ index various emission types (CO_2, SO_2, etc.), and the $\Phi(\cdot)$-function represents the emissions resulting from the generation profile. The $\overline{m}_{c,m}$ expression is the emission limit of emission type m in country c.

The dual values of these constraints can be interpreted as the marginal value of emission allowances. Given a value of an emission allowance one can effectively implement quotas as an emission tax, by assigning a price to tradable emission allowances. This is discussed further in section 9.3.

4.6 More on the Balmorel Model

For a more complete description of the model one can refer to the following documents: [1], [3], [2], [4], [5]. These and the Balmorel model itself can be downloaded from *http://www.Balmorel.com*.

CHAPTER 5

Flow Model

Interaction between a technical model describing flow and pressure with an economic model describing the costs and utilities, makes it possible to address two issues. The flow model ensures that commodities bought and sold on the market can actually be delivered. If not, it imposes restrictions and sheds light on the lost profit from such restrictions. This makes it possible to consider the effects of various policies such as capacity rationing and tariffs. Secondly it implies a value of additional capacity which, when held against investment costs could be used as a signal that investment may be financially sustainable.

The flow problem reflects on issues of supply security and efficiency from a capacity perspective, and is thus relevant in light of the stated objective of this project.

It is emphasized that the purpose of the flow model is not to derive an accurate description of exactly how natural gas is delivered to individual consumption points, or to be able to determine the precise pressure and flow rate in different parts of the transmission network. Rather, the purpose is to impose restrictions on the solution by modeling the flow in terms of the restrictions (mostly pressure related) of the transmission pipelines, and to give indications of capacity value. In short, the flow model ensures that the delivery which occurs falls within the technical limitations of the transmission system.

This chapter concerns the technical aspects of flow modeling. First, a brief introduction to the way in which fluids, and in particular compressible fluids such as natural gas, respond to the forces relevant to natural gas transmission. Next, a more practical modeling approach is introduced which has been the prime inspiration for the final model formulation. Finally, the flow model is formulated taking account of the special considerations and advantages of the Danish transmission network.

5.1 General One Dimensional Flow

There is the general agreement that fluid flow is described by four main conditions, expressed in a single spacial dimension along the length of a pipe. (see for example [14]).

Conservation of Mass

$$-\frac{\partial(\rho w)}{\partial x} = \frac{\partial \rho}{\partial t} \tag{5.1}$$

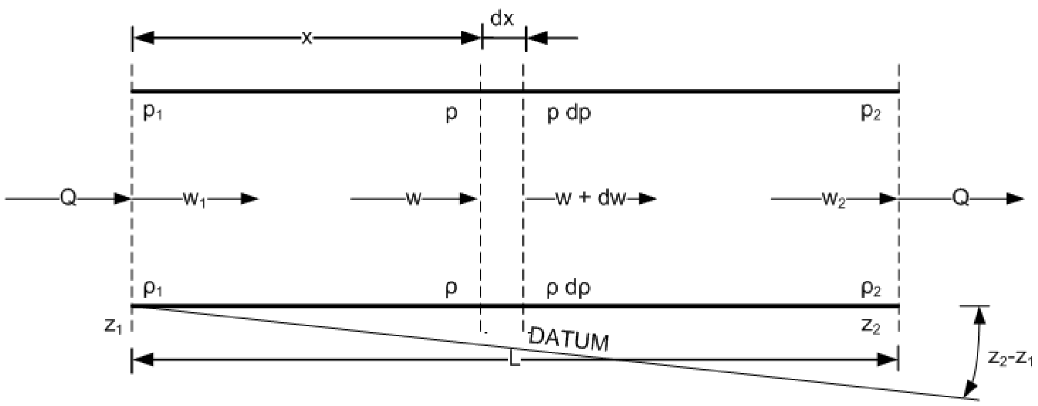

Figure 5.1: System for description of general one-dimensional flow.

Momentum Equation

$$\sum F_x = \frac{d}{dt}(mw) \tag{5.2}$$

Conservation of Energy

$$\Omega - W = \Delta E \tag{5.3}$$

State Equation

$$\rho = f(p) \tag{5.4}$$

In the above, m represents mass, ρ is an expression of density, F_x represents the net forces acting in direction x which is along the pipe, w is the flow velocity averaged over a cross-section of the pipe, Ω is added heat, W is the performed work, ΔE energy gain, p represents the pressure. Refer also to the nomenclature when necessary.

5.1.1 Conservation of Mass

The conservation of mass, or *continuity equation* in a pipe flow context, states mass may neither be created or destroyed. This means that accumulation of mass within a control volume must be equal to the net flow into the control volume. In other words what comes in, either goes out or stays in. The mass present within a control volume can be described by:

$$m = \int\int\int_V \rho dV \tag{5.5}$$

where V represents a control volume. Below two expressions are presented for the movement of mass into and out of the control volume.

$$dm = dt\int\int_A \rho w dA \tag{5.6}$$

$$dm = -dt\int\int\int_V \frac{\partial \rho}{\partial t} dV \tag{5.7}$$

The first equation (5.6) describes the change in mass by the mass-flux through the control surface. The second equation (5.7) described the change in density within the control volume over time in relation to the mass leaving the control volume. These together form the equation:

$$dt \int\int_A \rho w dA = -dt \int\int\int_V \frac{\partial \rho}{\partial t} dV \Leftrightarrow \quad (5.8)$$

$$\int\int\int_V \frac{\partial(\rho w)}{\partial x} dV = -\int\int\int_V \frac{\partial \rho}{\partial t} dV \Leftrightarrow \quad (5.9)$$

$$\int\int\int_V \left[\frac{\partial(\rho w)}{\partial x} + \frac{\partial \rho}{\partial t}\right] dV = 0 \quad (5.10)$$

Since the above must hold for any control volume, the initially presented formulation of the continuity equation is derived:

$$-\frac{\partial(\rho w)}{\partial x} = \frac{\partial \rho}{\partial t} \quad (5.11)$$

5.1.2 Momentum

Newton's second law of motion expressed in the direction of x, along the length of the pipe, adequately describes momentum of gas flow in pipes [17]. The net force in the direction of x on gas within the control volume is the algebraic sum of three individual forces projected on x. These forces are:

1. pressure forces
2. shearing forces (friction)
3. gravitational force

Since pressure is defined as force per area unit, the force induced by pressure difference over the pipe length dx is:

$$F_{pressure} = pA - \left(p + \frac{\partial p}{\partial x} dx\right) A = -\frac{\partial p}{\partial x} A dx \quad (5.12)$$

This of course in the direction of motion, as flow runs from high to low pressure. This pressure force is the component, which enables the transmission of gas through pipes, and pressure is the main control with which a transmission system operator is able to influence the rate of the flow at compressor stations or other pressure sources such as high pressure storage facilities.

The shearing force is caused by friction with the pipe and viscid forces within the gas.

$$F_{shear} = -\frac{Aw^2}{2} 4f \frac{dx}{D} \quad (5.13)$$

This is defined in the direction of the flow, hence the negative term. The origin of the term is Darcy's equation, which defines friction induced *head loss*. The concept of *head* is introduced in section 5.2.1 below. Darcy's equation states that the change in head due to friction can be described by:

$$dh_f = \frac{2f\rho w^2}{gD}dx \tag{5.14}$$

The f term is a function of the roughness of the pipe and the Reynolds number. This factor is discussed further later in the chapter.

The net body force on gas within the control volume can be formulated by:

$$F_{gravity} = g\rho A dx \sin\alpha, \tag{5.15}$$

where α is the vertical angel between the pipe's orientation and the horizon, and g is the acceleration of gravity.

The right hand side of the momentum equation empresses the flux of momentum through the control volume. This term can be reformulated by the following considerations.

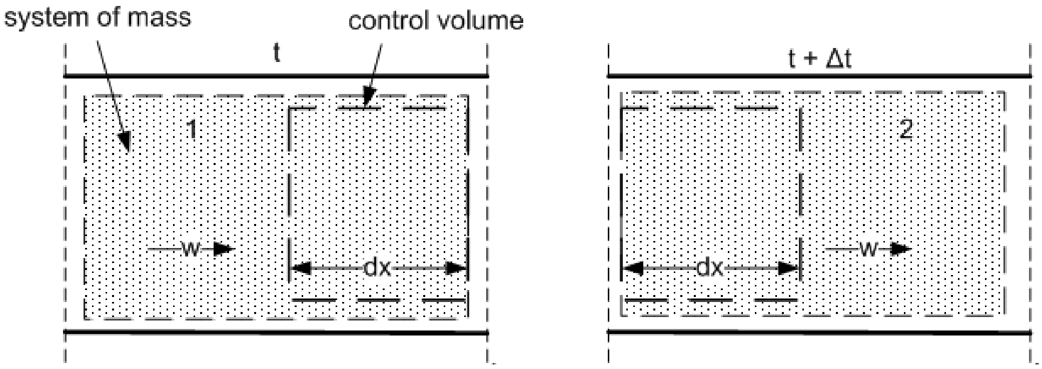

Figure 5.2: Closed system of mass moving within a pipeline, through a control volume.

By considering a system of constant mass flowing through a control volume it is possible to reformulate the momentum flux expression as follows: An initial state of the system is given at the time t. At this stage mass has entered the control volume, but no mass has yet left the control volume. As such the system is contained within the control volume or is approaching it. In the second state at the time $t + \Delta t$ all mass has entered the control volume and some of the mass has also left the control volume.

The momentum flux can be described by the limit for $\Delta t \to 0$ of the difference in momentum between the two states.

$$\frac{d(mw)}{dt} = \lim_{\Delta t \to 0} \frac{[(mw)_{cv} + (mw)_2]_{t+\Delta t} - [(mw)_{cv} + (mw)_1]_t}{\Delta t} \Leftrightarrow \tag{5.16}$$

$$\frac{d(mw)}{dt} = \lim_{\Delta t \to 0} \frac{[(mw)]_{t+\Delta t} - [(mw)]_t}{\Delta t} + \lim_{\Delta t \to 0} \frac{[(mw)_2]_{t+\Delta t} - [(mw)_1]_t}{\Delta t} \Leftrightarrow \tag{5.17}$$

$$\frac{d(mw)}{dt} = \frac{d(mw)_{cv}}{dt} + \lim_{\Delta t \to 0} \frac{[(mw)_2]_{t+\Delta t} - [(mw)_1]_t}{\Delta t} \tag{5.18}$$

The momentum of mass in the volume labeled 1 in the first state is in the limit the momentum of mass leaving that volume, hence the flux into the control area over the time Δt. This can be expressed as:

$$\lim_{\Delta t \to 0} \frac{(mw)_1}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta m_1 w_1}{\Delta t} = \rho_1 A w_1^2 \tag{5.19}$$

Identically the momentum of the mass which has left the control volume in the second state, labeled 2, can be described at the limit by the momentum flux out through the boundary of the control volume. This gives:

$$\lim_{\Delta t \to 0} \frac{(mw)_2}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta m_2 w_2}{\Delta t} = \rho_2 A w_2^2 \tag{5.20}$$

When the x component of the control volume goes towards 0.

$$\frac{dm}{dt} w_2 - \frac{dm}{dt} w_1 = \frac{\partial \rho A w^2}{\partial x} \tag{5.21}$$

Finally, the momentum contribution of mass within the control volume:

$$\frac{d(mw)_{cv}}{dt} = \frac{d}{dt}(\rho A w dx) \tag{5.22}$$

By combining (5.21) and (5.22) with (5.18) the following expression for momentum flux appears.

$$\frac{d(mw)}{dt} = \frac{\partial}{\partial t}(\rho A w dx) + \frac{\partial}{\partial x}(\rho A w^2) dx \tag{5.23}$$

Now the right hand side (5.23) and left hand side (5.12), (5.13), (5.15) of the momentum equation (5.2) is combined yielding the momentum equation expressed as:

$$-\frac{\partial p}{\partial x} A dx - \frac{A w^2}{2} 4f \frac{dx}{D} g \rho A dx \sin\alpha = \frac{\partial}{\partial t}(\rho A w dx) + \frac{\partial}{\partial x}(\rho A w^2) dx \tag{5.24}$$

$$-\frac{\partial p}{\partial x} - \frac{2f\rho w^2}{D} - g\rho \sin\alpha = \frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho w^2)}{\partial x} \tag{5.25}$$

The final equation is the general form of the momentum equation for one-dimensional flow [14].

5.1.3 Energy Conservation

The energy conservation equation is the first law of thermodynamics. The ΔE term in (5.3) represents the change in system energy. This energy can be divided into kinetic energy, potential energy and internal energy stored in the gas atoms and molecules (U).

$$\Delta E = \frac{1}{2} m w^2 + mgz + U \tag{5.26}$$

Using the system concept of the previous section, the thermodynamics can be described in the limit for $\Delta t \to 0$. The energy expression is thus divided into the content of the control volume plus the entering energy subtracted the leaving energy. On differential form, the energy conservation equation (5.3) can be formulated as:

$$\frac{d\Omega}{dt} - \frac{dW}{dt} = \frac{dE_{cv}}{dt} + \frac{dm_{out}}{dt}\left(\frac{w_2^2}{2} + gz_2 + u_2\right) - \frac{dm_{in}}{dt}\left(\frac{w_1^2}{2} + gz_1 + u_1\right) \tag{5.27}$$

The terms u are here the specific internal energies. The work can be divided into the flow work and every other kind (this includes friction). The flow work is performed at the up and down stream system boundaries. This work is formulated as

$$\frac{dW_{flow}}{dt} = \frac{dm_{out}}{dt}(p_2v_2) - \frac{dm_{in}}{dt}(p_2v_2) \tag{5.28}$$

The residual time rate of work is termed $\frac{dW_s}{dt}$. The energy equation can be expressed as

$$\frac{d\Omega}{dt} - \frac{dW_s}{dt} = \frac{dE_{cv}}{dt} + \frac{dm_{out}}{dt}\left(\frac{w_2^2}{2} + gz_2 + u_2 + p_2v_2\right) - \frac{dm_{in}}{dt}\left(\frac{w_1^2}{2} + gz_1 + u_1 + p_1v_1\right) \tag{5.29}$$

Introducing specific enthalpy h simplifies the expression as $h = u + pv$.

$$\frac{d\Omega}{dt} - \frac{dW_s}{dt} = \frac{dE_{cv}}{dt} + \frac{dm_{out}}{dt}\left(\frac{w_2^2}{2} + gz_2 + h_2\right) - \frac{dm_{in}}{dt}\left(\frac{w_1^2}{2} + gz_1 + h_1\right) \tag{5.30}$$

Finally in the limit of the length of the system $\Delta x \to 0$:

$$\begin{aligned}
\frac{d\Omega}{dt} - \frac{dW_s}{dt} &= \frac{dE_{cv}}{dt} + \frac{dm_{out}}{dt}\left(\frac{\left(w + \frac{\partial w}{\partial x}\right)^2 dx}{2}\right. \\
&\left. + g\left(z + \frac{\partial z}{\partial x}dx\right) + \left(h + \frac{\partial h}{\partial x}dx\right)\right) - \frac{dm_{in}}{dt}\left(\frac{w^2}{2} + gz + h\right) \Leftrightarrow && (5.31) \\
\frac{d\Omega}{dt} - \frac{dW_s}{dt} &= \frac{dE_{cv}}{dt} + \rho A w\left(\frac{1}{2}\frac{\partial w^2}{\partial x}dx + g\frac{\partial z}{\partial x}dx + \frac{\partial h}{\partial x}dx\right) \Leftrightarrow && (5.32) \\
\frac{d\Omega}{dt} - \frac{dW_s}{dt} &= \frac{\partial}{\partial x}\left[\rho A w\left(\frac{w^2}{2} + gz + u + pv\right)dx\right] && (5.33) \\
&+ \frac{\partial}{\partial x}\left[\rho A w\left(\frac{w^2}{2} + gz + u + pv\right)dx\right] && (5.34)
\end{aligned}$$

With respect to gas flow three cases are generally considered:

1. isothermal processes - where gas temperature is constant throughout the system.
2. adiabatic processes - if pipes are insulated so the added heat from the surrounds is zero.
3. polytropic flow, which is in essence the intermediary between the two aforementioned.

The adiabatic flow would assume $d\Omega = 0$. Isothermal flow simplifies the internal energy statements considerably. How depends on a number of issues, such as the type of gas and the state function.

In this project all flow is assumed to be stabilized by ground temperature and thus temperature is constant throughout the system. As such, flow is generally assumed to be isothermal. This is a common assumption for modeling natural gas transmission systems, but it is a simplification. In the article [17] Osiadacz and Chaczykowski conclude, unsurprisingly that significant errors occur when modeling a network where temperature fails to stabilize using an isothermal model. There is, however, no indication that temperature variations play a major role in the Danish transmission network.

5.1.4 Thermodynamic State

Two state equations were considered for different reasons. Literature regarding isothermal flow suggests a state equation where[14]:

$$\rho c^2 = p \tag{5.35}$$

However, elsewhere it is stated that the convention within the natural gas industry is to use the following state equation:

$$p = \rho Z R T \tag{5.36}$$

The Z-compressibility function can often be considered constant, and this is generally the case for natural gas where the calculation method is standardized according to [15]. This yields a compressibility of 0.95. This final method was selected mainly to avoid introducing too many non-linear terms.

5.1.5 Means of Transient Flow Analysis

The equations derived above for continuity, momentum, energy and state can with tedious effort and simplification be combined to form one second order partial differential equation. However, according to [14] this is highly impractical. Instead there are several other ways to reformulate the problem and by appropriate simplification devise models suitable for a variety of purposes. All usable transient formulations are, however, in the form of differential equations which must be solved numerically, making their use impractical for the purpose of this project. In the following we proceed to discuss how steady-state analysis can be used in an approximation effort to generate a set of flow constraints, which can be applied to the overall model.

5.2 Steady-State Analysis

A steady-state flow process describes a system state where the flow and pressure are constant in the individual locations in the system over time. Steady state analysis is very useful when there are no large variations in pressure and flow. Steady-state analysis is also generally used as a starting point for numerical solutions using transient flow analysis.

From the steady state condition we know that $\frac{\partial \rho}{\partial t} = 0$. Hereby the continuity can be expressed by:

$$\frac{\partial(\rho w)}{\partial x} = \frac{\partial u}{\partial x} = 0, \tag{5.37}$$

where u defines mass flow-rate, which is constant over the length of pipe under steady state conditions. When flow is expressed in terms of volumetric flow rate under normalized conditions it is often termed Q_n in literature. Here no distinction is necessary between u and Q_n as the applied unit of mass is Nm^3.

5.2.1 Head and Bernoulli's Equation

In the steady state case energy conservation can be formulated by the following equation which is Bernoulli's Equation at two points along the pipe adjusted for shearing head loss:

$$\frac{p}{\rho g} + \frac{w^2}{2g} + z = \frac{p + dp}{\rho g} + \frac{(w + dw)^2}{2g} + (z + dz) + dh_f \tag{5.38}$$

Bernoulli's equation gives rise to the concept of *driving head*. Each component of Bernoulli's equation is an expression of energy per unit of weight of the fluid. The first term represents the *pressure head*, which is the ability of per unit weight of the fluid to perform work. The *velocity head*, the second term, is an expression of kinetic energy per unit weight. Finally, the *potential head*, z, represents the potential energy per unit weight that can be transformed into work. Head loss has been described previously in section 5.1.2.

