COMBINED PRODUCTION AND DISTRIBUTION PLANNING

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Preface

To remain competitive, a constant development and optimisation of the supply network is necessary. For large networks, this is a difficult task, as many correlated parameters influence the problem - especially in an international production network.

In a production-distribution network with many interdependent activities, the global optimum is difficult to find and many companies tend to optimise the production allocation and distribution individually. This may lead to solutions less efficient than the results achieved through a combined production and distribution optimisation. To find the global optimum, the use of mathematical models for decision support is desirable.

In the following, a model for combined production and distribution planning, seen from an economical perspective, is developed. The model is based on a specific case - the supply network at Novozymes A/S - but can be considered general for a global network with production, storage and distribution activities. The aim of the project is to evaluate the potential of using optimization tools in the supply chain planning; it is not the aim to develop a tool for implementation in the daily planning process.

The tool developed in this report is intended to be used for two purposes. First as a decision support tool for evaluating the possibilities of obtaining profit improvements from the use of mathematical modelling and optimization of the production and the distribution plan. Second for evaluating different scenarios in the supply chain, and thereby identifying the potential savings. The model treats the problem for an international network at a tactical-strategic decision level, with the purpose of maximising the global enterprise profit.

The presented scenarios do not represent current or future plans for Novozymes, but are solely an evaluaton of the advantages in using the mathematical model implemented.

The project has been carried out as a MSc thesis by Peter Klitz and Jesper Pedersen in the period from September 2004 to April 2005, with the aim of achieving the MSc degree in Manufacturing and Management Engineering from The Technical University of Denmark (DTU). The project has been carried out in collaboration with the Department for Supply Chain Operations at Novozymes A/S (SCO), Informatics and Mathematical Modelling (IMM) and Department of Manufacturing Engineering and Management (IPL) at DTU.

We wish to thank the academic advisers at DTU: Assistant Professor Thomas K. Stidsen, IMM, for advice regarding operations research, model structure and implementation in GAMS. Associate Professor Aage U. Michelsen, IPL, for general input regarding operations management and production planning.

Special thanks to the employees in the SCO-department at Novozymes, in particular our daily contacts Master Planner Jørgen Hindhede Kjær and Director for Supply Chain Support Jesper Leth Espensen for their valuable input to help create and validate the model with respect to the real life supply net.

IMM, April 4th 2005.

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Abstract

In this project an evaluation of the potential of using optimisation in the supply network planning is considered. Existing literature in the fields of supply chain management and operational research is reviewed and a model consisting of general elements structured on the basis of the case is developed. The model has been linearised to ensure solvability if full scale problems need to be implemented. The model is set up as an acyclic digraph and considers production, distribution and storage costs, prices, transfer pricing, tax rates, exchange rates, import duties, export value added taxes, royalties and financial costs. The constraints include production and storage capacities, as well as demand satisfaction and bills of materials.

The general model is adapted to the specific case data, and demonstrates positive gains in using combined production and distribution planning in an international supply network. The model can be used as a decision support tool, when testing different scenarios with changes in parameters. In the scenarios modelled based on the case, a profit improvement is achievable. As only a part of the product portfolio is implemented, the results of the optimisation are not directly scalable though.

The project illustrates the importance of implementing profit maximisation and not cost minimisation in international networks, thereby enabling the model to include the associated fiscal flow.

The model is sufficiently general to be applied in other multi process production-distribution systems within an international intra-organisational network.

KEYWORDS: Combined Production and Distribution Planning, International Supply Chain Optimisation, Master Planning, Operational Research, Linear Programming, Multi-Facility Production Planning.

Sammenfatning

I dette projekt foretages en evaluering af potentialet i anvendelsen af optimering i planlægningen indenfor forsyningsnetværk. Eksisterende litteratur indenfor emneområderne supply chain management og operationsanalyse gennemgås og en model indeholdende generelle elementer udvikles med udgangspunkt i strukturen i casen. Denne model lineariseres, for at tilsikre modellens brugbarhed ved implementering af problemets fulde størrelse. Modellens netværk er en acyklisk digraph indeholdende produktions-, distributions- og lageromkostninger, priser, transferpriser, skat, valutakurser, import- og eksporttold, licensafgift samt finansielle omkostninger. Begrænsningerne består af produktions- og lagerkapaciteter samt opfyldelse af efterspørgsel og styklister.

Modellen tilpasses den case-specifikke data, og optimeringen efterviser potentialet i at anvende kombineret produktions- og distributionsplanlægning i et internationalt forsyningsnetværk. Modellen bruges også som et beslutnings-hjælpeværktøj, når forskellige ændringer i nøgleværdier afprøves som scenarier. I de afprøvede scenarier kan opnås en forbedring i profitten ved anvendelse af optimeringen. Eftersom kun en del af produktporteføljen modelleres er resultaterne af optimeringen dog ikke direkte skalerbare.

Projektet illustrerer vigtigheden af at implementere profitmaksimering og ikke omkostningsminimering i et internationalt netværk. Herved muliggøres medtagelsen af den til produktionen og distributionen tilhørende pengestrøm.

Modellen er struktureret så generelt, at den kan anvendes i andre flertrins produktions-, distributionssystemer i et intraorganisatorisk netværk i en international kontekst.

NØGLEORD: Kombineret Produktions- og Distributionsplanlægning, International Supply Chain Optimering, Master Planning, Operations Analyse, Lineær Programmering, Multi-Facilitets Produktionsplanlægning.

Contents

Chapter 1

Introduction

In the following chapter, a general introduction to the thesis, the framework for the thesis (the main definitions associated with the subject) and the structure of the report are presented, in order to clarify the points of view and aim of the project.

From a general perspective this thesis deals with the subject of using operations research (OR) techniques in the operations and supply chain management of production companies. Based on a specific case, the focus is on developing a model that suits the specific needs in the supply chain planning. The aim is neither to develop faster solutions algorithms for supply network problems nor to implement structural network changes from a business point of view. The aim is to create and evaluate a model that is usable in a rational planning process and suits the management strategies and organisation. The work is based on a specific case - the enzyme production and distribution at Novozymes A/S.

Supply chain and operations management as well as operations research cover a wide range of research areas. To clarify the framework of this thesis, some definitions linked to the project are presented in the following section 1.1; finally the structure of the report is presented in the succeeding section 1.2.

1.1 Framework

The function of integrating and managing business processes across the supply chain (SC) can be defined as supply chain management (SCM). In general, a supply chain within or between organisations is not a linear network of one-to-one business relationships, but rather a complex inter- or intra-organisational supply network. The supply chain function considers product flow, information flow, customer relations etc. One aspect of the supply chain management is the logistics management, considering the flows related to planning and controlling the flow of goods. The Council of Logistics Management (CLM) defines logistics management as

> Logistics Management is that part of Supply Chain Management that plans, implements, and controls the efficient, effective forward and reverse flow and storage of goods, services and related information between the point of origin and the point of consumption in order to meet customers' requirements. (source: www.clm1.org)

In the following, the term supply chain, supply network and logistic chain are used as synonyms, due to the fact that several authors do not differentiate between these terms. The mathematical models for supply chain optimisation presented in this thesis, consider the optimisation of flow of physical goods and the related fiscal flow.

A general model for a supply network is presented in figure 1.1. The supply chain considers suppliers, production- and distribution facilities as well as customers. In general the networks consider links between multiple processes and companies and their structures are highly dependent on the characteristic industry and the point of view (focal company).

The focus of the report is on a logistic network within a global organisation. Network relations exist between the different processes within the organisation and between the different regions. The inter-organisational aspects are not considered.

The planning and management of supply chains can be considered at several decision levels. In this report the following definitions are used:

- Operational planning: The short-term decisions, planning the operations at the individual production lines and plants, as well as the individual shipments.

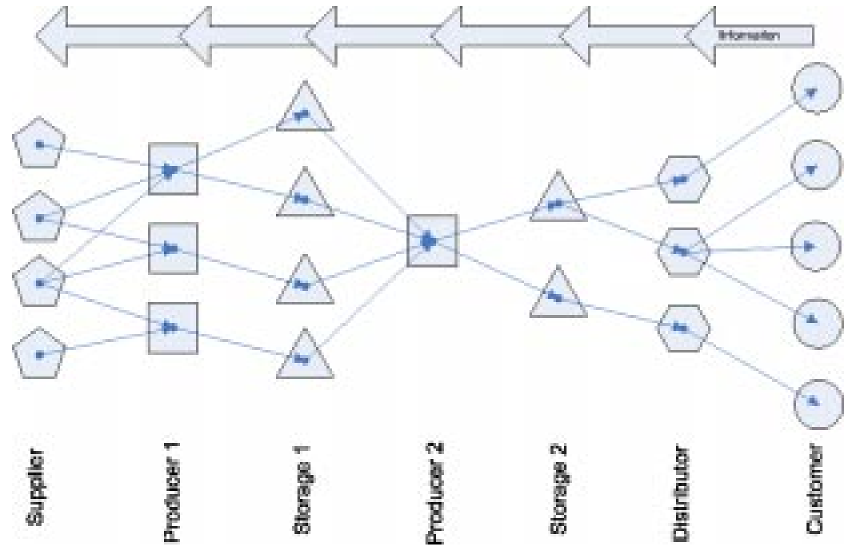


Figure 1.1: General Supply network

- Tactical planning: The middle-term decisions regarding production allocation among existing plants and the distribution in-between.
- Strategic planning: The long-term decisions regarding facilities location and choice of distribution channels.

In the following, the focus will be on supply chain models with a tactical and strategic perspective. The detailed production, storage and distribution plans, i.e. individual production orders, replenishment points and vehicle routes are not treated. The focus is on finding an overall master plan; the allocation of aggregated products to the individual regions and identification of the overall distribution quantities between the regions.

Vidal and Goetschalckx [17] distinguish between domestic supply chains and international supply chains. The domestic supply chain problem considers companies that produce and distribute within one country; where the international supply chain problem considers companies with facilities in several nations. The thesis deals with the international production-distribution problem with the intra-organisational network at Novozymes as a specific case.

1.2 Report structure

The report consists of five main parts:

1. Definition of the problem and objectives of the thesis. Presentation of the case situation.
2. A theoretical overview of the operations management and operations research areas, with relevance to the case, is presented. The operations management strategies are presented in order to identify the relationship between the organisation of the company and the planning task. The aim is to identify the relationship between the management process and the optimisation model. The operations research literature focuses on models for tactical and strategic supply chain optimisation.
3. Development of the mathematical optimisation model. A generic supply network model for global multi process production is developed. The generic model is afterwards adjusted to be able to handle the available data in the supply network at Novozymes.
4. Scenario (case) analysis. Different scenarios are analysed to evaluate the economical potential of using optimisation in the logistics planning process and to evaluate the influence of different parameters in international production distribution networks. Based on case specific data, the optimal and historical solutions are compared and evaluated. The results will be analysed to uncover the most promising future business strategies under the models assumptions and approximations.
5. Future perspectives. Perspectives on further model development perspectives from a general theoretical point of view are presented. Furthermore the future steps for development and a successful implementation of mathematical models with respect to the specific case are discussed.

The appendix contains an overview of the figures and tables in the report as well as the used notation and abbreviations. Furthermore, the mathematical formulations together with the verification scenarios and the implemented programmes for the developed models are presented.

The public CD contains the implemented models and the verification scenarios. Documentation of the case specific data and the results are only available on the company specific CD. The contents of the CDs are presented in appendix I (public CD) and appendix J (classified CD).

Chapter 2

Problem

The purpose of this chapter is to identify the nature of the problem. In section 2.1 some aspects of the thesis are identified. This leads to the problem definition for the thesis, presented in section 2.2.

2.1 Field of problem

This project is based on a case specific task at the company Novozymes A/S. With the case specific approach several limitations and external bounds are given and accepted as necessary in order to solve the problem. Seen in a broad perspective the problem is a matter identifying the optimal production and distribution plan seen from an economical perspective. In brief the purpose is to evaluate how to gain the highest profit from the limited resources available. Stated this way it may seem like a trivial problem but the underlying structure is complex. The complexity arises from two reasons; the size of the problem and the lack of obvious correlation between influencing factors.

First; the size of the problem depends on the number of input parameters and variables involved. For a large multinational company this quickly accumulates to a large amount of data; for individuals it is almost impossible to have a good grasp of all parameters. Secondly it is far from evident how the different factors affect each other when trying to solve a problem like this. In other words the foundation for a rational decision making process is missing.

With this problem structure, an attempt to solve the problem starts with limiting the size of the problem in order to get an overview. This leads to the attempt to aggregate parts of the problem, and to leave out sub problems not considered important. The reductions and restrictions should not result in any loss of significant information. In this specific case the restrictions lead to the need for a strategic-tactical viewpoint i.e. products are not considered in full detail and only the intra-organisational supply network is modelled.

2.2 Problem definition

The aim of the project is to evaluate the potential of using optimisation in the supply chain planning. In the context of this task a model of the existing production and distribution network will be developed.

Inputs for the model are sales forecasts regarding product type, amount and delivery time which are considered deterministic. Based on the forecasts the model should be able to:

- Maximize the net profit considering the production, storage and distribution costs and financial parameters.
- Function as a tool for the optimisation of the production and distribution on a tactical level, i.e. identify the allocation of production to facilities and distribution between facilities and markets.
- Uncover potentials for cost reductions in an optimal solution compared to the existing solution.
- Support sensitivity analysis of key parameters. Thereby help identifying the potential in eliminating bottlenecks.

The model will be used to conduct scenario analyses, testing the potential to gain savings using optimisation tools in the supply network planning and to test the effects of different key parameters in an international network.

The model will be developed with the use of applied mathematics using linear programming and operational research. The production and distribution plan must be optimised simultaneously.

To develop a model with respect to the above-mentioned criteria an understanding of the framework - case specific and theoretical - is necessary. In chapter 4 an overview of the most important case relevant theories within operations and supply chain management as well as operations research is presented. Initially the specific case is described in chapter 3.

Chapter 3

Case Description

The basis for developing a useable model is an understanding of the underlying production and distribution system at Novozymes. To achieve this understanding, a brief description of the case and the involved processes is necessary. In the following section the company and the case is characterised. In the succeeding chapters, the theories with relevance to the case and the solution approach is presented.

3.1 The Company and the Structures

Novozymes is an international group providing biotech-based solutions to industrial problems. They produce and market more than 600 different products for various industries in 130 countries. The most important markets are Europe, North America and Asia. The company is world leader in enzymes and micro organisms with a turnover of DKK 5.803 million and an operating profit margin of 16,9% (Annual report 2003, [42]). They have production facilities in Denmark, USA, China, Brazil, Switzerland and Sweden and several sales offices worldwide.

Novozymes has three areas of business; enzymes, biotech and micro organisms. By far the greatest part of the turn over, approximately 95%, originates from the enzyme business, whereas the biotech and micro organisms are seen as future areas of growth [42]. Based on the distribution of the turn over the enzyme business will be the area of focus for this case study. The supply network described is therefore only a representation of the enzyme production and distribution. As the range of the product portfolio is considerable, only a fraction of the products are actually implemented in the network. This is done to ensure an appropriate extent of the project, compared with the purpose of it.

It is the aim of this project to evaluate the potential benefits gained from an optimization of the production and distribution plans of the company. Currently, the master production and distribution plans are generated with a system developed internally in the company. The system is based on the use of databases in the ERP system and Microsoft Access combined with calculations in spreadsheets. As the project only considers the optimization in the framework of the existing facilities, only the intra organisational supply network is modelled. This means that no suppliers initiate a flow of products into the system and no flow of products beyond regional sales areas will take place.

The intra-organisational supply network can be described with the production facilities, storage facilities and customer regions (sales facilities).

Production facilities for the enzymes are located in five countries: Denmark, USA, China as well as Brazil and Switzerland. For each production facility there is a primary storage, where finished products are stored before they are sent to costumers. Sales take place either directly from the primary storage facility in the producing regions or via a secondary storage facility in the sales regions. The market is divided in approximately 12 sales regions covering approximately 130 countries. Among all sales regions world wide, the main sales regions are the EU, USA and Asia (see annual report [42] for details). Figure 3.1 illustrates a simplified overview of the worldwide production and storage facilities.

3.1.1 Enzyme Production

Enzymes are protein molecules that are found naturally in living organisms and used as catalysts in chemical reactions. The industrial production of enzymes can be divided into four process steps: fermentation, recovery, formulation and a final blending (see figure 3.2). The process steps differ from product to product and between production facilities, however a generalised overview of the process steps may be described as follows (Source: The Novozymes homepage, [43]):

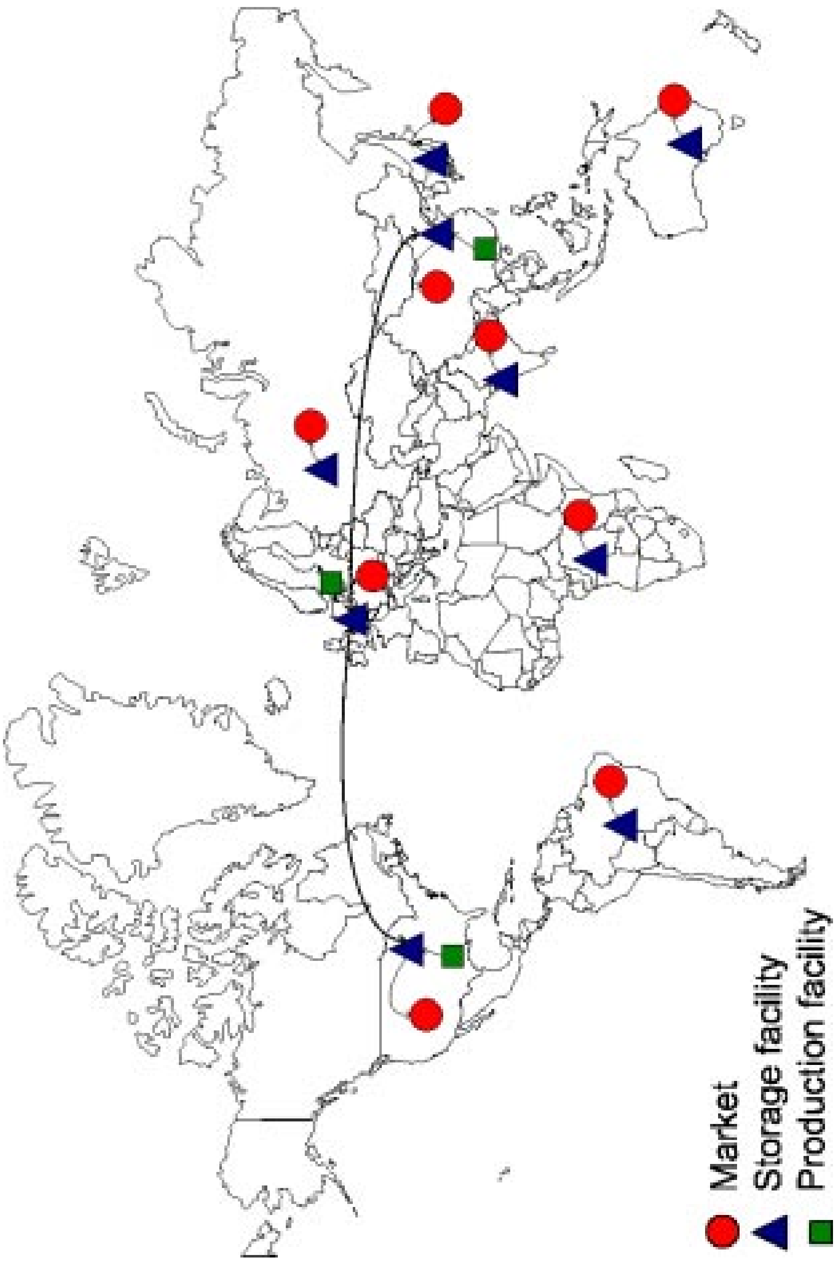


Figure 3.1: The worldwide production, storage and distribution network (simplified).

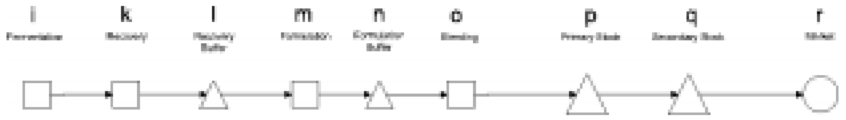


Figure 3.2: Manufacturing steps in enzyme production.

1. The fermentation process takes place in large tanks where micro organisms are grown to produce enzymes. The fermentation takes between 1-30 days.
2. Within 24 hours after the fermentation process the enzyme will go through the recovery process. Here the enzymes will be filtered or centrifuged to separate these from the unused nutrients and micro organisms. Then the enzymes are concentrated by evaporation or through the use of different membranes. The enzyme (concentrate) can now be stored in tanks until further processing.
3. The concentrate is transformed into either solid form (granulation) as granulates sprayed on particles or stabilised in liquid form (liquid standardisation) through the formulation process.
4. Finally the formulated products can now be mixed and a greater range of final products (blends) are produced in the blending phase.

Throughout the production steps both a convergent and a divergent product flow occur in the logistic network. For some of the products, it is possible to make several different enzymes from one fermentation batch; for other products it is possible to blend different enzymes together to one final product. The amount of product numbers rise through the production phases, from approximately 100 to approximately 1400. (The sum of product numbers is greater than the actual number of products as some products differ slightly according to end user specifications. For example extra analysis can be carried out in some instances, while in other cases this may not be necessary, but this creates new product numbers on the basis of the same products).

All products are made to stock based on forecasts. These points characterise Novozymes as an A-company with elements of V-type product flow (the company types are presented later, see section 4.2 for a description of the classifications).

The fermentation process takes place in large tanks. Due to high costs for production equipment and to achieve economies of scale, full tanks are always used when the components are fermented. This means that production always takes place in predefined batch sizes, depending on the facility. If the demand differs from the produced batch, the surplus production is stored.

3.1.2 Storage Facilities

The product flow between the process steps is controlled through the use of buffer storages. Buffer facilities exist after the recovery and the formulation process, whereas no buffer after the fermentation process is considered due to the need for completing enzyme recovery within 24 hours after fermentation. For finished products, there is a primary storage for each production facility where the final product is stored. The blending of enzymes takes place at the primary storage facility. From the primary stock, the finished enzyme will be sold to the customer, either directly in regions with production facilities or via a secondary storage in the regions without production facilities.

3.1.3 Distribution

Enzymes are generally distributed in containers (with big bags or 1000L IBCs) or in reefer tanks. Big bags are - as the name indicates - large bags for handling granulates (enzymes in solid form), IBCs and reefer tanks are large tanks for handling liquid products (See figure 3.3). Other types of packaging are used, however these are the most common for the distribution between facilities.

It is possible to distribute semi-finished products between some process steps, i.e. to send concentrates from one production facility to another for formulation. Furthermore, shipments of formulated and blended products take place. The shipments are done either by truck (e.g. within the EU) or by ship (e.g. between regions). The fiscal flow from these intermediate shipments within the intra-organisational network is regulated by transfer prices.

In this project, a simplified description of the system with production and storage facilities combined through distribution is used. For that reason

Figure 3.3: Packaging used for enzyme distribution.(IBC and Big bag)

only the intra-organisational supply network within the Novozymes organisation is considered. This means that individual customers as well as suppliers are not described.

3.1.4 Economical Aspects

This project focuses on the potential in an optimisation of the production and distribution plan from an economical point of view. Therefore, not only the flow of products between the production facilities, storages and sales regions should be considered but also the financial flow.

The economical aspects are closely related with the physical flow, i.e. all the regions create a turn over from their sales within their own sales areas but also from internal sales to other Novozymes facilities in other regions. The production, distribution and storage of products also have a cost. Furthermore there are some international aspects related with the movement of products between different regions, i.e. import/export duties and exchange rates. To balance expenses to research and development (R&D), royalties are paid from some regions to others. Also a cost in the form of the assets tied in products being stored or shipped should be taken into consideration.

The essence from the above description is the duality in the business case with the physical product flow and the interconnected fiscal flow. This aspect has to be considered in the development of the model for the logistic network.

In the following chapter the theories with relevance for the case are presented. As the optimisation model has to be developed with respect to the specific case, an overview of the operations management theories with specific relevance to the case company are presented.

Chapter 4

The Theoretical World

In order to develop a suitable model based on general elements for the case specific problem an understanding of the organisational structures and planning procedures is necessary. The use of optimisation techniques in production planning will depend on the existing planning system, and an understanding of the management aspect of the network planning is necessary to achieve an implementable model. In the following chapter, the framework and the context in which a mathematical model as considered here functions, is described.

Section 4.1 deals with the rational decision making process linked to complex network structures. Section 4.2 presents some existing planning procedures and tools associated with the company type and describes the conditions where an optimisation tool could be used. Finally the theoretical tendencies are summarised in section 4.3 and a hypothesis is established. In section 4.4 a review of the operations research literature within the tactical-strategic supply chain optimisation is given. The articles are evaluated to identify the most relevant parameters and structures linked to the case. The theoretical considerations lead to the considerations concerning the solution approach for the case presented in section 4.6.

4.1 Complexity vs. Decision Making

Seen in a broad perspective, an interesting question is whether there is a need for strategic-tactical optimisation tools in planning processes. In the following sections this aspect is discussed.

Strategic alliances, growth, mergers, acquisitions and more complex products needing a wider range of suppliers, are all factors prompting the structure of the supply networks to integrate and continually become more complex. The growth influencing the complexity of the supply network takes place for a number of reasons e.g. to ensure a competitive edge, gain access to adequate capital for further investments, maintain a sufficient knowledge base, enabling the development of cutting edge technology, secure market shares, achieve cost savings etc [37], [33], [38]. As the subject of growing complexity in the global supply networks lies outside the boundaries of this project, this trend is accepted as a fact without further examination.

Although there are many advantages leading to the growing complexity, the development also brings along some disadvantages. Rosenfield states,

> For companies to compete in world markets, they will need to manage their networks of production facilities and market channels in an organized way. [28]

However, with the growing complexity, the rational decision making is partially or entirely made impossible. The number of parameters and the following consequences are, in many instances, so numerous, that the human mind has no chance of coping with it. Nor can relative simple traditional management tools as e.g. scenario development on a qualitative basis, decision trees etc. yield answers with good reliability [10].

The problems to be resolved, in order to run the supply network in an optimal manner, do not need to be very large before it becomes too intricate to solve. A key issue which originates in these reflections is: How to manage the complexity and achieve the advantages?

In *The Managers Guide to Supply Chain and Logistics Problem-Solving Tools and Techniques* [35], Hicks presents four approaches for answering this question. The article describes the solution approach from the perspective of four commercial logistics problem-solving tools:

- ERP software

- SCM solutions
- Optimisation tools
- Analysis tools

The Enterprise Resource Planning (ERP) systems are viewed as tools for organising and executing the operation of a company - however with limited problem solving techniques. Supply Chain Management solutions are tools for modelling the supply chain - and especially coordinating the data management for all business processes. The SCM tools are used for the daily planning as an integrated part of the operations management. Optimisation tools are described as software for optimising supply chains through mathematical programming or simulation. Analysis tools are described as stand-alone tools for understanding system dynamics and are not integrated in the management.

The development within these areas has been towards an integration of the above-mentions tools since the publication of the article (October 1997). The strict differentiation between e.g. ERP-, SCM- and optimisation tools is not valid anymore, as modules for SCM-integration and optimisation are available for ERP-system etc. However, the considerations of how the problem-solving tools interact with the operations planning and management within a company are still valid.

The case company uses an ERP-based approach in their planning process. The use of optimisation tools to develop better (rational) solutions is therefore considered interesting under these circumstances.

The optimisation of the logistic network within a company has to be done with respect to the nature of the company, the strategies and the planning and management of the operations. A theoretically optimal solution that does not comply with requirements and structure of the company will probably not result in usable solution for real life. The link between the operations and supply chain management theories and the optimisation theories is therefore important.

In the following chapter 4.2, a brief examination of the theories concerning the possibilities for managing the logistic network is carried out, particularly with focus on the use of optimisation tools in combination with ERP-based solutions.

4.2 Production and Distribution Planning

How to plan for and control the production system depends on the type of company and on the type of controlling order. A common way of categorising manufacturing companies is through their product flow with the classifications A-, T-, V-, X-companies [31]. The characteristics of the different classes are:

A. Convergent product flow. Many subcomponents are transformed into few finished products. Finished products are stored and afterwards shipped to customers on order. The customer order decoupling point (CODP) is located at the storage for finished products. The production is therefore initiated by the storage level i.e. it is controlled by stock orders.

T. Subcomponents are transformed in a similar manner to products. Differentiation occurs late in the production process e.g. when packaging, labelling etc. The customer order decoupling point is located at the storage point before the final transformation. In this way the last production step is initiated by a customer order at this stage, while the production before this point is controlled by the stock order.

V. Divergent product flow. Few subcomponents are transformed into many different finished products according to customer specifications. The customer order decoupling point is located at the storage for raw materials. The production is initiated by the customer order.

X. Combination of A- and V-type. From a number of subcomponents relative few different modules are produced and stored. These modules are assembled into finished products according to customer orders. The customer order decoupling point is at the storage for product modules. As the CODP has been moved back to the module stock this production is a combination of customer orders and stock orders according to which level of the production is considered.

The convergent production flow and the proactive production to stock characterises the case company mainly as an A-company, however with elements of V-flow (see chapter 3 for a description of the production flow). For the different categories different production control approaches are used. For A-companies a common way of managing the production planning is by the use of the Material Requirement Planning (MRP) -systems. The case company uses ERP-systems in the production planning process; therefore

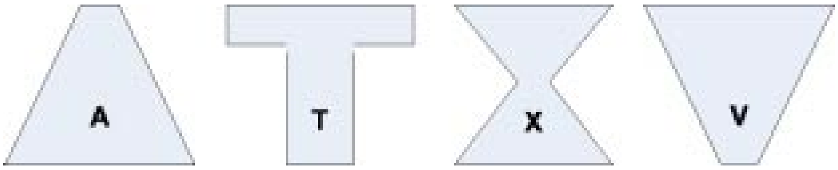


Figure 4.1: Company types

MRP- and ERP-systems are the context for the following section.

MRP- and ERP-systems

The MRP-systems were originally developed focusing on A-companies although the principles for MRP can be adapted and implemented for other types of companies. The systems are *not* used for optimising the production plan, but are used for controlling manufacturing to ensure a feasible production plan. The following description of the MRP-systems is based on [38], [41] and [31], chapter 3.

The MRP systems are traditionally hierarchical planning systems with three vertical levels and two horizontal dimensions. Through the vertical levels the planning process is performed in greater and greater detail. The three general steps are; the first level is the master planning, second level is the material- and capacity plan and the third level is the production activity and purchase plan. Throughout the planning steps, the planned material requirements are coordinated in the horisontal dimension with the available capacity to ensure a feasible solution. For each step the planning horizon is shortened and the details are increased. The steps are as follows (See figure 4.2).

First a master plan is developed from the sales forecasts. The master plan consists of an aggregate production plan, a corresponding resource requirements plan, a master production schedule and a corresponding rough cut capacity plan. The master planning is an iterative process involving the aggregate production plan and the resource requirements plan. This is done to ensure the adequacy of both materials and capacities. The same iterative procedure takes place when the aggregate production plan is detailed, transforming it to the master production plan, which is checked with the rough cut capacity plan.

With the master production schedule and rough cut capacity plan at hand further detailed plans, the material requirements plan and the capacity requirements plan, are developed. These plans are developed from the calculation of gross and the net needs with information from the bill of materials (BoMs).

At the third level; production activity control and vendor management is processed. This means that the need for in house production and purchasing from outside is planned. The results are a shop floor schedule and purchase orders.

It should be noted that, at the second level of planning, there may be included a distribution plan as a third horizontal level covering the available distribution capacity and the best way to employ this. The same applies for the third level where the distribution plan is detailed and converted into an actual plan for the transportation of raw material, semi finished goods, and finished products. These features are only included in the more advanced planning systems [41].

The MRP-systems have been further developed and with features like online continual data collection and analyses and the integration of the MRP with other functions like finance, purchasing, R&D etc. under the name Enterprise Resource Planning (ERP). As earlier mentioned the systems are not optimisation tools; their force however is the management and organisation of data for the planning processes. The systems consider the production planning at the individual facilities; transactional aspects between the regions are however traditionally not considered.

For ERP-based multi-facility planning three points have particular influence on quality of the plans. Many other data aspects and calculations of course also have an effect on the result, but these three points are viewed as the most general and significant sources of error.

1. The quality of the input (forecasts): Obviously the systems results are no better than the input provided. A faulty forecast, will most likely lead to an incorrect level of production and distribution. This generates unnecessary costs and decreases profits.
2. The quality of the master plan: Even with the right input in the form of precise forecasts, an incorrect solution when creating the master plan will have dire consequences for the solution of the whole ERP system, again generating unnecessary costs.

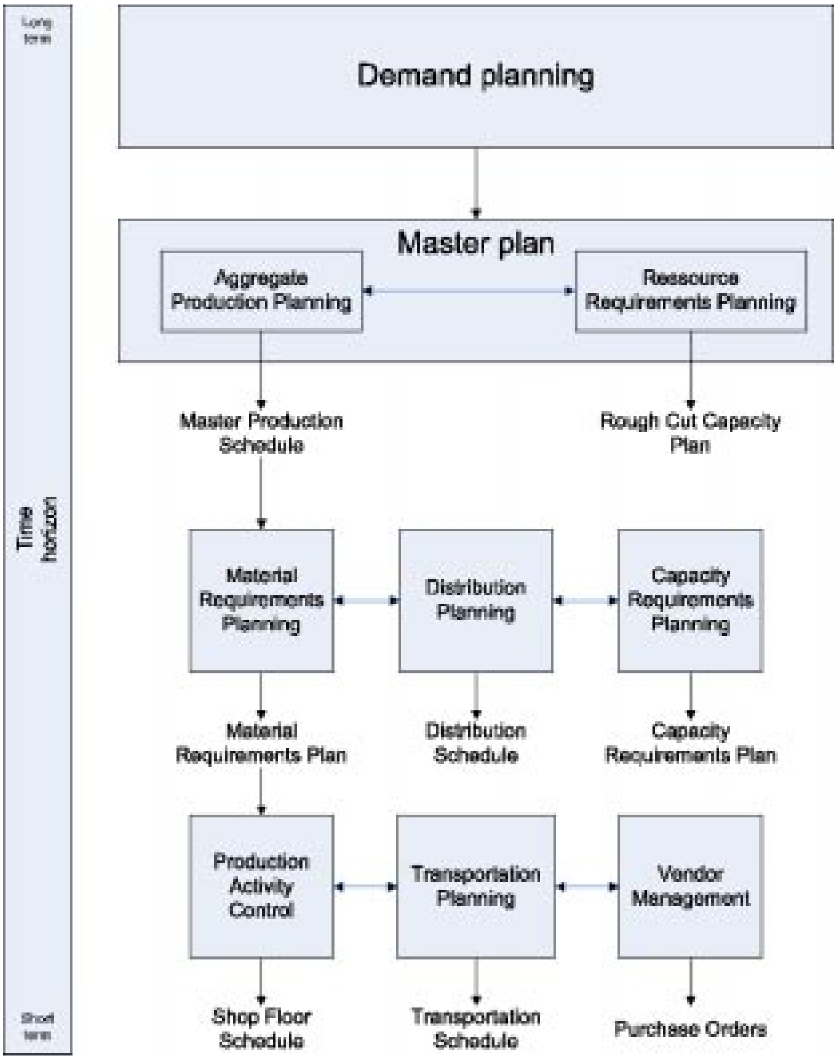


Figure 4.2: Framework for MRP-based Manufacturing Management. (Based on [31] and [41])

3. The lack of integration between production, warehousing and distribution: The lack of integration between all the nodes in the supply network, may lead to a series of optimal solutions for the involved sub-problems. The use of these solutions is sub-optimisation which does not necessarily yield an overall optimal solution, hence the overall profit will not be maximized.

The first point of interest, regarding the quality of the forecast, will not be discussed further in this paper. This point is perceived as a subject with great relevance from a statistical point of view, but in order to restrict the size of the problem the forecast will be treated as deterministic. As the precision in the specific case now has reached a level of approximately 80%, comparing forecasts with realised sales, this is not seen as a problem.

Point 2 and 3 regarding the quality of the solution and the lacking integration are recognized to be of significant importance for the aim of the thesis. In the following, the focus will be on these points, both in the theoretical considerations and in the case work.