At this point the assumption is made that potential head is negligible as Denmark is a rather flat country. Hence Bernoulli's equation (5.38) is simplified to the following:

$$-\frac{dp}{\rho g} = \frac{(dw^2)}{2g} + dh_f \Leftrightarrow \tag{5.39}$$

$$-\frac{dp}{\rho g} = \frac{(dw^2)}{2g} + \frac{2fw^2}{gD}dx \tag{5.40}$$

The change in kinetic energy due to a change in velocity is also assumed negligible.

$$-\frac{dp}{\rho g} = dh_f \Leftrightarrow \tag{5.41}$$

$$-\frac{dp}{\rho g} = \frac{2fw^2}{gD}dx \Leftrightarrow \tag{5.42}$$

$$-dp = \frac{2f\rho w^2}{D}dx \tag{5.43}$$

The continuity equation (5.37) gives the following usefull expressions:

$$\rho w = \rho_1 w_1 \Leftrightarrow w = \frac{\rho_1}{\rho}w_1 \tag{5.44}$$

The state equation yields the following as flow is isothermal:

$$\frac{p}{\rho} = ZRT \Rightarrow \frac{p}{\rho} = \frac{p_1}{\rho_1} \Leftrightarrow \rho = p_1\frac{\rho}{\rho_1}, w = \frac{p_1}{p}w_1 \tag{5.45}$$

Substitution into (5.43) gives the following equation:

$$-dp = \frac{2fp}{Dp_1}\rho_1\left(\frac{p_1}{p}\right)^2 w_1^2 dx \Leftrightarrow \tag{5.46}$$

$$-dp = \frac{2f}{D}\rho_1\left(\frac{p_1}{p}\right) w_1^2 dx \Leftrightarrow \tag{5.47}$$

$$-pdp = \frac{2f}{D}\rho_1^2 w_1^2 ZRTdx \Leftrightarrow \tag{5.48}$$

$$-pdp = \frac{2f}{D}\frac{\rho_n^2 Q_n^2}{(\pi(D/2)^2)^2} ZRTdx \Leftrightarrow \tag{5.49}$$

$$\tag{5.50}$$

The suffixed n indicates normal conditions. When using a mass flow rate in terms of $Nm3/h$ per time unit. ρ_n becomes $1Nm^3/m^3$ and the equation can be stated as:

$$-pdp = \frac{32f}{\pi^2}\frac{u^2}{D^5}ZRTdx \tag{5.51}$$

Since the flow is isothermal, the compressibility is constant, the friction factor is assumed constant, and the mass flow rate is constant over the length of pipe, both sides of the equation can be integrated.

$$-\int_{p=p_0}^{p_L} pdp = \int_{x=0}^{L}\frac{32f}{\pi^2}\frac{u^2}{D^5}ZRTdx \Rightarrow \tag{5.52}$$

$$\frac{1}{2}\left(p_0^2 - p_L^2\right) = \frac{32f}{\pi^2}\frac{u^2}{D^5}ZRTL \Leftrightarrow \tag{5.53}$$

$$p_0^2 - p_L^2 = \; = \frac{64f}{\pi^2}\frac{u^2}{D^5}ZRTL \tag{5.54}$$

Note that this is subject to the assumption that the flow direction is positive. For negative flow direction:

$$\frac{1}{2}\left(p_L^2 - p_0^2\right) = \frac{32f}{\pi^2}\frac{u^2}{D^5}ZRTL \Leftrightarrow \tag{5.55}$$

$$p_L^2 - p_0^2 = \; = \frac{64f}{\pi^2}\frac{u^2}{D^5}ZRTL \tag{5.56}$$

When these expressions are combined the following equation can be used for calculating the relationship between flow and pressure at the end points of a pipeline section:

$$p_0^2 - p_L^2 = \frac{64f}{\pi^2}\frac{u|u|}{D^5}ZRTL \tag{5.57}$$

If important observation is that if f is assumed, which in essence implies that the Reynolds number is constant and the roughness of pipelines are constant, a resistance coefficient can be defined for each pipe e termed c_e where:

$$c_e = \frac{64f}{\pi^2 D^5} ZRTL_e \tag{5.58}$$

This also implies that if a resistance coefficient is known for pipe e_1 the resistance of pipe e_2, which has the same internat roughness, can be found by:

$$c_{e_2} = c_{e_1} \frac{D_{e_1}^5}{L_{e_1}} \frac{L_{e_2}}{D_{e_2}^5} \tag{5.59}$$

This gives the simple equation for the pressure flow relationship:

$$p_0^2 - p_{L_e}^2 = c_e u|u| \tag{5.60}$$

This relationship generally holds for low pressure networks, however, when dealing with high pressure networks.

Unfortunately the friction factor f cannot always be considered constant. The terms $\sqrt{\frac{1}{f}}$ is often called the transmission factor. For fully turbulent flow this factor can be described by the following [14]:

$$\sqrt{\frac{1}{f}} = 6.872(Re)^{0.073} \tag{5.61}$$

Where Re is Reynolds number which again is described by:

$$Re = \frac{4Q\rho}{\mu D\pi} = \frac{4u}{\mu D\pi} \tag{5.62}$$

Where μ is the viscosity of the gas and is assumed constant. This is in accordance with the so called *smooth gas law*[14] for fully turbulent flow. This can naturally by substituted into equation (5.56), however first a final consideration is made.

In praxis the inner surface of a gas pipe reduces the flow actually incurred in comparison to that, which can be calculated using the smooth pipe law. *The rough pipe law* induces the concept of pipe efficiency. The efficiency of a pipe is the share of the theoretical flow which actually occurs over a pressure drop. As such it is applied to the transmission factor as follows:

$$\sqrt{\frac{1}{f}} = E\sqrt{\frac{1}{f_t}} = 6.872(\frac{4u}{\mu D\pi})^{0.073}E \tag{5.63}$$

Consequently the constants of this expression can be included in the derivation of the resistance coefficients c_e and the $u^{0.073}$ factor when squared, inverted and multiplied to the other $u's$ of (5.56) yields the following:

$$p_0^2 - p_{L_e}^2 = c_e u|u|^\alpha \tag{5.64}$$

where $\alpha \approx 0.854$[14]. This is the pressure flow relationship applied in the model.

Below the assumptions leading to this formulation are collected:

1. one-dimensional flow
2. steady state flow
3. flow is isothermal
4. constant compressibility of gas
5. constant specific gravity of gas
6. constant viscosity of gas
7. Darcy's head loss relationship is applicable
8. potential head is negligible
9. negligible change in kinetic energy across the pipe
10. friction is constant over the length of pipe
11. the rough pipe law holds

5.3 Gas Networks

The previous sections outlined how the flow process can be described generally and for the steady-state case in a single pipe. This must be extended to describe developments in the network as a whole. In the steady-state case, it is common to assume that flow-rate is constant throughout each pipe-leg and pressure is determined a both ends of the pipe. This implies that pressure is given in terms of variables connected to junctions between pipes.

By sustaining mass flow rate through nodes by a mass-balance equation (linear), and sustaining momentum over pipe legs through a relationship between upstream pressure, flow rate and downstream pressure (non-linear).

As such, the steady state flow can practically be described by two sets of equations, one linear and one non-linear, given that, the network is acyclic. The linear property of the flow through pipelines is actually a consequence of Kirchoff's first law. Given that at one location in the network the pressure is fixed, and given that all inflow and outflow are known (or controlled) the state of the network is uniquely determed[18].

If, however, the network is not acyclic a third set of equations come into play. As a consequence of Kirchoff's second law, the pressure drop over a full cycle is zero. Equations must be added for cycles in the network.

5.3.1 Practical Formulation

In representation of a transmission pipeline network, consider the graph $\mathbf{G} = (\mathcal{A}, \mathcal{E})$ where $i \in \mathcal{A}$ is vertex in the set of vertexes (nodes) and $e \in \mathcal{E}$ is an arc in the set of arcs (pipes) in the graph. A node-pipe incidence matrix $\mathbf{A}$ can formulated as:

$$a_{i,e} = \begin{cases} 1, & \text{if pipe } e \text{ comes out of node } i; \\ -1, & \text{if pipe } e \text{ goes into node } i; \\ 0, & \text{otherwise} \end{cases} \tag{5.65}$$

Given that p_i describes the pressure at node i and u_e describes the flow in network arc e, it is possible to express the two first sets of equations as follows. Mass balance equations:

$$\mathbf{Au} = \mathbf{s} \tag{5.66}$$

The steady-state pipe flow equations derived in the previous section:

$$p_i^2 - p_j^2 = c_e u_e |u_e|^\alpha, \tag{5.67}$$

where p_i and p_j are up- and downstream pressure respectively for the pipe with end nodes i and j. u_e is the mass flow rate in the pipe leading from i to j, and α is a constant nearly equal to 1. c_e are parameters describing the resistance of the specific pipe, which can be determined empirically calculated according to [14].

The supply variables s_i are amounts coming into, or leaving the system at i (assuming $\sum_i s_i = 0$). The network flow can be described as:

$$\begin{aligned} \mathbf{Au} &= \mathbf{s}, && (5.68)\\ \mathbf{A}^T\mathbf{p}^2 &= \phi(\mathbf{u}), && (5.69)\\ & && (5.70) \end{aligned}$$

where $\mathbf{p}^2 = (p_1^2, \ldots, p_{|\mathcal{V}|}^2)^T$, and $\phi(\mathbf{u}) = (\phi_1(u_1), \ldots, \phi_l(u_{|\mathcal{E}|}))^T$, and $\phi_e = c_e u_e |u_e|^\alpha$.

It holds that if the $\mathbf{s}$ vector is given (or otherwise determined) and pressure of a reference node is given, the system has a unique solution. The system can be reformulated by 1)using the node-pipe incidence matrix removing the reference node and 2) letting B_f be the reduced cycle matrix with respect to some spanning tree. In essence, this is removing the cycles of the system for separate treatment. The formulation is as follows:

$$\begin{aligned} A_f\mathbf{u} &= s_f, && (5.71)\\ B_f\phi(\mathbf{u}) &= 0, && (5.72)\\ A^T\mathbf{p}^2 &= \phi(\mathbf{u}). && (5.73) \end{aligned}$$

It can be shown[18] that the first two sets of equations have a unique solution, and since they are independent of the pressure this can be calculated after solving the optimization problem. Below a small example is presented to emphasize how this concept works.

Example 5.1 *The following example illustrates how the above described method can be used to calculate the network state.*

Given a directed graph G, which is depicted on figure 5.3, the node-arc incidence matrix A is given below.

$$A = \begin{bmatrix} 1 & & & & & & \\ -1 & 1 & & & & 1 & \\ & -1 & 1 & & & & \\ & & -1 & 1 & & & \\ & & & -1 & -1 & 1 & \\ & & & & & -1 & \end{bmatrix}$$

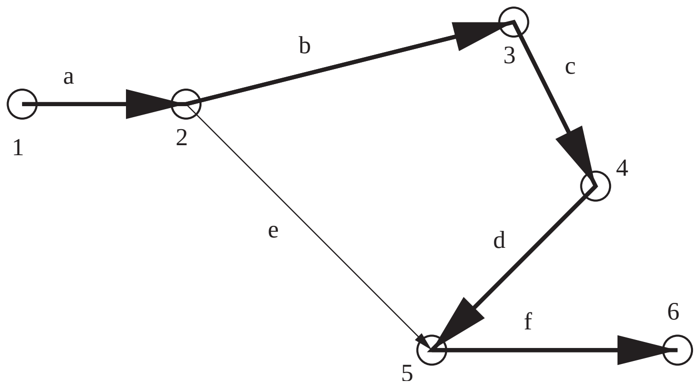

Figure 5.3: Small network example. Thick lines represent a spanning tree.

The rank of this matrix can be shown to be $n-1$ *where* n *is the number of rows. This means that one row is a linear combination of the others, and can therefore be omitted without loss in determining a unique solution. If the pressure at one reference node is given, and this node is omitted from the model mass balance equations, this does not change the feasible region of model. For the example it is additionally assumed that the source vector is given as* $s = \begin{bmatrix} 100 & 0 & 0 & -50 & 0 & -50 \end{bmatrix}^T$.

Hence the reduced incident matrix is defined for our example as:

$$A_f = \begin{bmatrix} -1 & 1 & & & 1 & \\ & -1 & 1 & & & \\ & & -1 & 1 & & \\ & & & -1 & -1 & 1 \\ & & & & & -1 \end{bmatrix}$$

The reference node (1) has thus been omitted, and the mass flow balance equations can be expressed as:

$$A_f \mathbf{u} = s_f$$

where s_f *is the source vector with the reference node omitted.*

A spanning tree $T = \{a, b, c, d, f\}$ *is emphasized by the thick lines on figure 5.3. From this a matrix containing all* fundamental cycles *can be formed. Fundamental cycles are cycles which appear when, given a spanning tree of a graph, one edge is added to the graph. It can be shown that, given all fundamental cycles of a graph, any additional cycles can be expressed as linear combinations of a number of fundamental cycles[16]. In the example there is only one cycle, which then naturally is the fundamental cycle, which appears when adding edge e to the spanning tree. It follows that:*

$$B_f = \begin{bmatrix} 0 & 1 & 1 & 1 & -1 & 0 \end{bmatrix},$$

when selecting the cyclic direction of edge b.

By theorem 2 of [16] it holds that $B_f A^T = 0$*. Hence, considering the pressure-flow relationship:*

$$\begin{aligned} A^T\mathbf{p}^2 &= \phi(\mathbf{u}) \quad \Leftrightarrow & (5.74)\\ B_f A^T\mathbf{p}^2 &= B_f\phi(\mathbf{u}) \Leftrightarrow & (5.75)\\ B_f\phi(\mathbf{u}) &= 0 & (5.76) \end{aligned}$$

The original formulation of the problem is thus equivalent with:

$$\begin{aligned} A_f\mathbf{u} &= s_f, & (5.77)\\ B_f\phi(\mathbf{u}) &= 0, & (5.78)\\ A^T\mathbf{p}^2 &= \phi(\mathbf{u}). & (5.79) \end{aligned}$$

The two first lines contain only flow variables and it has been shown in [16] that these have a unique solution. For the example these are expressed below.

$$\begin{bmatrix} -1 & 1 & & & 1 & \\ & -1 & 1 & & & \\ & & -1 & 1 & & \\ & & & -1 & -1 & 1 \\ & & & & & -1 \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_c \\ u_d \\ u_e \\ u_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50 \\ 50 \end{bmatrix} \tag{5.80}$$

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \phi(u_a) \\ \phi(u_b) \\ \phi(u_c) \\ \phi(u_d) \\ \phi(u_e) \\ \phi(u_f) \end{bmatrix} = 0 \tag{5.81}$$

The $\phi(\cdot)$ *function in the example is given as* $\phi(u) = cu|u|$ *where* $c = \begin{bmatrix} 1 & \frac{5}{4} & \frac{3}{2} & \frac{10}{9} & 2 & \frac{3}{4} & \frac{11}{10} \end{bmatrix}$.
By numerical solution of the system (5.80)-(5.81) the flow values are calculated and presented in the table below:

Flow Variable	*Value*
u_a	*100*
u_b	*46.064*
u_c	*46.064*
u_d	*-3.94*
u_e	*53.94*
u_f	*50*

This means that all flows follow their defined direction with the exception of the flow in arc d.

Given a reference pressure value $p_1 = 200$ *and the flow values pressure at the remaining nodes can be calculated iteratively. Pressure at the nodes are presented in the table below:*

Pressure Variable	*Value*
p_1	*200*
p_2	*173.205*
p_3	*165.371*
p_4	*155.45*
p_5	*155.505*
p_6	*146.396*

The pressure value in node 5 is used to confirm the result since this can be calculated from both arc d and arc e.

5.4 Quasi Steady-State Model

The developed flow model relies on the hypothesis that sufficiently accurate results may be obtained by considering flow within a time period from a node to the center of a pipe-leg to be steady. This is dependent on the assumption that variations in inflow and outflow of the network are in practice gradual to some extent, or they can be modeled with sufficient accuracy with a mean consideration near their occurrence.

The principle is that by lifting the flow conservation constraint on the pipe (which is defined implicitly by using one variable for mass flow rate per pipe-leg per time period) and introducing two flow variables per pipe-leg, the flow variables are interpreted in/out flow rates from each endpoint. Continuity is imposed by introducing a ”storage” variable describing the amount of gas present in the pipe at the beginning of each time period.

Pressure at the midsection of the pipe is derived imposing the state equation on the midsection. Pressure-flow relationships are formulated from both sides of the midsection of the pipes, so that the nodal pressure is bound using the flow and midsection pressure at each pipe incident to the node. The network is assumed to by acyclic. The only cycles in the Danish transmission system are a result of line duplications, where flow can be assumed to run in the same direction in both parallel lines. The possible implementation of a pipeline project such as the Nordic Gas Ring, would naturally require a reconsideration of this assumption. Algebraic formulation of this idea follows.

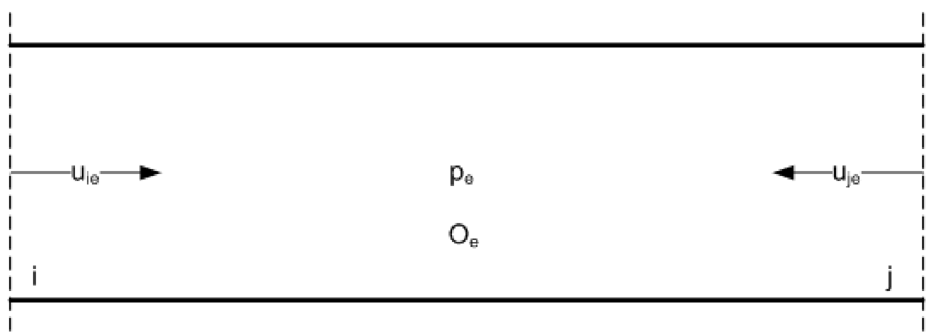

Figure 5.4: System described by the developed model

5.4.1 Conservation of Mass

Since pipes are broken down into two sections it is required that mass is conserved both with respect to edges and with respect to nodes. The edge mass balance constraint is formulated as follows:

$$O^{s,t+1} = O_e^{s,t} + u_{i,e}^{s,t} + u_{j,e}^{s,t}, \forall e, \forall s,t, i \to e, j \to e \tag{5.82}$$

The variable $O_e^{s,t}$ states the mass of gas present in the pipe at the beginning of time segment (s,t). Hence the net sum of mass entering or leaving the pipe is present in the pipe in the following time period. For nodes the mass balance constraint is analogous to the example 5.1.

$$\sum_{e \to i} u_{i,e}^{s,t} = \nu_i^{s,t} - \chi_i^{s,t} - d_i^{s,t} \tag{5.83}$$

for nodes, where $d_i^{s,t}$ is the demand into node i, $\nu_i^{s,t}$ is the input from an adjacent supply source, and $\chi_i^{s,t}$ is the rate of export.

5.4.2 Non-linear Momentum Constraints

Commencing with the steady-state expression of equation (5.64), the pressure flow relationship is given below:

$$p_i^2 - p_j^2 = c_e u_e |u_e|^\alpha, \tag{5.84}$$

Given a network with nodes i and j and edges incident to nodes indexed e. Consider a mean pressure of an edge (pipe) p_e, which in the steady state case is the average of p_i and p_j, which is the pressure at the end points. The pipe specific parameters c_e are derived.

The flow $u_{i,e}$ is the average flow-rate between node i and the midpoint of edge e. During a time period t we assume that the flow between node and edge midpoint can be considered near-steady (quasi-steady). Hence the relationship:

$$(p_i^{s,t})^2 - (p_e^{s,t})^2 = \frac{1}{2} c_e u_{i,e}^{s,t} |u_{i,e}^{s,t}|^\alpha \quad \forall i \to e$$

holds.

5.4.3 Pressure Drop over Pipe-length

Consider a network with pressure variables incident to nodes and flow variables incident to arcs. The network is acyclic. Assuming the flow is positive (running from an upstream node to a downstream node), the flow pressure relationship is as follows:

$$p_i^2 - p_j^2 = c_e u_e^{\alpha+1}$$

Assuming that the upstream pressure p_i is known, this means the downstream pressure p_j is:

$$p_j = \sqrt{p_i^2 - c_e u_e^{\alpha+1}}$$

Without loss of generality c_e is set to 1. This function is displayed in the 3-dimensional plot in figure 5.5.

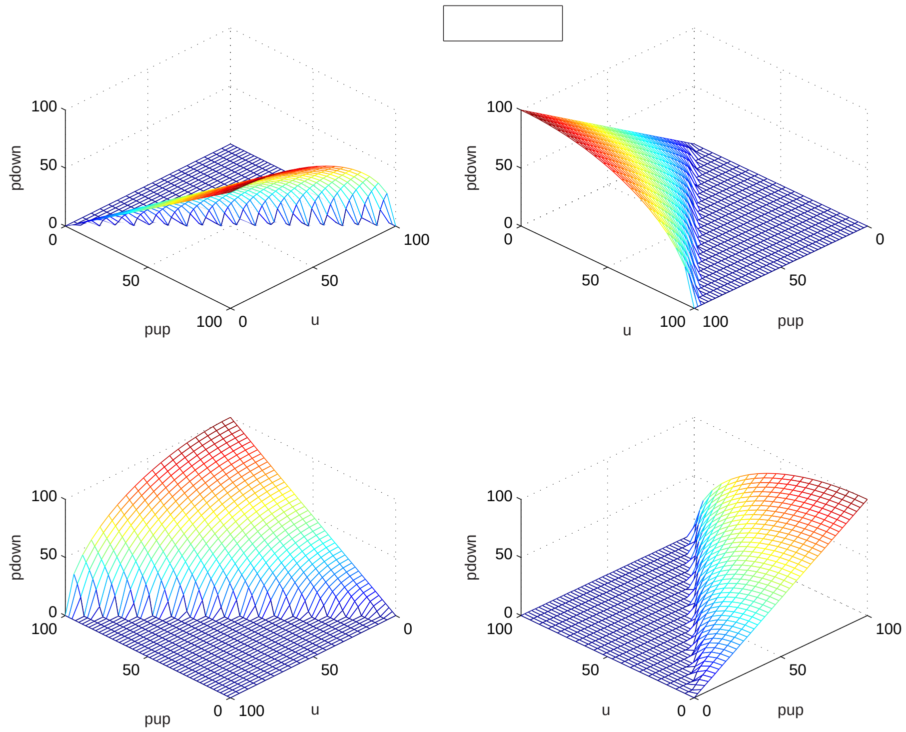

Figure 5.5: The relationship between flow-rate and pressure at each end of a pipe shown from four viewpoints. It is evident that as flow increase, so does the rate at which pressure drops.

As high pressure is desirable in the system, and the downstream pressure is concave as a function of both upstream pressure and flow-rates, this can be linearized. However, as flow can move in both directions in the pipe we return to the original formulation of flow and pressure relationships (reintroducing the absolute value for sign preservation on flows.

$$p_i^2 - p_j^2 = c_e |u_e|^\alpha u_e$$

Assuming $u_e < 0$ this gives the following expression for the downstream variable (physically the upstream variable).

$$p_j = \sqrt{p_i^2 + c_e u_e^{\alpha+1}}.$$

The combined function takes the form illustrated in figure 5.6

This function is neither convex nor concave over the entire span of feasible flow-rates. However, one can observe that when flow-rates become negative the original upstream down effectively becomes a downstream node and downstream becomes upstream.

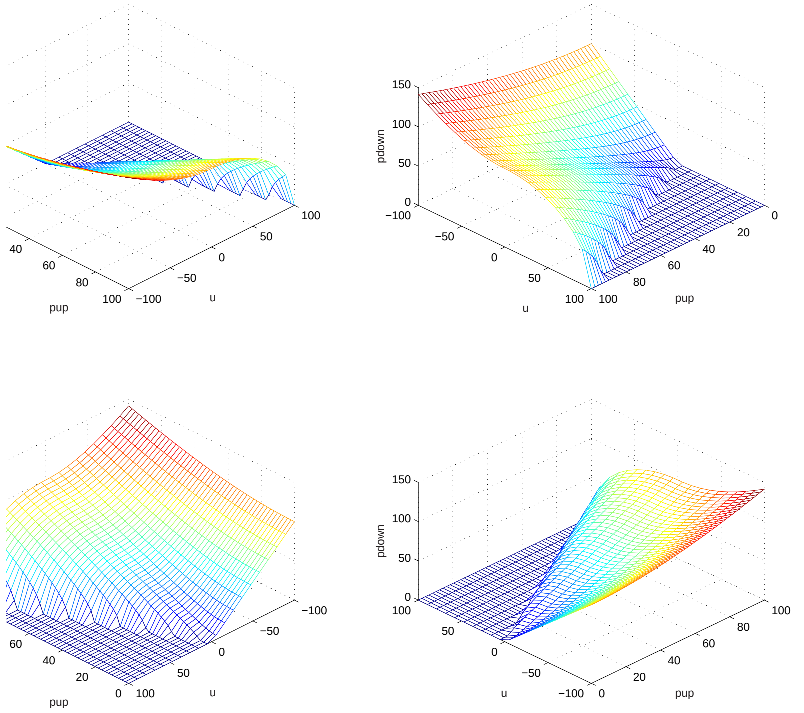

Figure 5.6: The relationship between flow rate and pressure at each end of a pipe. Allowing negative flow-rates, also shown from 4 views. It is apparent from the graph that at a flow rate of 0, the rate at which pressure drops/rises at the unfixed end of the pipe changes sign. This shows that the function of flow at one end of a pipe, given a fixed pressure at the other end, in relation to the flow rate is none convex/concave.

5.4.4 From Non-linear to Piecewise-linear

Now assume that for an edge-node incidence pair it holds that $u_{i,e} \geq 0$. This means that the flow-direction is from node to pipe, and therefore $p_i^t \geq p_e^{s,t}$. The energy relationship is then:

$$(p_i^{s,t})^2 - (p_e^{s,t})^2 = \frac{1}{2} c_e (u_{i,e}^{s,t})^{1+\alpha} \quad \forall i \to e$$

implying that:

$$p_i^{s,t} = \sqrt{(p_e^{s,t})^2 + \frac{1}{2} c_e (u_{i,e}^{s,t})^{1+\alpha}}$$

Obviously for $u_{i,e}^{s,t} = 0$ we have $p_i = p_e$. With fixed p_e, increasing the value of $u_{i,e}^{s,t}$ the second term becomes dominant and the relative distance to $p_i^{s,t} = \sqrt{\frac{1}{2} c_e u_{i,e}^{s,t}}$ decreases. For this solution area of $u_{i,e}^{s,t}$ and for fixed $p_e^{s,t}$ the function is convex.

Now assume $u_{i,e}^{s,t} \leq 0$. This leads to the energy relation:

$$(p_i^{s,t})^2 - (p_e^{s,t})^2 = -c_e \frac{1}{2} (u_{i,e}^{s,t})^{1+\alpha} \quad \forall i \to e$$

Solving for $p_i^{s,t}$ gives:

$$p_i^{s,t} = \sqrt{(p_e^{s,t})^2 - \frac{1}{2} c_e (u_{i,e}^{s,t})^{1+\alpha}}$$

Assuming again that $p_e^{s,t}$ is fixed, it is evident again that for $u_{i,e}^{s,t} = 0$ it holds that $p_i^{s,t} = p_e^{s,t}$. This function with respect to $u_{i,e}^{s,t}$ decreases as the numeric value of $u_{i,e}^{s,t}$ increases and at an accelerating rate. The function is concave with respect to $u_{i,e}^{s,t}$. Since the purpose is to restrict the throughput of the network in representation of technical limitations the two cases for flow direction have individually the correct functional form. Given is, either a value of upstream or downstream pressure. When upstream pressure is given, downstream pressure is a function of flow rate is concave. In this case a concave function sets the upper limit of the sustainable downstream pressure, given flow and upstream pressure. This can be approximated by a family of piecewise-linear functions. The other case, given downstream pressure, the upstream pressure as a function of flow rate is convex. This convex function is interpreted as a lower limit on the upstream pressure required to sustain the flow. Hence it can also be approximated by a family of piecewise linear functions.

Below the piecewise linearized versions of the energy relation are presented in the two cases of $u_{i,e}^{s,t}$.

For $u_{i,e}^{s,t} \geq 0$:

$$p_i^{s,t} \geq \underline{p}_e^l + \underline{\psi}_e^l u_{i,e}^{s,t} + \underline{\upsilon}_e^l p_e^{s,t} \tag{5.85}$$

For $u_{i,e}^{s,t} \leq 0$:

$$p_i^{s,t} \leq \overline{p}_e^l + \overline{\psi}_e^l u_{i,e}^{s,t} + \overline{\upsilon}_e^l p_e^{s,t} \tag{5.86}$$

The index $l \in \mathcal{L}$ refers to a point on surface of the original pressure function illustrated in figure 5.6 where the function is linearized. The parameters $\underline{p}_e^l$, $\underline{\psi}_e^l$ and $\underline{\upsilon}_e$ describe tangent planes to the original function, which form a lower bound for the physical upstream pressure which enables the flow $u_{i,e}^{s,t} \geq 0$. Likewise parameters $\overline{p}_e^l$, $\overline{\psi}_e^l$ and $\overline{\upsilon}_e^l$ describe the tangent planes to the original function, which form an upper bound on the downstream pressure which enable the flow $u_{i,e}^{s,t} \leq 0$.

5.4.5 Binary Flow Direction Variables

As it is obvious that both sets of constraints will not hold at once (except a 0-flow solution), a set of binary variables $B_{i,e}^{s,t} \in \{0, 1\}$ is introduced. By definition:

$$u_{i,e}^t > 0 \;\Rightarrow\; B_{i,e}^{s,t} = 1 \tag{5.87}$$

$$u_{i,e}^t < 0 \;\Rightarrow\; B_{i,e}^{s,t} = 0 \tag{5.88}$$

This is ensured by the introduction of the following constraints:

$$u_{i,e}^{s,t} \leq B_{i,e}^{s,t}\mathbf{M} \tag{5.89}$$
$$u_{i,e}^{s,t} \geq (B_{i,e}^{s,t}-1)\mathbf{M} \tag{5.90}$$

where **M** is a sufficiently large number.

The binary variables are also introduced in the energy equations such that:

$$p_i^{s,t} \geq \underline{p}_e^l + \underline{\psi}_e^l u_{i,e}^{s,t} + \underline{v}_e^l p_e^{s,t} + (B_{i,e}^{s,t}-1)\mathbf{M} \tag{5.91}$$
$$p_i^{s,t} \leq \overline{p}_e^l + \overline{\psi}_e^l u_{i,e}^{s,t} + \overline{v}_e^l p_e + B_{i,e}^{s,t}\mathbf{M} \tag{5.92}$$

5.4.6 Linking Time Segments

It has been repeatedly stated that the pressure at the midsection of a pipe is known, but how is this brought about? By continuity what enters a pipe in one time segment either leaves the pipe or is present in the next time segment. The state equation describes the static relationship between the present amount of natural gas and the pressure in the pipe. This property sustains the flow solution of the network between time segments.

Assuming the state equation is linear in terms of pressure and amount (constant compressibility Z) and defining O_e^t as the amount of gas in the pipe e at time t. From the state equation the following is derived:

$$p = \rho RTZ \Leftrightarrow \tag{5.93}$$
$$RZ = \frac{p}{\rho T} \tag{5.94}$$
$$RZ = \frac{p_n}{\rho_n T_n} \tag{5.95}$$
$$\rho = \rho_n \frac{p}{p_n}\frac{T_n}{T} \tag{5.96}$$

The mass present within the pipe is expressed by the volume of the pipe and the average density:

$$O_e^{s,t} = \rho_n \frac{p_e^{s,t}}{p_n}\frac{T_n}{T} V_e \quad \forall e, \forall t,$$

This expression is practical as $\rho_n = 1Nm^3$ by definition and the normalized conditions for pressure and temperature are $p_n = 1bar$ and $T_n = 273K$.

This system ensures that the value of $p_e^{s,t}$ is fixed from the previous time period. Hence the decisions taken at time s,t are p_i and $u_{i,e}^{s,t}$. Therefore if decisions are sequentially taken in order of t there are two unknowns to the energy relations in each time-step per edge-node incidence pair.

5.4.7 The Flow Model

The complete model now appears as follows:

$$
\begin{aligned}
p_e^{s,t} &= K_e O_e^{s,t} \qquad \forall e, \forall s, \forall t && (5.97)\\
O^{s,t+1} &= O_e^{s,t} + (u_{i,e}^{s,t} + u_{j,e}^{s,t})\Delta s,t, \qquad \forall e, \forall s, \forall t, i \to e, j \to e && (5.98)\\
\sum_{e \to i} u_{i,e}^{s,t} &= \nu_i^{s,t} - \chi_i^{s,t} - d_i^{s,t} \qquad \forall s, \forall t, \forall i && (5.99)\\
p_i^{s,t} &\geq \underline{p}_e^l + \underline{\psi}_e^l u_{i,e}^{s,t} + \underline{v}_e^l p_e^{s,t} + (B_{i,e}^{s,t} - 1)\mathbf{M} \qquad \forall s, \forall t, \forall (i,e) | i \to e && (5.100)\\
p_i^{s,t} &\leq \overline{p}_e^l + \overline{\psi}_e^l u_{i,e}^{s,t} + \overline{v}_e^l p_e^{s,t} + B_{i,e}^{s,t}\mathbf{M} \qquad \forall s, \forall t, \forall (i,e) | i \to e && (5.101)\\
u_{i,e}^{s,t} &\leq B_{i,e}^{s,t}\mathbf{M} \qquad \forall s, \forall t, \forall (i,e) | i \to e && (5.102)\\
u_{i,e}^{s,t} &\geq (B_{i,e}^{s,t} - 1)\mathbf{M} \qquad \forall s, \forall t, \forall (i,e) | i \to e && (5.103)
\end{aligned}
$$

The model has too much freedom with regard to the node pressure, but since the node pressure is only indirectly connected across time periods via the continuity equations for pipe-pressure, this freedom should not be exploitable.

5.5 Computational Intractability

Unfortunately the number of binary variables makes the model computationally intractable. Having around 50 edges with two flow sections each gives approximately 100 binary variables per time period. It is desirable to have at least around 144 time periods thus leaving the model computationally intractable.

Unfortunately it has been found impossible to reformulate in terms of binary variables without loosing either accuracy or operational functionality. It is therefore the logical step to search for a model reduction with least possible impact on functionality. Several ideas are put forward below.

- All pipe-legs to which all paths to source nodes are unidirectional should be locked in terms of flow direction. This forces a modest functionality reduction in order that line-pack near the "end" of the graph can only be used to supply demand from farther out. Examples affected: Lille Torup→Aalborg, Torslunde→Lynge.'
- Pipe-legs near a "strong" supply source can be directed. If one assumes that a supply source is always active, and demand near this source will always be supplied from this source, then flow direction can be fixed as flowing away from the source node assuming no relevant circuits are present. Example affected Nybro→Egtved.
- All pipe-legs have only 1 flow direction. This implies that flow cannot enter a pipe from both sides during a time period nor leave from both sides during a time period, thus forcing flow direction to change on the nodes instead of on pipes. This give a reduction by 50% on binary variables, and the functionality reduction appears to be irrelevant.
- Pipe-legs can be grouped. Instead of having full flexibility in terms of flow direction between metering stations one can group consecutive M/R stations making these

dependent on the same binary flow direction variable. This imposes the limitation that all demand points within the group must be supplied from the same direction. Examples groups: {Egtved, Lilleballe, Taulov} or {Viborg, Karup, Herning} etc.

Unfortunately these restrictions insufficiently reduce the problem complexity. There are still multiple binary variables within a time period and many periods in the model.

- Seasonal flow direction could also be imposed. By restricting flow direction to the season (e.g. the month) the number of binary variables could be reduced drastically. However, this may also give very bad results as the flow direction is likely to shift during the day, as demand is higher during certain peak hours than during nighttime.

All the described simplification possibilities have been attempted and though the model can be solved for a steady state scenario, the search tree of the branch and cut algorithm explodes exponentially once demand and supply variations are implemented. Multiple solvers for both the mixed-integer problem and the linear subproblem have been tested. Tampering with the types of cuts generated by the MIP solvers has also been attempted. The conclusion is that the problem is simply too large to solve as a mixed-integer problem.

5.6 A Solution

In order to achieve results from the flow model a simple heuristical approach is adopted. By first determining what is desired by the market, one can obtain a fair idea concerning the direction of flow in the individual time periods. Subsequently these directions can be forced into the model and the flow model can be solved linearly. This naturally rules out some flexibility in the model, but if the "guess" of flow direction makes sense, the results should be interesting enough. The three types of model execution can be termed *NO_FLOW*, *DIRECTION_IMPOSED_FLOW* and *FREE_FLOW*. The first model excludes the flow model and instead includes a supply=demand equation for natural gas on the individual metering stations. The second (*DIRECTION_IMPOSED_FLOW*) includes the flow model but with the binary flow direction variables fixed a priori. Finally, *FREE_FLOW* is the computationally intractable mixed-integer model. Using the first two models it is assumed that a fairly good estimation of the final model using the following iterative scheme:

Solution Scheme:	
1:	solve *NO_FLOW*
2:	determine directions and fix binary variables
3:	solve *DIRECTION_IMPOSED_FLOW*

This scheme could potentially be improved considerably by making iterative changes to flow direction as result of the resolution of the problem. This has not been made a priority for the project and has been left for future consideration.

5.7 A Conic Variation

This section investigates the possibility of applying conic programming in the pressure flow model (5.98)-(5.103). The section is mainly of mathematical interest and is included in the report to demonstrate an elegant alternative to piecewise-linear approximation.

In short, using conic programming makes it possible to extend the range of applicable constraints from linear programming slightly, by introducing conic constraints. Conic constraints are non-linear, but it is still possible to solve conic models using an interior point method. Thus the solution complexity is near that of a linear programming problem.

Basically two kinds of conic constraints can be modeled. These are the quadratic cone and the rotated quadratic cone.

$$C_t = \left\{ x \in \mathcal{R}^{n^t} : x_1 \geq \sqrt{\sum_j^{n^t} x_j^2} \right\} \tag{5.104}$$

$$C_t = \left\{ x \in \mathcal{R}^{n^t} : 2x_1 x_2 \geq \sqrt{\sum_j^{n^t} x_j^2}, x_1, x_2 \geq 0 \right\} \tag{5.105}$$

Revisit the non-linear formulations for the flow-pressure relationships from the past chapter. Here it is necessary to assume that $\alpha = 1$.

$$p_i^{s,t} = \sqrt{(p_e^{s,t})^2 + \frac{1}{2} c_e (u_{i,e}^{s,t})^2}, \qquad u_{i,e}^{s,t} \geq 0 \tag{5.106}$$

$$p_i^{s,t} = \sqrt{(p_e^{s,t})^2 - \frac{1}{2} c_e (u_{i,e}^{s,t})^2}, \qquad u_{i,e}^{s,t} \leq 0 \tag{5.107}$$

As conic constraints can be linked with linear constraints, consider the following.

$$p_i^{s,t} = \sqrt{(p_e^{s,t})^2 + (\overline{u}_{i,e}^{s,t})^2}, \qquad u_{i,e}^{s,t} \geq 0 \tag{5.108}$$

$$p_e^{s,t} = \sqrt{(p_i^{s,t})^2 + (\overline{u}_{i,e}^{s,t})^2}, \qquad u_{i,e}^{s,t} \leq 0 \tag{5.109}$$

$$\overline{u}_{i,e}^{s,t} = \sqrt{\frac{1}{2} c_e u_{i,e}^{s,t}} \tag{5.110}$$

Following the logic from the section 5.4.4 the equality signs can be replaced with inequalities, thus bounding the numeric values of the us.

$$p_i^{s,t} \geq \sqrt{(p_e^{s,t})^2 + (\overline{u}_{i,e}^{s,t})^2}, \qquad u_{i,e}^{s,t} \geq 0 \tag{5.111}$$

$$p_e^{s,t} \geq \sqrt{(p_i^{s,t})^2 + (\overline{u}_{i,e}^{s,t})^2}, \qquad u_{i,e}^{s,t} \leq 0 \tag{5.112}$$

$$\overline{u}_{i,e}^{s,t} = \sqrt{\frac{1}{2} c_e u_{i,e}^{s,t}} \tag{5.113}$$

The result is two cones which effectively bind the flow-rate by the allowed pressure. These cones eliminate the need for a piecewise linear approximation. Unfortunately the examined solver with capacity for introducing conic constraints is not currently able to combine this with a mixed integer programming interface, and as such the idea was abandoned. It has later become apparent that another solver may be able to combine these elements and this would be an interesting aspect to look into.