Master Planning

The master plan for the global production network defines the overall plan for which products are supposed to be produced in each region within the given time periods, under the given regional capacity constraints.

Several different procedures for finding solutions to create the master plan exist. Examples of solution techniques are presented by Michelsen in [31], chapter 3, and are divided into four categories:

- trial and error
- heuristics
- graphical methods
- analytical methods

These procedures span from the primitive way of searching for feasible solutions without any formalised procedures (trial and error) to the more sophisticated ways using formalised heuristics or algorithms to analytical methods using mathematical programming (linear, non-linear, integer, mixed integer etc.).

Allthough experienced employees can achieve very good results using primitive procedures, an optimal solution can not be ensured. This is particularly the case for complex global networks where several factors influence the decision process. The degree of optimality and the reliability of the results, increase considerably when using the more sophisticated tools.

If the problem can be described and solved as a linear program (LP) an optimal solution to the model can be guaranteed. If the model reflects the real life situation sufficiently a very good solution for the real life problem will be achievable.

The drawbacks of using the more sophisticated tools are the requirements for the model formulation and the data input. The model formulation should represent real life factors to ensure good solutions. The availability and in particular the maintenance of this often large amount of data is a major disadvantage for using mathematical models. These drawbacks are often the main motivation to use the more simple tools as heuristics or even trial and error techniques.

For behaviourists who believe in a rational-decision making process the use of mathematical models in the global master planning seems like on obvious choice. However, historical reasons may also be part of the explanation for the lack of these tools. In many businesses there is no tradition of using optimisation tools and the absence often leads to a lacking understanding of the possibilities and can therefore be a great barrier for implementation of these tools.

4.3 Hypothesis and Antithesis

The following has been established above:

- There is a growing complexity in the supply networks.
- There is a need to control the production and distribution to obtain a high degree of effectiveness and efficiency to maintain competitiveness.
- The decoupling of production and distribution planning may lead to a sub-optimization.
- Many tools from OR have been developed in order to solve several different types of problems in production and distribution planning and optimization, but no general conclusions regarding the use of OR can be made.

The intriguing question to be resolved in the perspective of the above mentioned four points is the cost-effectiveness in investing in the development of better planning and optimization tools.

It is a task of massive dimension to either prove or disprove the general hypothesis proclaiming great savings from investments in better tools for developing master plans in an aggregated form also containing distribution. As this task is too overwhelming it is instead chosen to try and find an indicator for this question by examining a specific case.

For the case at Novozymes A/S the case of evaluating the use of mathematical programming in the planning process is considered interesting. The optimisation model in this report should therefore be seen as a part of the hierarchical planning process. The combined production and distribution planning on the strategic-tactical level deals with the planning on the level of master planning or network planning. The optimal solution to the production-distribution problem gives the input for the operational planning of the production at the individual facilities and the distribution between them.

The **hypothesis** of this case study is: It is possible to gain substantial improvements in the profit from investments in the development of planning and optimization tools at Novozymes.

The **anti thesis** is: No improvements in the profit can be gained from investments in the development of planning and optimization tools at Novozymes.

The combined production and distribution planning in multi-facility networks has been treated with several approaches and by several researchers. In the following section 4.4, literature on the use of mathematical models for strategic-tactical supply network planning is reviewed in order to identify problems similar to the case.

4.4 Literature Review

The combined production and distribution problem has been treated by several researchers, however with different perspectives, degrees of generality and models. There are several ways of describing these models. One way is the size of the problem; some models consider only small-scale problems with few products and few echelons while other considers large-scale industrial problems. Other parameters are stochastic/dynamic aspects of demand forecasts, static or dynamic models etc. In some cases international aspects are considered, e.g. exchange rates, transfer prices and taxes, but most cases only deal with domestic production-distribution problems.

The literature examined for this project has been sorted according to research area and relevance and is presented in the reference list. A-publications are literature on mathematical modelling, with a high degree of resemblance to the case studied. B-publications are material on mathematical modelling with some resemblance and relevance to this project, while C-publications are texts on mathematical modelling with remote or no resemblance to the case. D-publications are literature on supply chain management theories and E-publications are case specific sources.

The following literature review focuses on the A-publications, with an emphasis on international models, as these are seen as the most relevant for this case study. These publications are further evaluated according to specific description parameters in the case compared to the published materials. The parameters chosen for this evaluation are multi nationality, number of time periods, demand characteristic, application level, characteristic of the objective function, number of facilities and the examination of production and distribution issues.

An overview of the articles and the description parameters is presented in table 4.1. The characteristic elements for this specific case are marked with **bold**. The degree of similarities between the case and the reviewed literature is found as the number of similar description parameters (presented in table 4.2). At end of the section five publications reviewing other articles of relevance are inspected. A more detailed description is presented below. The order of the publications is settled according to the degree of similarities, the most similar and therefore relevant material is examined first:

Cohen and Lee present a mixed integer non-linear program for resource deployment in a global manufacturing and distribution network in [5]. The objective is optimisation of the total global after tax profit with respect to purchasing, manufacturing and distribution as well as fixed cost for facilities and vendors. The model is used for evaluating global manufacturing strategies, defined by the plant charter strategy, the supply strategy and the distribution strategy. Results are presented from a case study for a global computer manufacturer, where different scenarios are tested over a five year period.

	Article						
	[6]	[13]	[16]	[8]	[1]	[18]	[11]
Domestic	x		x		x		x
International		x		x		x	
Single time	x					x	
Multiple time		x	x	x	x		x
Single product							
Multiple products	x	x	x	x	x	x	x
Deterministic demand	x	x	x	x	x	x	x
Stochastic demand							
Operational level					x		
Tactical level	x		x		x		x
Strategic level	x	x	x	x	x	x	x
Max Profit	x	x	x			x	
Min Cost				x	x		x
Multiple facility	x	x	x	x	x	x	x
Single facility							
Production	x	x		x		x	x
Distribution	x	x	x	x		x	x
	[14]	[7]	[15]	[9]	[3]	[5]	
Domestic		x	x	x	x		
International	x					x	
Single time		x	x				
Multiple time	x			x	x	x	
Single product							
Multiple products	x	x	x	x	x	x	
Deterministic demand	x	x	x	x	x	x	
Stochastic demand							
Operational level		x					
Tactical level		x	x	x	x		
Strategic level	x	x	x		x	x	
Max Profit			x			x	
Min Cost	x	x		x	x		
Multiple facility	x	x	x	x	x	x	
Single facility							
Production	x	x	x	x	x	x	
Distribution	x	x	x	x	x	x	

Table 4.1: Description parameters for the A-articles

Resemblance Level	Article						
	[6]	[13]	[16]	[8]	[1]	[18]	[11]
	8	9	8	8	6	8	8
	[14]	[7]	[15]	[9]	[3]	[5]	
	8	7	8	7	8	9	

Table 4.2: Number of similar parameters between the case and the article

Martel [13] develops a MIP-model for international production-distribution networks for converging products. The influence of minimizing costs or maximizing profit is considered as well as the planning horizon and model structure. The model considers facility location, inventory levels, taxes and transportation prices together with production cost and capacity constraints. Transportation costs are allocated to the sending unit. The transfer prices are assumed fixed. A solution on a large-scale problem is found with a heuristic approach; however no global optimum is sought.

Chen and Wang [6] develop a linear model for optimising the combined production and distribution planning in a Canadian steel company. The problem is based on a case and considers raw material and semi-finished product purchase, capacity allocation, customer demand and distribution. The objective is to maximize the total net profit considering domestic aspects. With an example problem, a sensitivity analysis is performed on costs of materials and transportation as well as fixed and variable costs. Furthermore the selling prices are evaluated. The sensitivity analysis is only based on individual, not combined, impacts and show that selling prices, fixed and variable costs have high influence on the total profit. The model has been used for a real life problem in the steel company, however no results are presented.

A LP-model for solving a case problem is developed by Smith et al. [16] for solving a large scale transportation and distribution problem at Delta and Pine Land Company. The model considers a domestic problem, maximizing the profit considering distribution and storage costs as well as lost sales influence. The model has been used to solve an industrial problem, however no results are provided.

Dhaenens-Flipo and Finke [8] consider a multi-facility, multi-product and multi-period industrial problem. They formulate a MIP-model minimizing the costs. At first a single period model is developed, considering production, switching and transportation costs. For the single period model warehouses are only considered as trans-shipment points and therefore no costs occur.

A model for optimising an international problem is considered by Vidal and Goetschalckx [18]. The model assumes transfer prices to be variable within an upper and lower bound. The model determines the optimal transfer prices within a time period. Furthermore the allocation of transportation costs between sending and receiving units is modelled as a decision parameter. The model optimises the after tax profit with respect to the non-linear effects of taxation and import duties as well as costs for transportation, production, inventory storage and fixed costs. The solution of a large-scale problem with a heuristic iterative LP-procedure is found by successively fixing one set of variables searching for a local optimum.

Glover et al. [11] describe a very comprehensive model, optimising production and distribution over a finite time horizon split in discrete intervals. The framework of the model comprises both short and long term decisions regarding production, distribution, acquiring and localising facilities at the company Agrico. This production, distribution and inventory (PDI) model, consists of a network involving mixed integer linear problems as well as linear problems. It was solved via an ARCNET code (Analysis Research and Computation Inc.), and the quick solution meant the model could be used as a tool in trying out different scenarios for demand etc. Considerable savings were obtained by Agrico, when implementing this model.

Some possible effects of inflation and exchange rates affecting the performance of the facilities of multinational companies are studied by Zubair [14]. The objective of the paper is to solve the problem where to produce what and when, under different scenarios with differing exchange rates and inflation rates. In this model the total turnover is considered constant as the price is constant and the demand has to be satisfied at all times. Therefore the profit is maximised by minimising costs. The solutions of the different scenarios yield an insight into the most important factors such as cost and capacities versus changes in exchange rates.

Philpott and Everett [15] develop a MIP-model for determining the optimal allocation of supplier to mill, product to machine and machine to customer. The model has been implemented in a paper mill cooperation and has been used for modelling strategic and tactical decision problems. The objective function maximizes the profit considering sales, procurement and process costs as well as fixed machine costs and annual investments. The model is case specific and considers domestic aspects only.

An industrial implementation of a linear program is presented by Brown et al. [3]. The model minimises long-term costs for production, inventory and distribution. Overtime is modelled through elastic goal constraints. Thereby capacity constraints can be violated by adding additional penalty costs. Furthermore, penalties for not meeting demand are modelled. The model is used in the Kellogg company for tactically and strategically decision support and the main focus of the article is on describing the interaction between the real-life decisions and the model results.

A multi-facility multi-product production system is the subject of the paper [7] by Dhaenens-Flipo. A decomposition is considered to handle the size of the problem. The model contains an integer linear problem closely related to the vehicle routing problem (VRP). This model is decomposed into smaller sub models for the lower levels of the multi facility enterprise. These smaller sub problems are solved using either CPLEX or a branch and bound algorithm. The algorithm turns out to produce the fastest solution times. Several advantages using the decomposition are revealed; better results are obtained compared to solving the large problem centrally. On the other hand, decomposing the problem is time consuming and the sum of the solution times for the sub problems is considerable longer than the solution time for the central problem.

Dogan and Goetschalckx [9] develop a MIP-model for determination of tactical production-distribution allocations for a multi-season problem. The model minimises the total costs for a domestic supply chain and include costs for supply, manufacturing, storage and distribution. The binary variables are used for modelling opened or closed status for facilities. A solution is found through primal decomposition, where the problem is divided into two sub-problems: a resource location and sizing problem and a transportation flow problem. The model is used on a case study with seasonal customer demand.

The integration of optimisation via a genetic algorithm and the collaborative planning and local planning at facilities is the subject of an article by Berning et al. [1]. A network of chemical multi purpose production plants manufacturing different products through multistage processes at Bayer AG is used as case. The result using the algorithm can be accessed by all the involved facilities allowing distributed decision making and an optimal information flow. This is accomplished through the use of models specific for the individual facilities communicating with a master model. In this way the paper describes a possibility for integrating long term and short term planning and the collaborative versus the individual plans.

The following articles ([17], [12], [4], [2] and [10]) present literature reviews on the production and distribution optimisation:

In [17] Vidal and Goetschaclkx evaluate the literature on strategic production distribution models. The main focus is on mixed integer programming models dealing with global and domestic supply chains respectively. The different models are compared on various description parameters, i.e. the objective function, linearity, fixed costs, capacity constraints, international features, solution methods etc. Finally, topics where further research is desirable are identified.

In [12] Goetschalckx et al. review the literature for global supply chains with emphasis on the impact of transfer prices. Furthermore, two models are introduced: a model focusing on the setting of transfer prices in an international supply chain and a model considering seasonal demands in a domestic supply chain. The article is a resume of the articles [17], [18] and [9].

An overview of the supply chain literature and models is presented by Chen in [4]. In this article, the focus is on models dealing with tactical to operational problems. Explicit production-distribution models are considered; this means that problems that integrate inventory replenishment decision across multiple stages as well as problems that integrate inventory and distribution decisions are not considered. Chen describes the models after their decision level, integration structure and problem parameters, which leads to five model classes:

1. Production-transportation problems
2. Joint Lot Sizing and Finishing Product Delivery Problems
3. Joint Raw Material Delivery and Lot Sizing Problems
4. General Tactical Production-Distribution Problems
5. Joint Job Processing and Finished Job delivery Problems

Finally, five directions for further research are presented. Bhatnager and Chandra in [2] review a broader aspect of supply chain models. They divide them into three categories:

1. Supply and production planning
2. Production and distribution planning
3. Inventory and distribution planning.

Furthermore the subjects of nervousness in the supply chain and lot sizing and safety stock issues are discussed with references to other articles. This discussion gives a good overview of the literature dealing with coordination on a very general level i.e. coordination between different functions as location and distribution planning as well as coordination inside the company between different plants.

Literature on network models in both strategic, operational and financial perspectives are treated by Geunes and Pardalos in [10]. The article starts with considerations concerning the motivation for exploring the possibilities in supply chain and financial optimization. The main reason is the potential to obtain savings considering the progress in hardware, software and solver techniques which have been developed. Models regarding strategic and operational perspectives are documented as well as financial models. Also the growing fusion between models in the fields of strategic or operational perspective with financial models is treated.

4.5 Literature Remarks

From the examination above no general conclusions regarding the use of OR in the production and distribution planning and optimisation can be made. This is in part due to the case specific aspects of all the papers and in part due to the fact, that different parameters and objectives are examined in each paper.

Although no general conclusions can be made based on this literature study, it is seen from the table 4.1 that all the studied A-articles deal with multiple products produced in a multiple facility set-up under deterministic forecast assumptions. Also common for all most all of these publications is the consideration of both production and distribution issues. There are some variations in the different models level of application, but generally speaking the chosen literature deals with the optimization issues on a strategic and semi-tactical level. None of the articles deal with real intra-organizational supply chain as no financial transactions between organisations are considered.

Several parameters from production, distribution and financial perspectives influence the optimisation of an international supply network. These are factors like production-, distribution and storage costs but also royalties, duties, transfer pricing or taxes. Several articles focus on mixed integer programs and only incorporate a fraction of these parameters. No general thoughts are made on which factors have the highest impact on the optimal solution - which parameters are essential and which are excludable.

Most models apparently concentrate on a domestic setting, not considering parameters arising from international production, distribution and sales. This indicates that a relative small part of the research focus specifically on subjects like transfer pricing, duties, exchange rates and taxes. Furthermore most of the global modelling work concentrates on minimizing costs not maximizing profit. Only three publications consider a global environment and works with maximizing the profit. Focus on cost minimizations prevents the correct implementation of several global aspects like taxes, duties, exchange rates etc. which can have a huge financial effect on the profitability. These aspects are not treated under a cost minimization perspective.

Apparently research in the use of mathematical modelling with a global perspective of after tax profit optimization is lacking. Furthermore the parameters influencing the global production allocation are not discussed and investigated thoroughly. This case study is an attempt to focus on the effect of implementing a truly global perspective in the optimisation of production and distribution on a strategic level.

4.6 Methodology consideration

4.6.1 Approach

The models in the literature consider different aspects. Some treat the production-distribution problem with a generic approach, while others treat the problem with a case specific approach. For this report, the problem is case specific, as the aim is to evaluate the potential of optimizing the production and distribution planning at Novozymes A/S. To build a model of a generic supply chain structure with an unspecified number of suppliers, production steps and storage facilities as well as regions is not relevant for this project. The model should therefore be based on the structure of the logistic chain at Novozymes and reflect the real-life production and distribution planning. The elements of the model though should be of a general character to make the model compatible with other similar cases.

The model should suit the current operations and supply net management, however the current planning method should not limit the quality of the model if avoidable and an automation of the current method may not be the optimal solution. It is well known, that automating existing processes is not the best solution when implementing it-based solutions. Hammer i.e. writes *Use computers to redesign - not just automate - existing business processes* [32].

With this in mind, the approach for developing a mathematical model is to develop a general model of the real supply chain. This model should give the best understanding of the solution and the highest precision on decision parameters - furthermore the generic approach helps to identify the ideal requirements to data structures. The model is also useful for identifying future needs to be able to develop better models. Since the project is case specific the general model may need some adjustments to be useful in the company.

To achieve a high degree of transparency in the results for the company, the measures and units used in the current planning procedures is found to the best way of communicating the results. This ensures that employees in the company can understand the solutions given by the model and evaluate them with respect to the current solutions.

Another aspect is the data availability. The best solution is to collect all the necessary data for the general model. Nonetheless, due to costs and time availability, adjustments of the model may be unavoidable. The use of existing data eases the comparison with current solutions, but can result in inaccurate results if the data are based on available data from an MRP-system that do not necessarily reflect reality.

4.6.2 Model Type

The supply chain at Novozymes, the manufacturing processes and use of standard software to solve the problem influence the choice of model type.

The manufacturing of enzymes is a batch production with pre-defined batch sizes, i.e. fermentation takes place in tanks and distribution in containers.

Production will always take place in full tanks and possible solutions are therefore of integer values, when planning how many tanks to fill. For distribution, the cost of distributing a container is independent of the capacity use. Consequently the use of full truck loads and containers is desirable when planning the distribution. The natural approach is therefore to develop a mixed integer model to achieve results as close to reality as possible.

The wish to develop a model of the real life supply chain which is solvable with commercial software indicates the need for a linear model. LP-models are solvable with commercial software - even for large scale problem. Due to the complexity of the problem, involving several production steps in several regions as well as several products, it has been chosen to develop a LP model. Though it is known that it may not be the optimal integer solution, the LP-solution should afterwards be evaluated to see if the rounding of the solution provides a feasible integer solution.

The approach for the model development is a four step approach with two main perspectives (see figure 4.3):

1. Development of a general model
2. Adjustments of the general model to the case specific situation

First a general model is developed from the case study. This mathematical model considers all details and mathematical elements as non-linearities etc. are included. This model is refined to ensure the softwares ability to solve the problem to optimality i.e. the model is linearized and the integer constraints are relaxed. In this way a linear continuous model (the full model) is attained.

In order to test of the wanted features a mini model for test purpose is developed. The complexity of this minimodel is further reduced by diminishing the network structure to an absolute minimum, resulting in a model environment where the results can quickly be verified and validated.

With all the features in place the next step is an implementation of these in the full network. Along with this implementation the last details regarding the form of data being used etc. are also implemented and the final model is checked by running different test problems and verifying the results.

4.6.3 Parameters

When modelling the specific case, the following factors influence the economical flow in the Novozymes supply network:

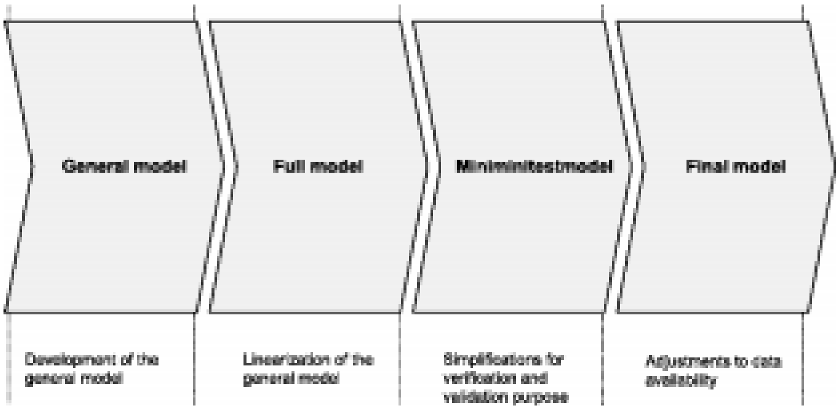


Figure 4.3: Modelling in a four step approach.

- production cost
- distribution cost
- storage cost
- transfer prices
- taxes
- exchange rates
- import duties
- export value added taxes
- royalties
- financial cost

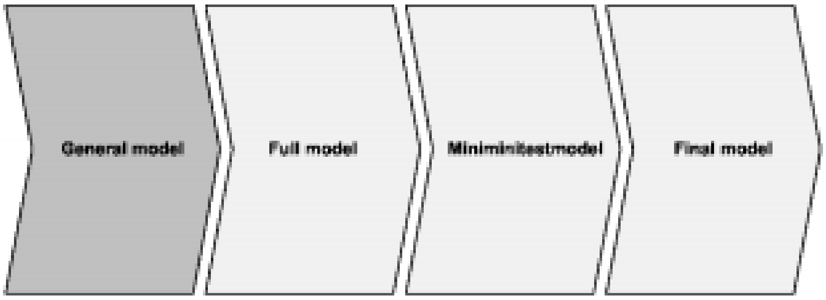
It is difficult to evaluate the influence of these factors for the optimal solution. Therefore, all factors will be incorporated in the model.

Chapter 5

The Model World

Based on the previously described four step approach, the first three steps of the model development are presented in this section. The general model is developed in section 5.1, the adjustments for the full model are presented in section 5.2 and the simplified mini model for verification is presented in section 5.3. The last development step - the final model - is presented in the succeeding chapter 6.

5.1 The General Model



In this section the general, generic model is described. The section is divided into the following subsections: Section 5.1.1 where the network structure and flow is described. Section 5.1.2 focuses on the constraints. In section 5.1.3 the economical flow in the organisation is described. Finally the elements in the objective function are described in section 5.1.4.

All these leads to the formulation of the model in a verbal form (section 5.1.5) which leads to the mathematical formulation (section 5.1.7). The notation used in the formulation is presented in section 5.1.6.

5.1.1 Network Structure and Product Flow

The production and distribution system described in chapter 3 consists of two fundamentally different processes; the non-stationary and the stationary processes. The stationary processes are the production phases, storage phases and the sales or demand phases. The non-stationary processes are all the instances where there is a flow of one or more products between the stationary processes. Using a network approach to describe the production and distribution processes will result in nodes representing the production phases, storage facilities and demand centres, while arcs represent flow of products between these nodes.

Based on this abstract description of the production-distribution system the use of a mathematical network based model seems straightforward (fig. 5.1). Also it should be noted that network structures are considered highly solvable and the visual aspects of the network representation assists in communicating and understanding the problem ([11], p.27).

The structure of the model is based on the actual production and distribution network as previously described (Chapter 3). This is done in order to ensure that the results of the model are meaningful in reality. Thereby it is possible to implement the abstract resource allocation and flow from the production-distribution system in reality and obtain results as predicted by the model. Ideally the results from the model and the application of the models allocation would yield the exact same result. This is of course not to be expected as approximations and exclusion of several factors is necessary to develop a solvable model. The structure though must enable the results to contain such a degree of precision that the model can be used as a decision support tool.

The following description of the network structure and flows is based on the single period network presented in figure 5.1.

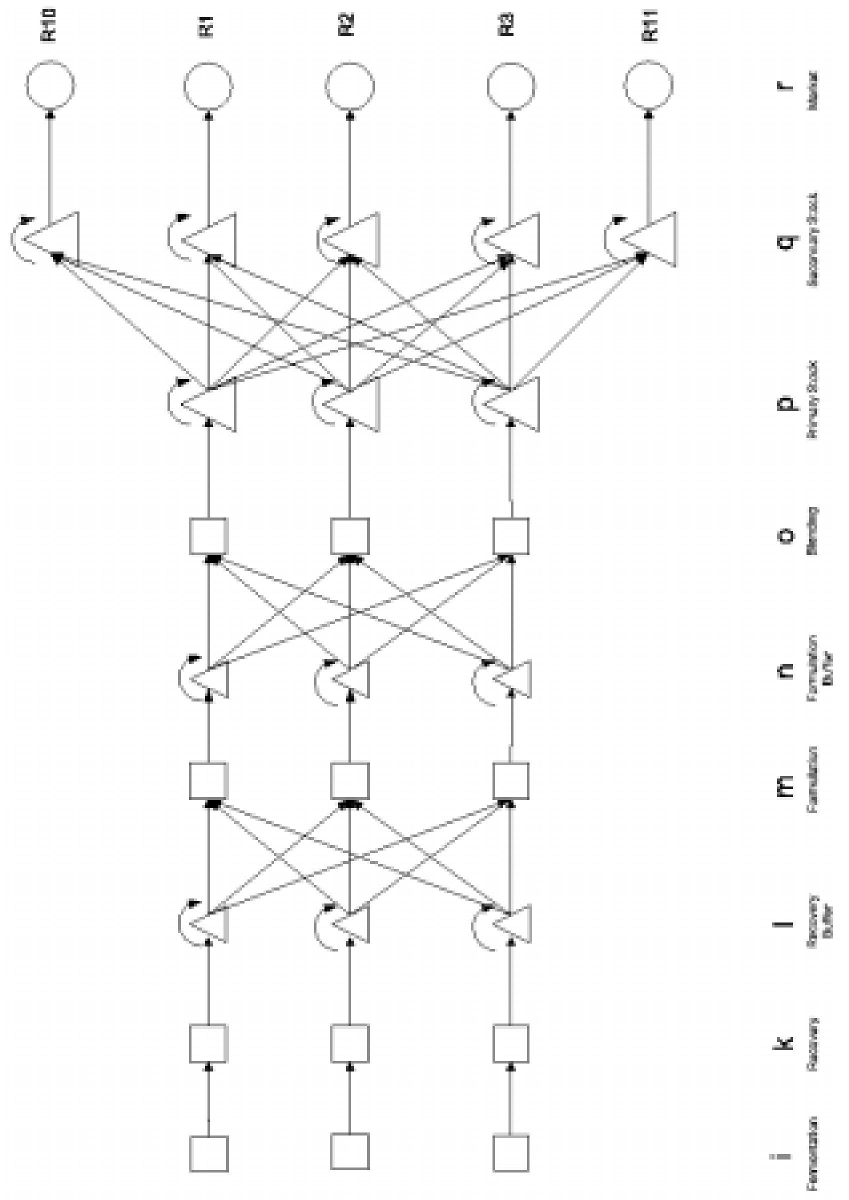


Figure 5.1: Supply network for a single time period. Three producing regions and two secondary regions are illustrated.

To keep the network structure close to reality it is chosen to model the production in the four mentioned steps: fermentation, recovery, formulation and blending, as these processes can be considered general for the whole range of the enzyme products. There are small differences in the production of the individual products, but these are either modelled as exceptions for the specific product or omitted due to insignificances.

Buffer storages after the production steps are modelled, and a transfer of semi-finished products is possible after the recovery buffer and the formulation buffer. The only possible flow from the actual production processes is through the corresponding buffer storage. This means that no direct flows between the regional production phases are allowed; this represents the real life situation.

In regions that do not contain production facilities, a secondary storage exists. Products from the primary storage facilities in the producing regions are shipped to these secondary storages, before being sold to the costumers. In the model it is chosen to model an imaginary secondary storage in the regions with production facilities. In reality these fictive secondary storage facilities are primary storage facilities. However, the use of fictive nodes is done to simplify the modelling of interregional product transfer between two primary storages and to ensure transparency when evaluating the model results.

It is assumed that each individual sales region is served by a single secondary storage. The final distribution from the secondary storage facility to the individual customers is not modelled. No direct shipments from a facility in one region to a customer in another region are allowed. The modelling of more markets will be done by expanding the number of secondary facilities.

The figure (fig.5.1) shows the network for a single time period, where the distribution arcs describe the product flow, given in the units used at Novozymes (L or kg). Only a directed flow from fermentation to the market is allowed. In reality a small amount of reverse flow from storage facilities to production facilities may occur, but due to insignificance and with respect to transparency of the model, this is not allowed. The reverse product flow for rework is an undesirable effect in the production system which should be eliminated.

In figure 5.2 the network structure for a multiple time period model is presented.

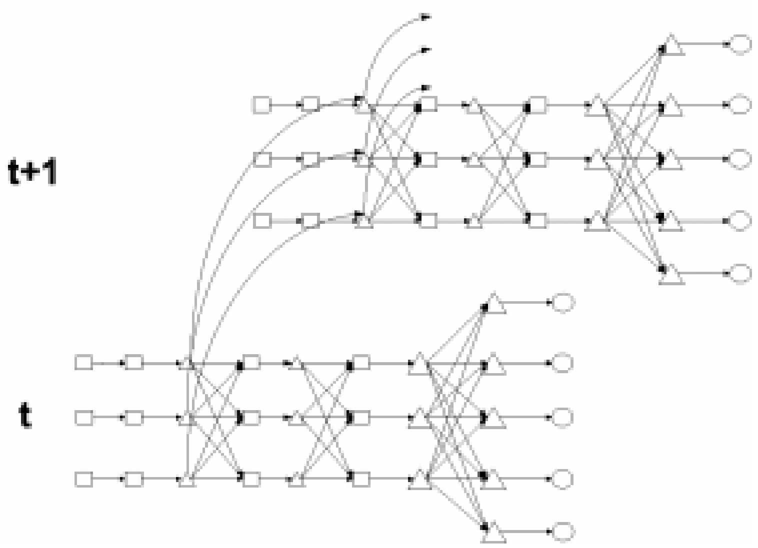


Figure 5.2: Supply network for multiple time periods. The single period models are linked through arcs between the storage nodes.

The model is an expansion of the single time period model, where the time dimension links different single time period models together. It is assumed that the production and distribution lead times are smaller than the length of a time period. Thereby the time steps are only used when modelling storage levels and do not have impact on the production and distribution times. The impact of time periods is illustrated with the arcs combining the individual storage nodes between the different periods.

5.1.2 Constraints

For each production node a bill of materials (BoM) is given. The BoM ensures that the flow out of a node corresponds to the ingoing flow. The conversion between different product units - e.g. L and kg - is implemented in the BoM. Products from earlier periods are not considered.

For each facility (node) a maximum capacity is given. The total amount of products produced within that facility is not allowed to exceed this limit within a given time period. The individual production lines are not modelled, but the overall capacity for the entire facility is used. This requires that every product can use every process line in a given facility. In real life this is not the case, however for rough cut capacity planning considering a long time period, this is assumed reasonable. The more detailed production allocation is left to the lower planning levels as described in chapter 4.2.

Fermentation only takes place in full tanks. This restricts the size of a production to a given batch size which may not correspond to the size of a given demand. The fermented quantity should therefore be restricted to reflect the batch size to achieve a feasible product flow. Furthermore, the available number of fermentation tanks limits the fermentation capacity.

For each storage facility a capacity is given in the unit of products to be stored - corresponding to the size of a facility. It is possible to buy additional storage capacity if necessary however this option is not implemented in the model. This scenario could be modelled through binary decision variables.

No hard capacity restrictions on distribution capacities are included. This corresponds to the real life situation, where the procurement of additional transportation capacity is uncomplicated. To achieve economies of scale, the use of full container loads when distributing is preferable. The distribution of more products within one container may be possible, depending on the packaging type. This option restricts the distribution to take place in batched sizes more or less corresponding to the container capacity.

5.1.3 Fiscal Flow

As mentioned in chapter 2, the aim of the model is to optimise the economical benefits by finding the right resource allocation. The fiscal flow can be divided into three main categories:

- the intra-organisational economical flow between regions, i.e. duties and cost or turnover contributions from product transfer
  and
- the flow from the company to other parties, e.g. production, storage and distribution costs to suppliers, employees etc or duties, taxes and values added taxes to authorities
  together with
- the less tangible cost factors like financial costs.

In figure 5.3 the money flow for a single region within the network is shown. The income contributions for the regions are illustrated with arrows pointing upwards, where cost factors are illustrated with arrows going downwards. The flow within the organisation is illustrated with dotted arrows and the flow across organisational borders with fully-drawn arrows. For royalties and taxes the economical flow between the regions is only indirectly dependent of the product flow. A more detailed description of the individual cost factors is given below, as they are incorporated into the objective function.

5.1.4 The Objective Function

To find the optimal economical production plan, a question is whether the objective should be maximising the profit or minimising the costs. In many article reviewed in chapter 4.4 the focus is on cost minimisation, i.e. [8] or [14]; only the articles [13], [18] and [5] deal with profit maximisation in international networks. However, cost minimisation does not seem sufficient for global supply networks.

Assuming that prices and customer demands are given and should be fulfilled at all times, the turnover will be constant and implicitly given by

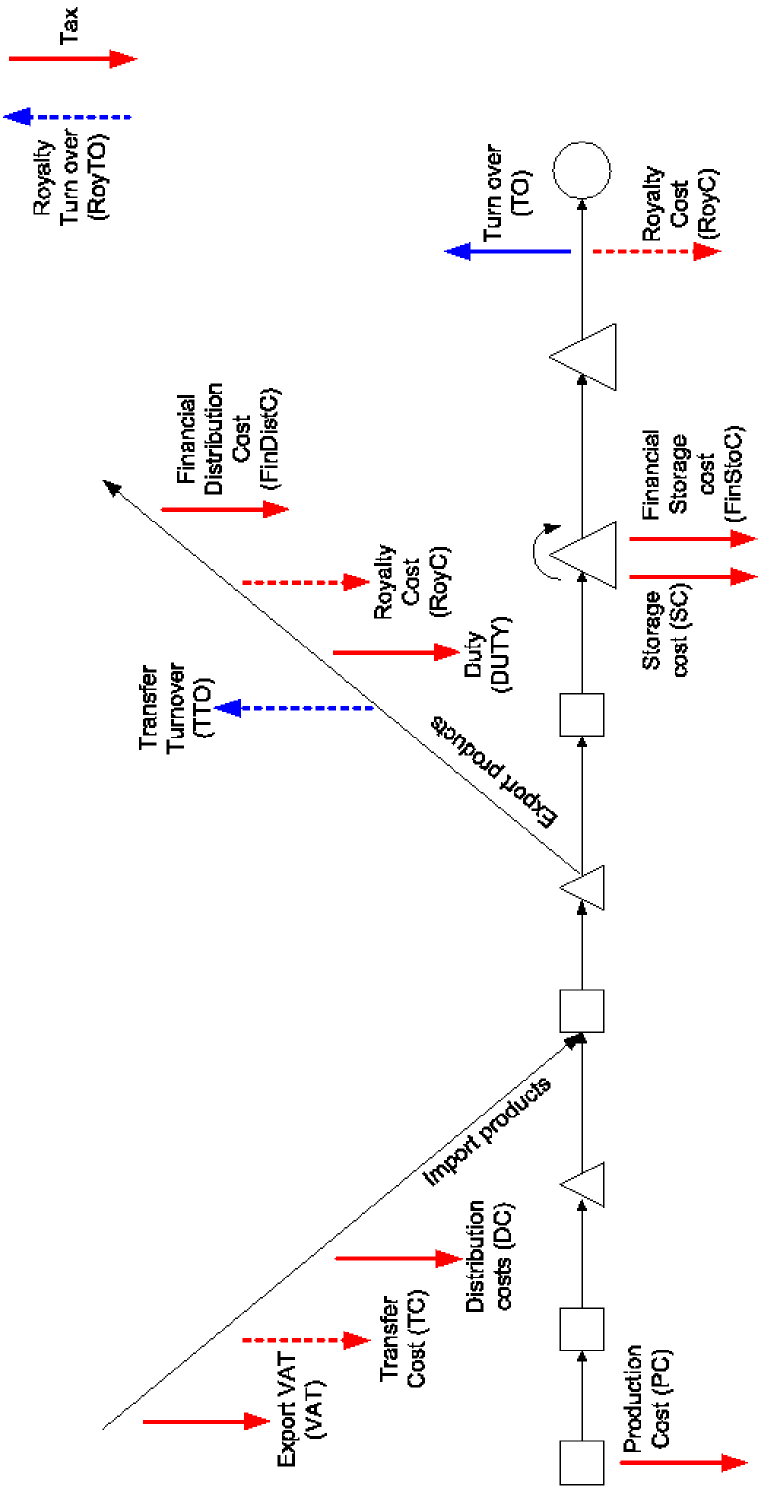


Figure 5.3: Money flow for one region. Cost factors are illustrated through arrows pointing downwards while turnover contributions are illustrated with arrows pointing upwards.

the input to the model. In this case it may seem irrelevant to include the turnover in the objective function and the straightforward conclusion will be that cost minimisation should be the objective. However, when optimising a global network with different taxation levels and exchange rates in different regions as well as duties, royalties and VAT contributions based on sales, the turnover has to be considered. Therefore, profit maximisation is the objective in this model. This leads to the following factors influencing the objective function:

Demand is assumed to be given and deterministic and all demand should be fulfilled on time. There might be short term cost savings when not replenishing all customers on time, and thereby an optimal capacity usage can be achieved. However, with long term customer relations late deliveries are not desirable. Furthermore, the company policy is to provide the required products on time. Therefore backlogging is not considered in the model. The individual customers are not considered, but sales forecasts are considered aggregated within the individual regions.

The *objective* is to maximise the global net profit after tax. The calculation of the net profit is based on the structure in the annual report [42], where the total profit is the sum of the regional profits. In the model, however a simplified calculation is used, only including parameters which are influenced by the results of the model. This means that income only considers sales and i.e. no financial or other special income factors are considered. Furthermore, the costs do not include administrative or fixed costs etc. The costs do include a financial cost on all products being stored and shipped, but all together the result provided by the model will not represent the real net profit.

For each region, a *regional profit* before tax is calculated, considering the income from sales - to customers and as internal transfer to other regions - as well as income from royalties from other regions. The cost factors considered are: *production*, *storage* and *distribution costs*, as well as *value added export taxes*, *duties*, *royalties* paid to other regions and *financial costs* of storing and shipping. A related approach for describing the objective function is used in [5], where the global after tax profit is the sum of the regional profits before tax reduced with an average global tax.

For transfer of intermediate products between facilities in the network, the corresponding turnover for sending facilities as well as costs for receiving facilities are given by the *transfer prices* (TP). The cost for one region (transfer cost, TC) corresponds to the income for the other region (transfer turnover, TTO) when value added taxes (VAT) and duties are not considered. In the model, the TTO for a sending region is reduced through an export VAT; where as the TC for an importing region is raised due to import duties. This means that the modelled transfer cost and turnovers before tax do not balance. The transfer prices are assumed to be fixed due to legal rights and are based on long-term historical values. An arbitrary optimisation of transfer prices is illegal and therefore not considered.

For each storage facility the *costs of storing* (storage costs, SC) are modelled. The storage costs for a facility are a function of the stored quantity within a time period and the storage cost per unit. The storage costs are related to the arcs combining the storage facilities between different time periods.

Distribution only takes place between given facilities in known regions. A common way of calculating the *distribution costs* (DC) is based on the distance (i.e. in facilities location where a cost factor is allocated to the distance, [47]). However, for this case more exact cost calculations can be done as cost factors can be assigned to the arcs in the network. This gives a less flexible model but more realistic results.

Costs are associated with distribution between facilities and storage in different regions (cross-regional distribution); however no costs are associated on arcs between a process step and the adjacent buffer stock in the same region. Furthermore no costs are associated between primary storage facilities and artificial secondary storage facilities in same regions; and no costs occur between secondary facilities and customer regions. The distribution costs are assigned to the receiving facility.

As production takes place in batches due to process requirements and equipment, the *production cost* (PC) will be a non-linear or piecewise linear function of the amount produced. In reality a fixed batch size is defined at each production facilities as fermentation is only done in full tanks. Therefore a fixed batch price should be used.

Royalties are in some regions paid on all sales - both within own sales region and on sales to other regions. Therefore, the royalty is calculated as a cost for the royalty paying region (RoyC), given as a percentage of the turnover and transfer turnover. For the royalty receiving region, royalties are considered as an additional turnover (RoyTO) - balancing with the cost for the paying region.

So far the model considers the variable costs of producing, distributing and storing as well as the impact of national differences, i.e. taxes and duties. However, in real life, many other parameters are important for decision making. Considering the storage of goods, several theories emphasise that the main costs for a company are not the direct handling and storage cost, but the value of the goods stored at the facility. (See i.e. Hillier, chapter 19 [46])

The value of tied up capital is considered important for the company in the specific case. This is a cost parameter that can not be directly seen in the budgets and annual reports. However, the impact on decision making is considered important. Using the perspective that tied up capital could have been invested in more profitable business areas - the cost of the investment can be calculated as the interests that could have been achieved on an alternative investment. In short it is the cost of not investing in the alternatives - the opportunity cost. Therefore the financial costs are implemented as the interests of the product values for a given time period.

This cost is fictive or at least semi-fictive as it does not appear in any of the budgets or the annual report. Nonetheless it is a very real factor to incorporate when modelling and optimising because this factor can have a significant influence on the optimal solution.

With all the given cost and income factors, the regional profit before tax is calculated. For regions with a positive profit, the net profit is found when the taxes are subtracted. For regions with a loss, no taxes are assumed. All regional profits are given in local currencies which, when calculating the global net profit, are turned into the currency of the primary region, R1, considering the *currency exchange rate* (ER). The financial costs for tied up capital in stored or distributed products (FinDistC and FinStoC) are subtracted from the global profit.

5.1.5 Verbal Model

Given the above mentioned network structure, product-, and economical flow a verbal formulation of the model is given to provide an overview and a general understanding of the model.

Objective
= maximise the sum of regional net profits after tax

- financial costs

where

Regional net profits after tax
= (Regional profit before tax - tax contribution) * Exchange rate

and

Regional net profit before tax
= Turnover from sales
+ Turnover from intermediate sales (transfer turnover)
+ Income from royalty payments
- Costs from intermediate procurement (transfer costs)
- Production costs
- Distribution costs
- Storage costs
- Cost from royalty payments
- Export value added taxes
- Import duties

with
Financial costs = interests on the value of products when stored or distributed

Subject to

Customer demand satisfaction
Capacity constraints for production facilities
Capacity constraints for storage facilities
BoM constraints for production facilities
Flow constraints for storage facilities
Batch size constraints for fermentation
Full container load constraints for distribution

5.1.6 Notation

The verbal formulation leads to the mathematical formulation in section 5.1.7. For the mathematical description of the model the following notation for sets and variables is used. For a description of each variable, please see appendix A.

The relation between the arcs and nodes in the network and the sets and variables in the model is shown in figure 5.4.

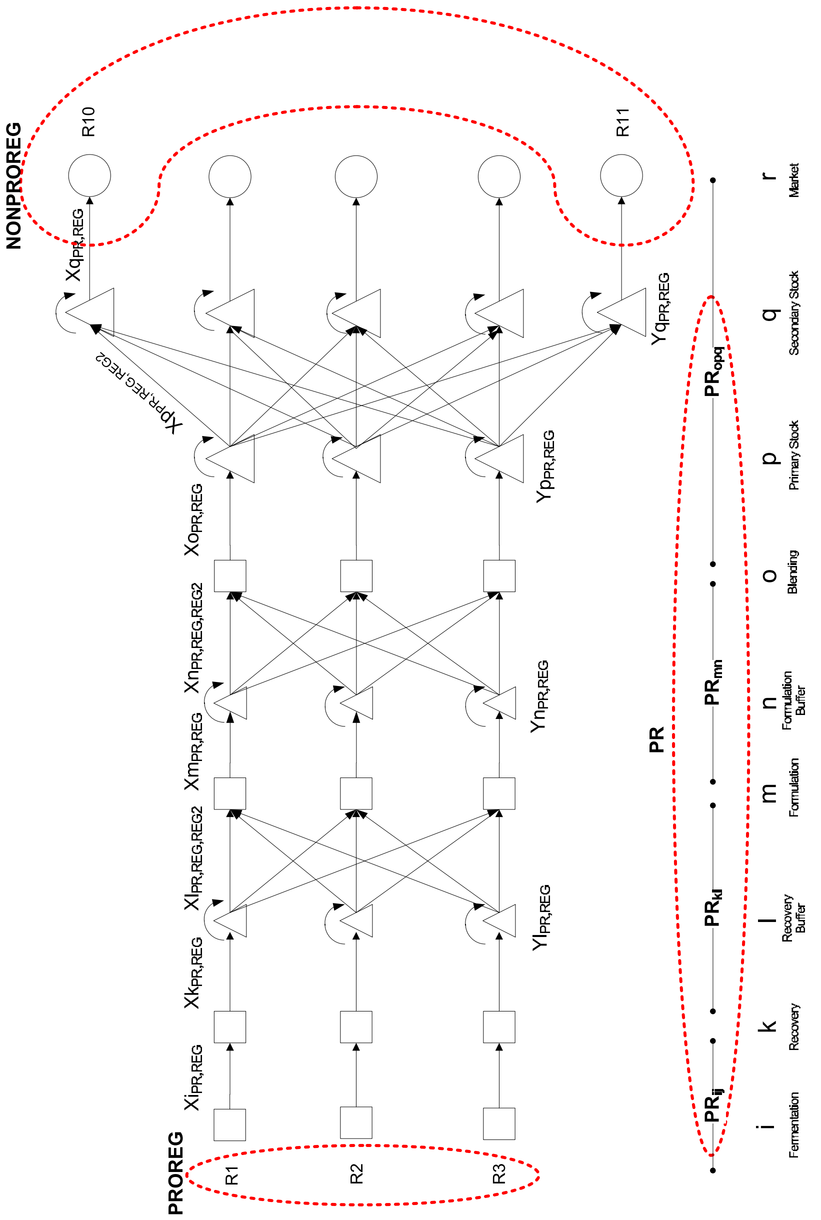


Figure 5.4: Sets and variables in the network model. Only five regions are modelled: the producing regions ($R1$, $R2$ and $R3$) and secondary regions ($R10$ and $R11$).

Sets and Variables

Sets

REG	Defines the set of all regions. Both primary production region and secondary sales regions.
$PROREG$	Defines the set of regions where production takes place. $PROREG$ is a subset of REG.
$NONPROREG$	Defines the set of secondary regions $NONPROREG$ is a subset of REG.
PR	Defines the set of all products in all phases This includes both intermediate and finished products.
$PRik$	Defines the set of intermediate products for the phases i to k. A subset of PR.
$PRkl$	Defines the set of intermediate products for the phases k to m. A subset of PR.
$PRmn$	Defines the set of intermediate products for the phases m to o. A subset of PR.
$PRopq$	Defines the set of final products for the phases o to q. A subset of PR.
t	Defines the discrete set of time periods.

Variables

Free variables	*In the objective function*
z	Defines the objective functions. International net profit after tax for all time periods. Given in the currency of region R1.
Npr_{REG}	The regional net profit in local currency.
$Regpr_{REG}$	The regional profit before tax in local currency.

Positive variables	*In the objective function*
$Regprpos_{REG}$	Auxiliary variable to observe a positive profit before tax in region REG.
$Regprneg_{REG}$	Auxiliary variable to observe a negative profit before tax in region REG.
TO_{REG}	Defines the turnover from sales for REG. Given in local currency.
$ProdC_{REG}$	Defines the total production costs for REG. Given in local currency.
$DistC_{REG}$	Defines the total distribution costs for REG Given in local currency.
$StoC_{REG}$	Defines the total storage cost for REG. Given in local currency.
TTO_{REG}	Turnover for region REG from intermediate product transfer. Given in local currency.
TC_{REG}	Costs for region REG from intermediate product transfer. Given in local currency.
$RoyC_{REG}$	Costs from royalty payments in REG. Given in local currency.
$RoyTO_{REG}$	Turnover from royalties paid to REG. Given in local currency.
$FinC$	Defines the total financial cost for storage and distribution.
$FinDistC_{REG,REG2,t}$	Defines the financial distribution cost per time period.
$FinStoC_{REG,t}$	Defines the financial regional storage cost per time period.

Variables

Positive variables	*Flow and auxiliary variables*
$ProV_{PR}$	Defines product value of a product in set PR There is a $ProV$ variable for each product set
$x_{PR,REG,t}$	Flow variable out of a production facility. Defines the production quantity of product PR given in kg for region REG. There are variables for each phase i, k, m, o and time period t
$x_{PR,REG,REG2,t}$	Flow variable between regions. Distribution flow from region REG to $REG2$ in period t Variables exist for all cross regional arcs in the phases l, n and p.
$xq_{PRopq,REG,t}$	Flow variable on the arc between the secondary storage and the sales nodes. Sales flow in region REG of product $PRopq$ for phase q in the time period t
$y_{PR,REG,t}$	Variable on the storage arc between time periods. Storage quantity of product PR in REG from time period t to time period $t+1$. Storage variables exist for the phase l, n, p and q

5.1.7 The Mathematical Model

In the following section the individual equations and constraints are defined based on the previous network and model description.

The objective [eq. 5.1] is to maximize the global after tax profit z defined by the sum of the regional net profits, Npr. Due to different currencies in the individual regions the regional net profits are converted into the same currency through the multiplication of an exchange rate ER. The financial costs $FinC$ for tied up capital are subtracted from the sum of the net profits.

$$maxz = \sum_{REG} Npr_{REG} ER_{REG} - FinC \tag{5.1}$$

For each region the profit after tax is given by the profit before tax minus the tax contribution. Assuming that the tax rate is constant for a positive income the net profit for each region, Npr, is given by the equation [eq. 5.2]. The positive auxiliary variables $Regprpos$ and $Regprneg$ ensures that taxes are only paid if there is a profit [eq. 5.3]. The regional profit before tax *Regpr* is given by difference between the sum of the turnovers and the costs [eq.5.4].

$$\begin{aligned} Npr_{REG} &= Regprpos_{REG}(1 - Tax_{REG}) \\ &\quad - Regprneg_{REG} \end{aligned} \tag{5.2}$$

$$Regpr_{REG} = Regprpos_{REG} - Regprneg_{REG} \tag{5.3}$$

$$\textit{with } Regprpos, Regprneg \geq 0$$

$$\begin{aligned} Regpr_{REG} = {} & TO_{REG} + TTO_{REG} + RoyTO_{REG} \\ & - TC_{REG} - ProdC_{REG} - DistC_{REG} \\ & - StoC_{REG} - RoyC_{REG} \end{aligned} \tag{5.4}$$

For each region the turnover is found as the sum of the turnovers for each product [eq. 5.5]. The turnover for each product is the product of the sold quantity xq and the average sales price $Price$.

$$TO_{REG} = \sum_{PRopq,t} xq_{PRopq,REG,t} Price_{PRopq,REG} \tag{5.5}$$

The production costs $ProdC$ for each region are given as the sum of production costs for all products at each production facility (phase i, k, m or o) [eq. 5.6]. For each facility a cost pr outgoing unit, PC, is multiplied with the outgoing quantity given by the flow variable x.

$$\begin{aligned} ProdC_{REG} = {} & \sum_{PRik,t} xi_{PRik,REG,t} PCi_{PRik,REG} \\ & + \sum_{PRkl,t} xk_{PRkl,REG,t} PCk_{PRkl,REG} \\ & + \sum_{PRmn,t} xm_{PRmn,REG,t} PCm_{PRmn,REG} \\ & + \sum_{PRopq,t} xo_{PRopq,REG,t} PCo_{PRopq,REG} \end{aligned} \tag{5.6}$$

The storage cost for each storage facility is defined by the product of the stored amount y and the corresponding storage cost pr unit SC [eq. 5.7].

The regional storage cost $StoC$ is the sum of costs for all storage facilities (l, n, p and q). The stored quantity between two time periods at a facility is given by the storage variable y.

$$\begin{aligned} StoC_{REG} = & \sum_{PRkl,t} yl_{PRkl,REG,t} SCl_{PRkl,REG} \\ & + \sum_{PRmn,t} yn_{PRmn,REG,t} SCn_{PRmn,REG} \\ & + \sum_{PRopq,t} yp_{PRopq,REG,t} SCp_{PRopq,REG} \\ & + \sum_{PRopq,t} yq_{PRopq,REG,t} SCq_{PRopq,REG} \end{aligned} \tag{5.7}$$

For each cross regional flow arc between a sending region $REG2$ and a receiving region REG a distribution cost is defined. The costs are allocated to the receiving region and is given as the product of the distributed quantity x and the distribution cost pr unit DC after conversion to the receiving regions currency. All distribution costs pr unit (DC) are given in the currency of region R2 which is converted into the receiving regions currency with the exchange rate ER. For each region the total distribution costs $DistC$ are given as the sum of the costs for each phase and each product PR.

$$\begin{aligned} Distc_{REG} = & \sum_{PRkl,REG2,t} xl_{PRkl,REG2,REG,t} DCl_{PRkl,REG2,REG} \alpha \\ + & \sum_{PRmn,REG2,t} xn_{PRmn,REG2,REG,t} DCn_{PRmn,REG2,REG} \alpha \\ + & \sum_{PRopq,REG2,t} xp_{PRopq,REG2,REG,t} DCp_{PRopq,REG2,REG} \alpha \end{aligned} \tag{5.8}$$

with $\alpha = \frac{ER_{REG="R2"}}{ER_{REG}}$

The cost and turnover contributions from product transfer between regions are given by the transfer turnover TTO and the transfer cost TC. For each product a transfer price TP between a sending region REG and a receiving region $REG2$ is defined and the transfer turnover is given by the product of the flow variable x and the transfer price TP. For some regions an export tax $ExVat$ is paid to the authorities in the sending region. The total transfer turnover for each region, TTO, is given as the sum of contributions for each product PR and phase (l, n or p) from all other regions $REG2$. For non-finished products (phase l) the transfer price is a function of the production costs and for finished goods (phases n and p) a function of the sales price. A conversion of the turnover currency for the phases n and p is therefore necessary.

$$\begin{aligned} TTO_{REG} = & \sum_{PRkl,REG2,t} xl_{PRkl,REG,REG2,t} TPl_{PRkl,REG,REG2} \gamma \\ & + \sum_{PRmn,REG2,t} xn_{PRmn,REG,REG2,t} TPn_{PRmn,REG,REG2} \gamma \beta \\ & + \sum_{PRopq,REG2,t} xp_{PRopq,REG,REG2,t} TPp_{PRopq,REG,REG2} \gamma \beta \end{aligned} \tag{5.9}$$

with $\beta = \frac{ER_{REG2}}{ER_{REG}}$
and $\gamma = (1 - ExVat_{REG,REG2})$

The corresponding transfer cost for a receiving region REG is the sum of the individual transfer costs for each product PR and phase (l, n or p) from all region $REG2$. The costs for the receiving region may be higher than the turnover from the sending region if import duties are paid. The import duties are given in the table Dut. For transfer costs, the currency conversion between regions is the opposite of the transfer turnover.

$$\begin{aligned} TC_{REG} = & \sum_{PRkl,REG2,t} xl_{PRkl,REG2,REG,t} TPl_{PRkl,REG2,REG} \zeta \beta \\ & + \sum_{PRmn,REG2,t} xn_{PRmn,REG2,REG,t} TPn_{PRmn,REG2,REG} \zeta \\ & + \sum_{PRopq,REG2,t} xp_{PRopq,REG2,REG,t} TPp_{PRopq,REG2,REG} \zeta \end{aligned} \tag{5.10}$$

with $\beta = \frac{ER_{REG2}}{ER_{REG}}$ *and* $\zeta = (1 + Dut_{REG2,REG})$

Royalties paid from one region to another are defined as a cost $RoyC$ for the paying region and a turnover $RoyTO$ for the receiving region. The royalties are given as a fraction of the sales - both from intermediate transfer to other regions (TTO) and from sales within the sales region (TO) - and are defined in the table $Royal$. As the TO and TTO are given in the receiving regions currency a conversion to the sending regions currency is ensured through the multiplication with the exchange rate ER between the regions.

$$RoyTO_{REG} = \sum_{REG2} (TO_{REG2} + TTO_{REG2}) Royal_{REG2,REG} \beta \tag{5.11}$$

$$RoyC_{REG} = \sum_{REG2} (TO_{REG} + TTO_{REG}) Royal_{REG,REG2} \tag{5.12}$$

with $\beta = \frac{ER_{REG2}}{ER_{REG}}$

The financial costs $FinC$ in equation 5.1 are the sum of the financial distribution costs $FinDistC$ and the financial storage costs $FinStoC$ given by the interests on the tied up capital.

$$FinC = \sum_{REG,t} (\sum_{REG2} FinDistC_{REG,REG2,t} + FinStoC_{REG,t}) \tag{5.13}$$

For each region the cost of storing a given product PR is found as a function of the product value $ProV$ multiplied with the stored amount y and the interest rate $Dfcost$ for a given time period $Stime$. The total financial storage costs $FinStoC$ are given as the sum of all product specific storage costs for each phase and region.

$$\begin{aligned} FinStoC_{REG,t} = & \sum_{PRkl} ProVkl_{PRkl} yl_{PRkl,REG,t} \kappa \\ & + \sum_{PRmn} ProVmn_{PRmn} yn_{PRmn,REG,t} \kappa \\ & + \sum_{PRopq} ProVopq_{PRopq} yp_{PRopq,REG,t} \kappa \\ & + \sum_{PRopq} ProVopq_{PRopq} yq_{PRopq,REG,t} \kappa \end{aligned} \tag{5.14}$$

with $\kappa = (1 + Dfcost)^{Stime} - 1$

The financial distribution costs, *FindistC*, are found as a function of the distributed amount, x, and the average distribution time, *Dtime*, using the same approach as for the storage costs.

$$\begin{aligned} FinDistC_{REG,REG2,t} = & \sum_{PRkl} ProVkl_{PRkl} xl_{PRkl,REG,REG2,t} \lambda \\ & + \sum_{PRmn} ProVmn_{PRmn} xn_{PRmn,REG,REG2,t} \lambda \\ & + \sum_{PRopq} ProVopq_{PRopq} xp_{PRopq,REG,REG2,t} \lambda \end{aligned} \tag{5.15}$$

with $\lambda = (1 + Dfcost)^{Dtime_{REG,REG2}} - 1$

The product values pr unit *ProV* for each phase in each region are defined as the sum of the production costs throughout the production steps. With a possibility of blending or dividing the products in the individual productions steps the cost will depend of the blend of products. Sub-components from different regions will have different cost, and therefore the product value of a final product will depend on its sub-components. Therefore the product values for products after the first phase are a function of the bill of materials and the costs at the facilities where they are produced. For each node the value of an incoming product is given as the weighted average of the regional product values from each sending region. The outgoing value is given as the initial product value multiplied with the product mixture *prodmix* and the production cost *PC* for the production phase (k, m or o).

$$ProVik_{PRik,REG,t} = PCi_{PRik,REG} \tag{5.16}$$

$$\begin{aligned} & ProVkl_{PRkl,REG,t} = \\ & \qquad \sum_{PRik} xi_{PRik,REG,t} \\ & \qquad \cdot Prodmixk_{PRkl,PRik,REG} ProVik_{PRik,REG,t} \\ & \qquad + PCk_{PRkl,REG,t} \end{aligned} \tag{5.17}$$

$$\begin{aligned} ProVmn_{PRmn,REG,t} = {} & \\ & \frac{\beta}{\sum_{REG2} xl_{PRkl,REG2,REG,t}} \\ & \cdot \sum_{PRkl,REG2} Prodmixk_{PRmn,PRkl,REG} \\ & \cdot xl_{PRkl,REG2,REG,t} ProVkl_{PRkl,REG2,t} \\ & + PCm_{PRkl,REG,t} \end{aligned} \tag{5.18}$$

$$\begin{aligned} ProVopq_{PRopq,REG,t} = {} & \\ & \frac{\beta}{\sum_{REG2} xn_{PRmn,REG2,REG,t}} \\ & \cdot \sum_{PRmn,REG2} Prodmixk_{PRopq,PRmn,REG} \\ & \cdot xn_{PRmn,REG2,REG,t} ProVmn_{PRmn,REG2,t} \\ & + PCo_{PRopq,REG,t} \end{aligned} \tag{5.19}$$

with $\beta = \frac{ER_{REG2}}{ER_{REG}}$

The above mentioned cost and turnover factors are all a part of the objective function and mainly describe the financial flow within the network. The following constraints ensure a feasible solution with respect to the product flow in the network.

For each region REG the demand for each product Dem should fulfilled within a given time period t [eq. 5.20]. The sold products are given by the flow variable xq linking the secondary storage with the sales region.

$$xq_{PRopq,REG,t} = Dem_{PRopq,REG,t} \tag{5.20}$$

For each production facility (i, k, m or o) there is a maximum production capacity. The total quantity of produced products for each region and facility - given by the outgoing flow variable, x - is not allowed to exceed the total capacity Cap within a time period t [eq. 5.21].

$$\begin{aligned}\sum_{PRik} xi_{PRik,REG,t} &\leq Capi_{REG,t} \\ \sum_{PRkl} xk_{PRkl,REG,t} &\leq Capk_{REG,t} \\ \sum_{PRmn} xm_{PRmn,REG,t} &\leq Capm_{REG,t} \\ \sum_{PRopq} xo_{PRopq,REG,t} &\leq Capo_{REG,t}\end{aligned} \tag{5.21}$$

Equivalent to the production capacity, the storage capacity Cap at each storage facility (l, n, p or q) should be respected [eq. 5.22]. The quantity of products stored within a time period is given by the storage variable y and the storage constraints are:

$$\begin{aligned}\sum_{PRkl} yl_{PRkl,REG,t} &\leq Capl_{REG,t} \\ \sum_{PRmn} yn_{PRmn,REG,t} &\leq Capn_{REG,t} \\ \sum_{PRopq} yp_{PRopq,REG,t} &\leq Capp_{REG,t} \\ \sum_{PRopq} yq_{PRopq,REG,t} &\leq Capq_{REG,t}\end{aligned} \tag{5.22}$$

For each production facility flow conservation with respect to the bill of materials should be fulfilled [eq. 5.23]. The bill of materials *prodmix* is given for each product as the ratio between in and outgoing product quantities. For each node the ingoing flow should balance with the product of the bill of material and the outgoing flow.

$$\begin{aligned}xi_{PRik,REG,t} &= \sum_{PRkl} xk_{PRkl,REG,t} Prodmixk_{PRkl,PRik} \\ \sum_{REG2} xl_{PRkl,REG2,REG,t} &= \sum_{PRmn} xm_{PRmn,REG,t} Prodmixm_{PRmn,PRkl} \\ \sum_{REG2} xn_{PRmn,REG2,REG,t} &= \sum_{PRopq} xo_{PRopq,REG,t} Prodmixo_{PRopq,PRmn}\end{aligned} \tag{5.23}$$

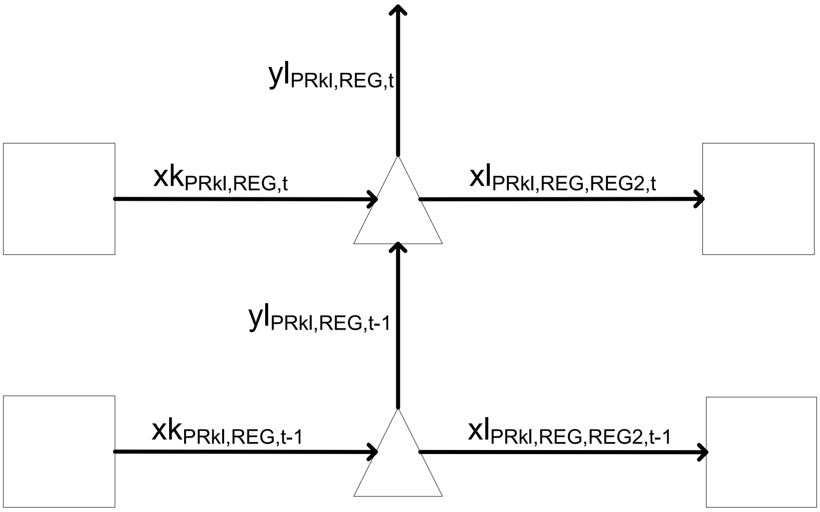


Figure 5.5: Flow conservation for a storage node.

For each storage facility the amount of products stored between time periods y is the difference between products stored from the previous period plus the products produced in the actual period and the products used within the given time period. The flow for a single storage node is illustrated in figure 5.5. Here the link between the product flow within a time period, x and the storage level between time period y is illustrated. For storage facilities, the sum of the ingoing products and the initial storage level at time $t-1$ should balance with the sum of the end storage level at time t and the outgoing products.

It is assumed that the time periods are cyclic and the storage level at the end of the last period should equal the level in the beginning of the first period (illustrated with the index $t--1$). This leads to the following flow conservation constraints [eq. 5.24] for each product in each time period within a facility and a region.

$$\begin{aligned}
yl_{PRkl,REG,t} &= yl_{PRkl,REG,t--1} + xk_{PRkl,REG,t} \\
&- \sum_{REG2} xl_{PRkl,REG,REG2,t} \\
yn_{PRmn,REG,t} &= yn_{PRmn,REG,t--1} + xm_{PRmn,REG,t} \\
&- \sum_{REG2} xn_{PRmn,REG,REG2,t} \\
yp_{PRopq,REG,t} &= yp_{PRopq,REG,t--1} + xo_{PRopq,REG,t} \\
&- \sum_{REG2} xp_{PRopq,REG,REG2,t} \\
yq_{PRopq,REG,t} &= yq_{PRopq,REG,t--1} \\
&+ \sum_{REG2} xp_{PRopq,REG2,REG,t} - xq_{PRopq,REG,t}
\end{aligned} \tag{5.24}$$

The enzymes production is a batch production, where full tanks are always used for fermentation. The flow variable xi modelling the outgoing quantity from the fermentation should be given as a multiple p of the batch size *batchsize*. The integer number of batches p represents the number of batches to be produces and is limited by the available capacity - given by the number of fermentation tanks p^{max}.

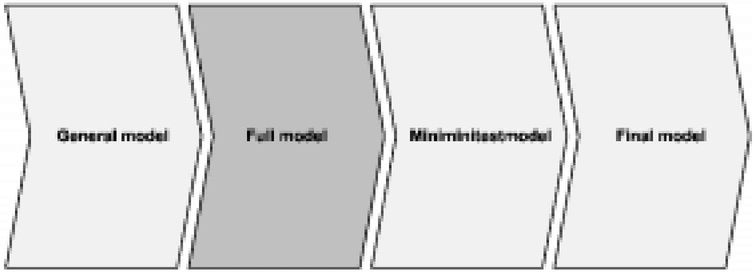
$$\begin{gathered}
xi_{PRik,REG,t} = p_{PR,REG,t} batchsize_{REG} \\
\sum_{PRik} p_{PRik,REG,t} \leq p^{max}_{REG,t} \\
p = 0, 1, ..., p^{max}
\end{gathered} \tag{5.25}$$

For distribution in containers between facilities the use of full container loads is preferable due to economies of scale. Allowing a mixture of different products PR within on container, the total amount of products distributed from region REG to region $REG2$ should be a multiple of the container size, *contsize* and the container filling factor, q^{real}. The number of containers, q^{int}, is the nearest integer value of the container filling factor. The distribution costs DC on a specific arc will depend on the number of containers q and the distribution price pr container *contprice*. The integer q should be either $q^{int}_{REG,REG2,t} - 1$ or $q^{int}_{REG,REG2,t}$.

$$\begin{aligned}
\sum_{PRmn} xn_{PRmn,REG,REG2,t} &= q^{real}_{REG,REG2,t} contsize \\
q^{int}_{REG,REG2,t} - 1 \leq q^{real}_{REG,REG2,t} &\leq q^{int}_{REG,REG2,t} \\
DCn_{PRmn,REG,REG2,t} &= q_{REG,REG2,t} contcost_{PRmn,REG,REG2,t}
\end{aligned} \tag{5.26}$$

The above described model can be considered as a general model for modelling the batch production in a logistic network where production, storage and distribution take place in an international context. The individual equations are case specific for the network presented in figure 5.1, however the individual classes of equations can be used for other logistic networks. In the above-mentioned description, the equations dealing with e.g. production cost or transfer cost and turnover etc are a generic way of treating the problem. The case specific approach - with respect to the exact cost factors and data availability - will be presented later 5.2.

5.2 The Full Model



The General Model described in chapter 5.1 represents a mixed integer non-linear problem (MINLP). Since the aim of this project is to develop a model that is solvable for the real-life optimisation problem of the logistic network at Novozymes a linearisation of the General Model is performed in this chapter. This is due to the fact that linear models even for large problems are solvable with commercial software (See also chapter 4.6 on consideration of the solution approach).

In section 5.2.1 considerations about the model size are presented. This leads to the linearisation in section 5.2.2.

5.2.1 Model Size

With 600 products disaggregating into more than 1400 product numbers, five production regions and 12 sales regions the specific case does not represent a small scale problem. With the given model structure, the size of the problem is a function of the sets defining the regions, products and time periods. The number of variables in the model is described in the following example and graphically presented in figure 5.6:

Example: Model Size

In the following example only the continuous equations 5.1 to 5.23 are considered. With REG_{max} defining the number of regions, PR_{max} defining the number of products and t_{max} defining the number of time periods, the number of variables is given by the following relations (For details, please see appendix D):

Flow variables (x):
$t_{max}PR_{max}(5REG_{max} + 3REG_{max}^2)$

Storage variables (y):
$4REG_{max}PR_{max}t_{max}$

Variables in the objective function:
$2 + 12REG_{max} + t_{max}REG_{max}(1 + REG_{max})$

Other variables:
$4PR_{max}$

The figure and example shows that even for i.e. six time periods with 250 products and ten regions the specific case the problem will have 586.782 variables. For the full scale problem with 1400 products the problem will have approximately 3,3 million linear variables. To evaluate the model size for other set sizes, please see appendix D and the CD (file: modelsize.xls).

The growth emphasize a need to either develop new solution algorithms for the real scale problem or use a linear program, to be able to solve this

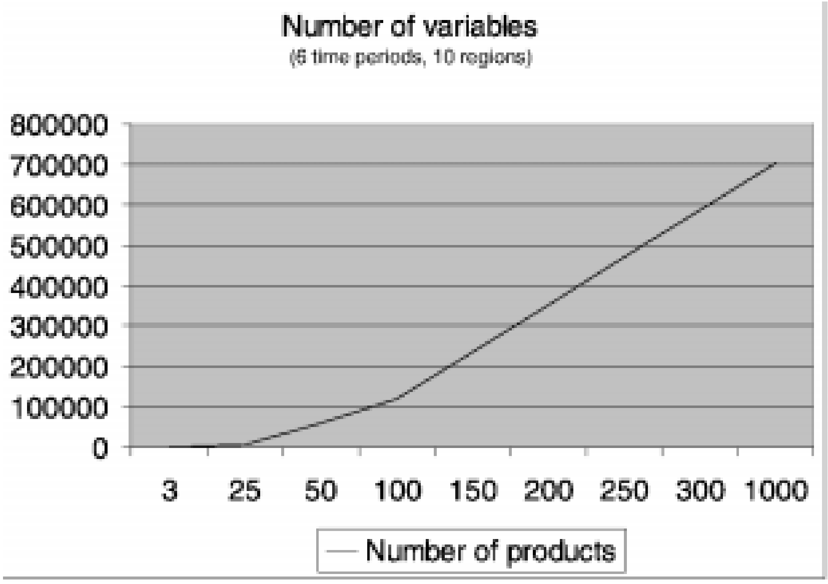


Figure 5.6: Model size as a function the number of regions.

to optimality. In this report it has been chosen to use a linear model as this enables the use of standard software. The non-linear equations and the integer constraints are treated in the following section.

5.2.2 Linearisation

Due to the definitions of the product values the calculations of the financial cost lead to non-linearities. Defining the product value as in the equations 5.16- 5.19 the value of the product will be a function of the optimal solution. The product value is the sum of all the production cost - and the production cost will depend on the mixture of different products and their origin. The case of different product values is illustrated with the following example.

Example: Product value as a function of the solution

The following example represents a simplified problem. For simplicity only a part of the network is considered (fig. 5.7) and all values are given in the same currency.

The value per kg of product PRi produced in region REG is given by the production cost. For the first production phase (i), the production value $ProVi$ will be.