5.8 Summary

With offset in general theory of compressible flow, a model has been developed by simplification and assumptions which describes the relationship between pipeline flow and pressure on a network level. Unfortunately it was necessary to implement a heuristical solution to the flow direction subproblem, and as such optimality cannot be guaranteed for the complete model as it is formulated mathematically.

The developed flow model serves the purpose of constraining flow in the transmission system in-line with what is technically feasible. Although the accuracy of the model can be questioned and the number of constraints which are introduced is considerable, it is possible to attain a solution within times which are deemed acceptable in relation to the modeled time horizon. Figures for time and resource consumption are presented in chapter 7, but first the derivation of the natural gas market model is described in chapter 6

CHAPTER 6

Gas Market Model

In this chapter, the main properties regarding the structure of the Danish transmission, distribution and storage facilities for gas, are outlined. The published policies of the system operators are interpreted to suit the model domain. From this a number of model constraints are defined and a cost contribution to the objective function. Also the implementation of the market for natural gas is described in terms of a domestic and an international market.

6.1 The Transmission System

The cost associated with the use of the transmission system is organized in a structure of tariffs. The current structure of natural gas transmission tariffs, is a combination of a tariff on capacity in the system, and a tariff on the actual amounts transmitted through the system. For capacity there is a tariff for reserving entry capacity and a tariff for exit capacity. The capacity system is basically an entry-exit system, which means that a tariff is payed for entry at one of three entry points, which are the gas treatment plant at Nybro, the Danish-German border at Ellund, and the Dragør border with Sweden. However, there is at present no actual entry possibility from Sweden. System exit tariffs are payed for exit in the exit zone, which is basically the whole country and the two international connections to Germany and Sweden again.

The transmission operator has developed a series of capacity products some of which, are implemented in this model. The basic product is an annual capacity contract, which gives the customer the option to use a certain amount of hourly entry or exit capacity for a year. This contract can be supplemented by monthly, weekly or daily contracts, which are relatively more expensive but provide flexibility. Since gas consumption is strongly correlated with temperature, it is hard to forecast accurately the capacity requirements for a year in advance. In the beginning of a given month one would have a better forecast of consumption in that month and one would perhaps choose to supplement one's annual capacity contract with a monthly contract. Weekly and daily contracts become relevant when the week-ahead weather forecast becomes reliable. In this model it is inconvenient to introduce weekly and daily contracts. This factor is attributed to the temporal resolution,

January	February	March	April	May	June
35%	35%	30%	15%	8%	8%
July	August	September	October	November	December
8%	8%	8%	10%	15%	30%

Table 6.1: Cost of monthly capacity as fractions of annual capacity contracts. (*SOURCE: Gastra A/S*)

which gives a picture of average hours within a month. Thus all days appear alike in the view of the model, and thus the more inexpensive monthly contracts would always be chosen.

It is assumed that market players have perfect foresight within the year, for purposes of capacity booking. Thus, finding the correct combination of capacity products between annual and monthly products is a linear subproblem. The following variables are introduced:

κ^Y_{EN} annual entry capacity booked
κ^Y_{EX} annual exit capacity booked
κ^s_{EN} monthly entry capacity booked in month s
κ^s_{EX} monthly exit capacity booked in month s

At present entry and exit capacity are identically priced at 15.55 kr./kWh/h/year [22]. Symbolically, the annual capacity tariffs are termed τ^Y_{EN}, τ^Y_{EX}, for entry and exit respectively. Monthly contracts are given as factional annual contracts ρ^s varying monthly. Table 6.1 shows cost of monthly capacity as percentage of annual contracts.

Introducing additionally the subset of areas $\mathcal{P} \subset \mathcal{A}$ where production of gas occurs or gas can be exported, the objective function contribution of transmission tariffs can be formulated as:

$$\underbrace{\tau_V \sum_{i\in\mathcal{P}} \sum_s \sum_t \nu_i^{s,t} \Delta^{s,t}}_{\text{volume}} + \underbrace{\tau^Y_{EN} \left(\kappa^Y_{EN} + \sum_s \frac{12\Delta^s}{8760} \rho^s \kappa^s_{EN} \right)}_{\text{entry}} + \underbrace{\tau^Y_{EX} \left(\kappa^Y_{EX} + \sum_s \sum_s \frac{12\Delta^s}{8760} \rho^s \kappa^s_{EX} \right)}_{\text{exit}} \tag{6.1}$$

The expression $\frac{12\Delta^s}{8760}$ ensures proper scaling when $|\mathcal{S}| \neq 12$. The following constraints ensure that sufficient capacity is always booked:

$$\sum_{i\in\mathcal{P}} \nu_i^{s,t} \leq \kappa^Y_{EN} + \kappa^s_{EN}, \qquad \forall s, \forall t \tag{6.2}$$

$$\sum_{i\in\mathcal{A}} d_i^{s,t} + \sum_{i\in\mathcal{P}} \chi_i^{s,t} \leq \kappa^Y_{EX} + \kappa^s_{EX}, \qquad \forall s, \forall t \tag{6.3}$$

Equation 6.2 states that for any given time period, the sum of ordered annual and seasonal entry capacity relevant to that time period must be greater than or equal to supply of that period. Equation 6.3 states that for any given time period, the sum of ordered annual and seasonal exit capacity, relevant to that time period, must exceed total deliveries and exports combined.

6.2 Distribution

There are five distribution network operators in Denmark. These are DONG Distribution, Hovedstadens Naturgas, Naturgas Midtnord and Naturgas Fyn. DONG Distribution operated the distribution networks of southern Jutland and most of Zealand (with the exception of the area of Greater Copenhagen). Midt Nord operates in the central and northern parts of Jutland. Naturgas Fyn operates on the island of Fynen and Hovedstadens Naturgas operates the network in Greater Copenhagen. All of the distribution network operators have presently opted for a tariff based on volume. There is no cost of securing capacity in this system.

The tariff structure is arranged so that the greater the demand from a single customer, the lower the unit cost of distribution becomes. This is unfortunate from a modeling perspective. As the market is modeled as a single player there is no distinction between individuals, and as such the impact of distribution tariffs cannot be distributed appropriately for a single plant. However, as generation technologies typically reflect a certain plant size, technologies are associated with a step on the distribution tariff ladder, or a linear combination of steps.

All operators have 8 price steps. A set Ξ is defined to represent these. The tariffs associated with a specific distribution area, $\delta \in \mathcal{D}$, in a given volume interval, $\xi \in \Xi$, is thus defined as τ_δ^ξ. For each generation technology type fueled by natural gas a weighting is estimated according to how much of the production capacity would normally fall within a certain distribution volume interval. This weighting is termed w_g^ξ. As such the economic impact of distribution tariffs, and thus their objective function contribution is formulated as:

$$\sum_s \sum_t \left(\sum_\delta \sum_{\xi \in \Xi} \tau_\delta^\xi \sum_{g \in \mathcal{G}} w_g^\xi \sum_{i \in \delta} F_{i,g}^{s,t}(\cdot) \right) \Delta^{s,t} \tag{6.4}$$

Where $F_{i,g}^{s,t}(\cdot)$ is a function describing the natural gas consumption of technology g in area i in time segment s, t.

6.3 Gas Storage

The Danish gas storage operator, DONG Lager, is in charge of the two Danish gas storage facilities at Stenlille and Lille Torup. These facilities have a combined workable storage capacity of 820 million Nm^3, roughly equivalent to 20% of the annual domestic consumption. The function of the storage facilities are two-fold. First, using gas storage it is possible to compensate for the large seasonal variables in demand, and second they provide a reserve option to sustain supply in case of a breakdown in supply. This reserve is scaled in order to maintain 60 days of non-interruptible supply and 3 days of interruptible supply. All customers who are not economically compensated for interruptibility are uninterruptible.

The pressure in the caverns of the storage facilities is kept around 160-220 bar and as such the gas must be compressed and chilled upon injection, while it must be heated upon extraction. This process limits the rate at which gas can be injected and extracted from the facilities.

DONG Lager supplies two different storage products, which may be combined at the user's discretion. One is more flexible than the other with regard to injection/extraction capacity and is as such also more expensive. Table 6.3 describes the two main storage products. A

	Volume cap.	**Extraction cap.**	**Injection cap.**	**Tariff**
Unit	$\overline{V}_\sigma$ kWh	$\overline{\varepsilon}_\sigma$ % of volume	$\overline{\iota}_\sigma$ % of volume	τ_σ kr./kWh
Product 1.	1	0.042	0.021	0.0295
Product 2.	1	0.209	0.104	0.0655

Table 6.2: Storage products from DONG Lager (*SOURCE: DONG Lager*)

σ is used to index storage products. The subset $\mathcal{I} \subset \mathcal{A}$ includes those nodes which contain a storage facility.

A linear combination of the two storage products is assumed to be able to cover most users' requirements, and for the purpose of this project it is assumed to be the only storage option. It is also possible to trade storage, injection and extraction capacity amongst storage users. As the developed model considers the market to be a single actor, such trading is assumed to be 100% efficient for practical reasons. There is additionally a variable cost of injection, currently $\tau^\iota = 0.00123$ kr./kWh.

For supply security reasons, storage contracts are subjected to a filling requirement. This is to ensure that all players do not fill at the last minute thus ensuring a smooth rate of filling in the autumn months. The filling requirements are dependent on the amount of storage purchased. Table 6.3 shows the percentage of total purchased capacity must be present by the first of each month. Γ_s is introduced to symbolically describe the fraction of purchased storage capacity, which must be in the storage facility by season s.

The following variables are used to describe the utilization of the storage facility.

$\Lambda_i^{s,t}$	stock of a facility at a given time period
$\iota_i^{s,t}$	injection rate into facility at given time period
$\varepsilon_i^{s,t}$	extraction rate from facility at given time period
ζ_σ	product units purchased of each storage contract

The variables are subject to facility specific technical upper bounds in terms of maximum capacity, and rate of extraction and injection. These are specified in table 6.3.

As a consequence the variables are bound as follows:

$$0 \leq \Lambda_i^{s,t} \leq \overline{\Lambda}_i^{s,t}, \qquad \forall i \in \mathcal{I}, \forall s, \forall t \tag{6.5}$$

	October	November	December	January	February	March	April
Γ_s	0%	20%	55%	60%	40%	10%	0%

Table 6.3: Filling requirements from DONG Lager (*SOURCE: DONG Lager*)

	Working gas volume	**Extraction cap.**	**Injection cap.**
	$\overline{\Lambda}_i^{s,t}$: mill. Nm^3	$\overline{\varepsilon}_i^{s,t}$: kNm^3/h	$\overline{\iota}_i^{s,t}$: kNm^3/h
Lille Torup	420	600	165
Stenlille	400	600	110

Table 6.4: Technical constraints on the storage facilities (*SOURCE: DONG Naturgas A/S*)

$$0 \quad \leq \varepsilon_i^{s,t} \leq \quad \overline{\varepsilon}_i^{s,t}, \qquad \forall i \in \mathcal{I}, \forall s, \forall t \tag{6.6}$$

$$0 \quad \leq \iota_i^{s,t} \leq \quad \overline{\iota}_i^{s,t}, \qquad \forall i \in \mathcal{I}, \forall s, \forall t \tag{6.7}$$

The equations relevant to the storage facilities are now presented. As mentioned, the market operates with only one virtual storage facility and as such, the constraints in relation to the products of table 6.3 are defined over the sums of the actual storage facilities. Energy unit conversion factors are omitted for clarity.

$$\sum_\sigma \overline{V}_\sigma \zeta_\sigma \quad \geq \quad \sum_{i \in \mathcal{I}} \Lambda_i^{s,t}, \qquad \forall s, \forall t \tag{6.8}$$

$$\sum_\sigma \overline{\varepsilon}_\sigma \zeta_\sigma \quad \geq \quad \sum_{i \in \mathcal{I}} \varepsilon_i^{s,t}, \qquad \forall s, \forall t \tag{6.9}$$

$$\sum_\sigma \overline{\iota}_\sigma \zeta_\sigma \quad \geq \quad \sum_{i \in \mathcal{I}} \iota_i^{s,t}, \qquad \forall s, \forall t \tag{6.10}$$

$$\Gamma_s \sum_\sigma \overline{V}_\sigma \zeta_\sigma \quad \leq \quad \sum_{i \in \mathcal{I}} \Lambda_i^{s,'t_1'} \qquad \forall s \tag{6.11}$$

The first three equations (6.8)-(6.10) ensure that adequate volumetric capacity, extraction capacity and injection capacity is reserved respectively. The forth (6.11) ensures that the filling requirement is upheld.

The storage system's contribution to the objective function is thus the cost of purchasing capacity products and the variable cost of injection.

$$\underbrace{\sum_\sigma \tau_\sigma \zeta_\sigma}_{\text{capacity products}} \quad + \underbrace{\sum_{i \in \mathcal{I}} \sum_s \sum_t \tau^\iota \iota_i^{s,t} \Delta^{s,t}}_{\text{injection}} \tag{6.12}$$

6.4 Gas Market

Two gas markets are implemented in the model. The first is the international market, where it is possible to buy and sell gas according to exogenous prices based on historical developments at the Zeebrügge gas hub in Belgium. A price difference of 0.02 kr./MWh is laid between the import and export price to account for transportation to and from the border. These prices are adjusted to follow the expectations with regard to developments in annual fuel prices. These expectations are described in chapter 7. This way of handling import and exports is selected as it is closely related to the method Gastra (now Energinet Danmark) uses to calculate a neutral gas price, which it employs in its balancing operations.

The domestic market works as a partial equilibrium market where the supply rate from the North Sea is price independent. This, by the assumption that the variable costs of production are negligible when compared to fixed costs and investments in offshore extraction. Demand is given by exogenous and inelastic demand from consumers with the exception of demand from heat and power generators. These consume the gas, which is available to them to make a profit. They are also able trade on the international market. As such a price indication can be derived when heat and power generators consume some, but not all the gas which is available to them.

Imports and exports naturally also have an impact on the objective function. When import prices are defined as $\pi_i^{s,IMP}$ and export prices are defined as $\pi_i^{s,EXP}$. Their contribution is:

$$\sum_s \sum_t \sum_{i \in \mathcal{P}} \left(\pi_i^{s,IMP} \nu_i^{s,t} - \pi_i^{s,EXP} \chi_i^{s,t} \right) \Delta^{s,t} \tag{6.13}$$

6.5 Link with the Balmorel Model

The link between the original Balmorel model and the gas flow model lies in a set of equations ensuring that the outtake of gas at the M/R stations matches the demand from the heat and power sectors and the residual demand (private homes, industry etc.). The residual demand is described in chapter 7, but for now it is enough to know that this demand is exogenous and defined as $R_i^{s,t}$. For clarity the consumption of gas by the electricity and heating sector is defined functionally as $F_{i,g}^{s,t}(\cdot)$.

$$d_i^{s,t} = R_i^{s,t} + \sum_{g \in \mathcal{G}} F_{i,g}^{s,t}(\cdot), \qquad \forall i \in \mathcal{A}, s \in \mathcal{S}, t \in \mathcal{T} \tag{6.14}$$

6.6 Summary

Presently the entire modeling effort has been described. The model described in the previous chapter dealt with the physical restrictions of natural gas transmission. This chapter outlined how considerations of a both technical and economic nature have been formulated into operator policy and prices/tarrifs, and these have been interpreted to suit the modeling domain. The market for natural gas has also been described briefly introducing both the international and domestic market functions. These will be considered further when discussing results of the model execution.

CHAPTER 7

Model Execution

This chapter concerns the configuration and execution of the model. Various input alternatives are described to give an impression of how the model can be configured to the needs of the user. The sources of standard data are presented, and the manner in which data is fitted to the model domain is disclosed. The impact of input alternatives in their implication on final results are discussed.

7.1 Geography and Time

At present the geography is fixed to describe Denmark by means of two electricity regions and a total of 50 areas within these regions for describing production of power and district heat and delivery of district heat and natural gas. On the long term the intension is to be able to aggregate areas into larger geographical sections according to requirements of individual model applications.

The years 2003, 2005, 2010, 2015 and 2025 are included in the model. Other years may be supported, but data may not be available in all respects. Data is included to support up to 12 seasons and 12 time periods. There is full flexibility for down-scaling the resolution as long as one is wary of the implications. Seasons and time variation profiles are implemented so that seasons basically represent months, while time periods represent different representative hours within the week. Table 7.1 shows the native intervals of for time segments. The flexibility of the temporal resolution gives rise to some interesting interpretations of which a few are listed below:

$\mathcal{S} = \{s_1\}, \mathcal{T} = \{t_1\}$ Model is executed using annualized averages for all data. The flow model describes the steady-state of the annual net flows.

$\mathcal{S} = \{s_1, s_7\}, \mathcal{T} = \{t_1\}$ As above, but with summer and winter simulations based on the variation between January and June.

$\mathcal{S} = \{s_1, \dots s_{12}\}, \mathcal{T} = \{t_1\}$ All individual months modeled with the steady-state flow conditions.

Time Element	Day Type	Hours
t_1	Weekday	00:00-09:00
t_2		09:00-11:00
t_3		11:00-11:30
t_4		11:30-15:00
t_5		15:00-18:00
t_6		18:00-18:30
t_7		18:30-21:00
t_8		21:00-00:00
t_9	Weekend	00:00-07:00
t_{10}		07:00-16:00
t_{11}		16:00-21:00
t_{12}		21:00-00:00

Table 7.1: The subdivision of an average week within a month into time segments.

$\mathcal{S} = \{s_n\}, \mathcal{T} = \{t_1, \ldots, t_12\}$ An annually averaged week with the variation profile of month n. Quasi steady-state flow modeling describe transmission.

$\mathcal{S} = \{s_1, \ldots s_{12}\}, \mathcal{T} = \{t_1, \ldots, t_{12}\}$ The full model with twelve months and as much variation as the data provides.

Numerous other combinations are possible each with its own interpretation and area of usefulness. Table 7.1 shows the hours which are represented by time periods within a week.

7.2 Fuel Prices

The latest published projection for fuel price developments by Danish Energy Authority of February 2003 [9] is applied in the model. The fuel prices applied are presented in figure 7.1. There are no seasonal variations in fuel prices, with the exception of natural gas which is treated apart from other fuels. Prices for natural gas are not used directly, as this price is determined by the model. The price development for natural gas is, however, used to project the border prices as mentioned in chapter 6.

7.3 Demands

Demands for network bound energy supply is derived for electricity, district heat and residual natural gas (gas not used for energy transformation purposes). It is assumed that the profile of these demands is sustained across years, while the annualized demands, to which the profiles are applied change over the years. Thereby each energy form a profile is found or derived, and a forecast for annual demand is found or derived.

7.3.1 Demand for Electricity and District Heat

Annualized demand for district heating is found through aggregation of annual deliveries of heat from heat producers within the respective areas. A total annual demand estimation can be made for each area in the model, which is coherent with the technology and capacity

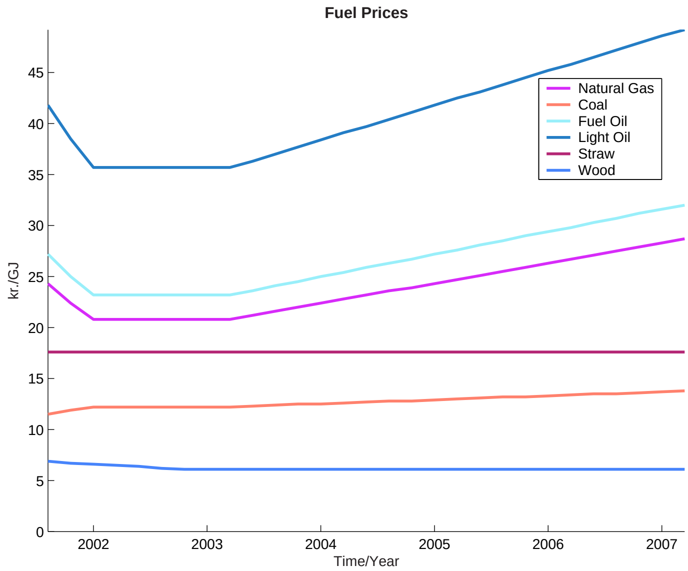

Figure 7.1: Fuel price development forecasts.

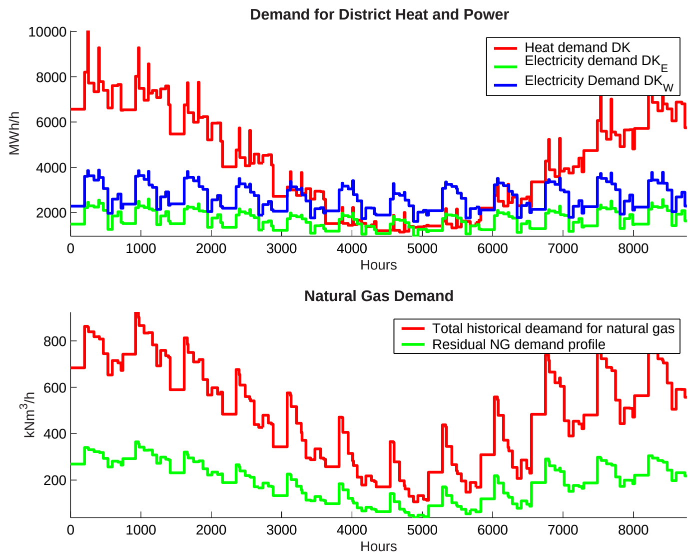

Figure 7.2: Exogenous input data relating to demands with full temporal resolution.

data. Annual demand is assumed to be constant. The variation profile for district heat in the Balmorel data set is employed.

Electricity annual demand is derived analogously to district heat demand. However, here an annual increase of 1.0% is assumed. Figure 7.2 (top) shows the variation profiles for district heating and electricity demand with the full resolution of 12 seasons and 12 time periods. Figure 7.3 (top) shows the same, but aggregated to 4 seasons and 4 time periods.

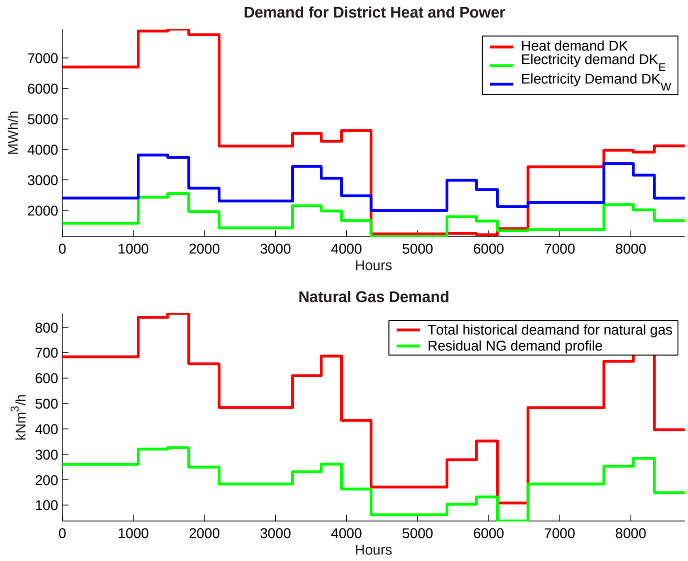

Figure 7.3: Exogenous input data relating to demands with four seasons and four time periods.

7.3.2 Natural Gas Demand

Demand data for natural gas is only available as total consumption over time. It is necessary to subtract that which has been used for heat and power generation. This data is, however, only available on an annualized basis. Using the profiles for heat an electricity production however, a decent estimation can be made. This estimation is illustrated on figure 7.2 (bottom) and in the time aggregated form with four seasons and four time periods on figure 7.3 (bottom).

7.4 Gas Production

Gas supply from the North Sea is assumed to be constant throughout the year. This reflects the economic advantage of operating at maximum capacity on the offshore rigs in order to reduce the long-run costs of sustaining the extraction operation. Figure 7.4 shows two possible trends in the North Sea production over the modeled time frame. One is the expected annual production given only presently confirmed reserves. The other shows and estimated contribution from future prospects.

The basis scenario, for which results are presented in chapter 8, considers only the confirmed resources and thus follows the rather pessimistic production curve. Section 9.2 includes results using the production curve which includes prospects.

7.5 Technology Data

Technologies and exogenous capacity with regard for energy transformation has been updated in relation to the data set distributed with Balmorel. The Balmorel transformation capacity is not distributed at a resolution sufficient for this project. It was necessary to generate a new capacity distribution.

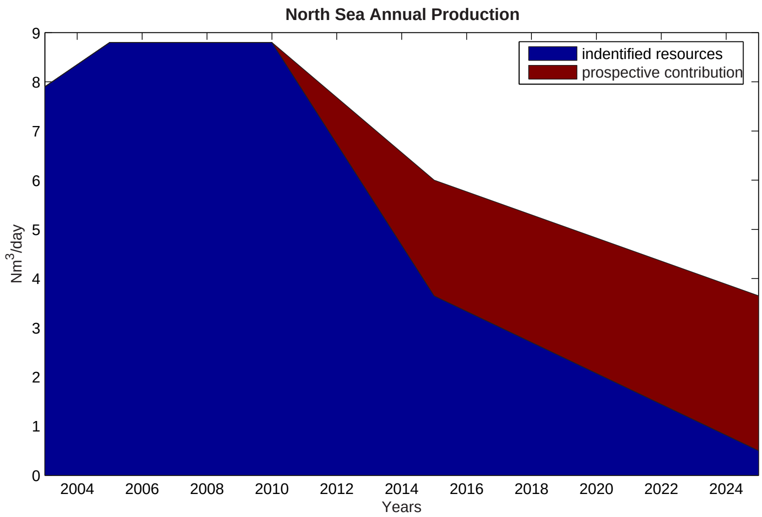

Figure 7.4: Natural gas production from the North Sea (SOURCE: Danish Energy Authority)

The root source of transformation capacity data used for the model is the Energy Producer Census 2003. Census of all Danish energy production units is performed annually by the Danish Energy Authority. This data set includes plant level information on unit location, electricity and heat generation capacity, annual electricity and heat production, annual fuel consumption and much more.

The generation capacity data set is aggregated by technology type within each area. Capacity is sorted by postal code and associated with the nearest metering and regulation station where applicable, using maps of the natural gas transmission and distribution networks in conjunction with a postal code map. Subsequently, generation capacity is aggregated to match the technology index.

A new technology index was also implemented. The Energy Producer Census contains plant by plant technological data, but this is too fine to be applied practically. Also, it lacks data with regard to operating and investment costs among other shortfalls. Instead the technology index created by Eltra, Elkraft System and the Danish Energy Authority [8] is applied, and plants are assigned a best match to technology types from the index.

It is important to note also that wind powered technologies are omitted entirely from this project. There has been no intention to make scenarios on this front, and since demand is generated by observing historic deliveries from thermal technologies only, it is assumed that there is a demand layer, which is being supplied by wind power. Since wind power is prioritized and production is fixed to an exogenous wind profile, the omission of a matching amount of supply and demand has no effect on the solution or associated shadow prices.

7.6 Model Complexity

In the basis scenario with 12 seasons and 12 time periods the model consists of 5 iterative solutions of two linear programming problems and a fast heuristic (linear time). The linear programming problem, which expresses the initial desires of the market, is a lot smaller than the second, which includes the technical considerations of the transmission system.

Excluding model generation time the solution of the model takes in the neighborhood of 43 hours or just short of two full days. Table 7.6 contains more detail about the model size and execution time. One interesting tendency is that the intial solution of both models takes noticeably longer than the subsequent iterations. This is the case for both models. This tendency must be attributed to solvers ability to "warm start", meaning that the second iteration commences at the solution to the first iteration. This property results in the rapid re-optimization as long as there is not too big a variation in the input data between the modeled years. The solver used is CPLEX 9.0 by ILOG.

Model Complexity

Model:	Variables	Constraints	Non-Zeroes	Execution Time seconds (min-max)
Initial model	480,000	250,000	1,500,000	500-9,700
Physical model	520,000	370,000	1,900,000	10,200-63,200
Total (5 years)				156,200

Table 7.2: Linear program dimensions and execution times.

7.7 Summary

The model is dependent on data from a number of sources, and the quality of sources and the method of data fitting is crucial for attaining reliable results. Although the generation of a realistic data set was not the focus of this report, even the generation of the imperfect data used to demonstrate the application of the model has been unreasonably time consuming. In the following chapter the strengths and weakness of the model, but also the applied data set will be disclosed.

CHAPTER 8

Simulation Results

The model produces a vast amount of data upon its execution. In this chapter it is shown how this output can be interpreted and illustrated in a context upon which it can be used for analytical purpose. Results are presented for a standard execution of the model as described in the previous chapter. Different temporal resolutions are employed in some of the different results. This is to demonstrate how effective of the flexible description of time is, but also due to the rather lengthy execution times of the full resolution model.

8.1 System Load

One of the more interesting results of a simulation is load distribution of the system. In the electricity market for example, there is a strong correlation between high load and high prices, as the generators with highest operating costs produce only during these periods. Due to these aforementioned operating costs they are unable to provide competitive bids in low and medium load situations, hence at peak hours they must bid to cover long term marginal costs. These are naturally also the most interesting hours with respect to investments.

For the transmission system operator (TSO) the peak load hours are also the most interesting. Gas transmission is a business where the cost share of investment is dominant over the cost of operations. Therefore, the signal most relevant to the TSO is a signal indicating capacity shortfalls; signals which could trigger investment. Load duration is highly indicative of how far the system is from running out of capacity.

Figure 8.1 (left) features the load duration curves for electricity production in the modeled years. Since electricity demand in this simulation is inelastic, the results are hardly surprising. On an annual basis a small increase in generation is required over time to supply the annually increased demand. The demonstration features maximum resolution with respect to time periods to give a better impression of the shape of the load-duration curve. The simulation which produced the graph incidentally featured 12 seasons and 12 time periods.

Figure 8.1 (right) similarly illustrates load duration with regard to exit from the natural gas transmission system. That is the sum of hourly exports and exit into the domestic market. The simulation uses only confirmed reserves as explained in section 7.4. Notably the variation in hourly values is less when there is plentiful supply. Excess available gas is exported, which means a more efficient use of the transmission system, and a higher utilization of capacity products. The low supply years reflect a difficulty in even providing for residual demand. Hence the load profile reflects mainly the residual profile, and not much gas is for other purposes (exports, district heating and power generation).

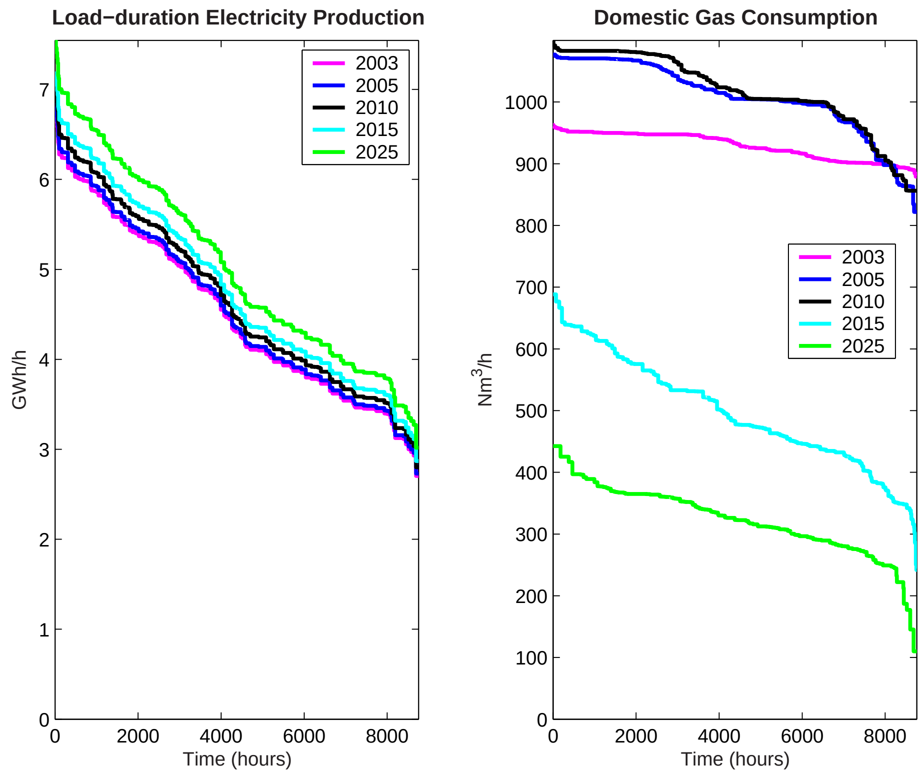

Figure 8.1: Load-duration curves for electricity demand (left) and natural gas transmission system exit (right)

8.2 Marginal Values

Marginal values of interesting interpretations in many areas of the model. The value of additional gas at some location in the model can be derived from the shadow price of the nodal mass balance equation (5.83) and can be interpreted as an area price consumers must be willing to pay to ensure supply. That is of course if gas was sold at area specific prices. Dual values to capacity constraints could be used to interpret a cost of additional capacity if the capacity constraint is binding. In the following the interpretation of marginal value of the supply variable are used to determine a gas price at the supply location. Following this, the derivation of an electricity price is explained.

8.2.1 Gas prices

In section 6.4 it was described how two forms of prices appeared in the model. Import and export prices are given by exogenous values with appropriate variation over seasons and time periods. An endogenous price appears from the domestic supply source in the North Sea.

The dual variable $\mu_i^{s,t}$ to the domain constraint for the supply variable $\nu_i^{s,t}$ is interpreted as the shadow price for an increase of supply rate by $1kNm^3/h$ for the duration of time segment s,t. Hence a gas price at the entry point is derived by:

$$p_i = \frac{\mu_i^{s,t}}{\Delta_i^s, t} \tag{8.1}$$

where i is the Nybro network node.

Figure 8.2 demonstrates an example of this for 144 time periods. The gas price is notably lower than the wholesale price of gas on the market today, which is currently around $1.8kr./Nm^3$ dependent on the supplying company. Interestingly enough this same calculation using only 4 seasons and 4 time periods gives the results on figure 8.3. Here the average price is definitely higher and more equal to the price observed in the market. This might indicate a weakness in the flow model as there is more variation in demand on the one with finer resolution. Supply issues resulting from this flow model could result in gas not being delivered to where it has the highest value, thus affecting the simulated market value.

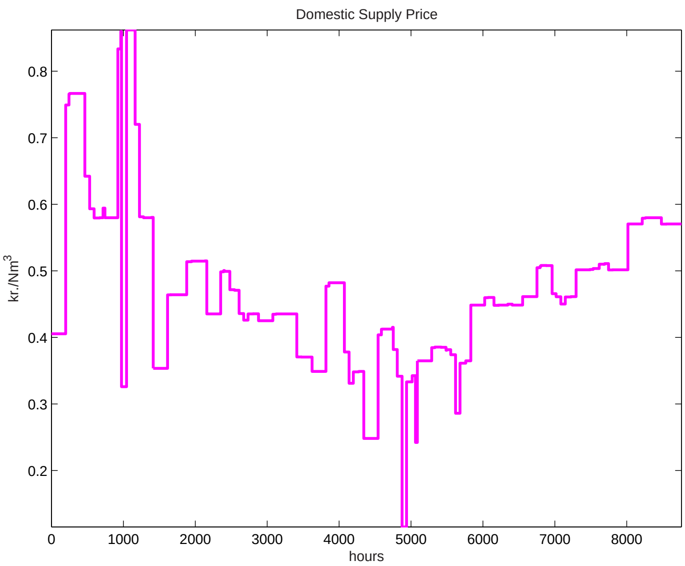

Figure 8.2: Gas price for the first year of the simulation with 144 time periods.

8.2.2 Electricity prices

The price of electricity in the model is reflected in the shadow prices of the electricity balance constraints. At optimality:

$$\pi_r^{s,t}\left(\sum_{g\in G(r)} e_{s,g}^{s,t} + \sum_{\rho\in R(c),r\neq\rho} x^{(r,\rho),s,t}(1-\epsilon^{x(\rho,r)}) - \frac{e_d^{r,s,t}}{1-\epsilon_r^e}\right) = 0, \forall r \in R, \forall s \in S, \forall t \in T \tag{8.2}$$

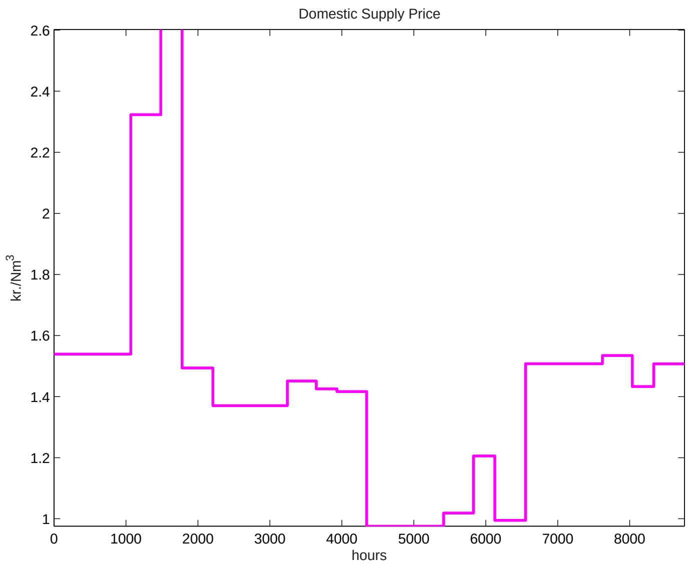

Figure 8.3: Gas price for the first year of the simulation with 16 time periods.

Here $\pi_R^{s,t}$ reflects the dual variable of the equation, which when optimal values of the primal variables are inserted, reflects the shadow price of power generation. In other words, the value of $\pi_R^{s,t}$ is normalized value of a marginal increase in available power, or the marginal generation cost of a unit of power. From partial equilibrium theory it is known that this reflects the market price under perfect competition, given that there is a market price for every region r in every time segment (s, t). Finally it is important to note that since the energy balances are given terms of power (MW), the electricity energy price is $p_e = \frac{\pi_R^{s,t}}{\Delta^{s,t}}$.