REG	$ProVi$
$R1$	12
$R2$	14
$R3$	6

For a product after the second production phase k, the value of the product depends on the product flow in phase j, xj. A product after phase two can be produced from a mixture of similar products from different regions. Thereby the product value $ProVk$ is a function of xj. If the costs for production phase two is 5 per kg products the values of different mixtures are:

Mix	From $R1$	From R2	From R3	$ProVk$
Mix 1	100 %	0 %	%	17,00
Mix 2	30 %	50 %	20 %	16,80
Mix 3	0 %	0 %	100 %	11,00

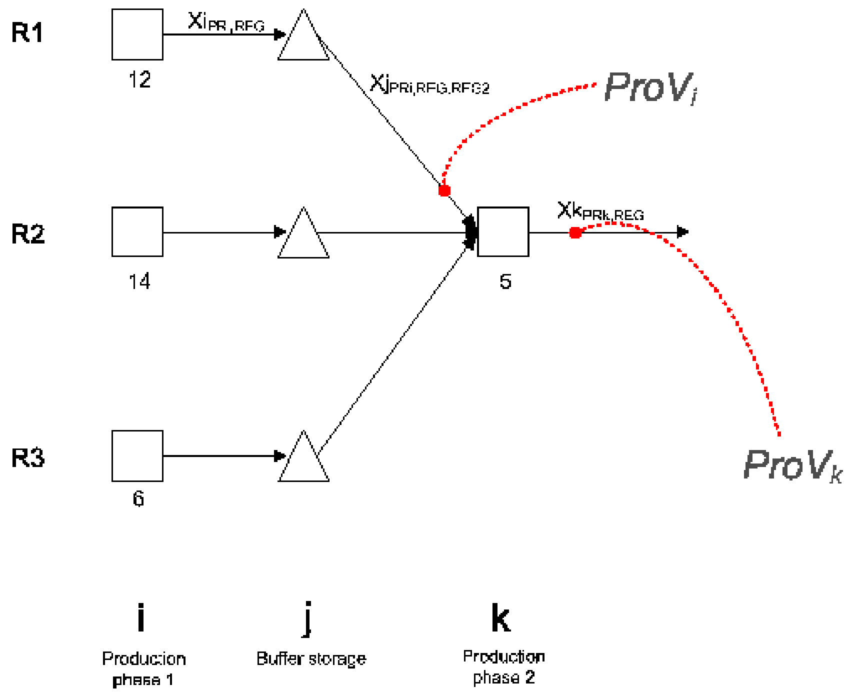


Figure 5.7: Product values. Production costs per unit are given below the production nodes in the phases i and k.

The financial cost for storage and distribution depends on both the product value, $ProV$, and the quantity of the product stored or distributed, y or x. As both the variables handling distribution x and storage y as well as the product value $ProV$ are found as a result of the optimal solution, it is obvious that the multiplication of the variables will lead to non-linearities in the equations 5.16 - 5.19.

Assuming that the cost of different products does not differ significantly between regions, an average product value - independent of the producing region - is used to model the financial costs. The average values of the products are calculated with the equations 5.27-5.30.

$$ProVik_{PRik} = \sum_{PROREG} \frac{PCi_{PRik,PROREG} ER_{PROREG}}{card_{PROREG}} \tag{5.27}$$

$$\begin{aligned} ProVkl_{PRkl} = \sum_{PROREG} (\sum_{PRik} & ProVik_{PRik} \\ & \cdot Prodmixk_{PRkl,PRik,PROREG} \\ & + \frac{PCk_{PRkl,PROREG} ER_{PROREG}}{card_{PROREG}}) \end{aligned} \tag{5.28}$$

$$\begin{aligned} ProVmn_{PRmn} = \sum_{PROREG} (\sum_{PRkl} & ProVkl_{PRkl} \\ & \cdot Prodmixm_{PRmn,PRkl,PROREG} \\ & + \frac{PCm_{PRmn,PROREG} ER_{PROREG}}{card_{PROREG}}) \end{aligned} \tag{5.29}$$

$$\begin{aligned} ProVopq_{PRopq} = \sum_{PROREG} (\sum_{PRmn} & ProVmn_{PRmn} \\ & \cdot Prodmixo_{PRopq,PRmn,PROREG} \\ & + \frac{PCo_{PRopq,PROREG} ER_{PROREG}}{card_{PROREG}}) \end{aligned} \tag{5.30}$$

The use of average product values leads to the following equations (5.31 - 5.32) for the financial storage and distribution costs:

The *Financial storage costs* are

$$
\begin{aligned}
&FinStoC_{REG,t} \\
&= \sum_{PRkl} \sum_{REG3} (\sum_{PRik} ProVik_{PRik} Prodmixk_{PRkl,PRik,REG3} \\
&+ \frac{PCk_{PRkl,REG3} ER_{REG3}}{card_{PROREG}}) yl_{PRkl,REG,t} \kappa \\
&+ \sum_{PRmn} \sum_{REG3} (\sum_{PRkl} ProVkl_{PRkl} Prodmixm_{PRmn,PRkl,REG3} \\
&+ \frac{PCm_{PRmn,REG3} ER_{REG3}}{card_{PROREG}}) yn_{PRmn,REG,t} \kappa \\
&+ \sum_{PRopq} \sum_{REG3} (\sum_{PRmn} ProVmn_{PRmn} Prodmixo_{PRopq,PRmn,REG3} \\
&+ \frac{PCo_{PRopq,REG3} ER_{REG3}}{card_{PROREG}}) yp_{PRopq,REG,t} \kappa \\
&+ \sum_{PRopq} \sum_{REG3} (\sum_{PRmn} ProVmn_{PRmn} Prodmixo_{PRopq,PRmn,REG3} \\
&+ \frac{PCo_{PRopq,REG3} ER_{REG3}}{card_{PROREG}}) yq_{PRopq,REG,t} \kappa
\end{aligned}
\tag{5.31}
$$

with $\kappa = (1 + Dfcost)^{Stime} - 1$

The *financial distribution costs* are:

$$
\begin{aligned}
&FinDistC_{REG,REG2,t}\\
&= \sum_{PRkl}\sum_{REG3}(\sum_{PRik} ProVik_{PRik}Prodmixk_{PRkl,PRik,REG3}\\
&+ \frac{PCk_{PRkl,REG3}ER_{REG3}}{card_{PROREG}})xl_{PRkl,REG,REG2,t}\lambda\\
&\\
&+ \sum_{PRmn}\sum_{REG3}(\sum_{PRkl} ProVkl_{PRkl}Prodmixm_{PRmn,PRkl,REG3}\\
&+ \frac{PCm_{PRmn,REG3}ER_{REG3}}{card_{PROREG}})xn_{PRmn,REG,REG2,t}\lambda\\
&\\
&+ \sum_{PRopq}\sum_{REG3}(\sum_{PRmn} ProVmn_{PRmn}Prodmixo_{PRopq,PRmn,REG3}\\
&+ \frac{PCo_{PRopq,REG3}ER_{REG3}}{card_{PROREG}})xp_{PRopq,REG,REG2,t}\lambda
\end{aligned}
\tag{5.32}
$$

$$\mathit{with}\ \lambda = (1 + Dfcost)^{Dtime_{REG,REG2}} - 1$$

The equations 5.14, 5.15, 5.16, 5.17, 5.18 and 5.19 in the general model (section 5.1.7) are replaced through the equations 5.31, 5.32, 5.27, 5.28, 5.29 and 5.30 in the linear full model.

As the financial costs only represent the interests of the tied up capital, the loss of precision in this linearisation is considered insignificant. This is illustrated in the following example where the financial costs based on the average values are compared with the solutions from the previous example.

Example: Financial costs for different product values.

The financial costs for storing or distributing the products are the interest of the product value. For a time period of three months with an interest rate of 12 % p.a. the financial costs, $FinC$, for the different products are given below.

Mix	$ProVk$	$FinC$
$Mix1$	17,00	0,49
$Mix2$	16,80	0,48
$Mix3$	11,00	0,32
$Average$	15,67	0,45

For this specific case, the use of the average product value will result in a 41% higher cost than reality if the real product is of mix3 and an 8% lower cost than reality if the mixture is mix1. This seems like a large difference, however a comparison of the absolute values shows a max deviation of 0,13 pr kg. Compared to the other cost parameters in the example this (i.e. the production costs for mix1: 17,00) value is rather low. Therefore the use of the average product value must be considered as an insignificant approximation that does not reduce the modelŠs result considerable.

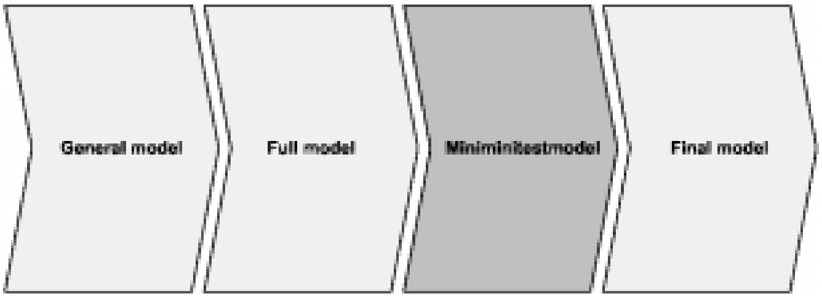
5.2.3 Relaxations

The batch size constraint in the general model (eq. 5.25) results in a mixed integer problem. To achieve a LP-problem this constraint is relaxed, assuming that within the given time period, the non-integer solution will result in feasible solutions. It should be noticed that the optimal solution to the batch-restricted problem may differ from the rounded linear solution (see i.e. [48], p.4 for a comparison of optimal integer and LP solutions). For the specific case dealing with the planning on tactical-strategic management level the relaxation is considered reasonable, as the detailed planning of batch sizes and production scheduling is done on lower levels. However, an analysis of the solution is necessary, to ensure that the solution is feasible at the lower levels.

The container load constraint (eq. 5.26) leads to another integer restriction. A relaxation of this constraint does not lead to non-feasibilities for the model. The result may be a solution, where the distribution costs per product unit are higher in reality than modelled. This should be evaluated in the solution to see if the costs should be adjusted.

The relaxation of the integer constraints and the linearisation of the financial costs lead to the full model; the model is presented in appendix E
.

5.3 The Miniminitestmodel



The full model described in the previous section 5.2 is a general linear model for international logistic networks with multi process production steps and buffer storages. Due to experience validation and verification of the full models performance is very complex. Even for reduced sets - four products, two producing regions and one additional sales region - the validity of the results are difficult and time consuming to prove. A reduction of the number of variables is necessary to achieve transparency. A simplified model for verification purpose has therefore been developed.

The test model does not represent all elements of complexity in the real network but all different kinds of equations are modelled in a simplified network. The model has all elements and parameters from the full scale problem, but the network is reduced as much as possible. This results in a model that is usable for validation and verification - since all parameters and equations can be double checked.

The model is described in the following steps: An overview of the network structure and flow is given in section 5.3.1. This leads to the mathematical formulation in section 5.3.3 using the notation from section 5.3.2. Finally the verification is performed as presented in section 5.4.

5.3.1 Network structure and Flows

The network for the miniminitestmodel is presented in figure 5.8. To test the flow between regions (economical and physical), the use of more than one region is necessary. In the model two producing regions (given by

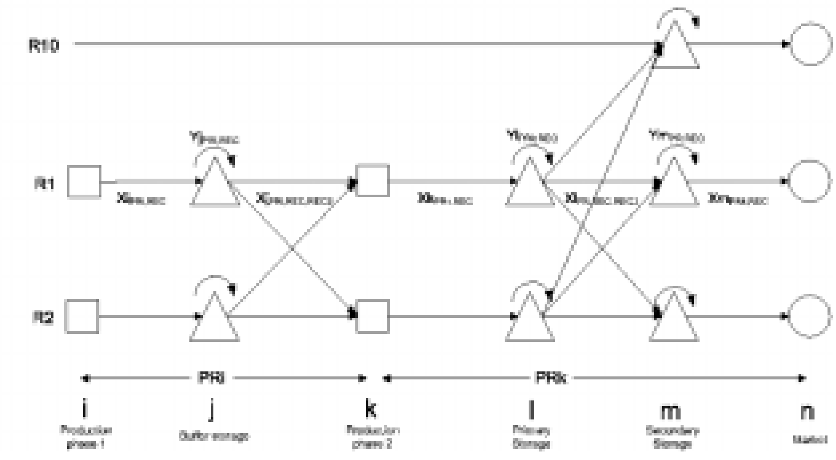


Figure 5.8: The miniminitestmodel.

the set *REG*) are considered. This enables transfer of semi-finished goods from buffer storage to another production facility. Furthermore both primary and secondary storage facilities are incorporated in order to model the flow between storage facilities. The cross-regional flows enables the verification of functions involving taxes, exchange rates, VAT, duties, transfer prices and royalties and distribution where an economical flow goes between different regions.

To verify the use of BOM-constraints, production and storage cost together with flow constraints, four final products (given by the set *PRk*) are modelled. They are aggregated and disaggregated based on two basic products (given by the set *PRi*). The simplified network provides the necessary overview to evaluate if the individual constraints function as intended.

The network consists of two production phases. When comparing with the full network, the two phases can be interpreted as an aggregation of the phases in the full scale model. Production phase one consists of the fermentation and recovery process and the product set *PRi* consists of the concentrates. Production phase two considers both the formulation and blending of products and the product set *PRk* is a set of final products. In the model it is possible to send concentrates between producing regions in phase *j* or to send finished products between producing regions and sales regions in phase l.

These features are considered sufficient to evaluate the main parameters of the full network.

5.3.2 Notation

The notation and variables used are similar to the previously described general and full models. The notation presented here only considers the main groups of variables. Each group can be divided into subgroups representing each process phase. The sets and variables are.

Sets

REG	Defines the set of all regions.
$PROREG$	Defines the set of regions where production takes place. $PROREG$ is a subset of REG.
PR	Defines the set of all products both intermediate and final products.
PRi	Defines the set of intermediate products for the phase i.
PRk	Defines the set of finished products for the phase k.
t	Defines the set of time periods.

Variables

z	Defines the over all net profit
Npr_{REG}	Defines the regional net profit in local currency
$Regpr_{REG}$	Defines the regional profit before tax
$Regprpos_{REG}$	Auxiliary variable to observe the positive regional profit in region REG
$Regprneg_{REG}$	Auxiliary variable to observe the negative regional profit in region REG
TO_{REG}	Defines the turnover for region REG in local currency
TTO_{REG}	Defines the turnover for region REG from intermediate product transfer between regions in local currency
TC_{REG}	Defines the costs for region REG from intermediate product transfer between regions in local currency
$ProdC_{REG}$	Defines the production costs for region REG in local currency
$DistC_{REG}$	Defines the distribution costs for region REG in local currency
$StoC_{REG}$	Defines the storage cost for region REG in local currency
$RoyC_{REG}$	Defines the royalty cost in local currency paid by region REG
$RoyTO_{REG}$	Defines the turnover from royalties paid to region REG

Variables

$FinC$	Defines the total financial cost
$FinDistC_{REG,REG2,t}$	Defines the financial cost of distribution
$FinStoC_{REG,t}$	Defines the financial cost of storage in REG
$ProV_{PR}$	Defines the product value pr unit product PR
$x_{PR,REG,t}$	Flow variable for product PR in REG. The variable is defined for the arcs i, k and m.
$x_{PR,REG,REG2,t}$	Flow variable for product PR from region REG to region $REG2$. Defined for the arcs j and l.
$y_{PR,REG,t}$	Flow on storage arcs for period t. Storage of product PR in kg in region REG The variable is defined for every storage arc i, l and m.

5.3.3 The Mathematical Model

Based on the full model, the corresponding mathematical formulation for the miniminitestmodel is presented below:

The objective function:

$$maxz = \sum_{REG} Npr_{REG} ER_{REG} - FinC \tag{5.33}$$

The regional net profit:

$$\begin{aligned} Npr_{REG} = {} & Regprpos_{REG}(1 - Tax_{REG}) \\ & - Regprneg_{REG} \end{aligned} \tag{5.34}$$

The regional profit before tax [eq. 5.35] with the auxiliary variable [eq. 5.36].

$$\begin{aligned} Regpr_{REG} = {} & TO_{REG} + TTO_{REG} + RoyTO_{REG} \\ & - TC_{REG} - ProdC_{REG} - DistC_{REG} \\ & - StoC_{REG} - RoyC_{REG} \end{aligned} \tag{5.35}$$

$$Regpr_{REG} = Regprpos_{REG} - Regprneg_{REG} \tag{5.36}$$

For each region the regional profit before tax [eq. 5.35] consists of the following cost and turnover factors:

The regional turnover:

$$TO_{REG} = \sum_{PRk,t} xm_{PRk,REG,t} Price_{PRk,REG} \tag{5.37}$$

The production costs:

$$\begin{aligned} Prodc_{REG} = & \sum_{PRi,t} xi_{PRi,REG,t} PCi_{PRi,REG} \\ & + \sum_{PRk,t} xk_{PRk,REG,t} PCk_{PRk,REG} \end{aligned} \tag{5.38}$$

The regional storage costs:

$$\begin{aligned} StoC_{REG} = & \sum_{PRi,t} yj_{PRi,REG,t} SCj_{PRi,REG} \\ & + \sum_{PRk,t} yl_{PRk,REG,t} SCl_{PRk,REG} \\ & + \sum_{PRk,t} ym_{PRk,REG,t} SCm_{PRk,REG} \end{aligned} \tag{5.39}$$

The regional distribution costs:

$$\begin{aligned} Distc_{REG} = & \sum_{PRi,REG2,t} xj_{PRi,REG2,REG,t} DCj_{PRi,REG2,REG} \alpha \\ + & \sum_{PRk,REG2,t} xl_{PRk,REG2,REG,t} DCl_{PRk,REG2,REG} \alpha \end{aligned} \tag{5.40}$$

with $\alpha = \frac{ER_{REG=''R2''}}{ER_{REG}}$

The turnover [eq. 5.41] and cost [eq. 5.42] contributions from intra-organisational product distribution:

$$\begin{aligned} TTO_{REG} = & \sum_{PRi,REG2,t} xj_{PRi,REG,REG2,t} TPj_{PRi,REG,REG2} \gamma \\ + & \sum_{PRk,REG2,t} xl_{PRk,REG,REG2,t} TPl_{PRk,REG,REG2} \gamma \end{aligned} \tag{5.41}$$

and $\gamma = (1 - ExVat_{REG,REG2})$

$$TC_{REG} = \sum_{PRi,REG2,t} xj_{PRi,REG2,REG,t} TPj_{PRi,REG2,REG} \zeta \beta + \sum_{PRk,REG2,t} xl_{PRk,REG2,REG,t} TPl_{PRk,REG2,REG} \zeta \beta \tag{5.42}$$

with $\beta = \frac{ER_{REG2}}{ER_{REG}}$ *and* $\zeta = (1 + Dut_{REG2,REG})$

The royalty costs and turnovers [eq. 5.43 and eq. 5.44]:

$$RoyTO_{REG} = \sum_{REG2} (TO_{REG2} + TTO_{REG2}) Royal_{REG2,REG} \beta \tag{5.43}$$

with $\beta = \frac{ER_{REG2}}{ER_{REG}}$

$$RoyC_{REG} = \sum_{REG2} (TO_{REG} + TTO_{REG}) Royal_{REG,REG2} \tag{5.44}$$

The financial costs for storing and distributing are a function of the product values. The equations are given by 5.45 - 5.49.

The total financial costs

$$FinC = \sum_{REG,t} (\sum_{REG2} FinDistC_{REG,REG2,t} + FinStoC_{REG,t}) \tag{5.45}$$

The financial storage costs

$$\begin{aligned}
&FinStoC_{REG,t} \\
&= \sum_{PRi} \sum_{REG3} \frac{PCi_{PRi,REG3} ER_{REG3}}{card_{PROREG}} yj_{PRi,REG,t} \kappa \\
&+ \sum_{PRk} (\sum_{PRi} (Prodmixjk_{PRk,PRi} \sum_{REG3} \frac{PCi_{PRi,REG3} ER_{REG3}}{card_{PROREG}}) \\
&+ \sum_{REG3} \frac{PCk_{PRk,REG3} ER_{REG3}}{card_{PROREG}} yl_{PRk,REG,t} \kappa \\
&+ \sum_{PRk} (\sum_{PRi} (Prodmixjk_{PRk,PRi} \sum_{REG3} \frac{PCi_{PRi,REG3} ER_{REG3}}{card_{PROREG}}) \\
&+ \sum_{REG3} \frac{PCk_{PRk,REG3} ER_{REG3}}{card_{PROREG}} ym_{PRk,REG,t} \kappa
\end{aligned} \tag{5.46}$$

with $\kappa = (1 + Dfcost)^{Stime} - 1$

The financial distribution costs

$$\begin{aligned}
&FinDistC_{REG,REG2,t} \\
&= \sum_{PRi} (\sum_{REG3} \frac{PCi_{PRi,REG3} ER_{REG3}}{card_{PROREG}} xj_{PRi,REG,REG2,t} \lambda \\
&+ \sum_{PRk} (\sum_{PRi} Prodmixjk_{PRk,PRi} \sum_{REG3} \frac{PCi_{PRi,REG3} ER_{REG3}}{card_{PROREG}}) \\
&+ \sum_{REG3} \frac{PCk_{PRk,REG3} ER_{REG3}}{card_{PROREG}} xl_{PRk,REG,REG2,t} \lambda
\end{aligned} \tag{5.47}$$

with $\lambda = (1 + Dfcost)^{Dtime_{REG,REG2}} - 1$

The product values are given by the equations:

$$ProVi_{PRi} = \sum_{REG} \frac{PCi_{PRi,REG} ER_{REG}}{card_{PROREG}} \tag{5.48}$$

$$\begin{aligned} ProVk_{PRk} = & \sum_{PRi} Prodmixjk_{PRk,PRi} ProVi_{PRi} \\ & + \sum_{REG} \frac{PCk_{PRk,REG} ER_{REG}}{card_{PROREG}} \end{aligned} \tag{5.49}$$

All together these equations 5.33 - 5.49 define the objective function for the miniminitest model. The constraints for the model are:

The customer demand satisfaction:

$$xm_{PRk,REG,t} = Dem_{PRk,REG,t} \tag{5.50}$$

The capacity constraints for the production nodes

$$\begin{aligned} \sum_{PRi} xi_{PRi,REG,t} = Capi_{REG} \\ \sum_{PRk} xk_{PRk,REG,t} = Capk_{REG} \end{aligned} \tag{5.51}$$

The capacity constraints for the storage nodes

$$\begin{aligned} \sum_{PRi} yj_{PRi,REG,t} = Capj_{REG} \\ \sum_{PRk} yl_{PRk,REG,t} = Capl_{REG} \\ \sum_{PRk} ym_{PRk,REG,t} = Capm_{REG} \end{aligned} \tag{5.52}$$

BoM constraint for the production facility j:

$$\sum_{REG2} xj_{PRi,REG2,REG,t} = \sum_{PRk} xk_{PRk,REG,t} Prodmixjk_{PRk,PRi} \tag{5.53}$$

The flow constraints for the storage facilities:

$$\begin{aligned}
yj_{PRi,REG,t} &= yj_{PRi,REG,t--1} + xi_{PRi,REG,t} \\
&- \sum_{REG2} xj_{PRi,REG,REG2,t} \\
yl_{PRk,REG,t} &= yl_{PRK,REG,t--1} + xk_{PRk,REG,t} \\
&- \sum_{REG2} xl_{PRk,REG,REG2,t} \\
ym_{PRk,REG,t} &= ym_{PRk,REG,t--1} - xm_{PRk,REG,t} \\
&+ \sum_{REG2} xl_{PRk,REG2,REG,t}
\end{aligned} \tag{5.54}$$

The model has been implemented in the software GAMS/CPLEX in order to verify the validity of the model. The implemented GAMS code is presented in appendix F.1. The validation and verification process is described in the following section H.

5.4 Validation and verification

To ensure that all elements and parameters in the model are correct, the simplified miniminitest model considering only the necessary complexity is validated and verified. This section contains a brief summary of the performed validation and verification. For details, please see appendix H.

The verification is performed by comparing the individual equations set up by GAMS in the output list files (see CD appendix for the individual scenarios) with the mathematical formulation given in the equations 5.33 - 5.54. The implemented GAMS code is given in appendix F.1 and the implemented data is presented in H.2.

Furthermore, a main part of the validation process consists of evaluating the model with respect to the described processes in chapter 3. This work has been done in cooperation with the employees at Novozymes through presentations of the model structures and elements.

A formalized verification is performed by changing individual parameters and evaluating if the results are as expected. The individual constraints are tested to see if the model performs correct calculations when changing a single parameter while keeping the other parameters constant. This is done with simple test data where different cases are tested. The simplicity of the problems ensures that it is possible to assure that the correct calculations are performed. Even in the very simplified model, the combined effects of changing several parameters concurrently are difficult to predict. The test data do not represent any real life data, but are constructed to the verification scenarios.

The details regarding the test cases, input and output data are presented in appendix H. The verification is performed with the miniminitest model (described in section 5.3) through the following cases. Though some of the equations are possible to evaluate in several cases (i.e. equation 5.33 for all scenarios) the focus has been on a selection of the equations for each case. For each case the specific equations tested are presented.

1. The simple case, where production takes place in individual regions only. (Verification of: 5.33-5.44, 5.50 and 5.53 - 5.54).
2. Transfer pricing and the effects of cross regional flow. (Verification of: 5.40, 5.41, 5.42 and 5.51).
3. International taxation and the non-linearities of tax functions. (Verification of: 5.34, 5.35 and 5.36).
4. Import duties. (Verification of: 5.41 and 5.42)
5. Export value added taxes. (Verification of: 5.41 and 5.42)
6. Royalties. (Verification of: 5.43 and 5.44)
7. Converging and diverging material flows with BoM constraints. (Verification of: 5.53)
8. Storage of products while considering multiple time periods. (Verification of: 5.39, 5.52 and 5.54)
9. Financial cost of storing and distributing. (Validation of: 5.45, 5.46, 5.47, 5.48 and 5.49)

This formalised verification shows that the above mentioned equations (5.33 - 5.54) are implemented as intended and the correct calculations are performed. The validation was made on the *miniminitestmodel*. However, for the *general model* and the *fullmodel* several of the same equations are reused and the test conclusion on the small scale model can be considered valid for the larger models as well. The relationships between the models are presented in the following section 5.5.

5.5 Comparison of the Theoretical Models

The similarities between the three models - the general model, the full model and the miniminitest model - are presented below. This enables to compare the verified equations from the previous section with the equations in the larger models.

In the larger networks the individual equations for nodes and arcs are used multiple times. This leads to i.e. production costs consisting of four similar elements in the full model (eq. 5.6)

$$\begin{aligned} ProdC_{REG} = & \sum_{PRik,t} xi_{PRik,REG,t} PCi_{PRik,REG} \\ & + \sum_{PRkl,t} xk_{PRkl,REG,t} PCk_{PRkl,REG} \\ & + \sum_{PRmn,t} xm_{PRmn,REG,t} PCm_{PRmn,REG} \\ & + \sum_{PRopq,t} xo_{PRopq,REG,t} PCo_{PRopq,REG} \end{aligned}$$

compared to only two elements in the miniminitestmodel. (eq. 5.38)

$$\begin{aligned} Prodc_{REG} = & \sum_{PRi,t} xi_{PRi,REG,t} PCi_{PRi,REG} \\ & + \sum_{PRk,t} xk_{PRk,REG,t} PCk_{PRk,REG} \end{aligned}$$

If the equations work for one production or storage phase, one should expect the same approach to work for several similar phases as well. In table 5.1 the relations between the main evaluated equations in the test cases and the corresponding equations in the larger models are shown.

Through the nine test cases, it was shown that the linear models of the general supply network problem perform as intended. Each feature was only tested with one test case with simplified data. A more detailed analysis of the performance will not explicitly be carried out in this report.

General model	**Fullmodel**	**Testmodel**	**Test case**
5.1	5.1	5.33	1
5.2	5.2	5.34	1, 3
5.3	5.3	5.35	1, 3
5.4	5.4	5.36	3
5.5	5.5	5.37	1
5.6	5.6	5.38	1
5.7	5.7	5.39	1, 8
5.8	5.8	5.40	2
5.9	5.9	5.41	2, 4
5.10	5.10	5.42	2, 5
5.11	5.11	5.43	6
5.12	5.12	5.44	6
5.13	5.13	5.45	9
5.14	5.31	5.46	9 ⋇
5.15	5.32	5.47	9 ⋇
5.16	5.27	5.48	9 ⋇
5.17	5.28	5.49	9 ⋇
5.18	5.29	-	9 ⋇
5.19	5.30	-	9 ⋇
5.20	5.20	5.50	1
5.21	5.21	5.51	2
5.22	5.22	5.52	8
5.23	5.23	5.53	1, 7
5.24	5.24	5.54	1, 8
5.25	-	-	
5.26	-	-	

Table 5.1: Relations between evaluated cases and the different models. The validation of (⋇) is only valid for the linear models: *the full model* and *the miniminitestmodel*.

Based on the structure of the generic full model for a supply chain, adjustments to the real life network is done in the following chapter 6. To preserve transparency the set of regions is reduced to only consider three regions. Furthermore, the product set is limited to only consider 20 products.

Changes to the model are not explicitly validated and verified through test cases, but a general evaluation of the resultŠs validity has been performed in order to check the model. For each model run in the real life scenarios (chapter 7) the results are evaluated and reviewed to ensure that feasible solutions are found. These evaluations are not explicitly described in the report but performed during the analyses of the results.

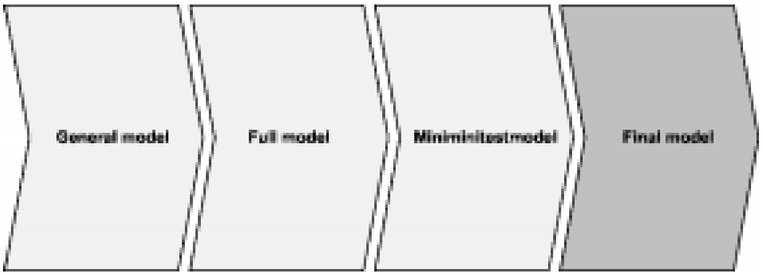
Chapter 6

The Real World

The three models described in the previous sections (*the general model*, *the full model* and *the miniminitestmodel*) are based on a theoretical perspective for the production and distribution structure. The models are generic for similar networks and implicitly assume that the necessary data for each process step, each facility and each product is available in the required input format. However, in real life necessary input data may not exist in the required form.

To get data, measurements are necessary. If the required data structure differs from the structure of the known data in a company, it might be difficult to gather the correct information. Due to the fact, that not all the necessary data are available in this specific case, and as it is outside the boundaries of this project to undertake the immense task of measuring and gathering data, the existing available data are used.

In the following sections the necessary adjustments to the theoretical models - based on the constraints in the available data - are presented. In section 6.1 the model changes are presented, leading to the final model. In section 6.2 the data analysis and preparation for implementation is presented. Finally the implementation in GAMS/CPLEX is shortly described in section 6.3.



6.1 Model Adjustments

To achieve a usable result with the existing data structures some changes are necessary in the model. All changes are case specific and are based on the available data. The changes will reduce the precision of the model but are necessary to obtain a solution for the real life problem.

The available data for 20 products are examined and evaluated in order to identify the necessary changes in the model and discover which data can be used directly and which data that will involve changes and approximations. The implemented changes lead to the final model that is used for modelling the specific case. In the following section the implemented changes are presented.

Bill of Materials (BoM)

Theoretically the process steps and the ingredients for the same product are assumed to be exactly the same across different facilities. But due to differences in the production equipment and the abilities and experience of the operators in the different regions this is not the fact. The BoM for identical products in different regions vary significantly and can therefore not be considered region independent. The bills of materials must consider not only the product but also the location of production.

Modelling this is done by adding the set of regions to the index of the BoM thereby making it specific for each region; i.e. $Prodmix_{PRik,PRkl,REG}$. The main consequence of this is a larger data input, where the amount of BoM data will be multiplied by the number of regions introduced in the model. This may seem like an unnecessary expansion of the model size, but this feature is considered essential in order to obtain realistic and usable results from the model. These changes influence the production flow constraints (equation 5.23) and the financial distribution and storage cost (equation 5.32 and 5.31) as the average product values are influenced.

The new equations for the financial storage costs (equation 6.1) and the financial distribution costs (equation 6.2) are:

$$\begin{aligned} FinStoC_{REG,t} &= \sum_{PRkl} ProVkl_{PRkl} yl_{PRkl,REG,t} \kappa \\ &+ \sum_{PRmn} ProVmn_{PRmn} yn_{PRmn,REG,t} \kappa \\ &+ \sum_{PRopq} ProVopq_{PRopq} yp_{PRopq,REG,t} \kappa \\ &+ \sum_{PRopq} ProVopq_{PRopq} yq_{PRopq,REG,t} \kappa \end{aligned} \tag{6.1}$$

$$\begin{aligned} FinDistC_{REG,REG2,t} &= \sum_{PRkl} ProVkl_{PRkl} xl_{PRkl,REG,REG2,t} \lambda \\ &+ \sum_{PRmn} ProVmn_{PRmn} xn_{PRmn,REG,REG2,t} \lambda \\ &+ \sum_{PRopq} ProVopq_{PRopq} xp_{PRopq,REG,REG2,t} \lambda \end{aligned} \tag{6.2}$$

$with\ \kappa = (1 + Dfcost)^{Stime} - 1$
$and\ \lambda = (1 + Dfcost)^{Dtime_{REG,REG2}} - 1$

For storage or distribution of products in the phases l, n, p or q, the product values depend on the origin of the sub-components. For product $PRkl$ in phase l the average product value is:

$$\begin{aligned} & ProVkl_{PRkl,t} \\ &= \frac{1}{card_{REG}} \Big(\sum_{REG} PCk_{PRkl,REG} ER_{REG} \\ &+ \sum_{PRik*} PCi_{PRik,REG} ER_{REG} Prodmixk_{PRkl,PRik,REG} \Big) \end{aligned} \tag{6.3}$$

$\forall_{REG=PROREG}$
$^{*}\forall_{PRik|Prodmixk_{PRkl,PRik,REG}\neq 0}$

In the succeeding phase, n, the average product value for product $PRmn$ is:

$$\begin{aligned}
&ProVmn_{PRmn,t}\\
&= \frac{1}{card_{REG}}(\sum_{REG} PCm_{PRmn,REG}ER_{REG}\\
&+ \sum_{PRkl^{*}} (PCk_{PRkl,REG}ER_{REG}Prodmixm_{PRmn,PRkl,REG}\\
&+ \sum_{PRik^{*2}} PCi_{PRik,REG}ER_{REG}Prodmixk_{PRkl,PRik,REG}))
\end{aligned} \tag{6.4}$$

$\forall_{REG=PROREG}$
$^{*}\forall_{PRkl|Prodmixm_{PRmn,PRkl,REG}\neq 0}$
$^{*2}\forall_{PRik|Prodmixk_{PRkl,PRik,REG}\wedge Prodmixm_{PRmn,PRkl,REG}\neq 0}$

Finally the average value of product $PRopq$ after production phase o is:

$$\begin{aligned}
&ProVopq_{PRopq,t}\\
&= \frac{1}{card_{REG}}(\sum_{REG} PCo_{PRopq,REG}ER_{REG}\\
&+ \sum_{PRmn^{*}} (PCm_{PRmn,REG}ER_{REG}\\
&+ \sum_{PRkl^{*2}} (PCk_{PRkl,REG}ER_{REG}Prodmixm_{PRmn,PRkl,REG}\\
&+ \sum_{PRik^{*3}} PCi_{PRik,REG}ER_{REG}Prodmixk_{PRkl,PRik,REG})))
\end{aligned} \tag{6.5}$$

$\forall_{REG=PROREG}$
$^{*}\forall_{PRmn|Prodmixo_{PRopq,PRkl,REG}\neq 0}$
$^{*2}\forall_{PRkl|Prodmixm_{PRmn,PRkl,REG}\wedge Prodmixo_{PRopq,PRmn,REG}\neq 0}$
$^{*3}\forall_{PRik|Prodmixk_{PRkl,PRik,REG}\neq 0}$
$_{\wedge Prodmixm_{PRmn,PRkl,REG}\wedge Prodmixo_{PRopq,PRmn,REG}\neq 0}$

The BoM constraints for production nodes are changed to consider the regional bills of materials. The changes lead to the following equations:

$$\begin{aligned} xi_{PRik,REG,t} &= \sum_{PRkl} xk_{PRkl,REG,t} Prodmixk_{PRkl,PRik,REG} \\ \sum_{REG2} xl_{PRkl,REG2,REG,t} &= \sum_{PRmn} xm_{PRmn,REG,t} Prodmixm_{PRmn,PRkl,REG} \\ \sum_{REG2} xn_{PRmn,REG2,REG,t} &= \sum_{PRopq} xo_{PRopq,REG,t} Prodmixo_{PRopq,PRmn,REG} \end{aligned} \tag{6.6}$$

Capacities

To reduce the complexity of the model the capacities for each facility in each production and storage phase are aggregated. In reality some products are dedicated to specific production lines however this is neglected in the model. Since the model only creates a rough cut production plan the detailed planning for each production line is not taken into account. At the tactical-strategic planning level incorporation of every detail from a diverse production and distribution system will result in unnecessary complexity. The detailed planning of the individual lines is therefore a task at lower planning levels (as described in section 4.2). A dynamic planning and re-planning procedure is therefore considered acceptable if the plan is not feasible.