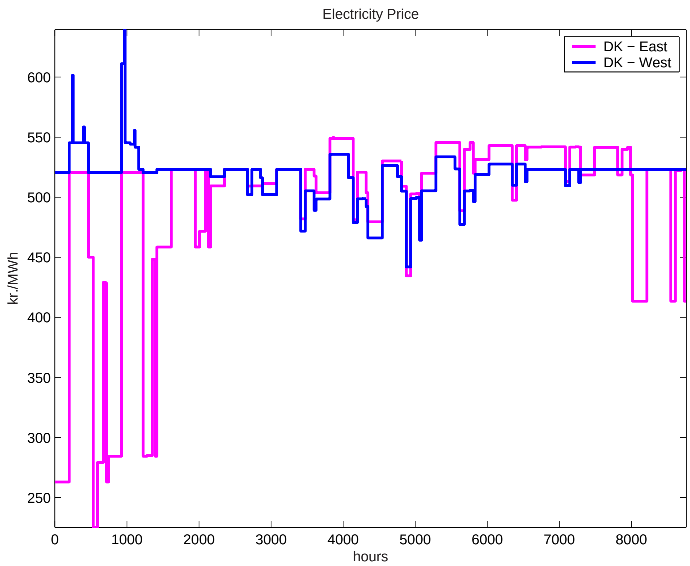

Figure 8.4: Electricity price for the first year of the simulation.

The electricity price, depicted on figure 8.4, remains fairly constant throughout the simulated year. It is interesting that the electricity price in the early months of the year hit the bottom mark. This is most likely due to the high heat demand making fixed ratio CHP generators of limited use. Theoretically this would imply that electricity to heat technologies such as heat pumps would possibly be an efficient investment. These are, however, not among the simulated technologies and therefore the price of electricity drops drastically.

8.3 Fuels for Energy Transformation

The fuel mix used for district heating and power production is another simulation result. The most deciding factors with respect to the fuel mix are fuel prices (exogenous and endogenous), transformation capacity and technology, competitiveness of investments and fuel potential within specific areas/regions. For natural gas, the supply system plays a deciding role. If fuel is used in one place it will naturally not be available for consumption at another. If prices in Sweden and Germany are favorable, the available supply for the domestic market dwindles at least until export capacity is reached. Bottlenecks in the transmission system would naturally also have an impact on the emphasis on natural gas in the fuel mix.

Results from a twelve season, twelve time period simulation are shown on figure 8.5 in annualized terms, and figure 8.6 shows the load duration fuel consumption. The simulations uses only the confirmed natural gas reserves (as described in section 7.4). It is as evident from the figures, as it is intuitive that gas is well represented in the first years where production is high. The share declines until all domestic production as well as import capacity is used to ensure the inelastic demand of residual customers. This is discussed further in section 8.4 below in context of transformation technologies.

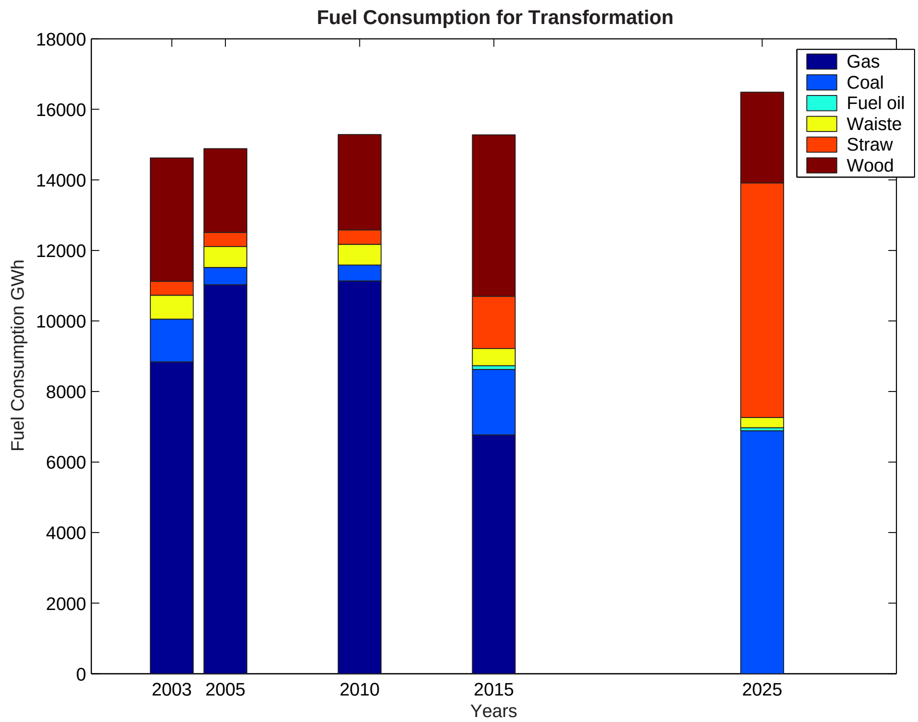

Figure 8.5: Fuel consumption by technologies in four simulated years.

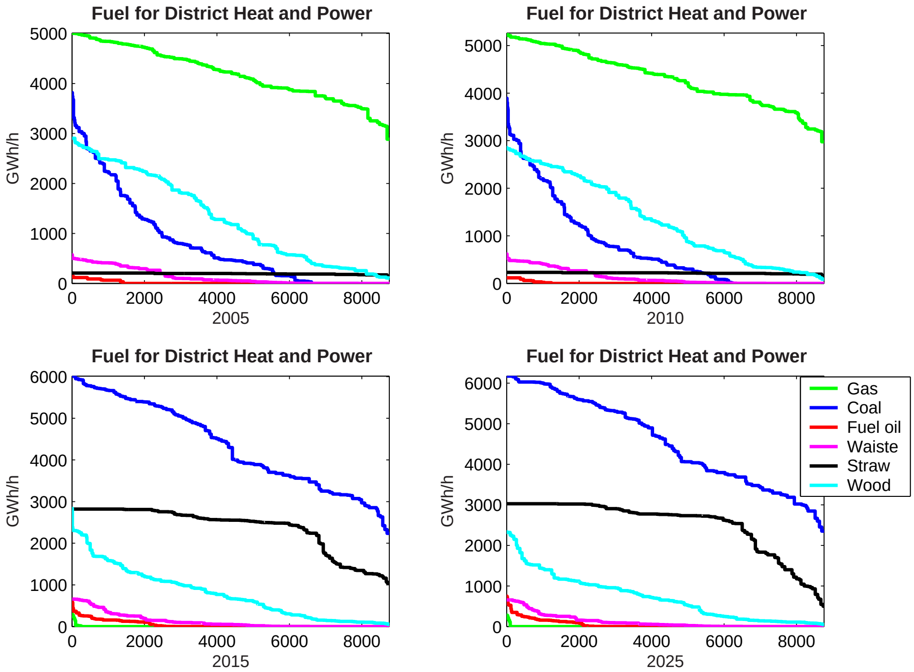

Figure 8.6: Fuel consumption by technologies in four simulated years.

The load duration for fuels (figure 8.6) also has some interesting implications. The shape of the duration curves can reveal details about which fuels are used for base load generation and which primarily are used in peak load hours, by considering the slope and curvature of the graphs.

In the first two simulation years illustrated, the natural gas consumption has an almost constant slope and no curvature. In the two final years there is almost no gas fired generation. It appears that coal and wood is used for high and peak load generation initially. As natural gas supplies disappear, coal takes over some of the base load and is supplemented by straw fired generation. Wood fired generation is still mainly used in the high/peak load hours.

8.4 Transformation Technologies

The model can demonstrate in different ways, which technologies are used to generate heat and power. One can view results in terms of annual distribution between technologies. How much production is on new or old capacity etc. One can also see how producing technologies are geographically distributed. Naturally, generation is separable in heat and power, and total fuel consumption can also be illustrated.

Each technology is identified on the figures by a label. To facilitate interpretation of the graphs these labels are described in table 8.4.

Figure 8.7 demonstrates how initially gas power is competitive in the simulated market. Especially with respect to power generation, the large scale combined cycle facilities are well represented.

The specific simulation is run with four seasons and four time periods and the North Sea production is limited to confirmed reserves only, as was described in section 7.4. The

Label	Description
APF-E01-CO	Large scale coal fired CHP with extraction capability
APF-E01-NG	Large scale natural gas fired CHP with extraction capability
W2E-B05-MW	Municipal waste fired back pressure CHP unit
GTLS-B06-NG	Large single cycle gas turbine (fixed ratio)
GTMD-B06-NG	Medium single cycle gas turbine (fixed ratio)
GTMN-B06-NG	Miniature single cycle gas turbine (fixed ratio)
CCLA-E07-NG	Large combined cycle facility with extraction capability
CCSM-B06-NG	Small combined cycle facility (fixed ratio)
GE-B08-NG	Gas engine (fixed ratio)
ST-B09-WW	Wood pellet or wood chip fired steam turbine (fixed ratio)
HO-51-WW	Heat-only boiler fired by wood
HO-52-NG	Heat-only boiler fired by natural gas
HO-54-MW	Heat-only boiler fired by municipal waste
ST-CO-FOsn	Power plant fired by fuel oil with de-NO_x and de-SO_2
ST-CO-COsn	Power plant fired by coal with de-NO_x and de-SO_2
HO-B0-FO	Heat only boiler fired by fuel oil
HO-B0-ST	Heat only boiler fired by straw
ST-B8-NG	Steam turbine fired by natural gas (fixed ratio)
ST-B8-CO	Steam turbine fired by coal (fixed ratio)
ST-B9-ST	Steam turbine fired by straw (fixed ratio)
ST-B8-FO	Steam turbine fired by fuel oil (fixed ratio)
G-HSTORE	Heat storage facility

competitiveness of gas power in the early years of the simulation thus reflects the high capacity of supply. In later years, as reserves are depleted, production is moved to straw and coal fired technologies, as can also be seen on figure 8.7. Evidently these constitute the best alternative when no cheap or technically feasible option for gas supply exists.

There is also a noticeable tendency to use mainly the large scale facilities, rather than the smaller facilities such as gas engines. This is in spite of the fact that these are the most numerously installed facility types in this country. Their individual capacity is, however, limited, so they were not expected to dominate the market, but one might have expected a larger market share for these technologies. This is discussed further in chapter 9. The extraction possibility is also a likely reason for the technology choice. Added flexibility to produce a higher share appears to be desirable.

8.5 Investments in Transformation Technology

There is a clear tendency towards investment in large-scale combined cycle facilities initially. This can be seen from figure 8.9. A few comments on this tendency. The Balmorel model uses a quasi-dynamic time representation. Perfect foresight is present for the duration of the currently simulated year, yet no foresight is present with regard for the following years. Therefore, the decision in the first year to invest in gas powered technology also reflects the view that the domestic production of natural gas will be sustained for the economic lifetime of the investment. When this is not the case it can result in a situation where an investment in gas fired technology will produce for the first couple of years, and then subsequently shut down due to supply shortage or a rise in gas prices before investment costs are covered.

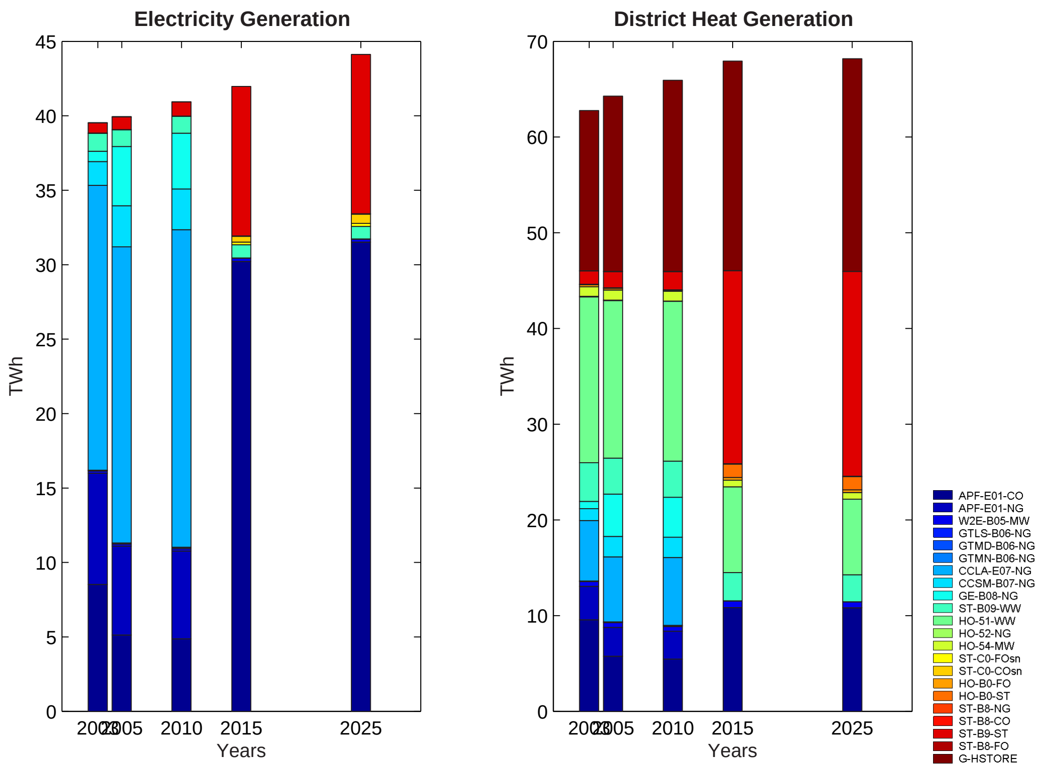

Figure 8.7: Generation of district heat and power by different technologies.

A masters thesis project, recently completed, investigated an alternative way of handling investments in Balmorel, and these ideas are likely to be implemented in some form in the not so distant future [27].

This first simulation was executed under the assumption that no new non-confirmed gas reserves are brought to production within the simulation period, as was stated in chapter 7.

It is also interesting to note that once the gas reserves are depleted, large investments are made in straw fired generation. This is likely due to the fact that straw is a CO_2 neutral fuel, and as such does not require CO_2 allowances.

8.6 Emissions from Transformation

Six graphs on figure 8.10 show the development in emissions resulting from the simulation. These are only from energy transformation, so any emissions from gas consumption in private homes etc. do not impact the results. One might add that these are by no means interesting in the context of the model, as emissions from residual consumption can be assumed linear with respect to consumption levels which were given exogenously.

The first five graphs show emissions of CO_2, SO_2, NO_x, CH_4 and N_2O. The final figure is the total energy generation. This is included in order to be able to see emissions developments in light of developments in energy consumption.

It is evident that the move towards more gas fired heat and power has a positive impact on most emissions, the one exception being methane. This is as can be expected since

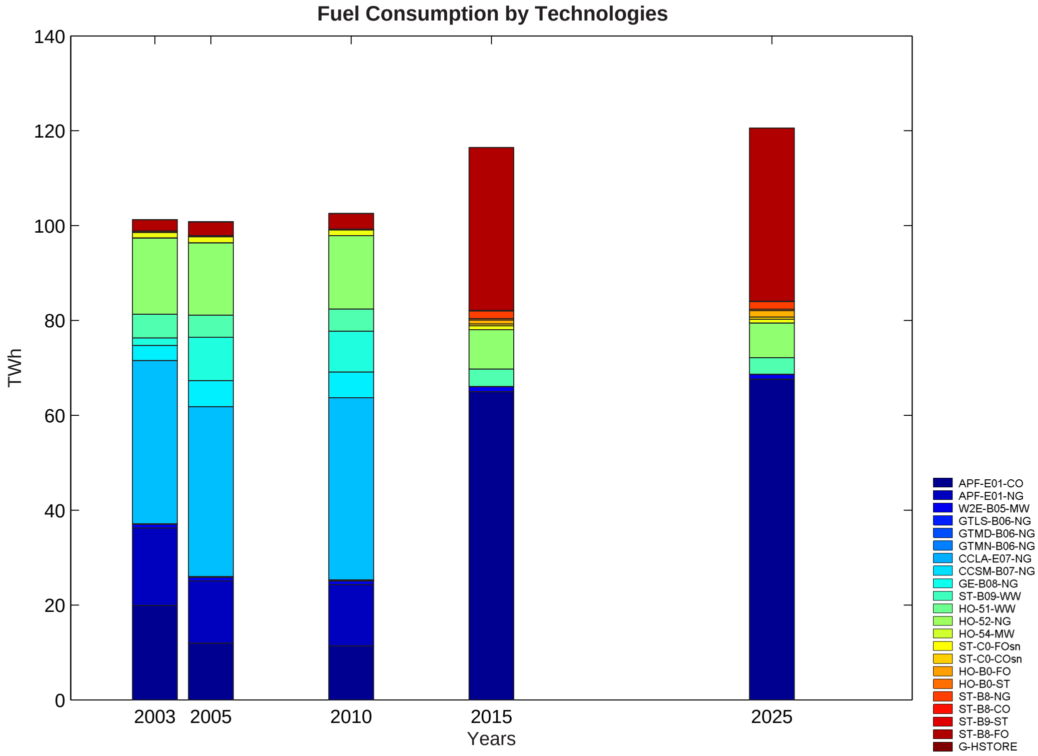

Figure 8.8: Fuel consumption by technologies.

firing of natural gas results in fewer emissions per generated energy unit than other fossil fuels. In 2015 more production is once again coal fired, resulting in an increase in all emissions except methane. Finally, in 2025 CO_2 emissions are reduced as a consequence of the aforementioned investment in biomass technology. In section 9.3 additional analysis is performed on the impact of CO_2 emissions allowances.

8.7 The Natural Gas Transmission System

Illustrating what happens in the natural gas transmission system in a comprehensive manner is rather complex. For each time period there is an abundance of data available describing the flows and pressure in different parts of the network. These is no final destination attached to gas flows. This makes it challenging to connect flow-values with the market effects or otherwise draw the connection for the broader picture. Pressure values are even more difficult to interpret clearly, since the pressure calculations are relaxed in formulating the pressure induced flow constraints. A falsehood, which is an otherwise intuitive interpretation, is that when pressure at the input sources is the maximally allowed, while the pressure at the far end of the network is minimal, then this system must be running at full capacity. This would be the case if the actual pressure was calculated. However, due to the piecewise linear approximation described in section 5.4.4, the only thing one can know for sure is that if the simulation result is feasible, the resulting supply is possible. In theory, the pressure results of the simulation are replicable if one were to use the line-valves in the transmission system to reduce pressure at certain locations along the transmission pipeline. This, however, has no practical operational purpose.