Production capacities are time dependent as vacation - especially during the summertime on the northern hemisphere - results in a reduced production capacity. This is implemented in the model by making the production capacities time dependent, $Capi_{REG,t}$. This change only influences the capacity constraints (equation 5.21).

Capacity utilisation

The theoretical model is based on a strict link between the quantities of produced goods (semi-finished or finished products) in mass or volume and the corresponding production capacity usages given in mass or volume. In the case the given data are not structured in this way.

In the available data, capacity usages are based on the amount of finished goods. This means that the corresponding capacity usage in i.e. fermentation and recovery is given per unit of finished product and not as a function of the actual amount of fermented or recovered enzymes.

With these measures, modelling a product flow that follows the current process steps within a region is straight forward. However, as all capacity usages are given per unit finished goods a different flow between regions is difficult to model, as capacity usages per unit finished goods are region dependent (due to BoM differences). Though some products are based on the same subcomponent the available data on capacity usage is dependent on the final product and not only the amount of sub-components at each facility.

For a product where all production takes place in region $R1$ all capacity usages are only given for the $R1$ production facilities. Therefore alternative plans are difficult to evaluate as the capacity usage at production facilities in other regions are un-known due to different BoMs and yields. Modelling capacity utilisation per finished goods is therefore not considered appropriate for this model.

This implies that a conversion from the capacity usage per unit finished enzymes to the corresponding capacity usage per unit sub-product is necessary. In the model this is implemented with the capacity utilisation factor, $Cuf_{PR,REG}$, given for every product, at every process phase in every region. The factor introduces the link between the flow of mass or volume, and the amount of capacity usage and influences the capacity constraint equations 5.21. The final production capacity constraints are:

$$\begin{aligned}
\sum_{PRik} xi_{PRik,REG,t} Cufi_{PRik,REG} &\leq Capi_{REG,t} \\
\sum_{PRkl} xk_{PRkl,REG,t} Cufk_{PRkl,REG} &\leq Capk_{REG,t} \\
\sum_{PRmn} xm_{PRmn,REG,t} Cufm_{PRmn,REG} &\leq Capm_{REG,t} \\
\sum_{PRopq} xo_{PRopq,REG,t} Cufo_{PRopq,REG} &\leq Capo_{REG,t}
\end{aligned} \tag{6.7}$$

The amount of storage space taken up by a product depends on the packaging. This means the same product packed differently, will have different storage capacity usages. To convert the given quantity of products stored (in kg or L) to the unit of available capacity, the capacity utilisation factor for storage facilities is implemented in the capacity constraints, equation 5.22. The final storage capacity constraints are:

$$\begin{aligned}
\sum_{PRkl} yl_{PRkl,REG,t} Cufl_{PRkl,REG} &\leq Capl_{REG,t} \\
\sum_{PRmn} yn_{PRmn,REG,t} Cufn_{PRmn,REG} &\leq Capn_{REG,t} \\
\sum_{PRopq} yp_{PRopq,REG,t} Cufp_{PRopq,REG} &\leq Capp_{REG,t} \\
\sum_{PRopq} yq_{PRopq,REG,t} Cufq_{PRopq,REG} &\leq Capq_{REG,t}
\end{aligned} \tag{6.8}$$

Transfer Pricing

The model uses fixed transfer prices between specific regions for a given product. Due to case specific policies for transfer pricing an exception to the transfer cost and transfer turn over is implemented. This ensures that transfer pricing on products shipped from region $R3$ to region $R2$ are settled via region $R1$. The triangular trade is implemented in the extended transfer equations 6.9 and 6.10.

$$
\begin{aligned}
& TTO_{REG} \\
& = \sum_{PRkl,REG2,t} xl_{PRkl,REG,REG2,t} TPl_{PRkl,REG,REG2} \gamma \\
& + \sum_{PRmn,REG2,t} xn_{PRmn,REG,REG2,t} TPn_{PRmn,REG,REG2} \gamma \beta \\
& + \sum_{PRopq,REG2,t} xp_{PRopq,REG,REG2,t} TPp_{PRopq,REG,REG2} \gamma \beta \\
& \\
& + \sum_{PRkl,t,REG=R3} xl_{PRkl,REG,''R2'',t} TPl_{PRkl,REG,''R1''} \gamma_R 1 \\
& + \sum_{PRmn,t,REG=R3} xn_{PRmn,REG,''R2'',t} TPn_{PRmn,REG,''R1''} \gamma_R 1 \beta_{R1} \\
& + \sum_{PRopq,t,REG=R3} xp_{PRopq,REG,''R2'',t} TPp_{PRopq,REG,''R1''} \gamma_R 1 \beta_{R1} \\
& \\
& + \sum_{PRkl,t,REG=R1} xl_{PRkl,''R3'',''R2'',t} TPl_{PRkl,REG,''R2''} \gamma_{R2} \\
& + \sum_{PRmn,t,REG=R1} xn_{PRmn,''R3'',''R2'',t} TPn_{PRmn,REG,''R2''} \gamma_{R2} \beta_{R2} \\
& + \sum_{PRopq,t,REG=R1} xp_{PRopq,''R3'',''R2'',t} TPp_{PRopq,REG,''R2''} \gamma_{R2} \beta_{R2}
\end{aligned}
\tag{6.9}
$$

$with\ \beta = \frac{ER_{REG2}}{ER_{REG}}$,
$\beta_{R1} = \frac{ER_{''R1''}}{ER_{REG}}$,
$\beta_{R2} = \frac{ER_{''R2''}}{ER_{REG}}$,
$and\ \gamma = (1 - ExVat_{REG,REG2})$,
$\gamma_{R1} = (1 - ExVat_{REG,REG2=''R1''})$,
$\gamma_{R2} = (1 - ExVat_{REG,REG2=''R2''})$.

$$\begin{aligned}
&TC_{REG}\\
&= \sum_{PRkl,REG2,t} xl_{PRkl,REG2,REG,t} TPl_{PRkl,REG2,REG} \zeta \beta\\
&+ \sum_{PRmn,REG2,t} xn_{PRmn,REG2,REG,t} TPn_{PRmn,REG2,REG} \zeta\\
&+ \sum_{PRopq,REG2,t} xp_{PRopq,REG2,REG,t} TPp_{PRopq,REG2,REG} \zeta\\
&\\
&+ \sum_{PRkl,t,REG=R1} xl_{PRkl,"R3","R2",t} TPl_{PRkl,"R3",REG} \zeta_{R3} \beta_{R3}\\
&+ \sum_{PRmn,t,REG=R1} xn_{PRmn,"R3","R2",t} TPn_{PRmn,"R3",REG} \zeta_{R3}\\
&+ \sum_{PRopq,t,REG=R1} xp_{PRopq,"R3","R2",t} TPp_{PRopq,"R3",REG} \zeta_{R3}\\
&\\
&+ \sum_{PRkl,t,REG=R2} xl_{PRkl,"R3",REG,t} TPl_{PRkl,"R1",REG} \zeta_{R1} \beta_{R1}\\
&+ \sum_{PRmn,t,REG=R2} xn_{PRmn,"R3",REG,t} TPn_{PRmn,"R1",REG} \zeta_{R1}\\
&+ \sum_{PRopq,t,REG=R2} xp_{PRopq,"R3",REG,t} TPp_{PRopq,"R1",REG} \zeta_{R1}
\end{aligned} \tag{6.10}$$

$with\ \beta = \frac{ER_{REG2}}{ER_{REG}}$,
$\beta_{R1} = \frac{ER_{REG2="R1"}}{ER_{REG}}$,
$\beta_{R3} = \frac{ER_{REG2="R3"}}{ER_{REG}}$,

$and\ \zeta = 1 + Dut_{REG2,REG}$,
$\zeta_{R1} = 1 + Dut_{"R1",REG}$,
$\zeta_{R3} = 1 + Dut_{"R3",REG}$

6.1.1 Mathematical Model

The implemented changes result in the final model. The entire model is not presented here but can be seen in appendix G.1. The model is a linear model that represents the specific case and consists of the previously described equations. As summary is given below:

- Overall objective: equation 5.1
- The regional net profit: equation 5.2
- The regional profit before tax: equation 5.3 and 5.4
- Turn over: equation 5.5
- Production costs: equation 5.6
- Storage costs: equation 5.7
- Distribution costs: equation 5.8
- Transfer pricing: equation 6.9 and 6.10
- Royalties: equation 5.11 and 5.12
- Financial costs: equation 5.13, 6.2 and 6.1
- Average product values: equation 5.27, 6.3,6.4 and 6.5.
- Customer demand: equation 5.20
- Storage capacity constraints: equation 6.8
- Production capacity constraints: equation 6.7
- BoM constraints: equation 6.6
- Storage flow constraints: equation 5.24

6.2 Data Analysis

The final model enables the use of the available data. However the structure, quantity and quality of the data necessitate analyses and adjustments before implementation is possible. In the following section the input data are generated based on sales data and BoM data from the ERP-system, as well as the historical used capacity utilisation rules. In the following section only examples of the analyses are given. The data and the individual analyses are only available in the company specific appendix.

Products

For data input 20 final products are modelled, given the names $P10001$ to $P10020$. The products are chosen from the company portfolio and represent products with high sales quantities and with production and sales in more regions. All data are based on available historical information which enables the comparison of the optimal solution with the historical. Between each production phase a different product set is implemented, i.e. $PRmn$ for formulated products between the phases m and o. The product set PR represents all products throughout all phases,

$$\begin{aligned} PR &= PRik, PRkl, PRmn, PRopq \\ &= P1...P21, P101...P121, P1001...P1021, P10001...P10021 \end{aligned}$$

Similar products with different product numbers are evaluated one-by-one and aggregated to represent the same product in $PRopq$. Furthermore the different product numbers are evaluated to identify the subcomponents and their origin (factory ID). The product name-number key is given in the company specific appendix.

Time periods

Each time period in the set $t = \{t_1, t_2, t_3, t_4\}$ represents a three month period. It is assumed that production and distribution of a product within the given time period is possible without the use of storage. This indirectly implies that a demand that occurs in the beginning of a time period can not be fulfilled through a production within the same time period due to lead times. The production initiation times in order to fulfil the demands should therefore be displaced in time in order to achieve feasible solutions.

As the model deals with strategic and tactical resource planning, the time displacement is not considered further. As demand and capacities do not vary significantly between the individual time periods the assumption that both production plans and demand fulfilment within a time period is feasible is considered as a minor approximation. The time displacement is a planning problem at lower levels and is not considered further in the model and the data.

BoM data

A typical bill of material is a region specific description of the individual components used in the production in the individual process steps. For different regions and different products the process steps vary. The previously described four production steps (see section 3) are sufficient for modelling some products while other BoMs are divided into fewer or more steps. The different BoMs are treated the following way:

- If a product goes through more phases than the four steps (fermentation, recovery, blending and formulation), the individual phases are analysed and aggregated. This implies that i.e. a two-phase recovery process will be considered as one.

- If a product goes through less phases than four the BoM for the additional phases are considered as one-to-one ratio between the input and output. The result corresponds to a name change of the product, i.e. from $P1001$ to $P10001$.

In some cases the same product name has different product numbers and BoMs in the same region. In these instances the BoM of the material with the highest sales volume is used. Furthermore it should be noted that the BoMs are not only dependent on the region but also on the origin of the semi-finished products used in the production. In example the yield of the recovery process ($PRkl$) to produce a given amount of finished products ($PRopq$) can differ significantly between regions. If the semi-finished products are shipped between regions with significant different BoMs this will result in flaws in the capacity usage and costs. This could be solved by expanding the BoM data to include the origin and thereby adding an extra dimension to the *prodmix* tables. However the BoM differences are assumed of minor importance and the approximation of only producing region dependent BoMs is considered acceptable.

The bills of materials are given per 10.000 kg finished enzymes. This is not considered appropriate in the model as the interpretation of the actual flows between regions will become difficult. To enable a combination of semi-finished products with different origins a bill of materials for each production phase is necessary. For all products the output per finished quantity is converted into the corresponding output per production phase. One example of the conversion is presented below.

Example: conversion of the BoM for product P10005

Figure 6.1 illustrates the analysis of the bill of materials for product $P10005$ in a specific region. All flow quantities are given pr 10.000 kg $P10005$.

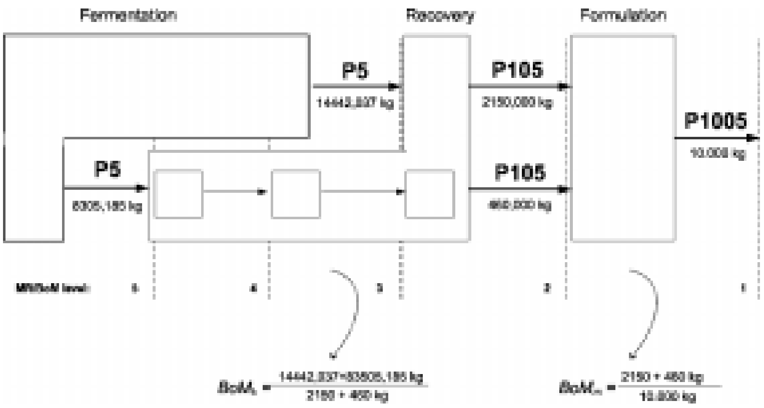


Figure 6.1: BoM conversion for $P10005$

Through analysis of the detailed production steps an aggregation of some of the flows is performed, i.e. a recovery process in several steps. Furthermore, the parallel flows indicate a divergent and convergent flow for this product. However, for this specific case is has been chosen to model the separate flows in and out of the recovery process as two single in- and outgoing flows.

The bill of material is the ratio between the ingoing and outgoing quantity for a production phase. The BoM for the recovery phase, is i.e. the ratio between the ingoing quantity (14442 kg + 8305 kg) and the outgoing quantity (2150kg + 460 kg). The recovery BoM is implemented as the parameter $Prodmixk_{PRkl,PRik,PROREG}$. The BoM for the formulation is found in a similar way.

There is no blending phase for this product. The bill of material for the blending phase o, $Prodmixo_{PRopq,PRmn,PROREG} = 1$, therefore only results in a name change from $PR1005$ to $PR10005$ for $PR1005 \in PRmn$ and $PR10005 \in PRopq$.

Capacities

To define the capacity utilisation factors $Cufx$ and the capacity limits $Capx$, the historical BoMs and capacity utilisation rules are used.

As only a smaller part of the product portfolio is implemented the use of real life capacities is not reasonable. Therefore the available capacities are reduced to represent the part of the capacity available for the chosen product range. As products which can only be produced in one specific region are not modelled, the corresponding capacity taken up by this production is deducted from the aggregated capacities.

The capacity usage and limits are stated in different kinds of units and given per 10.000 kg finished product. To be able to handle the individual flow between the nodes, the corresponding capacity usage per semi-finished product is found. This way the capacity usage for each product at each phase is linked with the amount of goods moving from phase to phase.

The capacity usages are available in different kinds of units, i.e. numbers of $40m^3$ tank-hours for the fermentation phase. These units are used for a number of reasons - primarily historical. All the fermentation tank facilities are build in $40m^3$ units or a multiple of $40m^3$, i.e. $80m^3$ or $160m^3$ and the staff therefore all uses this unit as a basis for their calculation of capacity use. This means that the capacity usages are defined in a certain number of $40m^3$ tank hours pr 10.000 kg finished good.

The historical use of $40m^3$ tank hours for the capacity usage is used in the model in order to be able to communicate the results. Defining the outgoing flow variable from fermentation in this unit raises the question of how much storage a number of $40m^3$ tank hours takes up. For this reason it is considered necessary to model the actual flow of mass or volume and link the corresponding capacity use to this flow. This will ensure a model structure much closer to reality and with a greater degree of transparency.

Several factors influence the capacity utilisation time pr unit. The total lead time for one process step will depend on the campaign and batch sizes. The given data represent an average time including the set up times between batches and clean out times between campaigns and are based on the historical production set up.

The given capacity usages in $40m^3$ tank hours per 10.000 kg finished product $PRopq$ are converted into the corresponding tank hours per unit fermented product $PRik$. The conversion is based on the same principle as the above-mentioned BoM-conversion and an example for product $P10005$ is presented below.

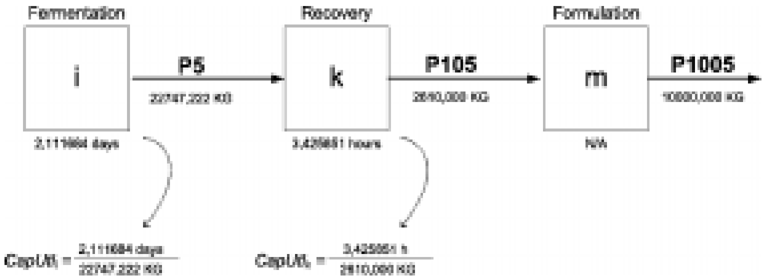


Figure 6.2: Capacity utilisation conversion for $P10005$

Example: conversion of the capacity usage factor for product P10005

The fermentation and recovery usage pr 10.000 kg is 2,111684 $40m^3$ days and 3,425851 recovery hours respectively. To find the corresponding capacity utilisation factors for the individual phases, the ratio between the capacity usage and the output flow is found. For product $P10005$ the outgoing flow from each production node is the batched quantities from figure 6.1. The calculations of the Cuf-factors for $P10005$ is shown in figure 6.2. For the formulation process the capacity utilisation is measured in the product flow unit.

Demand

Demand is assumed to be deterministic and is based on the historical sale data for the given products $PRopq$. The sales data are extracted for the different regions $REG = \{R1, R2, R3, R10\}$ and the individual product numbers are grouped for the individual products in $PRopq$ in order to identify the total quantity within a given region. The sales data are generated for three month periods - corresponding to the time steps given by the set t in the full model. In reality a further product diversification than modelled is achieved by the different packaging. This process step is not modelled and the given demands are therefore aggregated to present the corresponding number of finished products.

Financial Cost (DFcost)

This input represents the monthly interest rates for the financial cost on tied up capital in stored or distributed products. This rate is the interest rate possible to obtain for the company when investing in other portfolios than the products being stored or distributed. In short it is the cost of not investing in the alternatives.

It is considered that an interest rate of 12% p.a. is always achievable on alternative investments. This corresponds to an interest rate of 0,0095% per month, which is incorporated into the model.

Distribution cost

The cost of distributing is generated from the shipping costs for the different types of transportation (as presented in chapter 3) between different regions. For all products in all phases where distribution is possible, the transportation costs per product unit is calculated and used as input data, DCx. For the reefers tanks there is an additional daily cost per tank per day. Since the transportation times for given routes are known the cost is added evenly to the total transportation price per tank. With the assumption that all distribution takes place in full tanks, the corresponding cost allocation of fixed cost per product volume is added to the given transportation prices pr kg.

It is assumed that each product can be related to a specific transportation type and the relation is found through analysis of the BoM information in order to identify what modality is used. The use of one specific transportation form for a product is a minor assumption as there may be more possible ways of distributing the product. The following approach is used to relate transportation form and product:

- All concentrates (products from the set $PRkl$) - where the finished products is in liquid form - are shipped in reefer tanks; if data for reefer tanks are not available, 40R schutz are assumed to be used. All concentrates for granulation are assumed to be transported in 40R schutz, except to region $R2$ where reefer tanks are used.

- Finished products in solid form (from the set $PRmn$ or $PRopq$) which are either formulated or blended can be shipped with 20 feet containers in big bags. Each big bag contains 1000kg and the container can hold 20 big bags. If data for 20D big bags are not available for a specific combination of regions, data for 40D schutz are used instead.

- Finished products in liquid form (from the set $PRmn$ or $PRopq$) are shipped by 40 feet containers in refrigerated schutz tanks (40D schutz), except to region $R2$ where reefer tanks are used. Each schutz contains 1000kg products and there can be 20 schutz in each container.

Distribution time

In the model only distribution by ship is implemented. Distribution by air is not desirable because of the costs and is therefore not modelled as a planning option. The time for land distribution is neglected due to the transportation time compared to the length of the model time periods (3 months).

The distribution time used when shipping goods between facilities in the model is a rough average of the sailing times in month. Due to the length of time periods used in the model the exact sailing time in days is not important. The distribution time is defined between specific regions. Since distribution time is only used to calculate the financial distribution cost and no production lead times are implemented, it is considered reasonable to use the rough time measures.

Duty

Import duties between the different regions are given as a fraction of the import cost for the receiving region and are well documented in the data provided.

Exchange rate

Information concerning the currency exchange rates is based on the companys historical standard rates. It can be observed that these rates do not necessarily correspond to the latest exchange rates.

For modelling of future scenarios based on forecast, the use of standard rates will be implemented. The reason for this is the companys use of currency hedging. In short these hedging contracts fix the future rate of a currency in a given time period. The idea is to protect the company against sudden and/or great changes in the future exchange rates and thereby minimizing the risks when creating budgets. Furthermore, strategic and tactical decisions are not based on the day to day currency rates.

Exvat

Export value added taxes for the different regions are given as a fraction of income from export. The export value added taxes are well documented.

Prices

Prices for the different products are not fixed but will depend on the individual sales orders. To use reasonable sales prices in the model average sales prices are generated. For each sales order for a given region the relation between the turnover and the sold quantity is found - representing the individual sales prices. The average sales prices, $Price_{PR,REG}$, are generated as the average over all periods for each region.

Production Cost

The production costs at the individual facilities for different production phases exist as the costs of raw materials and energy consumption. These cost factors only represent the (direct) variable costs, but several other cost factors may be important as a decision parameter. The calculation of indirect costs, fixed and variable, can be important when modelling longer time periods and may have a high influence when comparing different regions. The implemented costs of production can be viewed as an abstract measure of production costs serving the purpose of making it possible for the model solver to pick the cheapest way to produce. The costs are not a true expression of the actual costs of producing.

The cost pr 10.000 kg finished product is provided in the bill of materials for each individual product number. These costs are converted to represent the cost pr unit of product flow between the individual nodes. The conversion is based on the product specific bill of materials and follows the same patterns as the conversions of BoM and capacity data. An example of the conversion for product $P10005$ is given in following example.

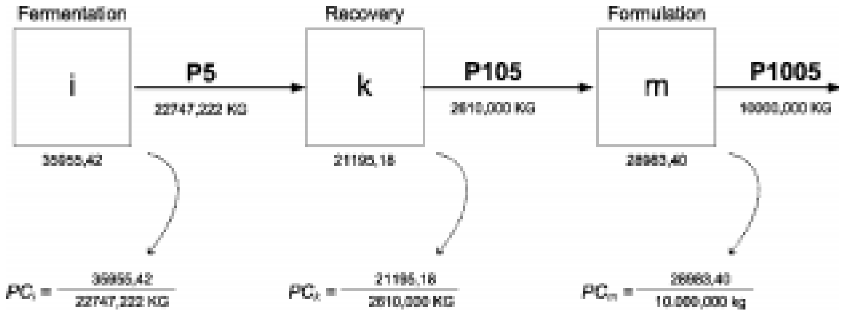


Figure 6.3: Production costs PC conversion for $P10005$

Example: conversion of the production costs for product P10005

Figure 6.3 shows the conversion of the production cost pr unit finished product, $P10005$, to the corresponding production cost pr process phase. The production cost pr 10.000 kg finished product $P10005$ are presented below the individual nodes, i.e. 35955,42 for phase i. Since the flow quantities between the individual nodes corresponds to 10.000 kg finished good, the costs pr node are found as the ratio between the production cost and the flow quantity. Since product $P10005$ only goes through the fermentation, recovery and formulation processes there are no production costs for blending, $PCo = 0$.

Royalties

The royalties serve as a way of balancing the R&D costs between regions where no or little research and development takes place and regions where products are developed.

Storage time

Products are made to stock and the storage time for the individual products vary between time periods. The storage level and time depends on the quality of the forecasts, the detailed operations management and the realized orders. The company bases their production management on the lean philosophy focusing on lead time and stock reductions.

Exact storage times and levels for the individual products are not implemented in the model as this will highly depend on the detailed planning on lower planning levels. The model is developed around time periods of three months with the possibility of producing and distributing the products within a time period. When demand within a time period exceeds the production capacity the use of storage facilities may be initiated. As the real storage times are not implemented, the additional costs do not represent real storage costs but can be seen as a penalty cost for storing products. This complies with the lean philosophies of the company where reductions of the storage quantities are desired.

All products stored are assumed to be stored for an average of three months, $Stime = 3$.

Storage cost

The storage costs are stated for all products in all regions at all storage facilities (phase l, n, p or q). The costs represent the quarterly cost corresponding to the above mentioned three month period. The annual costs pr unit (kg or L) in region $R1$ currency are approximately 1 for normal storage and 3 for refrigerated products. This corresponds to a storage cost of $SC = 250$ respectively $SC = 750$ pr 1000 kg product pr quarter. The regional storage costs in local currencies are generated based on the companys standard currency exchange rates.

The storage costs are generated for each product through an analysis of the individual products. All products in liquid form are supposed to require cooling while solid products do not have this requirement. Some product do not fall into these two categories, however this is a smaller insignificant approximation.

The amount of storage space taken up by a product depends on the packaging. This means the same product packaged differently will have a different storage cost. The distribution of how the products are packaged is not available. There will of course be a minor approximation of storage costs involved in this process, as the predicted distribution of how a product are stored differs slightly from the actual situation at the stocks. However as storage costs in different regions do not differ significantly this will not have a high impact on the production allocation in an optimal solution.

Tax

Tax rates for the individual regions are assumed to be linear and fixed for all positive profits. The rates are available for each region and no special data treatment is necessary for implementation.

Transfer prices

The transfer prices are fixed for individual products sent between specific regions and are calculated based on the inter-regional rules. The transfer prices are generated for all products and regions in phases where product transfer is possible.

For concentrates, $PRkl$, the transfer prices are a function of the variable costs given by the production costs within a region. The prices are given in the sending regions currency.

For finished goods, $PRmn$ and $PRopq$, the transfer prices are a function of the sales prices (based on the generated average sales prices). The transfer prices for formulated and blended goods are treated as finished goods given in the receiving regions currency.

6.3 Implementation

The final model described in section 6.1 and the input data described in section 6.2 have been implemented in the General Algebraic Modeling System (GAMS). The GAMS models - corresponding to the previously described mathematical models - are presented in appendix F.1(miniminitest model) and appendix G.2(final model).

GAMS is specifically designed for modelling linear, non-linear and mixed integer optimisation problems (Source: [44] and [45]). The programme is powerful for validating and verifying different models under different constraints and furthermore enables the evaluation of different scenarios. However, as a tool for delivering optimal production plans on daily basis it is less suitable due to lack of user interfaces and integration with production planning systems.

In the GAMS the mathematical problem is specified, given by the

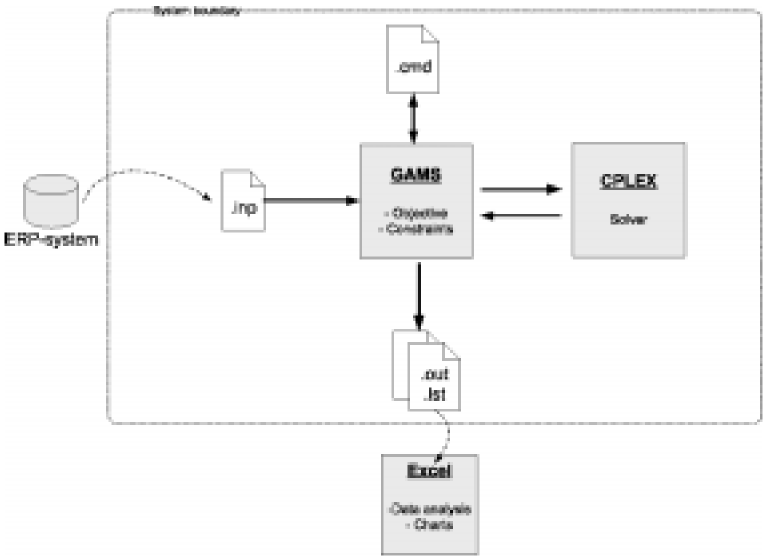


Figure 6.4: Programme structure

- sets,
- variables,
- equations (objective function and constraints)

and the problem type - i.e. maximize the linear problem. Data are entered in lists and table forms and the models are described in algebraic statements. GAMS does not solve the problem but generates the input - in form of each constraint equation - to a solver. The output of the solver can be evaluated to identify the level of each variable, their marginal values and the constraining equations. For the case problem, the linear solver from CLPEX has been used. The structures and links between programmes used to solve the problem is presented in figure 6.4.

The relation between the individual steps and system boundaries in figure 6.4 are as follows:

- The company specific data from the ERP-system are manually evaluated and converted into input sets for the model (.inp-files).
- The GAMS programme creates the input equations to the solver (CPLEX). For a better overview of the programme commands for

output and calculation of key parameters are located in external files (.cmd-files).

- Based on the result from the solver GAMS creates an output file with the model result (.lst-files) and several custom defined outputs with the key parameters (.out-files). The output files are output in text format.
- To evaluate the results the output files are imported with Excel and the different results are sorted and organised. For key parameters graphical results are presented. The Excel tools enable evaluations of new scenarios through import of the new output-files.

A description of the contents of the individual GAMS files (.inp, .cmd, .out) and the different Excel files is provided in appendix C. The model has been implemented in GAMS 21.5 and solved with the linear solver in ILOG CPLEX 9.0. In the following section the results of the different scenarios are described, based on the analyses in the Excel tools.

Chapter 7

Scenarios

With the dual objective of the thesis:

- evaluation of the perspective of using optimisation tools in the production planning at Novozymes and
- evaluation of the individual factors influencing strategic-tactical production and distribution planning in a global supply chain,

different scenarios have been evaluated. In the following sections the scenarios are presented and evaluated in order to identify the answers. The results do not represent the real production plan, but should be seen as extreme scenarios where the potential is tested and different key parameters are evaluated. All input and output data frmo the GAMS programmes are documented on the company specific CD, see the company specific appendix for details on the content of the CD.

Generally the scenarios can be divided into two categories. One category contains isolated optimisations of the existing network. The other category contains structural changes i.e. changes in the bounds, which in reality would represent capacity expansions at existing sites, or development of new facilities.

The first category (Cat.1) is represented by scenario 1 (section 7.2.1). Here the potentials of using optimisation tools in the planning process in the existing supply network are identified. Scenario 5, 6 and 7 (section 7.4.1, 7.4.2 and 7.4.3) do also belong to this first category. Here changes in different external parameters are tested.

In the second category (Cat.2) the restrictions are relaxed to identify potentials in a new production-distribution system - this is i.e. done in the scenarios 2-4 (section 7.3.1, 7.3.2 and 7.3.3). Finally the number of time periods is expanded to evaluate the influence of seasonal capacity restrictions (scenario 8, section 7.5.1).

The scenarios are as follows:

1. Optimisation of the real life situation with given capacity constraints and regional product limits. (Cat.1)
2. Unlimited capacities. Products restricted to historical regions. (Cat.2)
3. All products allowed in all regions. Capacities restricted. (Cat.2)
4. All products allowed in all regions. Unlimited capacities. (Cat.2)
5. Influence of currency fluctuations. (Cat.1)
6. Influence of taxation. (Cat.1)
7. Changes of royalties. (Cat.1)
8. Seasonal capacity effects. (Cat.2)

If the currency is not explicitly given in this chapter, the values are given in the currency of $R1$. All numbers have been indexed; therefore the presented results in absolute values do not correspond with the actual results from the modelling. The real values are available on the company specific CD.

7.1 Overview of the Scenario Analysis

For each scenario a partial conclusion is drawn, outlining the characteristics for the specific case. A general summary for all scenarios is presented in section 7.6. To create an overview of the different scenarios, the most significant output and the changes are presented in table 7.1. All the changes are stated as relative changes, comparing the optimal solution to the historical. Positive percentages are improvements in the objective value while negative percentages represent a rise in the costs, i.e. a negative effect on the profit.

The percentages stated for the distribution levels are based on the fraction of the total production distributed between regions. In table 7.1 the difference of these fractions between the historical and optimal solution is stated. It should be noticed that this is not an exact calculation of the changes as the flow differs for two solutions due to differences in the regional BoMs.

In this instance, where the general tendencies in the changes are sought, this imprecision is not seen as a problem.

The changes for scenario 6 (changes in the exchange rates) are based on the scenarios with a combined rise of the $R2$ and $R3$ exchange rates. Results are presented for $ER_{R2} = 6,5$.

Scenario 8 is a comparison of two optimal solutions, evaluating the costs of a seasonal reduction in capacities. Therefore no comparison with historical solutions are possible and presented.

As it appears in table 7.1, there are large differences in the relative values. These differences should of course be evaluated with the absolute values in mind, in order not to focus excessively on relative differences.

Scenario no	1	2	3	4	5	6	7
Output parameters							
Objective value	7,6%	14,5%	8,2%	14,9%	8,4%	4,8%	7,4%
Distribution (Across regions)							
Phase l	14,6%	3,0%	22,7%	8,6%	15,5%	14,7%	11,5%
Phase n	12,5%	42,3%	15,2%	30,1%	23,8%	11,3%	10,1%
Phase p	-1,3%	18,5%	-1,7%	30,6%	-10,0%	-8,8%	-4,4%
Production cost	-3,9%	-5,7%	-4,2%	-5,9%	-6,0%	-3,2%	-4,3%
Distribution cost	199,8%	912,7%	247,7%	938,8%	234,2%	96,6%	130,7%
Storage cost	0,0%	0,9%	0,0%	0,0%	0,0%	0,0%	0,0%
Financial cost	63,9%	140,2%	82,2%	153,7%	76,0%	17,0%	22,3%
Duty	13,0%	127,0%	11,3%	122,2%	-18,7%	-45,3%	27,1%
VAT	509,0%	2132,0%	576,8%	2163,1%	232,9%	48,0%	161,8%
Tax	-21,0%	-55,0%	-24,6%	-55,9%	-15,1%	-8,2%	-16,4%

Table 7.1: Changes in the scenarios output (overview)

The objective value is improved in all instances. There are great variations in the size of the improvements, and obviously the scenarios with the largest potential for improvements are the more relaxed scenarios 2 and 4 with the largest feasible regions. The improvements mainly originate from lowering the production costs together with savings obtained from lowering tax contributions and in some instances lowering the duties. These savings are countered by large relative increases in the distribution and VAT costs. Associated with the increases in distribution levels are a growth in the financial costs as more goods are shipped.

7.2 Improvements in the existing system

The first point to be tested is the potential of using optimisation tools in the planning process at Novozymes. The potential is tested through modelling of the real life supply network.

7.2.1 The Real Life Constraints

To evaluate the potential of a different production-distribution plan in the existing network, the historical data for the 20 products presented in section 6.2 have been implemented in GAMS. Furthermore the historical production plans for the implemented periods have been generated through analysis of the sales data for each product number. This enables the comparison of the linearized optimal solution and the historical solution.

The aim of the case is to evaluate the short term improvement potential, when optimising the production allocation in the existing production system. The case is the most constrained where products can only be produced where historical plans exist. Capacities are reduced to reflect the real life restrictions. The historical plans exist for 50 product-region combinations.

The production capacities are reduced to reflect the available capacity for the chosen product portfolio. As the company operates close to the capacity limits at some facilities the model has been run with the historical production plan and the resulting capacity use has been evaluated. The capacity constraints at each facility are then limited to represent the historical capacity utilisation.

	Optimal solution	Real life solution
Overall profit z	5.544.494	5.154.503
Netprofit pr region		
R1	3.493.571	3.281.859
R2	0	137.291
R3	2.829.091	1.531.156
R10	496.289	426.992
Profit before tax		
R1	4.990.815	4.688.370
R2	0	228.818
R3	2.829.091	1.531.156
R10	708.984	609.989

Table 7.2: Absolute profit distribution for scenario 1. All profits are in regional currencies, except the overall profit (in R1 currency).

Limited capacities implies that movement of production of a product from one facility to another initiates a reverse movement of other products, in order to comply with the constraints. In this case it is interesting to evaluate which products are moved, what characterise the individual products and what initiates their allocation. Furthermore the economical benefits from the changes are evaluated.

The results of the optimisation are presented below. The results are documented in the output file (finalmodel.lst) and the Excel spreadsheets (Scenario 1 analysis.xls, Product flow.xls and Marginal values.xls) in the folder *sc1 existing products capacityconstraints* under Scenarios output on the company specific CD.

Economical results

With the given products, costs and model structure the profit can be improved with 7,6%. The regional distribution of profits for the historical and optimal solution is presented in table 7.2.

The results show that a higher profit is achieved, mainly through movement of the profit from a high taxation region $R2$ to the other regions. The highest profit growth is achieved in region $R3$, where the profit before and after tax is improved with 84,8%. The main contribution to the overall

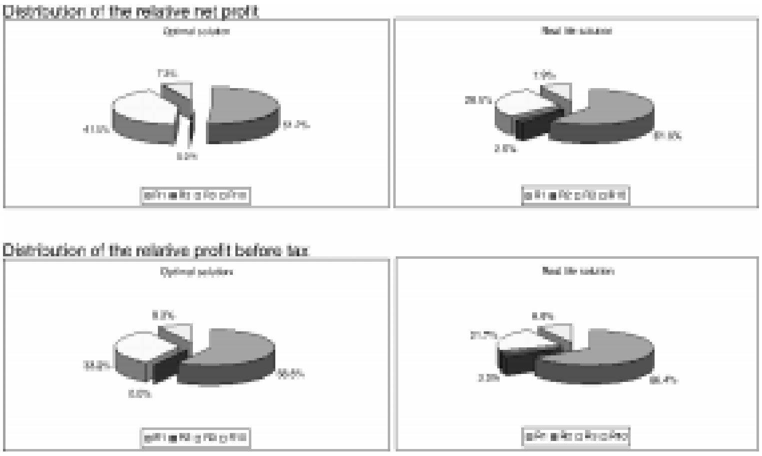


Figure 7.1: Regional profits relative to the total accumulated profit for scenario 1

profit is however still generated in region $R1$. The profit before tax in region $R1$ is 58,8% of the total profit before tax and after tax, the profit is 41,2% of the overall profit. For region $R2$ costs and turnovers are balanced to achieve zero profit. The total tax reduction is 21%. (see figure 7.1 for details)

Production and Distribution

The profit improvement is generally achieved through a higher production quantity in the regions $R1$ and $R3$, while the production in region $R2$ is reduced correspondingly. For region $R1$ the quantity is nearly maintained while the fermentation quantity in region $R3$ is almost doubled. Figure 7.2 illustrates the capacity utilisation for the recovery, fermentation and granulation phases as the fraction of the total capacity used and thereby indirectly the quantity produced within the regions.

With a lower production rate in region $R2$ a higher distribution level is necessary in order to fulfil the regional demands.

For finished products the distribution from region $R1$ to $R3$ is reduced considerably, while distribution from region $R1$ to $R2$ and from $R3$ to $R1$

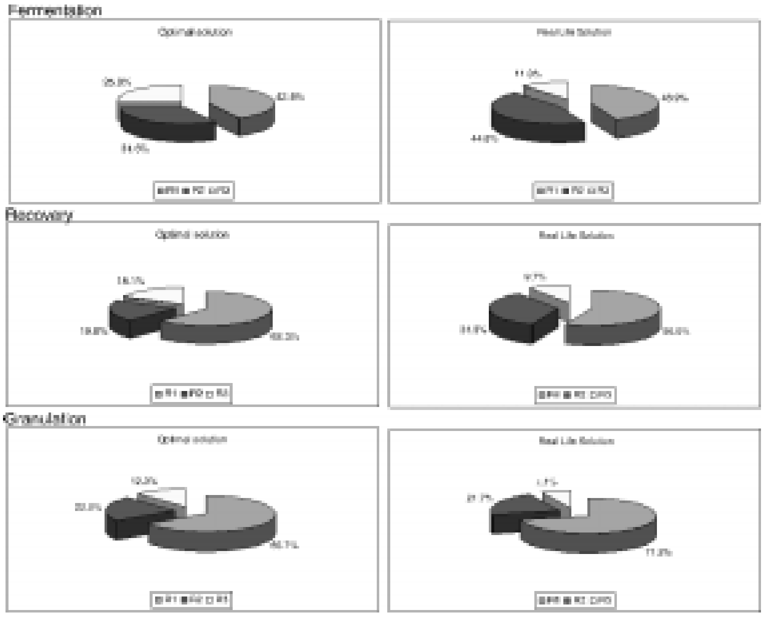


Figure 7.2: Regional capacity utilisation relative to the total capacity utilised for scenario 1

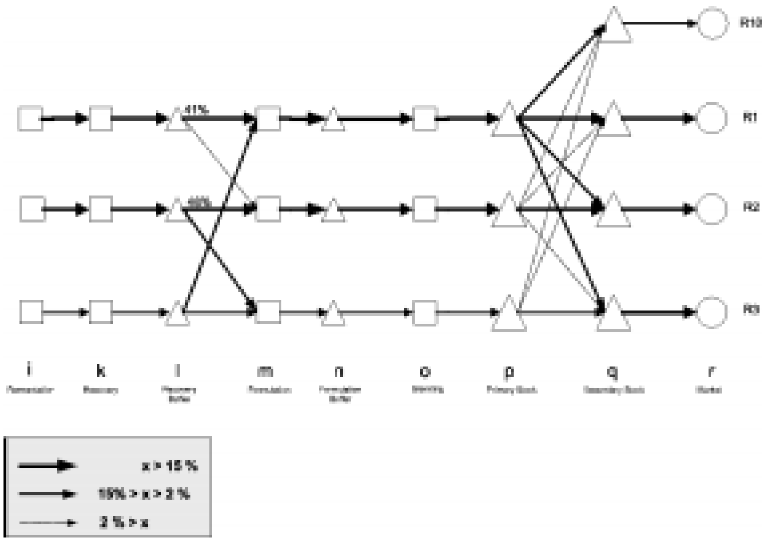


Figure 7.3: Historical product flow in the supply network

and $R2$ is raised. Region $R10$ is mainly supplied by $R3$. For concentrates the distribution from $R3$ is reduced while the distribution $R1$ is raised significantly - especially from $R1$ to $R3$. No storage is initiated. A general overview of the average optimal flow is presented in figure 7.4, and for the historical solution in figure 7.3.

Turnovers and Costs

The sales turnovers are not influenced by the altered product flow. However, the intra-organisational turnovers and costs are influenced by the result.

The total production costs are reduced with 3,9% through the different production allocation, mainly achieved through the lower production quantity in region $R2$. The total distribution costs on the other hand are increased with 199,8% due to higher product transfer quantity. However, a comparison of the absolute values shows reduction of the total production and distribution costs with 1,5%.

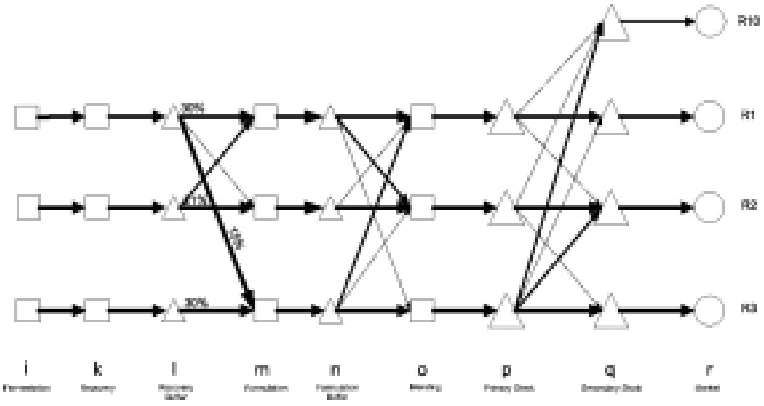


Figure 7.4: Product flow in the supply network for scenario 1

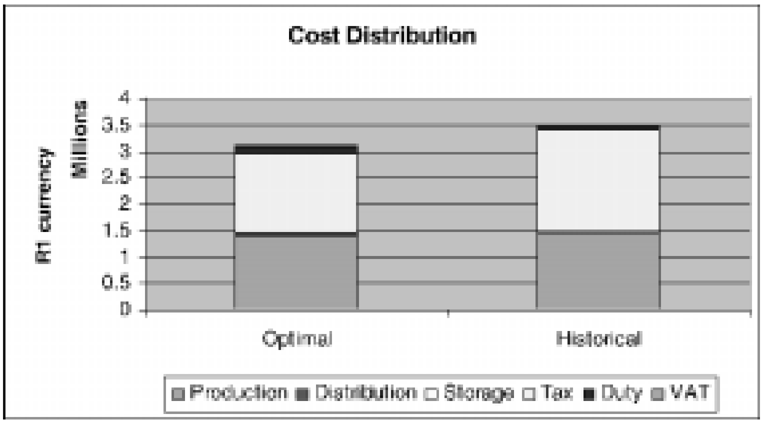


Figure 7.5: Costs for scenario 1

Period	Region	Fermentation	Recovery	Granulation
1	R1	50,0%	56,1%	91,9%
1	R2	61,1%	51,7%	93,7%
1	R3	98,4%	**100%**	**100%**
2	R1	84,8%	99,5%	86,7%
2	R2	67,2%	57,5%	64,9%
2	R3	**100%**	**100%**	**100%**

Table 7.3: Capacity utilisation for the optimal solution in percentage of max capacity for scenario 1.

The transfer turnovers from internal distribution are increased for all producing regions. Transfer costs are raised for region $R1$ and $R2$ while they are reduced for region $R3$ and $R10$. In the model, transfer turnovers include export VAT and transfer costs include import duties. Therefore a profit raise is achieved in e.g. region $R10$ due to lowered import duties on transferred products. In general the transfer costs are increased with 99,5% and the transfer turnovers are increased with 102,4% compared to the historical values. The distribution of costs are shown in figure 7.5.

Capacity Constraints

For the optimal solution the capacity utilisations for the phases i, k and m are presented in table 7.3. The limiting capacities are the capacities in region $R3$, where a utilisation of 100% is achieved for all processes and time periods (except for time period $t = 1$, phase i).

The marginal levels for the capacity constraints for fermentation, recovery and granulation in region $R3$ show the economical improvements of investments in higher production capacities (table 7.4). The expansion of capacities for fermentation will result in an after tax profit improvement of 426,1 R1 currency pr $40m^3 days$ additional capacity. For recovery capacity the potential is 3194,48 R1 currency pr additional recovery hour and for granulation 4095,54 R1 currency pr 1000 kg.

For recovery the average capacity utilisation factor, $CapUtli_{R3} =$ 1,84 hours pr 1000 kg. This leads to a marginal profit of 1736,13 pr 1000 kg concentrate. For fermentation the average capacity utilisation factor is 0,20

Period	Fermentation	Recovery	Granulation
1	-	3.300	4.096
2	426	3.089	4.096
Average	-	3.194	4.096

Table 7.4: Marginal level for the capacity constraints in region $R3$ pr capacity unit for scenario 1. All values are in $R1$ currency.

$40m^3 days$ pr 1000 kg and the average bill of material factor for the recovery phase is 5,83; therefore the marginal profit for fermentation expansion corresponds to 365,4 R1 currency pr 1000 kg concentrate. For formulation of granulates the average BoM factor is 0,17 which leads to the marginal values of approximately 10200 for the recovery process and 2150 for fermentation.

Only considering the profitability and not the investment costs the first step for expansion should be the recovery capacity. These values are only valid within marginal capacity expansion levels. Increasing capacity beyond those levels for one facility may result in another distribution of the marginal profits.

The marginal profits are based on average BoMs and aggregated capacities. Due to large differences between the individual products bills of materials and capacity utilisation factors a more detailed analysis of the individual products influence should be performed to identify the exact marginal profit on specific lines. An evaluation of the consequences of free capacity is carried out in the free capacity case below (section 7.3.1).

Differences in production

The general changes which differ significantly from the historical plan are presented in table 7.5. For more details, see the Excel sheet (*Product flow*) for scenario 1 on the company specific CD. Here the exact quantities are presented.

The reason for the different production plan is difficult to identify, as several factors are incorporated in the model. The optimal solution will be a complex trade off between among others the different cost factors, the capacity utilisation factors on limited capacities and regional taxation sys-

Product	Changes
$P1$	Fermentation and recovery is moved from $R1$ and $R3$ to $R2$ Concentrates distributed from $R2$ to $R1$
$P2$	Production moved from $R2$ to $R3$
$P4$	Production moved from region $R2$ to $R1$
$P5$	All fermentation and recovery moved to $R1$ Concentrates distributed from $R1$ to $R2$ and $R3$
$P6$	All production moved to $R3$
$P7$	All production moved to $R3$
$P8$	Production moved from $R1$ to $R2$ and $R3$
$P9$	Fermentation and recovery moved to $R1$ Concentrates distributed from $R1$ to $R2$ and $R3$
$P10$	Production moved from $R1$ to $R3$
$P12$	All fermentation and recovery moved to $R1$ Concentrates distributed from $R1$ to $R2$
$P14$	All production moved from to $R1$
$P15$	Fermentation and recovery moved to $R1$ Concentrates distributed from $R1$ to $R2$
$P17$	Fermentation and recovery moved to $R2$ Concentrates distributed from $R2$ to $R1$
$P18$	All production moved to $R1$
$P20$	All production moved to $R2$

Table 7.5: Changes in the production and distribution plan for scenario 1

Product	$R1$	$R2$	$R3$
$P3$	2.704	2.277	3.478
$P5$	2.898	2.326	3.362
$P9$	2.717	2.326	3.405
$P3$	259 units	211 units	290 units
$P5$	985 units	152 units	626 units
$P9$	248 units	109 units	484 units

Table 7.6: Production cost and optimal production allocation for scenario 1

Product	Average marginal value	Capacity utilisation
$P3$	24.172	1
$P5$	27.874	1
$P9$	22.968	1

Table 7.7: Average after tax profit (in $R1$ currency) per demand constraint and the capacity utilisation for the granulation (formulation) process in region $R3$. (scenario 1)

tem. Each factor can indicate a tendency for decision making however their mutual effects are more complex.

The bills of materials at facilities have an influence on the product flow. If the yields between different regions vary significantly, the flow in predecessor nodes can be reduced through cross regional distribution. This may lead to lower production costs and capacity uses - depending on the specific production costs and capacity utilisation factors. These savings are not realistic as the model utilizes an illusionary advantage.

A minimisation of the costs pr limiting factor does not yield the optimal production plan. Table 7.6 shows the production costs (in the same currency) for the granulation phase for the products $PR3$, $PR5$ and $PR9$ and the optimal product allocation for each region. Though production costs are lower in region $R2$ and region $R1$ and available capacity exists, a high production quantity still exists in region $R3$.

Theoretically the after tax profit pr limiting factor should be maximised at constraining facilities, in order to achieve the highest profit on the throughput. However, with tax effects, VATs and duties this factor is difficult to evaluate. The marginal level for the demand in each region defines the profit growth per additional product demand. For the above mentioned products the marginal demand level does not differ significantly between the regions.

In table 7.7 the average profit growth per additional demand is presented. The product with the highest marginal value, $P5$, does also have the highest capacity use on the constraining facility in the optimal solution. This indicates that products with a high profitability should have first priority on constraining facilities.

Limitations

There are some limitations on the conclusions that can be drawn upon in this case.

The costs only include the variable production cost and not fixed cost or i.e. salaries. With the tendencies to achieve a contribution margin of zero in high taxation areas fixed cost will not be covered. However, as the total product portfolio is not implemented other profit contributions will exist in reality. Furthermore the real production costs between regions will be different if i.e. salaries are included. This could move the optimal solution since the cost trade off would be different.

Regional BoMs are implemented with the assumption that all sub components can be sent between regions and finished at other facilities with the receiving regions BoM. In reality the BoM will also depend on the origin of the sub components. This allows the model to create a fictive reduction of the total net quantity if those products are distributed to other regions. The flow variables of products where large differences exist between the regions have been evaluated to see if this leads to a fictive cost reduction. For the optimal solution only small fictive reduction are registered, these are not considered to influence the validity of the result significantly.

Other aspects are the not quantified aspects like i.e. strategic needs or customer requirements. Some customers may demand products from a specific region, which limits the solution space and may reduce the optimal solution. Therefore an analysis of the real life feasibility of the results is necessary if the result from a model should be implemented.

Conclusions

The case proves that an improvement potential from using optimisation tools exists within the production-distribution system. The profit improvement of 7,6% is achieved mainly through movement of the profit before tax to a low taxation region, $R3$ (0%). The total contribution to authorities is reduced with 19%.

A trade-off between costs and contribution margins of the individual regions leads to a zero profit level on the product portfolio in the high taxation region, $R2$ (40%). This does not necessarily result in zero profit in region $R2$, as the product portfolio in reality covers more products that contribute to the overall profit, however as an indicator for the tendencies in the model, the tax rates have a high impact.

The result shows, that a minimisation of the individual cost factors does not lead to the highest global profit. The distribution - and thereby distribution costs, export value added taxes and import duties - is increased. However the benefits from lowered production costs and tax contributions give an overall profit improvement. The optimal result is a trade off between the different cost factors and not just a minimisation of the individual cost factors.

Production is mainly moved to region $R3$, where maximum capacity is used for fermentation, recovery and formulation (granulation). A higher profit is achievable through capacity investments in granulation and recovery capacity in region $R3$. The profit for secondary facilities can be increased through reduction of import duties from alternative sourcing.

7.3 Improvements in a new production distribution system

The strength of applying the combined production and distribution planning is also tested when the production system is improved.

Improvements in the production system can from a strategic perspective be categorised in two different scenarios. These relaxations enlarges the size of the feasible region.

Relaxations

- no capacity constraints (facility investments)
- all products can be produced in all regions (product approvals in other regions, facility investments)

These relaxations are tested to see the effect on the models ability to improve the planning, when the feasible region of the problem is enlarged. In a less abstract formulation it is an evaluation of the potential in non specialisation and expansion of the production sights.

7.3.1 No Capacity Constraints

First the effects of unlimited capacities are tested. The data set for the final model is changed so the available capacities are infinite. The results can be used as supporting information, when analysing costs and locations for expanding the capacities. This information is valuable in a situation where the current capacities on the sites worldwide are expected to be fully utilised.

The results of the optimisation are presented below. The results are documented in the output file (finalmodel.lst) and the Excel spreadsheet (scenario 2 analysis.xls, Product flow.xls and Marginal values.xls) in the folder *sc2 Existing products no capacityconstraints* on the company specific CD.

Economical results

With the given products, costs and infinite capacities implemented the profit can be improved with 14,5%. The profit distributions for the historical and optimal solutions are presented in table 7.8.

Again the results show that a higher profit is achieved, mainly through movement of the profit from a high taxation region $R2$ to the other regions. This trend is even more pronounced when the capacities are infinite. Now the profit grows with 252,7% in $R3$ compared to a growth of 84,8% in the optimal solution for scenario 1. With infinite capacities the main contribution to the overall profit in the optimal solution is generated in $R3$ with 68.4% compared to 41,5% in scenario 1 with constrained capacities. For region $R2$ costs and turnovers are balanced to achieve zero profit. (see figure 7.6 for details).

	Optimal solution	Real life solution
Overall profit z	5.899.390	5.154.503
Netprofit pr region		
R1	2.002.407	3.281.859
R2	0	137.291
R3	5.401.028	1.531.156
R10	496.289	426.992
Profit before tax		
R1	2.860.582	4.688.370
R2	0	228.818
R3	5.401.028	1.531.156
R10	708.984	609.989

Table 7.8: Absolute profit distribution for scenario 2. All profits are in regional currency, except the overall profit (in R1 currency).

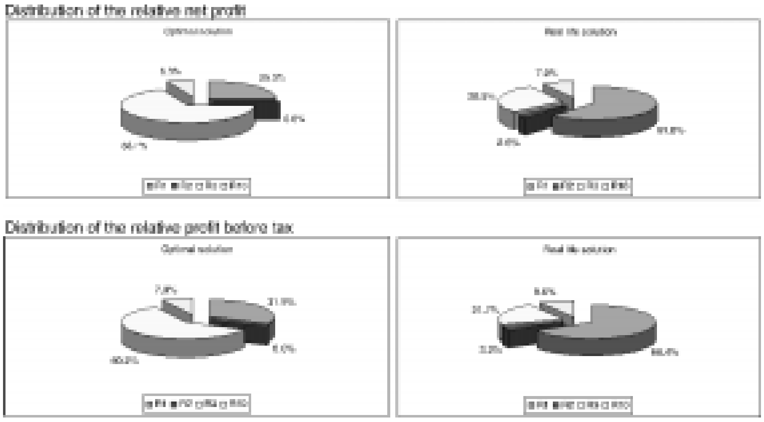


Figure 7.6: Regional profits relative to the total accumulated profit for scenario 2

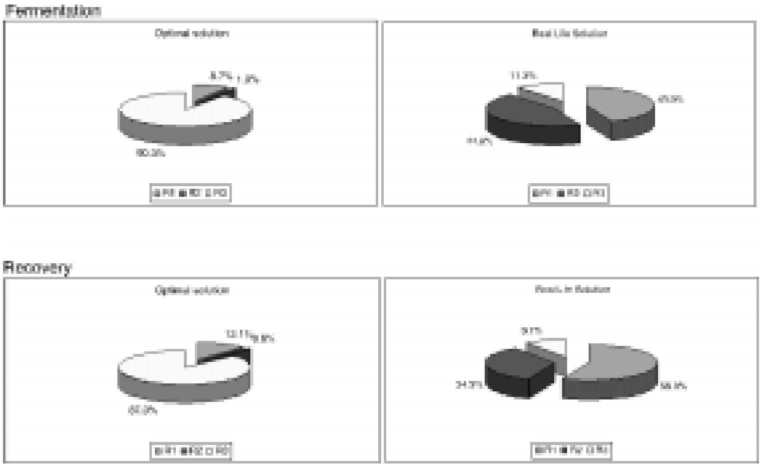


Figure 7.7: Regional capacity utilisation relative to the total capacity utilised for scenario 2

Production and Distribution

The profit improvement is achieved through a concentration of production in region $R3$ while the production in both region $R1$ and $R2$ is reduced correspondingly. The production is generally drastical reduced in $R1$ and $R2$ and dramatically increased in $R3$, e.g. the relative production flow in the phases i, k, m and o increases between seven and ten times in $R3$. Furthermore there is an increased profit in $R10$ due to smaller distribution and duty costs for $R10$.

Figure 7.7 illustrates the capacity utilisation for the recovery and fermentation phases as the fraction of the total capacity used and thereby indirectly the quantities produced within the regions.

The lower production rates in region $R1$ and $R2$ increase the need for distribution from $R3$ to $R1$ and $R2$ in order to fulfil the regional demands. Historically, $R3$ has only had a minor distribution to $R1$ and $R2$ - corresponding to 2,0% in phase l and 0,1% in phase p of the total amount distributed. In the optimal solution $R3$ now represents 41,5% in phase n and 30,0% in phase p of the total amount distributed. Also the inter regional distribution of goods in $R3$ grows from 6,5% to 82,0% in phase l, from 8,7% to 47,2% in phase n and from 7,6% to 17,3% in phase p.

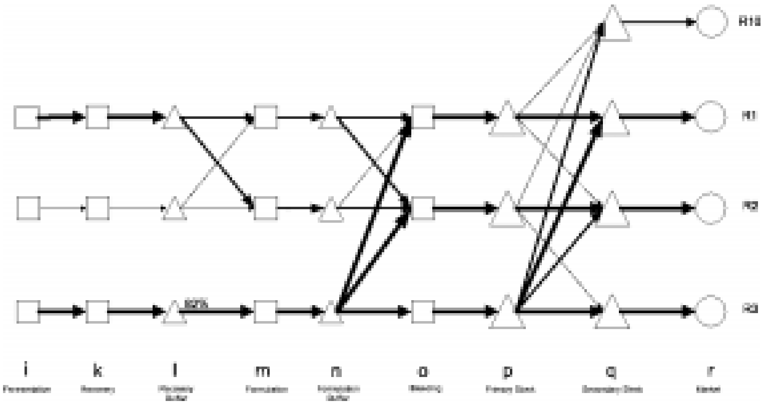


Figure 7.8: Product flow in the supply network for scenario 2

A general overview of the average flows is presented in figure 7.8.

Turnovers and Costs

The total production costs are reduced with 5,7% through the different production allocation. The total distribution costs on the other hand are increased with 912,7% due to the growth in product transfer quantity. All in all this results in an increase in production and distribution costs of 5,0%. The total duties paid rises with 127% and the VAT with 2132%. These extra costs are offset by a lowering of the tax paid by 55%, yielding a better overall result.

The transfer turnovers from internal distribution are increased with 29,7% for $R1$ and with 2132,1% for $R3$, while it drops by 36,5% for $R2$. The transfer costs are raised with 1476,6% for $R1$ and with 672,7% while this cost decreases with 97,2% for $R3$ and 2,4% for $R10$. This corresponds to the data above, describing a growth in production in and distribution from $R3$. Again it should be noted that the decrease in transfer costs for $R10$ is due to a decrease in the duties paid as the transfer prices are fixed with the sales prices in the region.

Differences in production

The following general changes in the production are suggested by the model.

Quantities of the products $P1$ - $P3$, $P5$ - $P11$, $P13$, $P16$ and $P18$ are all moved to $R3$ while quantities of the production of $P4$, $P12$, $P14$ and $P15$ are moved from $R2$ to $R1$. Only the production of $P17$, $P19$ and $P20$ are wholly or partially moved to $R1$ from $R2$. The overall flow of the optimal solution is represented in figure 7.8

When evaluating these changes in the production and the consequences associated, it should be noticed that the model in some instances can take advantage of differences in the regional BoMs, which may not be realistic, see section 7.2.1. In example, the movement of $P19$ to be made partially in $R2$ in the optimal solution compared to the historical solution, shows that a reduced net quantity of semi finished products are necessary to fulfil the demand. This is obtained in the model by moving a small part of the production to $R2$ in phase n, in order to take advantage of the BoM in $R2$. Compared to other regions the $R2$ BoM requires a lower input to generate the equivalent output.

This is not realistic, and the solution obtained must be checked to see if the model *cheats* in this way. Apparently it is not a general problem, probably because distribution and financial costs - due to movements - must be offset by the savings from using other regions BoMs.

One way to get around this unrealistic use of the differences in the BoMs is to make an average BoM for each product valid in all regions. These BoMs would not be as precise, but may create more realistic solutions. Another way to solve this problem would be to implement a factor in the production considering where the semi finished goods originate; in this way a true expression for the input needed is obtained.

Marginal analysis

As expected the marginal profits for all the capacities are zero as these are defined as infinite in this scenario. The capacity usages in the optimal solution can be perceived as the optimal adjustment of the different sites capacities. This would entail significant changes in the capacities, e.g. a reduction in $R1$ of up to 86,9% and in $R2$ of up to 98,1% in phase i. In phase k, region $R1$ the reduction is up to 86,0% and in $R2$ 97,7%. For phase m the relative reductions are, in $R1$ 98,9% and in $R2$ 100%. In $R3$ the capacity usages grow significantly, in phase i with up to 804,7%, in phase k with up to 1081,9% and in phase m with 1385,3%. These changes in capacities are of course not, at least in the short run, realistic, and all the fixed and variable costs of changing these have not been calculated in this simulation. But it does provide a good indicator for capacity expansions in the long term.

The marginal profits have also been calculated for the demand. These values differ very much i.e. from zero to 124500 in $R1$ currency pr 1000 units product, with most of the products having marginal profits between approximately 20000 and 35000 in $R1$ currency. Five of them are especially interesting. This is $P4$ in $R3$, $P14$ in $R1$, $R2$ and $R3$ and $P14$ in $R10$. Here the marginal profits are well over average with values between 88.000 and 124.500 in $R1$ currency pr 1000 units. This would motivate a boost of the sales effort for precisely these products. In other words the models results also contains valuable information regarding earnings pr. product and can in this respect be seen as a supporting tool for the sales staff. As far from all costs in the production have been implemented, further analysis of the marginal profits are needed to ensure their validity.

Limitations

When evaluating the solution with infinite capacities, the consequences for the fixed and variable costs have not been considered. Therefore the quite extreme changes should be perceived as an indication of which possibilities to focus on, not as a complete conclusion ready to be used as the foundation for the next production and distribution plans.

Conclusions

The trend observed in scenario 1 is further accentuated, moving a greater part of the production to $R3$. Furthermore this movement of products entails a much greater inter-regional distribution of goods. This distribution will of course result in higher distribution and financial costs, as well as a growth in duties and VAT. These are offset by the savings in production and the financial savings in tax, all in all producing a growth in profit of 14,5%.

	Optimal solution	Real life solution
Overall profit z	5.578.486	5.154.503
Netprofit pr region		
R1	3.394.556	3.281.859
R2	0	137.291
R3	3.014.040	1.531.156
R10	519.177	426.992
Profit before tax		
R1	4.849.366	4.688.370
R2	0	228.818
R3	3.014.040	1.531.156
R10	741.682	609.989

Table 7.9: Absolute profit distribution for scenario 3. All profits are in regional currency, except the overall profit (in R1 currency).

7.3.2 Enabling Production at All Sites

The effects of being able to produce all products in all regions are tested. This is done by using a common BoM for all regions whether production of the specific product historically has taken place or not. In this manner a cost benefit analysis by investing in product approvals and facilities can be carried out for one or more products. In this section, the production of all products at all sites with the given capacity constraints is evaluated. The possible solutions are expanded with ten product-region combinations - now allowing all 20 different products in all three producing regions.

Economical Results

The relaxations of the regional production constraint lead to a possible profit improvement of 8,2% compared to the historical solution (see table 7.9). This adds less than one percent more of the historical solutions profit to the possible improvement attainable, when comparing to the improvement obtained in scenario 1.

The tendencies from scenario 1 are further supported from the relaxed solution, as profit has been increased even more in region $R3$ and $R10$ and reduced in the other regions. The profit distribution between the regions

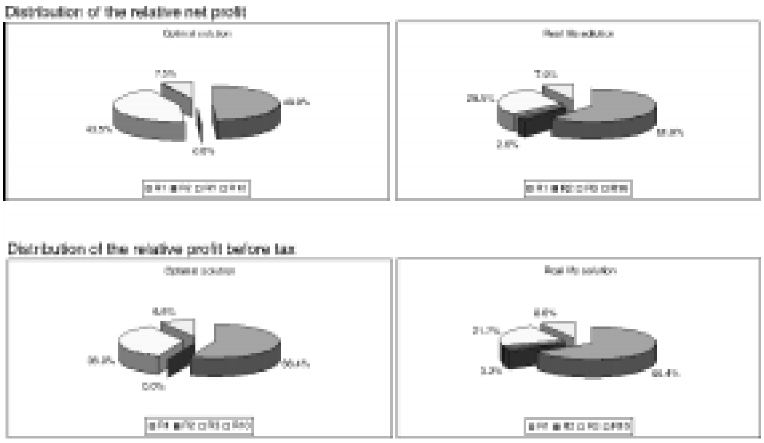


Figure 7.9: Regional profits relative to the total accumulated profit for scenario 3

is presented in figure 7.9. The main contribution for the overall profit - before and after tax - is still from region $R1$, however it is reduced with a few percent. The $R3$ share has been increased correspondingly.

Production and Distribution

As in the previous scenarios, the production has been increased in region $R3$, however limited by the capacity constraints. This leads to a higher distribution of concentrates between the regions, where 27,2% of the total concentrate quantity is distributed between the producing regions (the corresponding for scenario 1 is 19,1% and for the historical plan 4,5%). Especially from $R1$ to $R3$ the concentrate distribution has been increased.

For finished products the distribution to region $R3$ is low, however the export from the region is increased. The $R10$ market is supplied solely from region $R3$. In general more finished products are distributed than in the historical solution. The model distributes 27,2% of the total product quantity which is considerably more than the historical solution (13,7%). An overview of the flow is presented in figure 7.10.

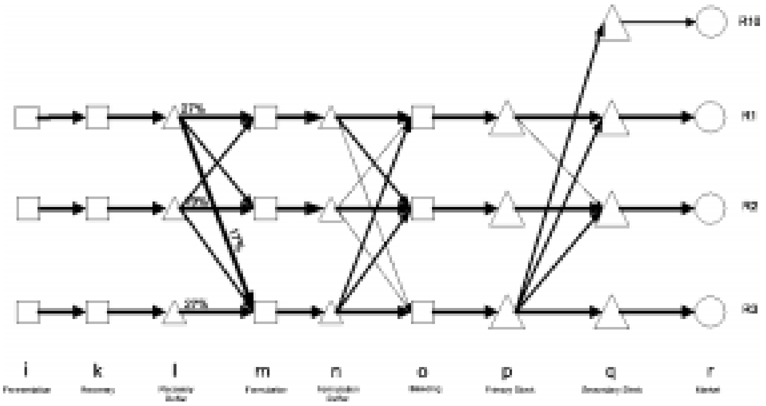


Figure 7.10: Product flow in the supply network with relaxed regional production constraints (scenario 3)

Turnovers and Costs

The increased distribution quantities lead to higher distribution costs for all regions, except for $R10$ where all products are distributed from $R3$. Though the total distribution costs are increased with almost 250% the reduced production costs reduces the total production and distribution costs. In total distribution and production costs are reduced with 1,2%.

The total costs for value added export taxes and import duties are increased due to the higher level of distribution. The import duties for $R3$ and $R10$ are reduced - however this cost reduction is levelled through raised import duties in region $R1$ and raised export taxes in region $R3$. The total costs are raised with 42%. The financial reduction however is achieved through reduced tax contributions. The total tax contribution is reduced with 24% which corresponds to a 21% reduction of the total VATs, duties and taxes.

The total cost reduction is thereby achieved through reduced production costs and taxes, see figure 7.11.

Capacity Constraints

The constraining factors are the same as for scenario 1; the fermentation, recovery and granulation process in region $R3$. Compared to scenario 1

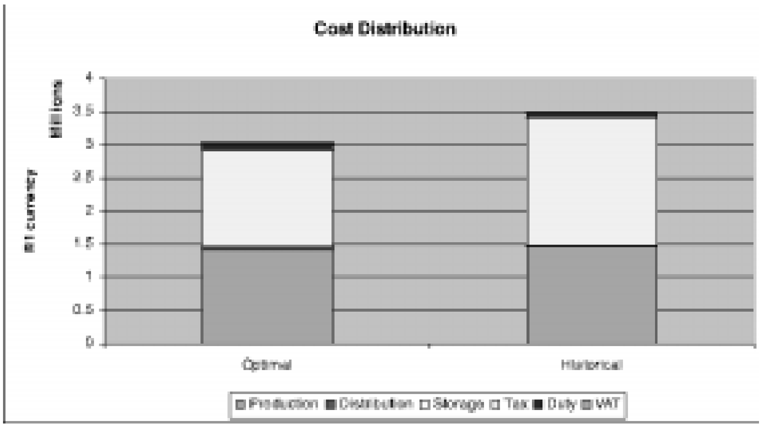


Figure 7.11: Costs for scenario 3

Period	Region	Fermentation	Recovery	Granulation
1	R1	49,6%	55,1%	91,9%
1	R2	60,0%	55,3%	93,7%
1	R3	**100%**	**100%**	**100%**
2	R1	82,3%	95,9%	86,7%
2	R2	71,9%	67,7%	64,9%
2	R3	**100%**	**100%**	**100%**

Table 7.10: Capacity utilisation for the optimal solution in percentage of max capacity (scenario 3)

full capacity utilisation is achieved on those facilities in all periods. For the other regions only small differences are achieved on the capacity utilisation. The regional capacity utilisation levels for the process phases i, k and m are presented in table 7.10.

The marginal profits on the capacity constraints for region $R3$ are presented in table 7.11. With a broader product portfolio they do not differ considerably from the scenario 1 case. A small reduction in the marginal levels can be seen. This indicates a better capacity utilisation with respect to the overall profit, if other products are produced in other regions.