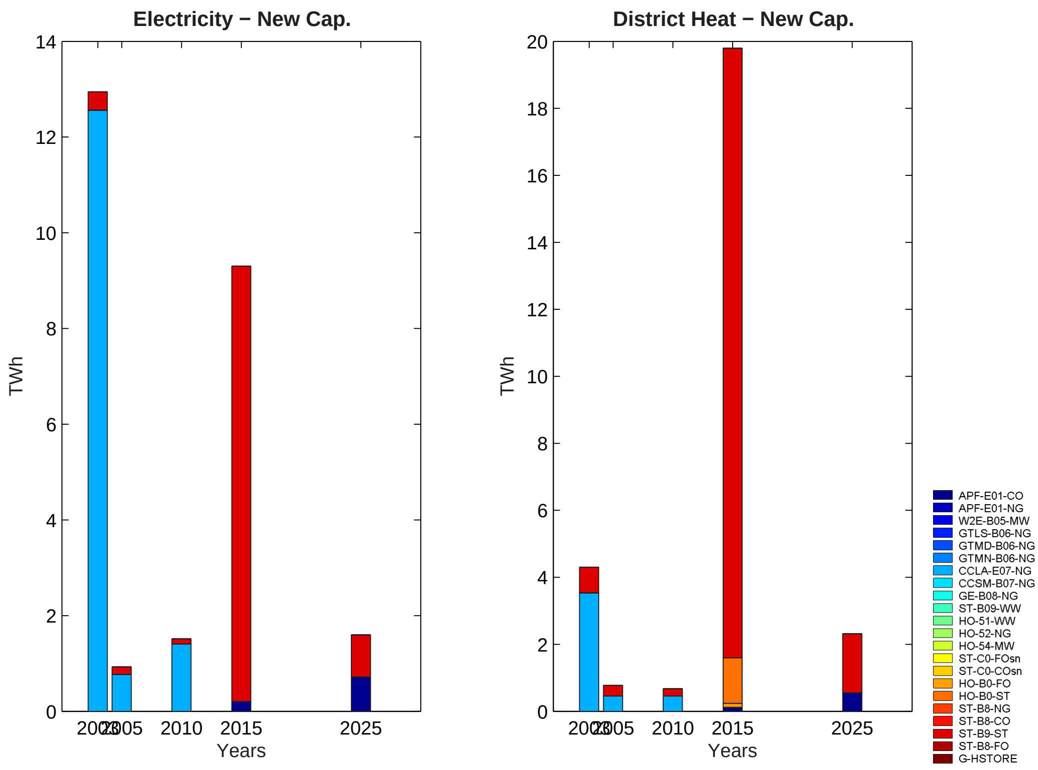

Figure 8.9: Generation on newly purchased technology.

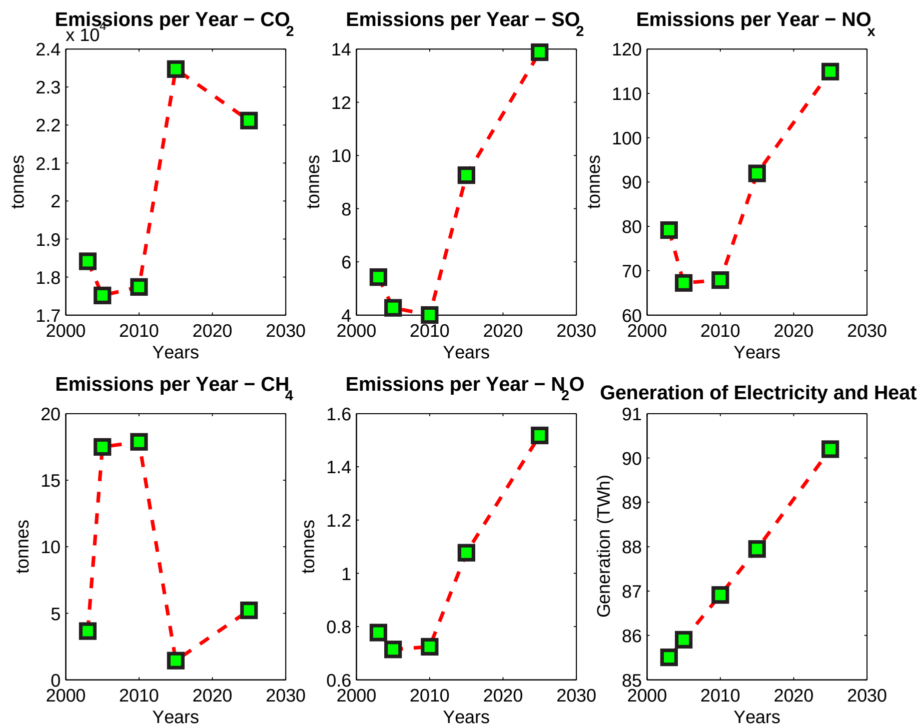

Figure 8.10: Emissions from transformation in relation to total transformation.

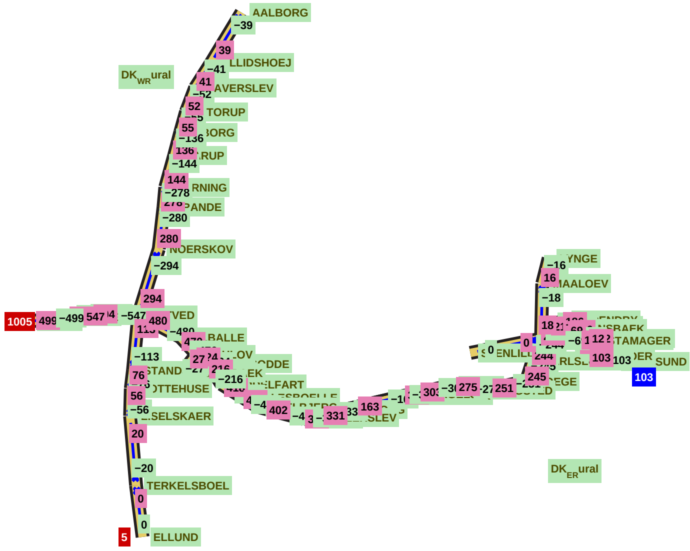

Figure 8.11: Flows in the transmission system.

Figure 8.11 shows the flows during one time period of a simulation. The figure contains too much information to be of practical use in this scale, but is included to provide an overview of how the flow solution can be visualized. The teal and pink labels are flow rates at the ”entry” and ”exit” of each pipe section. The red labels indicate production, import or extraction from storage facilities. The blue labels represent export or injection into storage facilities.

Figure 8.12 shows an enhanced image of the transmission system in Western Jutland. Here pressure values are also labeled. Green labels indicate nodal-pressure while yellow labels indicate mean pipe pressure. On this specific simulation it is interesting to note the flow values in the parallel pipe sections. If the system was running far from full capacity, one would expect the linear program solver to suggest flow in only one of the parallel pipes besides supplying the few small M/R stations located on the northern pipeline. If, on the other hand, the system was at full capacity, the flow would be divided evenly between the two pipes (again taking the intermediaries into account). As such, one is led to the assumption that the flow solution is not at full capacity, but one would not be able to supply the same amount without the redundant pipeline.

8.8 Area Distribution

Other results can also be specified with detailed geographical resolution. This capacity is very useful, if not analyzing results in detail, at least for getting a sense of what is going on. A complete absence of gas fired generation, in a larger area for example, might alert one to a capacity issue, or modeling error, that one would misinterpret as a sign of general reduction in gas fired transformation, should one only consider the annually aggregated results. Only one additional example is demonstrated to avoid clouding the chapter with results. Figure 8.13 shows fuel consumption by transformation technology with area level resolution. Note that the scaling of abscissa are independent between areas, so nothing can be read from this picture concerning the scale of investments.

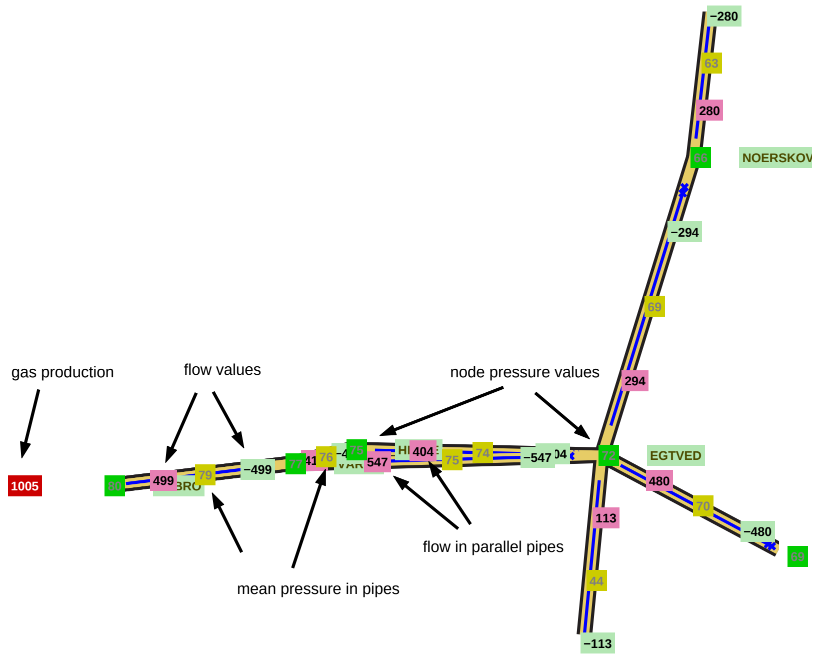

Figure 8.12: Flow and pressure in a portion of the transmission system.

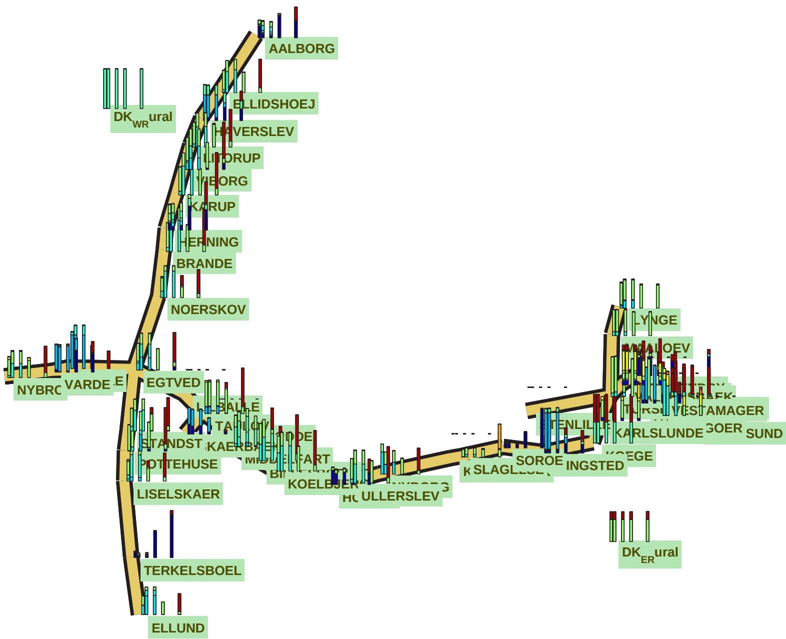

Figure 8.13: Investments distributed by area.

8.9 Summary

Hopefully this chapter has succeeded in illustrating how interesting results can be interpreted from the model output. There is an abundance of output not presented, which has equally interesting potential for interpretation. The trick is not as much how one can find, but rather to determine what one is looking for.

CHAPTER 9

Demonstration Cases

In this chapter the model is used to analyze three demonstration cases. The purpose is to illustrate how the model can be used to evaluate relevant energy policy issues and issues pertaining to systems analysis. Input variations are described where applied, and a selection of output deemed appropriate is presented and discussed. These interpretations are intended to be inspirational with respect to model application, rather than judgemental with regard to the issues described in the demonstration cases. The selected cases are:

1. De-centralized CHP facilities operating on market terms and their impact on the natural gas system.
2. Prospective reserves in the North Sea and the implication investments in heat and power.
3. Market prices for CO_2 allowances and their impact on emissions from the heat and power sectors.

9.1 De-centralized Combined Heat and Power

All electricity generators not located at one of the 15 central plant locations are by definition de-centralized. In 1986 it was decided to subsidize the conversion of a number of district heating boilers into CHP units [25]. Subsequently, in 1990 the decision to convert all larger district heating boilers to either CHPs or biomass fired boilers was taken [26]. The incentive was that electricity produced by de-centralized co-generation units could be sold at a fixed and favorable feed-in tariff with priority over the large central plants.

The feed-in tariff became known as the 3-stage tariff due to differential tariff levels for the three stages of low, high and peak load generation. As of January 1st 2005, this structure has been replaced by subsidy on the heat side of the energy mix for larger de-centralized producers. The rest are assumed to come on similar terms within a short time frame. Eventually all CHPs will likely compete on the electricity market, with the possible exception of municipal waste fired facilities.

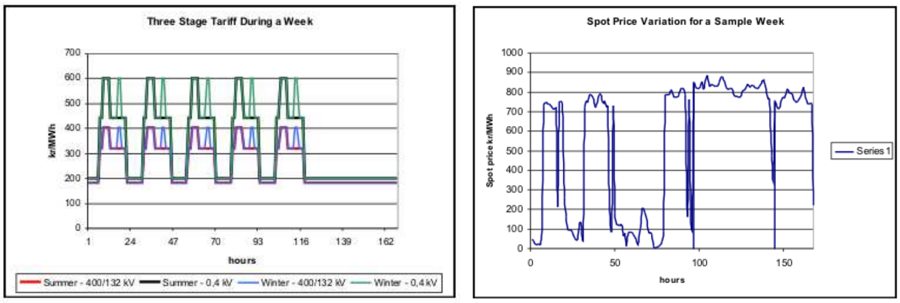

Figure 9.1: Left: The three stage tariff at different transformer levels and for different seasons. Right: The variation in spot prices for the first week of January 2003.

9.1.1 Fixed Tariff vs. the Spot Market

There is a huge difference between producing against known prices and bidding against an uncertain spot market. The fixed tariff case one can plan against an expected heat demand in the production optimization problem becomes a fairly deterministic problem involving the decision of when to production on the CHP unit, when to use a heat boiler, and how to use the heat storage facility, in order to generate a surplus for the heat customers. Now they are faced with the challenge of bidding against a highly volatile and uncertain market price. Figure 9.1 illustrate the 3-stage tariff side by side with a sample week on the spot market.

The uncertainty is passed on to the natural gas system, which provides the fuel for most de-centralized CHPs. Here it is especially interesting how the freely operating CHPs will respond to the spot market in the peak hours in the gas system. If the electricity price is high simultaneously with heat demand, there is a potential that the last free capacity in the transmission system, could come under pressure.

9.1.2 Technologies

One unfortunate property of the model is that there is no distinction between central and de-centralized electricity generators. This results from an unfortunate lack of foresight during the categorization and aggregation process. This data generation process was unfortunately incredibly time consuming and is only partly automated. Once all CHPs operate on market terms this will be less of a problem, as the need for distinction is decreased. For now, the strategy is to consider the technologies most common for gas fired CHPs. These are described in the following sections.

Engine Driven Plants

Engine driven plants are the most common form of de-centralized CHP. Basically a combustion engine powers a turbine to generate electricity. Heat recovery systems enable the use of heat from cooling water, lubricants and exhaust in the district heating network. Engines are generally either natural gas or diesel fired. These plants account for about 85% in terms of number of installed plants, and 45% of the electricity generation capacity in Denmark [13].

Steam Turbines

A steam turbine is basically a boiler, which heats water forming steam. This steam is pressurized over the length of a turbine driving a generator. Traditionally exit steam has been cooled by an intake of seawater, but by replacing this with a district heating condenser, the heat is transferred to the district heating system. Steam turbines can either be back pressure units, which supply electricity and heat at a constant ratio, or extraction units giving added flexibility from a full back-pressure mode to a full condensing. The extraction unit type of steam turbine combines the seawater condenser with the district heating condenser to get the desired ratio between electricity and heat. Approximately 6% of de-centralized CHP units in Denmark are steam turbines, but being generally larger units they cover near 20% of installed electricity generation capacity[13].

Gas Turbines

A gas turbine can be coupled with a generator to generate electricity. Again heat can be recovered from exhaust gas. The advantage of gas turbines is that heat can be recovered without reducing the electricity production efficiency. However, gas turbines have low regulatory ability in that efficient is drastically reduced when operating below nominal effect[13].

Combined Cycle Technology

The combined cycle technology encompasses the combination of a gas turbine and a steam turbine. Instead of the steam turbine being supplied with steam from a boiler, it is driven by the high pressure exhaust from the gas turbine. Combined cycle facilities are usually of a rather large size and account for 6% of facilities in Denmark and an electricity generation capacity share of 35%[13].

9.1.3 Distribution of Initial Capacity

Among the technologies which make up the de-centralized power generation capacity the distribution of initial capacity is illustrated on figure 9.2. This distribution results from the aggregation and categorization of heat and power generators listed in the energy producers census. The capacity shares deviate notably from those presented in section 9.1.2. This is unfortunately the result of inaccurate methods for categorization, as some of the capacity illustrated here is not de-centralized. The technologies were listed in table 8.4.

9.1.4 Model Results

An interesting result of the initial simulation was the increased emphasis on gas power. This implied investments in large-scale combined cycle facilities. The relatively large capacity share of gas engines is not put to much use by the model. One could interpret that these are being replaced by more efficient technologies. Unfortunately the model does not accurately describe the advantages of gas engines. Normally of small scale they are able to use the heat demand of a relatively small district heating system, however, this advantage is not evident since the geographical resolution does not achieve a distribution level.

An estimation of the historic consumption load of CHPs is generated by distributing annualized data for gas consumption by de-centralized CHPs across a load duration variation curve

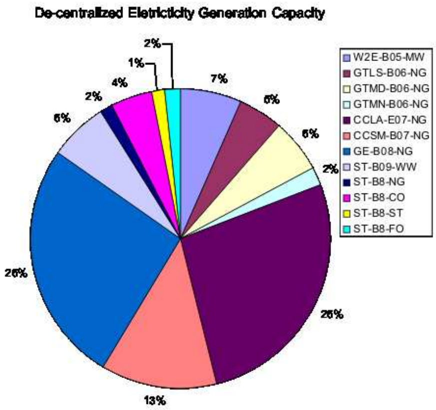

Figure 9.2: Distribution of electricity generation capacity among de-centralized technologies as implemented in the model.

based on the electricity generation variation for CHPs published on the Elkraft System web site (*http://www.elkraft.dk*). The load duration curve of the natural gas fired technologies described above is presented alongside the historic combined CHP consumption on figure 9.3. It is clear to see that the two depictions are incommensurable. The simulated results obviously include a larger than expected share of central generation capacity.

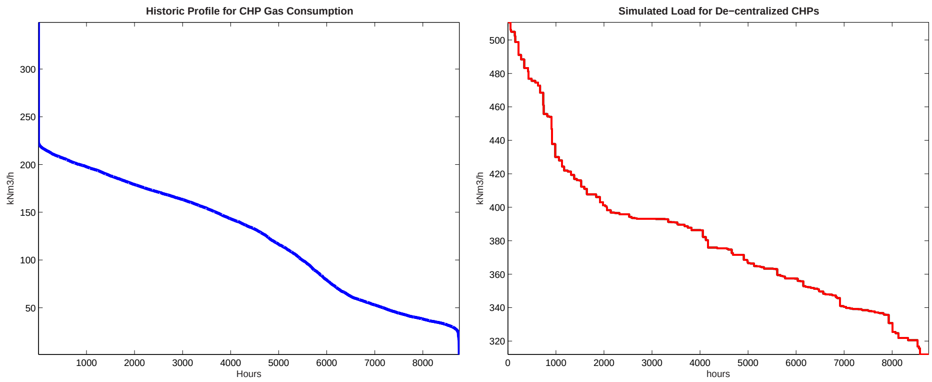

Figure 9.3: Left: Estimated historic consumption of natural gas for de-centralized CHPs. Right: Simulated consumption from de-centralized CHPs

An alternative method of analysis which attempts to salvage the case by considering only the consumption patterns of the individual plants. A load duration curve is generated for each technology individually in relation to the maximum load generated by that technology. The load duration profiles are the normalized in relation to the peak load hour of that technology.

For the historical profile the highest peak value is leveled as this likely represents a specific extreme incident. Now the variation is normalized in relation to the highest remaining peak. This is compared with the load variation of the simulated technologies on figure 9.4.

The relative load distribution of the individual technologies in relation to the historic load of the de-centralized CHPs can be viewed in light the existing capacity and share of the generation mix.

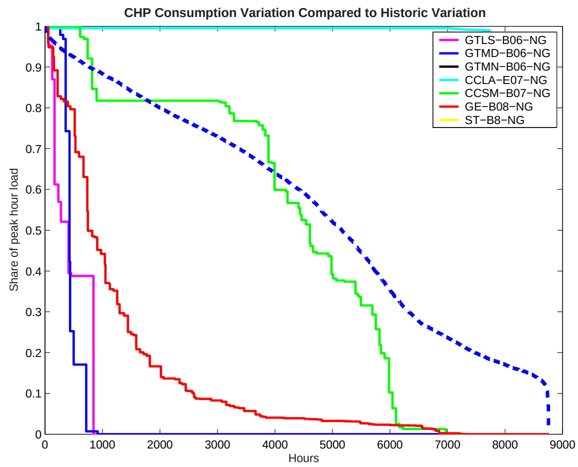

Figure 9.4: The historic variation profile compared with variation of simulated technologies.

Analysis of the demonstration case is inconclusive. Given a better aggregation of capacity data, it would be possible to derive more relevant results. The important issue of scaling is lost, and thus it is not possible to determine what happens from de-centralized CHPs in the peak hours. This is essentially what is interesting from the gas systems perspective. The case does, however, demonstrate some of the possibilities for performing analysis and selecting amongst the model output in order to shed light on a particular issue.

9.2 Prospective Natural Gas Reserves

The rapid depletion of domestic reserves for natural gas is not a certainty as mentioned in chapter 7. The Danish Energy Authority operates with an expectation of prospective discoveries, which can extend natural gas production some years into the future. This was illustrated on figure 7.4. It is now considered how this prospective contribution would influence the energy situation. The results presented in chapter 8 indicated the rather obvious fact that when natural gas supplies were depleted and import options limited, the market moved towards alternative technologies for heat and power generation.

The difference between the original simulation described in chapter 8 and the following is only the domestic supply. However, many other situations could be modeled. Other possible supply options could be the implementation of the Nordic Gas Ring project. It remains a possibility that the BGI consortium will construct a transmission pipeline connecting Germany with Denmark and Sweden. The pipe is expected to branch out to Skåne and Zealand and make landfall at the Avedøre plant.

The intention of this case is to demonstrate that alternative supply situations can be described. Not to provide thorough analysis of their precis impact. Therefore, only the prospective increase in North Sea will actually be implemented in this project (also due to the unfortunately rather long model execution times as mentioned in chapter 7).

Figure 9.5 shows the domestic fuel consumption by heat and power technologies given the new supply situation on the left and the original setting on the right. One can see when comparing with the original, where it was previously necessary to convert to coal and straw towards the end of the simulation period. Now gas power sustains a large share in 2015 but subsequently is still lost from heat and power generation.

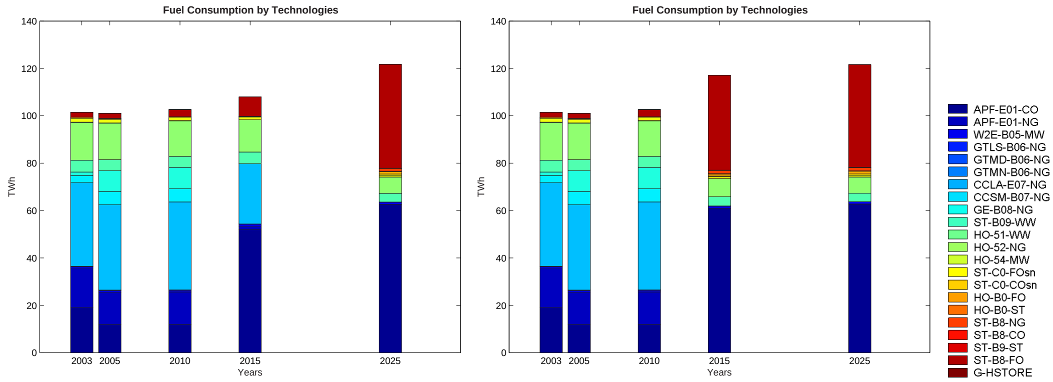

Figure 9.5: Fuel consumption by technologies. Left: Given additional supply from the North Sea. Right: Given only confirmed reserves.

9.3 Emissions of CO_2

As a consequence of the adoption of the Kyoto Protocol for reduction of greenhouse emissions, quotas of CO_2 have been implemented. Emission allowances are tradable and this can be done on the Nord Pool power exchange. The clearing price of EUA (European Union Allowances) is published on *http://www.nordpool.com/marketinfo/co2-allowances/allowances.cgi*. Currently EUAs trade at around 18.60 EURO/ton.

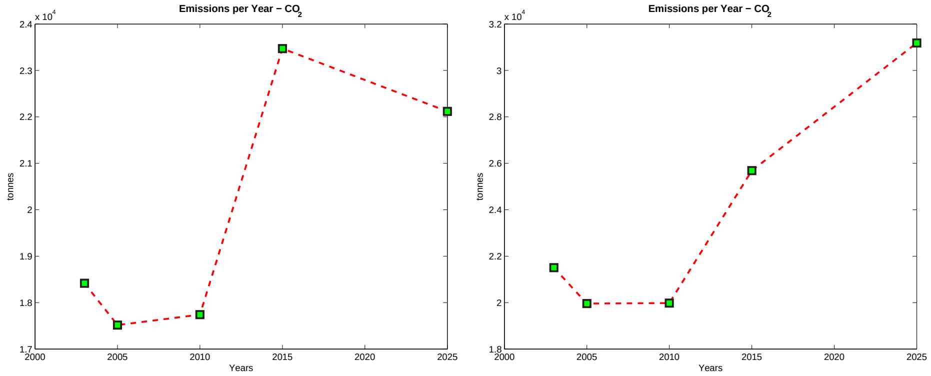

Figure 9.6: CO_2 emissions. Left: With priced CO_2 allowances. Left: Without restrictions on emissions.

9.3.1 Implementation

There are two ways to incite the model to reduce emissions of CO_2. One can implement impose an emission quota explicitly. Alternatively one can assign a price to emissions allowances. The last option is selected. This is done as the impact on solution time should be minimal as additional costs in the objective function have less impact than adding constraints containing a summation over a large number of variables. Quotas would naturally be attached to all generation variables across all time periods.

Subsequently it will be possible to evaluate whether the market price for emission allowances is reasonable. Actually the CO_2 price has been included in all scenarios so far. Consequently they are omitted from this final simulation. The comparison is made with the simulation from the previous section, the gas supplies are net depleted so rapidly. The CO_2 emissions from the heat and power generation are depicted on figure 9.3. The difference in emissions in 2005 is approximately 2,500 tonnes and in 2025 approximately 9,100 tonnes.

9.4 Summary

There are many possible applications for the developed model. However, it is necessary to be extremely aware of the data which is applied. The data set generated for this project needs to be refined before one is able to base conclusions with due confidence on the basis of a simulation. Hopefully the chapter gives an impression and inspiration for what and how one could use the model to address a variety of issues.

CHAPTER 10

Conclusion

The focus of this project has been to investigate the market structure and technical facilities supporting the Danish supply of natural gas. And to create a model description which could take the adjacent supply systems for electricity and district heating into account. Thus creating a tool to analyze the interplay between the newly liberalized markets of natural gas and electricity.

Secondly, the applicability of this tool for performing analyses of relevant energy policy issues should be demonstrated.

To accomplish this a mathematical model has been developed and implemented, which describes the natural gas supply system and market structure. This has been integrated with the Balmorel model which describes the markets for electricity and district heat. The main challenges have been:

1. Describing technical limitations of the facilities, which constitute the natural gas supply system, thus ensuring the technical feasibility of a supply solution.
2. Identifying and describing mathematically the aspects of natural gas supply to which the market responds.

The first challenge was addressed by a thorough review of existing models and theory of fluid dynamics in general and more specifically models and theory of natural gas transmission. Major pitfalls were immediately identified as the behavior of compressible gasses subjected to forces are mathematically complex. Existing models and formulations were found either too general and thereby a direct implementation would be computationally intractable in the desired context, or too specific and dependent on assumptions, which were unsound for the defined purpose. A compromise between accuracy and tractability was finally reached. The underlying assumptions are disclosed so the reader may judge their merits with regard to this project and for the purpose of future application.

The second challenge was simpler to address, yet here it was more complex to attain the desired results. Certain fundamentals of the domain, restricted in part by the flow model and in part by the structure of and assumptions of Balmorel, made implementation of the conditions under which gas transmission, distribution and storage services are provided difficult to model with the desired level of detail. The simplifications are reasonable with respect to the scope of the model, yet as a modeling exercise, the many interesting features regretfully had to be omitted. Issues such as balancing, dynamic capacity booking under uncertainty and under-/overdelivery are lost due to assumptions of perfect foresight and competition.

The resulting model uses a quasi steady-steady flow modeling scheme to provide a technical bounding on the use of the transmission system. This achieved by the formulation of a relaxed steady-state model within a number of discrete time intervals, and relying on the principal of mass conservation and a thermodynamic state equation to provide an offset for each time period.

The domestic gas market is implemented by the interpretation of the conditions for access to the transmission, distribution and natural gas storage facilities and how they induce tariffs on the party attaining access. Coupling with an import and export market function using exogenous prices and a domestic production generating fixed exogenous supply creates a framework for a partial equilibrium domestic market using the Balmorel model does generate an endogenous demand for natural gas. The result of the integrated effort is a means for simulating the integrated and liberalized markets for electricity and natural gas working in symbiosis by partial equilibrium, where technical limitations are an explicit part of the model.

The model is formulated as a mixed-integer programming problem but solved as a linear programming problem using a simple heuristic to fix the integer variables a priori. The model is implemented in the high level modeling language GAMS and has been solved for annual instances using 144 time periods per year for five simulated years.

10.1 Evaluation of the Product

The resulting model is implemented in the GAMS modeling language and is fully integrated with the Balmorel model. By this integration the collective program represents a description of the three network bound energy supply forms available in Denmark. The economics of three supply forms are interdependent. Technical considerations are undertaken to ensure supply or demanded energy commodities. The impact of these technical aspects are registered in the behavior of the market. Domestic prices are determined endogenously. Endogenously priced natural gas is used for the district heating and power generation in competition with exogenously priced alternatives. Energy commodity prices reflect developments from all three sectors.

Considering scope, scale and number of details included, the integrated model is quite comprehensive. Needless to say that interpretation of all the output data is an overwhelming task in itself. It has been attempted to demonstrate some of the diversity of the model output, but limitless effort could be put into extracting, interpreting, and appropriately presenting results. This, saying nothing of accuracy or commensurability with the real world. There is also a considerable potential for development of more specific simulations and case studies.

The model describes a market setting which is still under development. The perfectly competitive scenario is a baseline objective of market liberalization. This model can be used to shed light on the value of this baseline. It can potentially be used to determine a bounding of values of public and private energy and supply commodities in the energy market.

The expectation with regard to its utilization is somewhat less ambitious. Hopefully it will be a subject of discussion how this and other models of similar nature can be applied in energy planning. This is expected to some extent in the project "A Model of and Analyses of an Integrated Gas and Electricity System", which was mentioned in section 2.8. Even though models do not always produce accurate results, their value as means of communication is rarely overestimated. A key result of this project is to be able to shed light on issues, but the issues are left open for discussion.

10.2 Soundness of Assumptions

A number of model assumptions are presented in this report. Some are made out of necessity, some for reasons of practicality and finally some are inherent of the Balmorel model. In general, the assumptions of course influence the results, but they do not hamper the general picture. In some ways limiting the scope by assumptions has been necessary in order to close the model, thus making it possible to evaluate the system-wide effects. Unfortunately the drawback is that certain interesting areas of analysis are excluded by this. The most important assumptions are reviewed in the following.

10.2.1 Perfect Information

Perfect information is assumed for the trivial reason that it is simpler to model and describe than uncertain information. A stochastic approach falls outside the scope of this project. It is, however, very relevant for future developments. The model assumes perfect information with respect to input data, and effect of decisions made. This causes in super-optimal results, as much information is inherently unreliable. Decisions taken in practice by market players will reflect the individual player's expectations and interpretation of available information, as well as the individual player's reservations on information quality. Imperfect information could, as mentioned, be introduced by adding stochastic input data on a market wide level. This would bring better into light issues as seen by observant market players, and thus result in more realistic aggregated decisions on a system-wide scale.

Modeling the market as a single actor also takes some of the punch out of such analysis. Risk management is performed by single actors independently of the rest of the market. Any stochastic uncertainty modeled under the assumption of perfect competition and with the market as a single actor would assume also a perfect trading of risk in solidarity among market players. This seams unlikely in a competitive scenario.

Another related issue is information symmetry. All players are assumed to have equal access and equal confidence in information (not mentioning equal response). This, in conjunction with linear programming has an "all-or-nothing" characteristic to solutions. Had elastic demands been included, the issue would be all the more important. With elasticities one would be obliged to consider at which level the consumers had practical access to price signals and were able/incited to respond to these.

10.2.2 Perfect Competition

Perfect competition gives a baseline for the liberalization effect. For competition to be perfect, however, it is necessary to either have enough competing market players of which none have a dominant position, or to have effective competition policy. While the level of competition may improve over time, it is essential to keep in mind that subject to a simultaneous privatization process, there is an initial structure with one or few dominant players. Therefore markets cannot be expected to be perfectly competitive immediately following a liberalization of the market. The disadvantage of the perfect competition assumption is that an accurate verification of results is not possible.

10.2.3 Quasi Steady-State Modeling

Quasi steady-state flow modeling is a practical construction. Rates of network input and output vary quite a bit over the course of the simulation, and as such the steady-state criteria is not upheld. However, practical experimentation with the model indicates that the model is not able to take advantage of this. Generally flow rates are the same (in absolute terms) at each end of pipes. This indicates two things. Firstly, that a steady flow process is energy efficient. Secondly, the model does not grant a degree of freedom that the optimization algorithm can exploit, in that the behavior of the flow solution is in line with what one would expect.

It was evident that transient flow modeling would be extremely difficult to put in context of an economic optimization problem, especially a linear one. Even if such a model could be formulated, the required linear relaxations would most likely make it possible for the model to unrealistically increase capacity due to foresight. The advantage of the quasi steady-state formulation is that the system will achieve the most when performing realistically.

10.2.4 Demand Inelasticity

There are arguments both for and against inelastic demands with respect to energy commodity prices. The original reasoning for not including elasticities was that consumers do not react to changes in prices. People might raise an eyebrow when receiving the electricity bill, but take little practical action. This is one of the key challenges facing the liberalization project today: Getting consumers to express preference in reaction to prices. This challenge in itself would be interesting to study, but is not the focus of the project and as such inelastic demand was selected 1) to avoid spending time estimating elasticities and 2) to avoid introducing extra variables and constraints to the model.

It was found, however, that from a practical sense elasticities might have saved time. Inelastic demand will often result in infeasibility of an executed model, should there by supply issues (bottlenecks, insufficient generation capacity, shortage of reserves etc.). In the real world any of these issues may well result in not a large shift in prices. A shift of a scale where demand reduction would actually occur and thus help the supply system get back on its feet. Hence even if elasticities were not very accurate, they could have helped in making the model more robust. Without elasticities one must be careful to balance supply and demand on the input side to prevent infeasibility.

10.3 Outstanding Issues

There are many areas where additional effort is desirable in order to attain a more accurate model. From an assessment of relevance of the issue and the amount of work, which would be required on each, has led to the following non-exhaustive list of issues, where work ought to continue.

10.3.1 Time Delay

One issue, which has not been dealt with in this project is the time delay in natural gas system. Generators are, with the exception of three central plants, located some distance from the M/R stations where their consumption is registered. This means that when the generators go to full capacity production it takes a while before effects are felt in the transmission system. This has an obvious impact with regard to the timing of peaks in the natural gas transmission system and the electricity system. Since individual generators are located at different distance to the M/R station, this could have a stabilizing effect on the throughput in the M/R station. This delay should be investigated and somehow implemented if one is to use the model for analyzing the timing of peaks in the modeled systems, especially if one is to determine the possibilities that the systems may somehow be able to alleviate problems by synergic reaction.

10.3.2 Exogenous Prices

An important data concern in this project is prices. All services which are priced must be comparable. If one assumes that only efficient technologies are modeled, the differences in the strategic position may not vary all that much. Hence a discrepancy in the way prices for fuels are derived can be the deciding factor between using all available capacity of one technology or using none of it. A lot of effort has been put into ensuring that prices are consistent. All prices are index regulated, and data sources have, where possible, been found from the same year (mostly 2003 data).

Fuel transportation cost is an outstanding issue. The cost of supplying fuels depend greatly on the transportation expenses. These in turn depend on the location of plants. It should therefore be reflected in a model that power plants using coal are able to get coal to the plant cheaper when located on one of the central plant locations than when located at an inland location. Likewise, it makes little sense if plants using straw are located far from agricultural areas. The scale of this issue can be exemplified using the Danish Energy Authority's assessment concerning fuel prices[9] of 2003. It is estimated for fuel oil that 0.06 kr./GJ be added for transportation to central plants while 4.46 kr./GJ for other plants. This seen in relation to the price of 22.4 kr./GJ for the energy source itself (2003).

Since the model includes the supply cost for natural gas so that location impacts on price of the fuel, it would be reasonable to implement something equivalent for other fuels.

10.3.3 Heat Demand

Accurate heat demand data is hard to acquire. In this project the demand variation from the Balmorel data set was used. This data appears to have one drawback. When considering the variations over a day, there are two extremely high spikes in heat demand. This does not appear to make sense from a demand point of view. It does, however, make sense if the data actually represents a production profile! The three-stage tariff incited de-centralized CHPs to produce heat during peak load times in the electricity sector. Then using heat storage facilities, excess heat was saved for later consumption. This is not a bad idea, but when using production data as demand data, it is unfortunate that one could be lead to conclude that optimal production in the post-liberalized market matches that of the pre-liberalization setting. It would be prudent to find new data for heat demand based on actual demand.

10.3.4 Residual Natural Gas Demand

Another outstanding issue is more accurate data for residual natural gas demand. This has unfortunately been unavailable, as consumption data has not been separated by consumer type. Good data on this field and an estimation of residual elasticity would be a great improvement.

10.3.5 Heuristic Quality

The quality of the direction heuristic is debatable. The amount of effort put into this area is very limited. The problem with the heuristic is that it responds only to the economics in relations to natural gas. This is of course because the flow modeling is in effect shut off while determining flow direction. This has the unfortunate effect that the heuristic may choose direction according to an expectation that all supply can be satisfied from one end of the country. This can for example effectively put the Stenlille gas storage facility out of business. This in turn may result in infeasibility. Another reason to revisit the heuristic is that it is tailored to the current supply situation. Any new sources of supply would require a new heuristic.

10.4 Contribution

Few models have been found to combine the technical considerations of natural gas transmission with relevant economic structures in an optimization context. Those which do exist generally look at a single economic parameter. This could for instance be economic fuel consumption in compressor stations. Although the accuracy of the flow modeling is questionable, it does impose flow limitations on the merits of technical considerations relevant to the specific simulation. The economic impact of these limitations are measurable. The flow is described in terms of a linear programming problem. No other references to such a model have been located.

The integrated tool is a contribution, which enables a look at correlation between a liberal natural gas market, and a liberal market for electricity. Analysis in this area is highly relevant.

10.5 Summary

It proved possible to formulate, implement and execute a model which simultaneously performs optimization across three markets in partial equilibrium while taking into account the non-linear and non-convex pressure-flow relationship in the natural gas transmission network. By this accomplishment a tool has been created, which combines the technical and economic aspects of network bound energy supply. This will hopefully be possible to refine further in the interest of holistic modeling.

Finally, in completion of a general education in engineering it is satisfying on a personal level to be able to present a thesis which, though essentially specialized in the area of mathematical modeling, also draws upon diverse fields such as thermodynamics, fluid mechanics and industrial economics.

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