Period	Fermentation	Recovery	Granulation
1	423	3.090	4.096
2	425	3.090	4.095
Average	424	3.090	4.096

Table 7.11: Marginal level for the capacity constraints in region $R3$ pr capacity unit

Product	Changes
$P4$	The main production is moved to region $R1$
	Some concentrates are sent from $R1$ to $R2$
	Production for the $R10$ market is moved to region $R3$
$P14$	The main production is moved from $R2$ to $R1$
	Production for the $R10$ market is moved to region $R3$
$P17$	Concentrates are sent from $R2$ to $R1$ and $R3$
	Formulation for the $R3$ and $R10$ market is moved to $R3$

Table 7.12: Changes in the production-distribution plan for additional production-region combinations in scenario 3

Differences in the Production

The production and distribution only differs slightly from the optimal solution under the real life constraint. From the ten additional regional production possibilities the main changes are for the products $P4$, $P14$ and $P17$. In this relaxed scenario production in region $R3$ is enabled.

The changes compared to the historical plan is presented in table 7.12. Compared to the optimal solution in the real life, the main influence is that products for sales in region $R3$ and $R10$ are produced in region $R3$. Through the movement a reduction of import duties and transportation costs is achieved.

As capacities are limited in region $R3$ the movement of products to the region initiates a reverse movement of other products. This is achieved through a movement of the fermentation and recovery of $P7$, $P11$ and $P13$ from region $R3$. The concentrates are shipped to region $R3$, since the formulation and blending process is un-capacitated for those products.

The products moved to $R3$ ($P4$, $P14$ and $P17$) are characterised by the

Product	$R1$	$R2$	$R3$	$R10$	Average
$P4$	71.613	71.133	102.665	79.918	81.332
$P14$	88.001	87.772	121.555	100.091	99.355
$P17$	40.658	70.456	66.032	164.200	85.337
$P7$	9.427	8.476	17.032	0	11.645
$P11$	23.590	0	22.361	31.699	25.883
$P13$	29.804	14.260	26.501	31.633	25.549

Table 7.13: Marginal profit levels on additional demand for selected products in scenario 3. Average for all products: 32414. All profits are in $R1$ currency.

highest marginal profit on additional demand - in particular in the regions $R3$ and $R10$ which are supplied by $R3$ in the optimal solution. Thereby it can be seen that the restricted capacities to a higher degree are occupied by products with a high after tax profit.

The products which are moved from region $R3$ are however characterised by a considerably lower marginal profit. For all products the marginal profit is below the average for all products. The products do not have the lowest marginal profit from all products. Capacity utilisations, interregional factors and the balance mechanisms between profit and costs in the model result in more complex decision parameters than a narrow marginal consideration. The profit contribution does however have influence on the use of restricted capacities.

The marginal profits for the moved products are presented in table 7.13. No sales prices and demands are implemented for products where no historical sales data exist. Therefore no marginal values exist; the non-available values are not included in the average marginal values presented. For details on all products, please see the Excel file *Marginal Values* in the folder *sc3 all products allowed* on the company specific CD.

Other minor changes in the product flow occur to balance capacities, costs and turnovers between the regions. (For details see the company specific CD; Folder SC3: *Scenario3 analysis.xls*, *product flow.xls* and *Comparison Sc1 and Sc3.xls*).

Limitations

To enable production in all regions the production costs have been converted into another currency, and bill of materials and capacity utilisation factors have been copied. This does no reflect the reality as differences in costs and production equipment exist between the regions. Therefore other factors may influence the solution in a real life solution. This of course limits the validity of the results if no further analyses are done. However, as an indicator of which products may be interesting to move and the reason for their movement the validity is sufficient.

Only few additional products are added (ten extra production-region combinations). This only result in a marginal improvement compared to the more restricted scenario. In the product portfolio many other products are restricted to production in only one region. Modelling the full product portfolio without the regional restrictions would allow an analysis of which products should be produced in other regions to obtain a higher profit.

Conclusions

Though only few products are restricted, a higher profit is achievable from enabling production in other countries. The profit is mainly achieved through movement of production for the $R10$ and $R3$ markets to the production facilities in region $R3$. This reduces the distribution cost and import duties for the given products and reduces the total tax contribution. In total the profit has been improved with 8,2% compared to the historical solution and tax contributions have been reduced with 24%.

Compared to the historical solution and the first scenario (production in the existing network) the distribution quantities are increased. For both finished products and concentrates higher quantities are shipped between the regions; finished products mainly from region $R3$ and concentrates from region $R1$ to region $R3$. The concentrates are distributed to $R3$ due to capacity constraints in the region; finished product are shipped from the region to achieve maximum profit in a low taxation region. The additional distribution and financial costs are exceeded through lower production costs and tax contributions.

Due to capacity restrictions a reverse movement of other products is necessary when allocating new products to the $R3$ facilities. The products moved to $R3$ are characterised through a high marginal profit on additional demand fulfilment, where the replaced products have a considerably lower marginal profit. This is however just one of the key parameters in the production allocation problem.

7.3.3 All products produced in all regions with no capacity constraints

In this scenario the effects of fully relaxing the bounds are tested, i.e. production of all products are allowed in all regions and no facilities have any limitations on their capacities. The data set for the final model is changed so the available capacities are infinite and constraints disallowing production of specific products in some regions are relaxed.

The results have little relevance for the short term decision making but can be seen as an analysis of the potential in expanding the capacities and making the production more flexible, both regarding the processing equipment and the necessary documentation and permits.

The results of the optimisation are presented below. The results are documented in the output file (finalmodel.lst) and the Excel spreadsheet (scenario 4 analysis.xls, Product flow.xls and Marginal values.xls) in the folder scenario4 on the company specific CD.

Economical results

With no bounds on either products or capacities implemented the profit can be improved with 14,9%. The profit distributions for the historical and optimal solution are presented in table 7.14.

The results do not differ significantly, compared to the semi-relaxed scenario 2, where only the capacities where unbounded. The overall improvement in profit rises from 14,4% to 14,9% of the historical solution comparing scenario 2 and 4. The higher profit is again achieved, mainly through movement of the profit from a high taxation region $R2$ to the other regions. Compared to the basic case, scenario 1, with constrained capacities and production the net profit grows to 69,1% of the total profit in $R3$ compared to 28,5% in the real life solution. The share of the profit generated in $R1$ drops from 61,0% to 24,4%. Roughly speaking the production seems to

	Optimal solution	Real life solution
Overall profit z	5.922.360	5.154.503
Netprofit pr region		
R1	1.946.097	3.281.859
R2	0	137.291
R3	5.510.859	1.531.156
R10	519.177	426.992
Profit before tax		
R1	2.780.138	4.688.370
R2	0	228.818
R3	5.510.859	1.531.156
R10	741.682	609.989

Table 7.14: Absolute profit distribution for scenario 4. All profits are in regional currency, except the overall profit (in R1 currency).

shift from $R1$ to $R3$. For region $R2$ the model still balances costs and turnovers to achieve zero profit. (See figure 7.12 for details).

Production and Distribution

The main improvement in profit is still through a concentration of production in region $R3$, while the production in region $R1$ and $R2$ is reduced correspondingly.

$R10$ is supplied from $R3$ in the optimal solution, instead of from $R1$ and $R2$ in the real life case. The model probably picks this solution due to the savings obtained from the zero duties between $R3$ and $R10$.

Figure 7.13 illustrates the capacity utilisation for the fermentation, recovery and granulation phases as the fraction of the total capacity used and thereby indirectly the quantity produced within the regions.

The general tendencies observed in scenario 2 regarding the lower production rates in region $R1$ and $R2$ (which increases the need for distribution from $R3$ to $R1$ and $R2$ in order to fulfil the regional demands) can again be observed in scenario 4. Compared to scenario 2 the distribution, represented by the fraction of the total amount produced, is in $R3$ reduced from 41,5% to 28,9% in phase n and rises from 30,0% to 43,3% in phase p. This

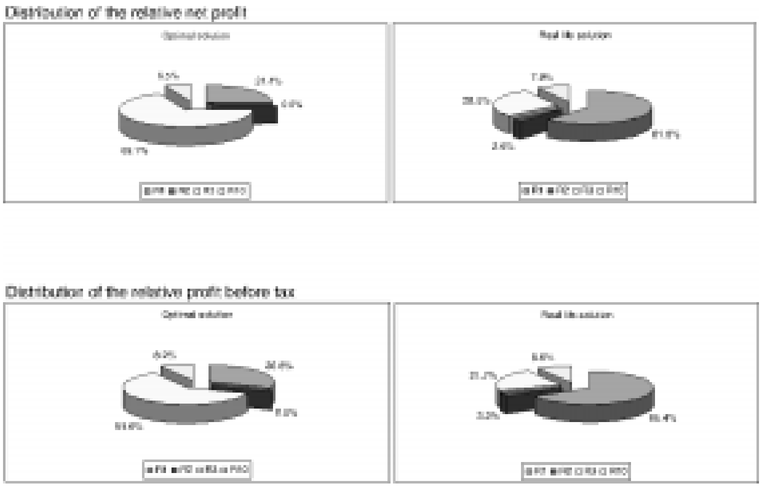


Figure 7.12: Regional profits relative to the total accumulated profit for scenario 4

seems to be a result of the allowed production of all products which calls for less distribution of semi finished products, and therefore a more centralised production. This leads to an increased distribution of the finished products. The total amount distributed from $R3$ in phase n for the time period 1 and 2 drops by 2995,58 tonnes, but for phase p the quntity rises with 3270,6 tonnes.

A general overview of the average flow is presented in figure 7.14.

Turnovers and Costs

The total production costs are reduced with 5,9% compared to the real life solution. It is reduced 0,2% more compared to scenario 2. The total distribution costs are increased with 938,8% compared to an increase of 912,7% due to the growth in product transfer quantities. All in all this results in an increase in the production and distribution costs of 5,2% which is 0,2% more than in scenario 2. The total duties paid rises with 122% and the VAT with 2163%. These extra costs are offset by a lowering of the tax paid by 56%, yielding a better overall result.

Figure 7.15 shows the distribution of the costs.

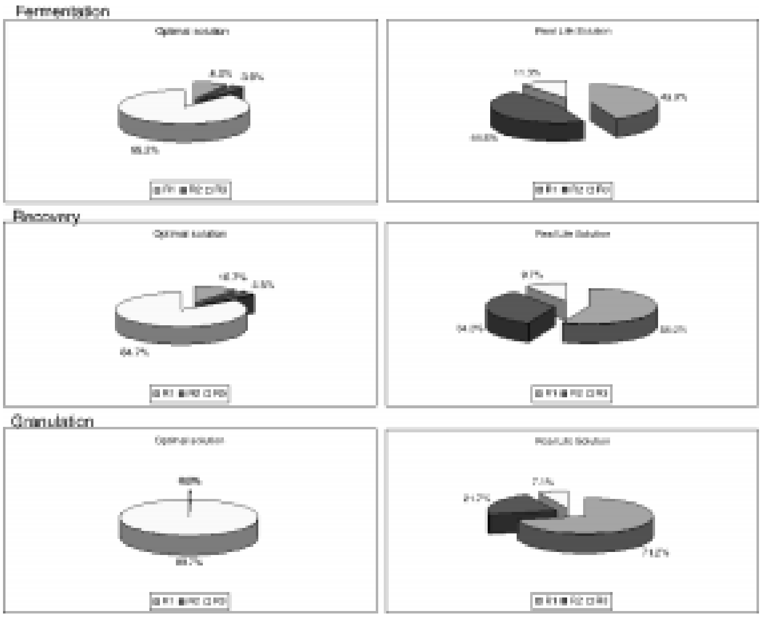


Figure 7.13: Regional capacity utilisation relative to the total capacity utilised for scenario 4

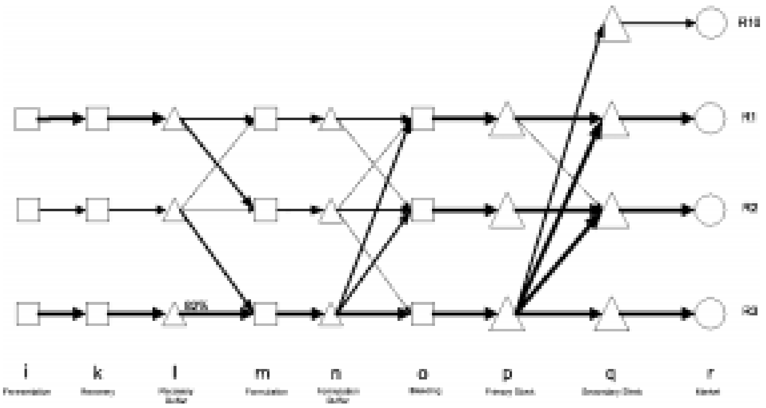


Figure 7.14: Product flow in the supply network for scenario 4

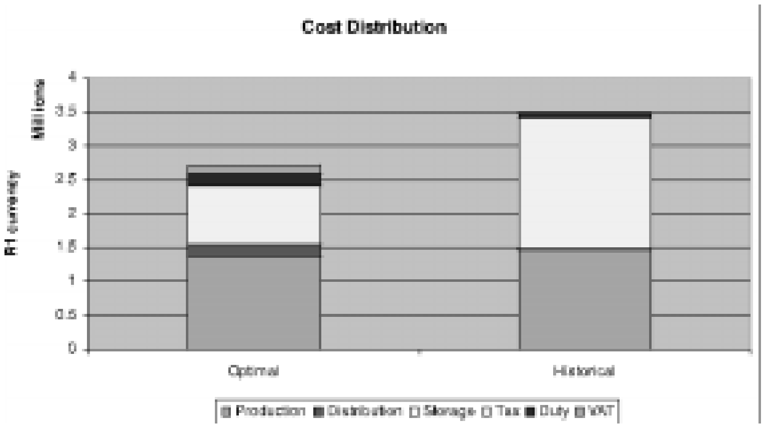


Figure 7.15: Costs for scenario4

Differences in production

The changes in production and distribution suggested in the optimal solution do not differ considerably from the optimal solution to scenario 2. The only significant changes are found in the production of $P11$ and $P13$ which are produced in $R2$ and not in $R3$. The product flow of $P15$ should also be observed, as the solution takes advantage of the differences in the BoMs in the regions. The product is produced in $R1$ in the phases i and k, then shipped to $R2$ and produced in phase m. Finally it is shipped to $R3$ and the production is finished. The product is then shipped back to $R1$ and $R2$ to be sold.

This is in reality not a good solution as the solution takes advantage of some at least partially artificial benefits in the model. The solution must be checked to see if it contains significant flows benefiting from these falsities. If so the variables in question should be fixed to zero and the model solved once more. In the long run the problem should of course be solved permanently as described above in section 7.2.1 and section 7.3.1.

Marginal analysis

The marginal profit for all the capacities are zero as these are defined as infinite in this scenario. The capacity usages in the optimal solution are even more pronounced compared to scenario 2. Again it is not realistic to adjust the capacities in this way i.e. moving the greatest part of production to $R3$, but the indication from scenario 2 is further accentuated.

The marginal profits have also been calculated for the demand in this scenario and the result does not differ significantly from the solution in scenario 2. Of the products with the highest marginal profits i.e. $P4$, $P14$ and $P17$ there is a change in production from $R2$ to mainly $R1$ and partially $R3$. This seems logical in order to balance the costs with the profits in $R2$, as the apparently most profitable products are moved away from this region.

Limitations

When relaxing the bounds and allowing production of all products at all sites, the BoMs for other regions are implemented where historically no BoMs are available. This is clearly an approximation as there in most instances are regional differences in these BoMs, but it is not seen as a critical source of error in the model at the current stage, where an evaluation of the general tendencies in the behaviour of the solutions is taking place.

Conclusions

Not surprisingly, the trend observed in scenario 1 moving a greater part of the production to $R3$, which was further accentuated in scenario 2 with free capacities, is developed to the utmost in the fully relaxed scenario 4. The movement of products entails a much greater inter regional distribution of goods. This distribution will of course result in higher distribution and financial costs, as well as a growth in duties and VAT. These are offset by the savings in production and the financial savings in tax, all in all producing a growth in profit of 14,9%.

7.4 Influence of key parameters

The previous scenarios focused on both the improvement potentials in the specific case within the existing network (by changing the production and distribution) and on which structural changes to analyse first in order to raise the profit level. The scenarios 1 to 4 are carried out under the assumption, that only the bounds on the capacities and the possible production are varied while all other parameters are constant

For global supply networks another interesting aspect is the influence of the different parameters that affect the decision making. This implies in particular the influence of external decision parameters like currency fluctuations and changes in tax rates that will not have influence on regional networks but only global supply nets. As the parameters are outside the influence of the company an understanding of their effects is essential in order to compete in a dynamic world. Furthermore the internal decision parameters, like the effects of royalty changes on the optimal production plan are considered interesting. In the following sections some of the individual parameters are evaluated.

7.4.1 Currency fluctuations

In this section the influence of currency fluctuations is evaluated. Through different scenarios the exchange rate for region $R2$ is changed to identify the consequences for the production-distribution system. In the scenarios the $R2$ currency is influenced and the fluctuations in the $R3$ currency are linked to the $R2$ level. The main purpose is to evaluate the economical consequences of the fluctuations as well as the general tendencies in the optimal solutions. The scenarios are based on the real life case with restricted production-region combinations and capacities. All scenarios are available in the folder *scenarios output* on the enclosed CD.

Economical results

The model shows an increased profitability with increased exchange rates for the $R2$ and $R3$ currencies. This is the case for both the historical and the optimal solution. This is mainly caused by the fact that more than 50% of sales are in region $R2$ and $R3$ with the given portfolio and time periods (see table 7.15).

The development of the optimal and historical solutions with changing currencies are presented in figure 7.16. The figure shows the overall profit after tax, when the exchange rates for $R2$ and $R3$ are correlated. The relative

Region	Share
R1	43,6%
R2	36,2%
R3	17,6%
R10	2,6%

Table 7.15: Regional sales distribution

deviations between the historical and optimal solutions do not differ significantly with the changing currencies. The optimal solutions improve the profit between 7,6% and 8,4% for the given currency range. This indicates that the currency fluctuations do not have a high impact on the relative improvement obtained by the optimal solution. The absolute profit however is highly dependent on the exchange rates.

Production and Distribution

The distribution of production levels between regions changes with the currency. Figure 7.17 shows the capacity utilisation for the region as the fraction of the total capacity utilisation. The tendencies clearly show that a high exchange rate for the $R2$ and $R3$ currencies results in a higher capacity usage in region $R1$, where as the opposite is the case for lower currency levels. Thereby the production is allocated to the regions wirh a low exchange rate, where production costs are relatively low. For scenarios with low exchange rates the distribution of concentrates is higher than for scenarios with high exchange rates.

For high $R2$ and $R3$ exchange rates a higher production level is achieved in region $R1$. This also initiates a higher distribution level from the region, while import quantities are reduced. To region $R2$ mainly finished products are distributed, while the distribution to region $R3$ is concentrates. For $R3$ maximum capacity is used for recovery and granulation. In region $R1$ maximum fermentation capacity is used. This also initiates the use of storage facilities, where smaller amounts are stored at the buffer storages for recovery and formulation in region $R1$.

For lower $R2$ and $R3$ exchange rates a larger fraction of the production is moved to $R3$. From $R3$ finished products are distributed to the regions $R1$ and $R2$. A high share of the production still takes place in region $R1$,

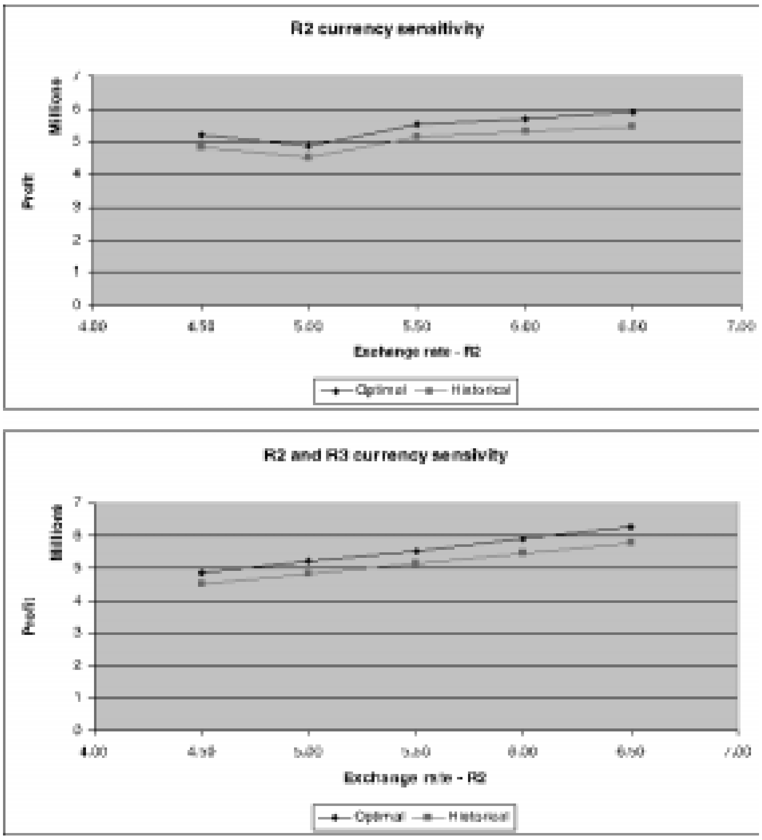


Figure 7.16: Net profit development with exchange rate changes. (Scenario 5)

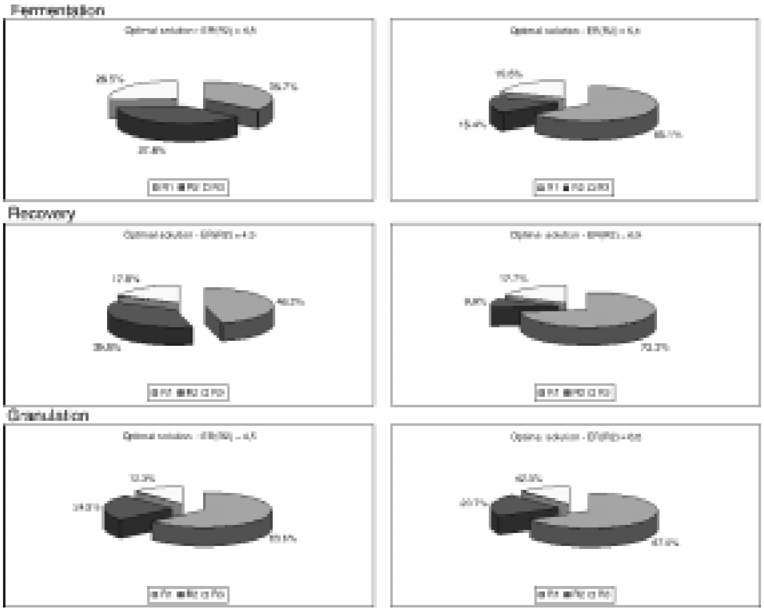


Figure 7.17: Regional capacity utilisation relative to the total capacity utilised for $ER_{R2} = 4,5$ and $ER_{R2} = 6,5$ (scenario 5)

from where primarily concentrates are distributed to $R2$ and $R3$. For $R3$ maximum capacity is used for fermentation, recovery and granulation. For $R2$ maximum capacity is used for the recovery. This initiates the use of the recovery buffer in both regions, where smaller quantities are stored from time period 1 to 2.

The use of buffer storages in the regions - although penalised due to financial and storage costs - indicates that the maximum usage of production facilities in *low currency* regions is economical. The capacity utilisation levels are available on the CD, folder *Sc5 existing capacities exrates* in the file *scenario analysis.*

Turnovers and Costs

Though the relative deviation between the overall profits for the historical and optimal solution is quite constant for the different exchange rates, different cost factor reductions influence the result.

For the situation with a high $R2$ exchange rate, the optimal solution differs from the historical through a reduction of the production costs. The total production cost reduction is 6% and is mainly achieved in region $R2$. The total reduction of production and distribution costs is approximately 3%. Contributions to authorities are reduced with 15% achieved through reduced taxes and duties.

For low $R2$ exchange rates only a 4,8% reduction of the production costs is achieved, this leads to a reduction of the total production and distribution costs with 1,1%. The production cost reductions are achieved in region $R1$ and $R2$. The total reduction of duties, VATs and taxes is 22%, achieved through reduced tax contributions, but higher VAT and duties.

For the low exchange rate scenario, the costs occur due to a higher distribution level compared to the high exchange rate scenario.

Conclusions

The case shows that profit improvements are achievable through optimisation - also under currency changes. The improvement ratio however is nearly constant for different exchange rates; approximately 8% compared to the historical solution. The overall profit in absolute values does however depend strongly on the currency fluctuations. The profit contributions are achieved through allocation of production to regions with a low currency exchange rate. Thereby the overall production costs are reduced.

With different exchange rates the allocation differs significantly between the regions. Though the company uses currency hedging, fluctuations are not deterministic. The real life application should be based on forecasts for the exchange rate development. One approach could be the use of stochastic programming or simply through a trade off between the different optimal solutions.

7.4.2 Changes in Tax Rates

The consequences for the profit and production, in scenarios with more similar tax rates for the different regions, are examined in this section. The capacities as well as possible production are bounded as for scenario 1. The only parameter changed is the tax rate. The tax in $R1$, $R2$ and $R10$, seem to be stable but the rate in $R3$ may change over time. The rate in $R3$ is therefore raised to 20%.

The results of the optimisation are presented below. The results are documented in the output file (finalmodel.lst) and the Excel spreadsheet (scenario 6 analysis.xls, Product flow.xls and Marginal values.xls) in the folder scenario6 on the CD.

Economical Results

With a tax rate of 20% in $R3$, comparing the historical solution with the optimal solution, the profit can be improved with 4,8%. The profit distributions for the historical and optimal solution are presented in table 7.16.

The results clearly show a smaller difference in the optimal and the real life solution to the problem, when comparing to the results of scenarios with no tax in $R3$. Some of the characteristics are still found e.g. the solution is still seeking to minimize the profit in $R2$, but overall the profits before and after tax of the optimal solution is much closer to the profits in the historical solution.

The share of the net profit generated in $R1$ decreases from 64,7% to 60,3%, in $R2$ it decreases from 2,7% to zero, in $R3$ it increases from 24,2% to

	Optimal solution	Real life solution
Overall profit z	5.172.562	4.934.017
Netprofit pr region		
R1	3.719.243	3.281.859
R2	0	137.291
R3	1.997.136	1.224.924
R10	456.205	426.992
Profit before tax		
R1	5.313.205	4.688.370
R2	0	228.818
R3	2.496.420	1.531.156
R10	651.722	609.989

Table 7.16: Absolute profit distribution for scenario 6. All profits are in regional currency, except the overall profit (in R1 currency).

Tax rate	Optimal solution	Real life solution	Relative difference
0%	5.544.494	5.154.503	7,57%
10%	5.354.697	5.044.260	6,15%
15%	5.263.032	4.989.138	5,49%
20%	5.172.562	4.934.017	4,83%
25%	5.083.383	4.878.895	4,19%
30%	4.997.253	4.823.774	3,60%

Table 7.17: Profit according to different tax levels ($R1$ currency, scenario 6)

32,4%. There is still a shift in the production from $R2$ to $R3$, but the trend is much weaker than in the scenarios 1 to 4 (see figure 7.18 for details).

The development in the net profit as a function of different tax rates in $R3$ has also been tested.

The relative difference of the solutions seems to decrease with an increasing tax rate i.e. the real life solution comes closer and closer to the optimal solution. It must be noted that this is stated as an apparent observation, not as a matter of fact, as the profit as a function of the tax has not been examined thoroughly. Table 7.17 with figure 7.19 illustrates this trend.

This trend brings focus to the question of which parameters are most im-

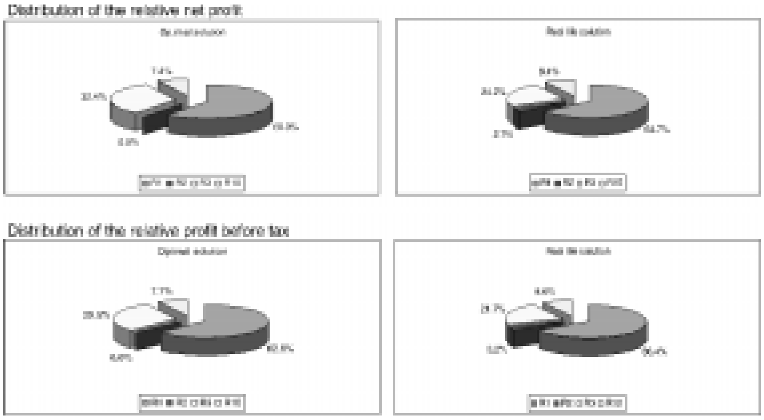


Figure 7.18: Regional profits relative to the total accumulated profit for scenario 6

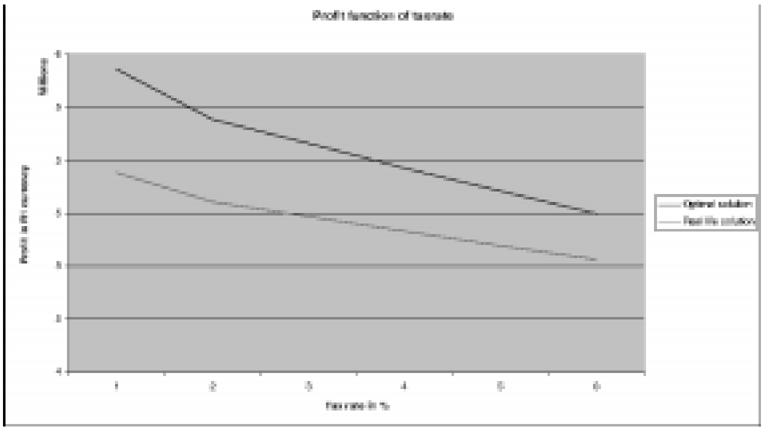


Figure 7.19: Profit according to different tax levels (scenario 6)

Cost	Optimal solution	Real life solution
Production	40,9%	39,5%
Distribution	1,0%	0,5%
Storage	0,0%	0,0%
Tax	56,6%	57,7%
Duty	1,3%	2,2%
VAT	0,2%	0,1%

Table 7.18: Cost distribution for scenario 6

portant to optimise. With the observed trend it should be considered if the tax parameter does not deserve more attention. At least it should be considered to examine this problem further.

Production and Distribution

The main improvements in profit are still achieved through a concentration of production in region $R3$ while the production in region $R1$ and $R2$ is reduced correspondingly. This change in production again calls for a greater distribution, but this tendency is now dampened considerably in comparison to the earlier scenarios.

Figure 7.20 illustrates the capacity utilisation for the fermentation, recovery and granulation phases as the fraction of the total capacity used and thereby indirectly the quantity produced within the regions. From this it can be seen how the optimal and the real solution corresponds much closer than in the earlier scenarios.

Turnovers and Costs

The total production costs are reduced with 3,2% compared to the real life solution. The total distribution costs are increased with 96,6%. All in all this results in a lowering of the production and distribution costs of 2,0%. The total duties paid decreases with 45% and the VAT rises with 48%. These extra costs are offset by reducing of the tax payments with 8%, yielding a better overall result. The semi fictive costs, the financial costs, also rise with the increased distribution. The effect is small though as it is less than 0,1% of the total production and distribution costs.

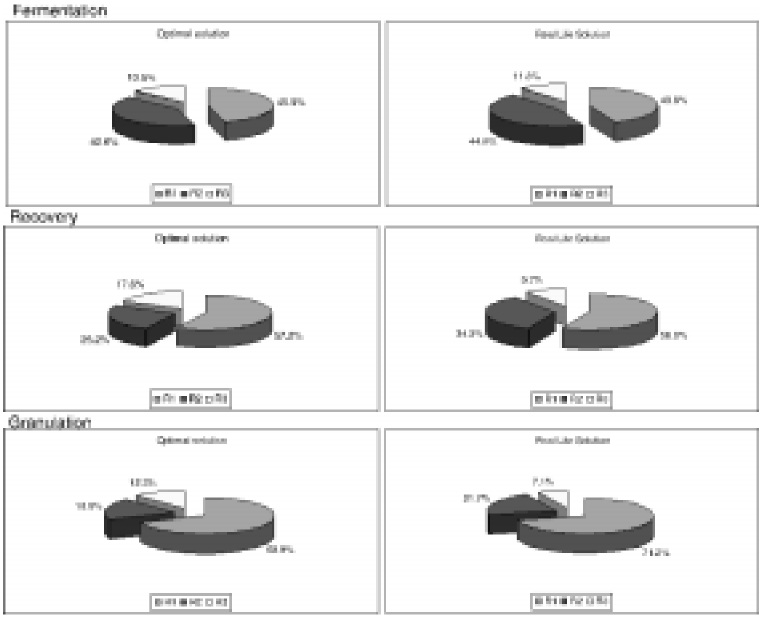


Figure 7.20: Regional capacity utilisation relative to the total capacity utilised for scenario 6

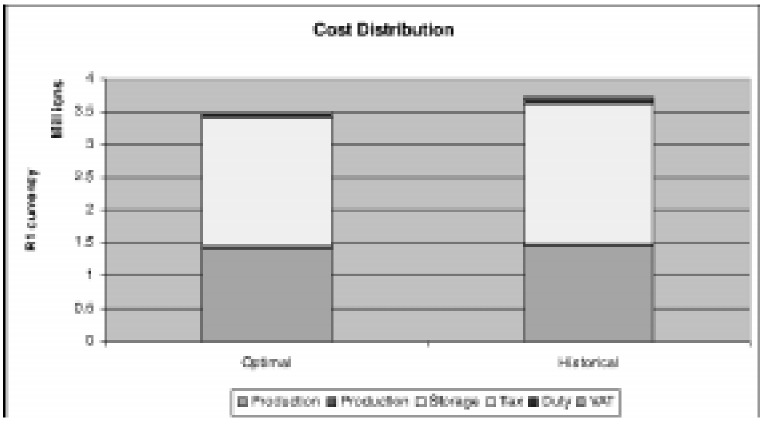


Figure 7.21: Costs for scenario 6

Figure 7.21 shows the different cost levels.

Differences in Production

Many changes in the production, moving products between facilities, are suggested by the optimal solution compared to the historical solution, but the overall capacity usage is roughly the same across the different regions.

The main changes is found in the production of products $P2$, $P8$, $P10$, $P17$ and $P20$, which are partially or wholly switched from production in region $R1$ to $R2$, while products $P4$, $P14$, $P15$ are moved the opposite way. Products $P7$ and $P9$ are moved from $R2$ to $R3$ and $P1$ vice versa.

All in all these movements do not, as aforementioned, have a significant impact on the capacity usage, but definitely the costs of production and distribution are affected as well as the taxes, VAT and duties paid.

Marginal Analysis

The only region with a marginal profit on the capacities is $R3$. Here there is a marginal profit on both phase k and m in the production in both time periods. This points to which capacities should be examined first and considered developed first in the case of a rising tax level in $R3$. The level of increase should also be examined meticulously, as it is very likely to be a deciding factor for the potential in improving the capacities. Should the level of taxation rise above 20% in $R3$, it is likely not necessary to increase the capacities in this region.

The marginal profits on the demand of the different products do not change significantly compared to the previous scenarios. It is still product $P4$, $P14$ and $P17$ having the greatest marginal profits although it is lowered for $P4$ and $P14$ compared to scenario 1.

Conclusions

The consequences of raising the taxation level in $R3$ is a decrease in the difference between the historical solution compared to the optimal solution. Most of the products are moved around the facilities, but overall the capacity usage is the same. The distribution cost rises and therefore also the financial costs, but the production costs are lowered together with the total VAT, duties and taxes paid all in all yielding an improvement in profit of 4,8%.

The overall trend of a decreasing difference between the historical and the optimal solution should be investigated further to verify if there is too little focus on the financial factors in the current planning process.

7.4.3 Changes in Royalties

The consequences for the profit and production, with raised royalties payments between the regions, are examined in this section. The capacities as well as production possibilities are bounded as for scenario 1. The only parameter changed is the royalty rate. As the royalty rate is already fixed in reality in both $R1$ and $R2$ the only interesting rate to change is in $R3$. The rate in $R3$ is therefore raised to 10%, as this is seen as the most realistic future development.

The results of the optimisation are presented below. The results are documented in the output file (finalmodel.lst) and the Excel spreadsheet (scenario 7 analysis.xls, Product flow.xls and Marginal values.xls) in the folder scenario7 on the company specific CD.

	Optimal solution	Real life solution
Overall profit z	5.461.963	5.083.880
Netprofit pr region		
R1	3.860.238	3.446.646
R2	0	137.291
R3	2.203.820	1.204.198
R10	456.079	426.992
Profit before tax		
R1	5.514.625	4.923.780
R2	0	228.818
R3	2.203.820	1.204.198
R10	651.541	609.989

Table 7.19: Absolute profit distribution for scenario 7. All profits are in regional currency, except the overall profit (in R1 currency).

Economical Results

The implementation of a 10% increase in royalty in $R3$, yields a possible improvement of 7,4% in the profit when comparing the optimal and the historical solution. The absolute value of the profit is decreased as expected, as the model is now forced to move a part of the profit from a zero tax region to a nonzero tax region. The profit distributions for the historical and optimal solution are presented in table 7.19.

The absolute profits are reduced, but the relative difference is the same as in scenario 1. As expected, the absolute reduction in the historical solution when comparing scenario 1 and 7 is the same as the increase in total taxes paid. For the optimal solution the reduction in the profit is only 52% of the increase in taxes paid. (See *scenario 7 analysis.xls* in folder sc7 and *scenario 1 analysis.xls* in folder sc1). The historical solution behaves as expected, and the optimal solution illustrates the ability to move around the production and distribution in order to minimize the effect of the increase in taxed profit. In this way it seems possible for the model to continually achieve the same relative improvement in the profit, independent of the level of royalty in $R3$.

This has been tested further as indicated in table 7.20, where the development in the net profit when applying different levels of royalty rate in $R3$

Royalty rate	Optimal	Real life	Relative difference
0%	5.544.494	5.154.503	7.57%
10%	5.461.963	5.083.880	7.44%
20%	5.388.346	5.013.257	7.48%
30%	5.316.129	4.942.635	7.56%

Table 7.20: Profit according to different royalty rates ($R1$ currency, scenario 7)

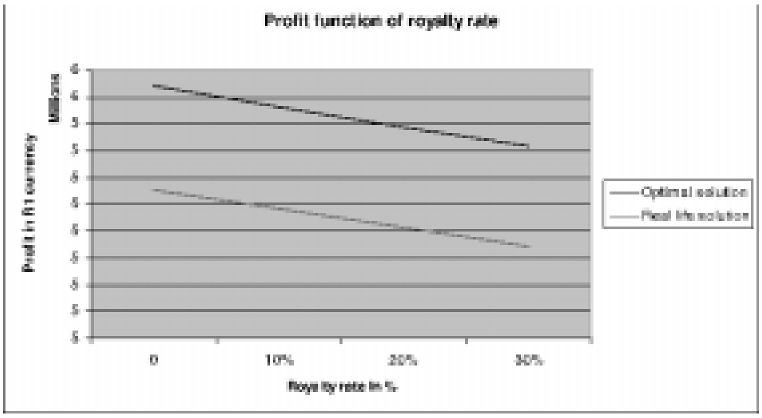


Figure 7.22: Profit according to different royalty levels for scenario 7

is shown.

The relative difference of the solutions seems to be very constant with an increasing royalty rate. It must be noted that this is stated as an apparent observation not as a matter of fact as the profit as a function of the royalty has not been examined thoroughly. Figure 7.22 illustrates this trend.

Not surprisingly there seems to be a general problem in maintaining the overall level of profit, when raising the royalty level for regions with lower taxation levels. The result is a higher average taxation rate.

Production and Distribution

The changes in capacity utilisation is very much like seen in scenario 6, where the optimal and the real solution corresponds much closer compared

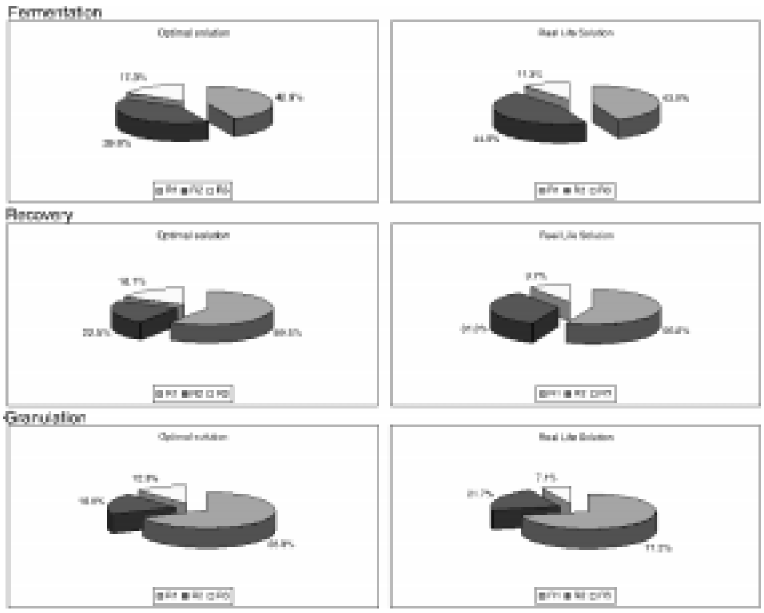


Figure 7.23: Regional capacity utilisation relative to the total capacity utilised for scenario 7

to earlier scenarios. This is illustrated in figure 7.23.

Turnovers and Costs

The total production costs are reduced with 4,3% compared to the real life solution. The total distribution costs are increased with 130,7%. All in all this results in a reduction of the production and distribution costs of 2,8%. The total duties paid decreases with 27% and the VAT rises with 162,0%. These extra costs are offset by a lowering of the tax paid by 16%, yielding a better overall result.

Figure 7.24 shows the different cost levels.

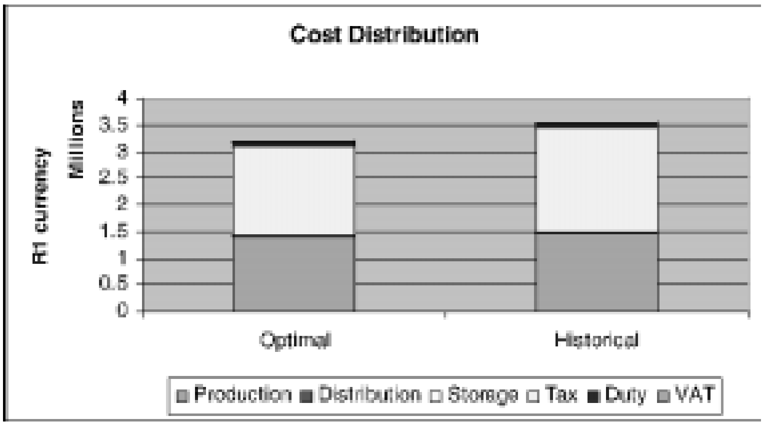


Figure 7.24: Costs for scenario7

Differences in the Production

In the optimal solution a number of changes in the production and the corresponding distribution levels, compared to the historical solution, are suggested. However, the overall capacity usage is roughly the same across the different regions.

All in all these movements do not, as aforementioned, have a significant impact on the capacity usage, but definitely the costs of production and distribution are affected as well as the taxes, VAT and duties paid.

Marginal Analysis

As seen in scenario 6 the only region with a marginal profit on the capacities is $R3$. Here there is a marginal profit on both phase k and m in the production in both time periods; this is also the case for scenario 7. The increase is quite large as the average over both time periods for the marginal value in phase k is approximately 212% and in phase m approximately 403%. When evaluating an increase in the royalty, these capacities become considerably more interesting than in the previous scenario 6, as the increase in marginal profit is quite significant.

The marginal profits on the demand of the different products do not change significantly compared to the previous scenarios. The products $P4$, $P14$ and $P17$ do still have the greatest marginal profits.

The sensitivity of the marginals are not considered. Due to the effects of the BoMs through the production chain the effect of the marginal considerations are difficult to interpret. The purpose of considering the marginals in this perspective is solely to gain knowledge of where to start a further analysis of the capacities. It should not be perceived as a true expression for the size of an investment exactly, balancing the extra profit obtained.

Conclusions

Implementing royalties in $R3$ paid to $R1$ definitely has a negative influence on the profit. The overall level of taxation rises for the company reducing the absolute profit in the model with approximately 8 million in $R1$ currency. Most of the products are moved around the facilities, but overall the capacity usage is the same. The distribution cost rises and therefore also the financial costs, but the production costs are lowered together with the total VAT, duties and taxes paid all in all yielding an improvement in profit of 7,4%.

There is a trend of a constant difference in the historical and the optimal solution when raising the level of royalties paid in $R3$.

7.5 Changes in Capacities Over Time

7.5.1 Decrease of capacities

The capacities in the production at the different sites decrease in some periods according to holidays. To fulfill demand requirements this emphasize the usage of storage or a different global production sourcing, The cost of this reduction in capacities due to the increased need for storage or distribution is tested by expanding the number of time periods from 2 to 4 and recalculating the capacity in time period 3. The capacities are multiplied by a workforce utilisation factor in period 3 to reflect the holiday season. See Excel spreadsheet *bom and capacity usage for implementation* on the company specific CD. Through this workforce utilisation factor, the capacities are lowered to a level in period 3, forcing the use of storage.

	Decreased capacity	Constant capacity
Overall profit z	11.064.658	11.088.987
Netprofit pr region		
R1	7.032.827	6.987.142
R2	0	0
R3	5.565.603	5.658.183
R10	989.740	992.578
Profit before tax		
R1	10.046.896	9.981.631
R2	0	0
R3	5.565.603	5.658.183
R10	1.413.915	1.417.968

Table 7.21: Absolute profit distribution for scenario 8. All profits are in regional currency, except the overall profit (in R1 currency).

The scenario is run once more with the usual bounds on the capacities in all four periods to see the difference in the objective values.

The results of the optimisation are presented below. The results are documented in the output file (finalmodel.lst) and the Excel spreadsheet (scenario 8 analysis.xls, Product flow.xls, Marginal values.xls and excel comparisons.xls in the folder constant data) in the folder scenario8 on the company specific CD.

Economical Results

The implementation of a reduction in the capacities in time period 3, yields a result which is slightly worse compared to having constant capacities. There is a relative decrease in the overall profit of 0,22%. A decrease is of course expected, as a decrease in the capacities forces the use of storage or distribution - adding an extra cost. The profit distributions for the constant and decreased capacities are presented in table 7.21.

Apart from the minor decrease in the overall profit, there seems to be very little difference between the regional profits before and after tax in the two cases.

Production, Distribution and storage

The production and distribution changes when the capacities are decreased but the changes seem relatively small. In period 2 and 3 the greatest number of changes take place. This seems reasonable as the solution has to store products from period 2 to period 3 as the production is forced to change due to the further constrains.

The only significant changes are found in the production and distribution of $P5$, $P16$ and $P18$. In time period 1, the production of $P5$ in $R1$ is moved from production in time period 2 and production of $P16$ in region $R1$ is in the same manner pushed one period back from time period 3 in $R1$ to time period 2 in region $R2$. Production of $P18$ is transferred from region $R1$ to $R2$ in time period 2.

An evaluation of the total capacity usage fits nicely with the analysis of the changes in production. The total capacity usage shows a greater tendency to utilize the capacities in time period 1 and 2, while more of the so far unused capacity in $R2$ is employed. The solution with the decreasing capacities in time period 3 behaves as expected - by employing a greater capacity up till the decrease - and furthermore the extra available capacity utilized in $R2$ is balanced with the cost of storing.

With the implemented decrease in capacities in period 3 the need for storage is limited. As an example, not more than approximately 9,6% of the total production in phase m, is stored from time period 2 to 3. (1 unit of product equals 1000 kg or l) (See scenario 8 excel comparison.xls on the company specific CD, folder Sc8, Constant data). All in all the changes of the costs in storage, production, financial, VAT, duties, distribution and taxes yield a decrease of the objective function of 24330 in $R1$ currency. The results are summed in table 7.22, showing the absolute deviation between the two scenarios. The greatest part of the decline in the solution stems from the rise in taxes paid as a greater part of the production is moved to a region with a higher tax. The extra taxes paid with a decrease in the capacity is -19515 in $R1$ currency, amounting to approximately 80,2% of the total reduction in the objective value.

Marginal Analysis

With the lowered capacities several more bounds generates a marginal profit different from zero. Generally speaking the marginals appear mainly in the

Variable	Absolute difference
Production cost	-4755
Distribution cost	2036
Storage cost	-5063
Financial cost	-3207
Duty	3329
VAT	2845
Tax	-19515
Total	-24330

Table 7.22: Differences between costs for scenarios with constant capacities compared to the periodic reduced capatities. (scenario 8)

phase k and m in region $R3$ when the capacities are constant in all periods. When the capacities are lowered in period 3, the marginals appear both in $R1$ and $R3$ in phase k in several time periods, and in phase m all producing regions has positive marginals in period 2 and 3.

The observation of the marginals, and the development when lowering the capacities, points to phase m as the most profitable place to develop the capacities. Region $R3$ definitely seems to have the greatest potential when looking at the marginals, but also in $R1$ and $R2$ there is a profit to be gained. The marginals for the earlier production phases in $R3$, phase i and k, are far higher than the marginal levels for phase m in region $R1$ and $R2$.

Conclusions

Testing the influence of implementing a seasonal reduction of capacities yields, not surprisingly, a worse result than when keeping the capacities constant. The solutions leads to a different sourcing of production and the use of storage. The demands are balanced through storage rather than distribution; the different sourcing leads to reduced distribution costs. What may seem more surprising is the basis for the reduction which is found in the regional differences in taxation levels, and not as much in the extra costs caused by the need for storing.

These extra storage costs, together with the associated financial costs, are more or less offset by the reduction of duties and VAT in the solution. Most of the additional costs occur from the production which is forced to a higher taxation region. These extra costs represent approximately 80% of the reduction in the net profit.

Finally it should be noted that the need for storing is limited and the realism in this is doubtful. The exact level of capacities and the seasonal reduction in these should be investigated further to enable a more detailed and realistic modelling of this feature.

The results emphasize that a higher profit is achievable when capacities are increased in the holidays. However it should be noticed that additional costs for i.e. capacity expansions or fixed costs are not considered in this scenario, and a more detailed analysis should be performed in order to draw any conclusions on this aspect.

7.6 Conclusions from the Scenario Analysis

Modelling the eight scenarios based on the Novozymes supply chain structure indicates an economical potential from using OR-tools in the planning process. The optimal solutions generated in the different scenarios ranges from displaying improvements in the profit from 4,8% to approximately 14,9% compared to the historical solution. In the case of scenario 1 containing all bounds, the optimal solution is 7,6% better than the corresponding historical solution. For more relaxed scenarios (scenario 2, 3 and 4), an improvement of up to 14,9% is achievable. In relationship to the relaxations it should be noted that the removal of capacity bounds yields massive improvements compared to the removal of restrictions in the regions production portfolios. However, the costs of investments in order to realise the scenarios in real life have not been evaluated.

When testing changes in external factors as currencies, taxes and royalties paid, the achievable improvement is 8,0% when changing the exchange rates, 4,8% when raising the level of taxation and 7,4% when raising the level of royalties.

Optimal solutions are generated in the case of modelling 4 time periods with and without a reduction in the capacities, due to the holiday season. Comparing the optimal solutions in the two cases show an improvement of only 0,2% under constant capacities. Reduced capacitites lead to a different sourcing and storing. Thereby additional production costs and taxes occur; distribution costs are reduced due to the different sourcing.

7.6.1 Structural changes - changes of the bounds

The optimal solutions are characterised through a trade off between the different cost factors. For the optimal solutions increased distribution costs and import duties are accepted if the overall costs are reduced. This indicates that optimal solutions are not achieved through narrow minded minimisations of the individual cost factors. The main contributors to profit improvements are reductions in tax contributions and production costs due to different product allocation.

Tax is a fiscal flow to the authorities and not a flow within the organisation like i.e. transfer pricing. The tax levels have a high influence on the production allocation. This clearly shows that optimisation models for international supply chains with focus on cost minimisation are too simple. They are not sufficient to ensure a valid model under international aspects as one of the main aspects is left out. A general tendency is movement of production to low taxation regions (here $R3$). For limited capacities the products are characterized by a high marginal after tax profit pr quantity. The result is a higher profit share for the low taxation region and a reduced overall production cost. For less profitable regions turnovers and costs are balanced to avoid losses.

The distribution between regions is increased - both of concentrates and finished products. The concentrates are mainly shipped due to capacity restrictions on fermentation and recovery. Finished products are distributed from low cost and taxation regions to improve the profit for the export region.

7.6.2 External parameters in the existing network

The model shows a low sensitivity to currency fluctuations. The relative improvement for the optimal solution compared to the historical is almost constant under currency fluctuations. The absolute profit however does have a considerable dependency on the levels of the currency exchange rates. The production allocation differs under different currencies with a tendency of allocating production to regions with the lower exchange rates.

Changing the taxation level in $R3$ yields an overall lower result and also a smaller relative difference between the optimal and the historical solution. The trend when raising the taxes in $R3$ further is a lower absolute profit and a further reduction in the difference between the optimal and the historical solution. This leads to the need for further analysis of the effects of differences in the regions taxation levels.

Implementing royalties in $R3$ decreases the overall profit as a regional profit in a zero tax region is moved to a region with nonzero taxes. The optimal solution seems to constantly improve the historical solution with approximately 7,5% independent of the level of royalty implemented.

The influence of the individual parameters is easy to evaluate, the correlated effects of all parameters are however difficult to understand and evaluate. A rational decision process in the search for an optimal production and distribution plan is therefore difficult without the aid of it-based tools for scenario analysis. Furthermore the model can serve as a support tool in the marketing process as marginal levels on product sales can be identified.

The conclusion on the scenarios is that the LP model for the supply net leads to better, more profitable solutions. These solutions are not necessarily applicable in reality but as a decision support tool the model gives insight into several important aspects, i.e. the influence of key parameters. Using optimisation in the planning process is therefore only considered profitable as a tool for scenario evaluations, as several other non-financial aspects should be considered. In the model other parameters should be implemented to give a human planner full understanding for the solutions, i.e. possibilities for comparison of marginal values across process steps.

Chapter 8

A Better World

The following chapter focuses on the future perspectives from three different points of view:

1. A case specific viewpoint
2. A theoretical viewpoint
3. A practical viewpoint

The case specific approach covers future aspects for improvements in the model and data structures in relationship to the specific case discussed in this project. The case specific point of view is presented in section 8.1.

The theoretical approach focuses on general modelling theory and on the general usability of the developed model in relationship to different industries. The theoretical perspectives are presented in section 8.2.

Finally the future perspectives of a practical implementation are discussed. A rough outline for an implementation plan is set up and an approximate cost-benefit analysis is drafted in section 8.3.

8.1 Improving the Case Specific Model

The work of modelling a complex and widely ramified production and distribution network and fitting the available data over a short time is a highly iterative process. Improvements and changes have been implemented continuously, but there are some general points where the modelling work and the model can be improved.

In the following section the possible model improvements and adjustments to achieve a better model are presented (section 8.1.1). Also the specific need for data has been defined more precisely during the development of the model. In section 8.1.2 the data improvements are presented. Finally a structural change of the network moving the CODP (section 4.2) backwards in the network is suggested (section 8.1.3). This suggestion relates to the present technological possibilities in the enzyme production, as these incur strict boundaries on the options available in this respect.

8.1.1 Model Improvements

The model can be improved in the following ways:

- Loss of optimality when using batch and campaign sizes.
- Strength / concentration adapted when shipping between regions.
- Distribution and storage costs associated with indexes.
- Using mixed integer programming.
- Introducing stochastic elements.
- Enabling sensitivity analyses of parameters.
- Capacity Usage Measure.

Loss of Optimality

As the model is linear, the loss in the objective function from rounding the production plan to the nearest higher batch size is not taken into account. As the production in reality is run with quite large batches and campaign sizes this could have an effect which is far from negligible. In this case only a fraction of the product portfolio is implemented, however if all products are implemented the batch size constraints should be considered.

In GAMS an implementation approach is to identify the number of batches. The results from the optimal solution could be divided with the pre-defined batch sizes for the different products and thereby a real number of batches will be given. The nearest higher integer value could be found using the ceil function in GAMS.

The model should be run again with fixed variables for the first production step (corresponding to the batch size). Comparing the objective values from the linear optimal solution and the fixed solution would give an idea of the loss in optimality when using correct batch sizes as the linear problem can be considered as an upper bound for the maximisation problem.

Strength / Concentration Adaption

There are in some instances quite large differences between the regional bills of materials. Due to different process equipment similar products have different strengths in different regions. For products with a higher strength a smaller quantity is necessary in the succeeding process steps. This is used by the solver to generate artificial savings in the solution which realistically are very doubtful (see section 7.2.1 and 7.3.1). To counter this source of error several possibilities could be considered.

One approach is to use the same BoMs for all facilities. The BoMs could be created with average values from the different regions. This would result in a loss of precision in the BoM but also corroborate the realism in the solutions.

Another approach is to implement a conversion factor when using subcomponents from other regions. Using this kind of *strength/concentration* coefficients could equal the differences in the BoMs and in this way obtain a true expression for the input needed, when semi-finished products are shipped across regions.

Distribution and Storage Costs Associated with Indexes

In the current model the costs of distributing and storing are modelled as costs associated with each product in each phase. This involves a lot of redundant data as many of the products have the same distribution or storage characteristics. A more clever way of modelling the costs could be done by associating additional indexes with each product in each phase for the type of distribution and storage taking place. In this way only a small dataset representing the different costs of storing and distributing is needed. This would also minimize the amount of data needed to be maintained.

Using Mixed Integer Programming

It should be considered to implement the use of mixed integer programming in order to model further features such as fixed cost (binary programming), batch and campaign sizes and full loads when distributing.

The structure of the model could resemble reality much closer when using integer programming. This would enable the instant generation of more realistic and credible solutions, which would need less manual rework. This could enable the direct use of the results as the basis for an actual production and distribution plan or in analysis of the scenarios. This is in particular interesting if all products are implemented in the model.

The size of the model should be thoroughly evaluated in case integer programming is implemented. To find optimal solutions within reasonable solution times could turn out to be a problem for the full size problem (See section 5.2.1 regarding the size of the model). The solvability when implementing integer programming is very hard to predict, as the ability of the model to handle the problem is entirely dependent on the structure of the specific problem. The only way really to find out is to test the specific case as a mixed integer problem and see if solutions can be generated inside a reasonable time frame.

Introducing Stochastic Elements

An interesting feature to implement is the use of stochastic elements. Most of the parameters involved in the model are not constant over time. This variation could be modelled more precisely by trying to find appropriate distributions representing the variation of external parameters i.e. the demand, prices or exchange rates.

The distribution of the parameters could be incorporated by describing possible future developments by scenarios. The overall profit should be maximized in relationship to the probability of the scenarios taking place. This would not give the optimal solution to the best case but it would maximize the profit according to the incorporated probabilities. In the long run it is therefore expected to create better solutions.

Enabling Sensitivity Analyses of Parameters

It would also be interesting to enable analysis of the marginal values and sensitivities of parameters like exchange rates, transfer prices, royalties, duties and taxes. This could be achieved by modelling the parameters as variables with defined intervals. The problem of course is that this makes the model non-linear as these parameters are associated with the product variables.

The size of the model should also in this case be observed to ensure solvability.

Capacity Usage Measure

It is not only desirable to model the actual flow of products through the production nodes it is also seen as a clear advantage to model a concurrent capacity utilisation per unit finished good. This concurrent modelling will enable a comparison of the marginal values across process steps, and help identifying the most profitable investments.

The current marginal values on the production bounds are very difficult to compare as the scale is different for each phase.

Concluding Remarks

As seen from the above section there are a number of possible model improvements. The motives in the mentioned improvements are twofold:

- First of all it is a matter of bringing the model and the flow in the model as close to reality as possible to ensure reliability in the result. This reliability also contains a validation and verification that the optimal solution does not utilize illegal advantages as described in section 7.2.1 and 7.3.1.
- Second it is a matter of attaining transparency in the output which is generated. Transparency in the meaning that the results can be analysed and the reasons behind the result can be uncovered. In this way the model and the solution can be seen as an argument generator for evaluating future production and distribution strategies.

8.1.2 Data Improvements

The need for high quality data when modelling is of great importance as the results will not reflect reality any better than the input data does. It is therefore an incontestable fact that the success of the modelling is mainly dependent on the ability to incorporate reality in the structure and features of the model and on the data input.

The data material can be improved on the following points:

- Expansion of production costs.
- Detailed storage costs and usage.
- Capacity usage and bounds on blending.
- Scale of capacity usage and bounds.
- *Strength/concentration* factor.

Expansion of Production Costs

Production costs must be expanded to include all of the costs and not only energy and material consumption. Speculations on this subject will probably lead to the conclusion that the prices of production equipment, raw materials and energy is fairly constant worldwide whereas costs regarding wages are much more dependent on the location. When only including the costs of energy and raw materials the true differences in the regions production costs are likely not reflected.

To obtain a greater reliability in the results, a further analysis of the overall production cost contributions from the individual cost factors should be performed, in order to identify the mandatory costs and the negligible costs.

Detailed Storage Costs and Usage

The cost of storing implemented in the model at this stage is an estimation of the actual costs. These costs should therefore be studied closer in order to be verified.

The storage usage must also be studied further as the usage is dependent on the packaging of the products. At the current stage the use of storage capacity and therefore also costs are calculated directly from the units stored on a one to one basis. This is not absolutely true as the packaging of the products determines the actual storage space used. The model is set up to implement this feature via the capacity utilisation factors - also for the storage phases.

One fairly simple and fast way to get around this problem is to utilize the existing knowledge of each of the products distribution over different packaging types, combined with existing knowledge of the storage used for the individual packaging types. In this way a weighted average of the storage usage could be calculated for each product. This weighted average could be used as the capacity utilisation factor in each of the storage phases.

Capacity Usage and Bounds on Blending

Currently no bounds or usages in the blending phase have been introduced. The blending phase is merely a process combining products from the previous production phases. As far from all products need a blending phase it can be questioned if this phase belongs in a tactical or strategic model. Actually it is probably more relevant to use a sales forecast for products on a level where the blending has not taken place yet. This would also move the CODP a small step backwards in the internal network, which must be seen as a strong advantage, see 8.1.3. The reliability of the forecasts also stands a good chance to be improved in this way as the forecasts will now be generated over a partially aggregated portfolio of products, which can be transformed into different finished products.

Scale of Capacity Usage and Bounds

The capacity bounds and usages should be measured pr unit product in the matching phase, thereby avoiding the need for using capacity utilisation factors calculated from other data. In the existing model, data regarding the capacity usages per 1000 units product in the production phases, are generated from several other data sources. This undermines the reliability, as the unit size in the primary data is different and the capacity use is stated per unit of finished goods. The recalculation of the capacity usage according to the actual flow of products through the node has not been validated.

Therefore it is desirable to develop a whole new set of measurements in which the capacity use is generated directly per 1000 units of product flow in the specific phases.

Strength / Concentration - Factor

The *strength/concentration* the products must be introduced as a factor applied when the transfer of semi-finished goods takes place (as described in section 8.1.1). This will ensure the right production costs and capacity use when shipping intermediate products between facilities with different BoMs.

8.1.3 Structural Improvements

As Novozymes A/S is primarily defined as an A-company (see section 4.2) moving the CODP backwards in the chain presents a great potential for cutting cost and thereby improving profit. This possibility has been examined by the company, but the limitations with the present production technologies impede the opportunities in this area to a great extent. From a theoretical point of view there is no doubt it would be the right thing to do, but it is only possible to implement small changes in this area. The options here - though limited - should be utilized.

When planning on a strategic level it is advantageous to aggregate similar products into families, with a forecast on the accumulated demand. This leads to the wish for excluding the blending phase, as this is just a process of mixing finished products to create new products. In this way only forecasts for the ingredients in the blending phase are needed. Aggregating the products which are blended with the unblended ones may conceal a cost factor incurred on the blended products, but this cost appears neglectable.

Other products can be aggregated as these only contain a very slight differentiation in the end phase of the production. As an example several products differ solemnly in the number of final analysis performed, according to the market where it is sold. These products should also be aggregated when planning on a strategic level. Again a small cost is neglected, but the costs of these differentiations is considered insignificant.

These two types of aggregations can be implemented with the current technologies available. It is seen as an advantage, both when forecasting and when planning strategically; furthermore it generates a slightly more flexible production system if the last production steps are made to order.

Concluding Remarks

The quality of the data is just as essential as the structure of the model, when trying to generate solutions with a high degree of reliability.

Concerning the data there are a number of points where improvements can be attained. Considering the amount of data implemented, the precision of this data and the lack of actual data verification, the reliability of the model is weakened considerably. The model and the solutions serve as a proof of concept in regards to the profitability in using this kind of optimisation tools but the results generated does not contain adequate reliability to be used as a basis for decision support in the production planning.

It is not possible to fully utilize a change of the CODP, but aggregations in the product portfolio can be implemented generating at least small improvements in respect to the forecast quality, the planning process and the flexibility.

8.2 Improving the theoretical model

Through the project an insight in some of the general theoretical issues, when modelling, has been obtained. The section below is a description of two points; the first section (8.2.1) deals with the experiences regarding general modelling techniques, while the second section (8.2.2) deals with the general application of such models.

8.2.1 Modelling Technique

The single most important observation is the massive consequences of the fiscal flow when optimizing the profit in a global supply network. This case clearly shows this importance, as the fiscal factors are dominant when evaluating where savings can be obtained (see section 7.6.1). This observation only becomes more relevant in relationship to the amount of modelling work relying solely on cost minimisation - even in global supply networks (see section 4.4). Focusing on cost minimisation reduces the amount of necessary information when modelling but entails the risk of grossly simplifying the model, ruining the reliability of the results completely. In the current situation with a growing globalisation tendency the importance of this observation is emphasized.

Depending on the type of industry the implementation of batch and campaign size constraints in the model may be very relevant. This emphasises the importance of developing mixed integer programming models. In relationship to this subject it should be noted that implementation of all products and their sub-components is necessary to achieve reasonable batch sizes in the model. This is especially important in the case where different finished products are based on the same batch of sub-components. If the feature is not implemented in this manner the cost of batching will rise and the reliability of the result will be weakened.

Depending on the industry the implementation of fixed costs may be very relevant, as this feature according to its dominance in the model, may move the optimal solution. The fixed cost should be implemented as integer constraints to model the cost of opening/closing production facilities or secondary storage facilities. An implementation on production line level (instead of aggregated plants) could indicate the profitability of the individual production lines at each facility. Also an implementation of more detailed costs, i.e. setup/clean out costs etc would be useful. The level of detail must be decided in relationship to the purpose of the model. Clearly a strategic model dealing with the enterprise overall planning, should not go into the same detail as a more operational model.

Modelling realistic storage levels emphasize a very precise implementation of lead times and distribution times, e.g. through a continuous time approach or through smaller discrete time steps. Modelling with shorter time steps may enable a more precise description of the constraining factors entailing solutions with a higher degree of reliability. The purpose of the model is decisive in this respect as a model focusing on optimisation of storage levels compared to a model focusing on profit will be structured very differently.

Finally the use of soft demand constraints with a more detailed market description is perceived as very interesting. This feature enables an analysis of where to achieve the highest profit pr customer. Of course implementing a feature like this involves the problem of quantifying the long term costs of not supplying customers.

8.2.2 General Use of the Model

With the attained experiences regarding the general modelling technique as described above in section 8.2.1 the possibility to generalise the use of the developed model is considered.

The model initially was build as a structure with nodes representing production or storage phases and arcs representing transportation inside a facility or across regions to other facilities or customers. Next the physical flow of goods and the associated fiscal flow was setup. The elements in these flows are quite general as all production step have appended costs and distribution flows will have a corresponding fiscal flow of duties, VAT etc. As these features are common for all production and distribution networks it is definitely possible to adjust the elements in the model to fit other cases. The elements implemented in the model can be seen as building blocks which can be put together to suit other specific business cases. If the business case in question does not make use of internal royalty payments, the royalty equation can be omitted etc.

The structure of the model though, does prescribe a series of points which should correspond with the cases, if the model should be applied successfully. The model is seen as compatible in situations where the business case has the following characteristics:

- Production companies with multi-step production.
- International transfer of products and semi-finished products.
- Production of identical products at different facilities internationally.
- Similar production structure for the modelled products.
- Intra-organizational network.
- Tactical-strategic level of planning.

In order to implement the model it must be possible to split up the production in multiple stages with clearly defined bounds in time and place. As the model is using profit optimization it is clearly most relevant to use in situations where an international distribution is taking place. To utilize the potential in the optimisation, production of identical products should be possible at different facilities, ensuring the solutions ability to change the production and distribution plan as freely as possible. The product portfolio must have a similar production structure to make certain the whole range of products fit in the modelled network. The model functions currently as a depiction of an intra-organisational network without any actual borders to other organisations. In inter-organisational networks the fiscal flow between organisations will have a high influence; this is not considered in the developed model. In this respect it is an isolated entity. The model functions well on a strategic level but could also be used on a tactical level, although the relevance definitely is reduced. Due to aggregations and neglecting of lead times, the model is not suitable on an operational level.

In situations where the above characteristics are not present the model needs to be restructured or re-engineered.

In relationship to the above considerations an interesting subject of research is the characteristics of generic strategic supply network models for optimising production and distribution planning. A further evaluation of the general usability of the model would entail such an examination.

8.2.3 Concluding Remarks

Developing the model has brought attention to both the importance of some general features when optimizing, and to the elements characterising the case from a theoretical perspective.

First and foremost, it is seen how important it is to optimize the profit in a case where there is an international fiscal flow attached to the product flow. Second it is shown that the model - though composed of general elements - do require a series of characteristics in order to obtain any gains from using it. The different characteristics and the associated models should be further examined, enabling a detailed analysis of the generality and usability of this specific model and other model types.

8.3 Implementation

Through the project of developing a model of the internal production and distribution system in the company and comparing the historical solutions with the optimal solutions generated it is concluded that it is possible to obtain considerable advantages using this kind of tools, see section 7.6. From the company perspective, this project can be seen as a proof of concept regarding the potential in using optimisation tools in the planning process. The potential in using this kind of optimisation is not seen as a system generating the basis for a master plan, but as a decision support tool, used when evaluating the consequences of different long term strategies.

The next logical question is of course how to transform this potential to an actual improvement on the bottom line. In the following section a short description of the necessary steps for implementation and the economical aspects are evaluated, even though the effect of a decision support tool can be very hard to identify and quantify. The described costs and time frames are rough estimates and should be analysed further for an implementation. The cost calculations for the individual time periods are avaliable on the enclosed public CD (see the Excel file *Costs of implementation.xls*).

The section contains a rough outline of the four phases seen as essential in order to attain the full advantages of using supply network modelling and optimization. The phases are:

1. Purpose and specifications
2. Data collection, verification and modelling.
3. Implementation of solutions.
4. Maintenance of the system.

This outline also describes the necessary resources and the approximate timeframe to complete the implementation. It must be emphasized it is a very rough outline and the proposed costs as well as the proposed financial benefits are merely assumptions as these are hard to quantify. The purpose of including these costs in the outline of an implementation plan is to attain the possibility of evaluating the approximate costs in comparison with the potential improvements gained, establishing the basis for rating the appeal of the project.

8.3.1 Purpose and Specifications

Having established the proof of concept; the purpose of the model and the data requirements must be defined. The organisation must be able to uncover and state precisely and unambiguously, the needs which are sought solved with the application of the model. The quality of the specifications are decisive in the process of successfully implementing the use of supply network planning using OR optimization tools.

Resources: A project team of at least two people fulltime are needed in the opening phase when establishing purpose and requirements. The project team needs reference groups in the management of the Supply Chain Organisation (SCO) and in the management of Novozymes A/S, in order to assure ownership of the management and influence on the project. External consultants may also be needed in this phase.

Timeframe: It is estimated that this phase can be carried out over the course of approximately six month.

8.3.2 Data collection, Verification and Modelling

Second an iterative process of data collection and verification in combination with the further development of the model must be carried out. The data must be collected verified and integrated with the ERP system database structures, ensuring that the modelling and use of OR does not require additional implementation and maintenance of database systems.

Resources: A project team of four or five people is needed, as the collection and verification of data in this scale is the most labour intensive part of such a project. A reference group in the SCO department is needed to ensure the implementation of the correct features and the usability of the solutions generated from the model. External consultants may also be needed in this phase.

Timeframe: The time needed is estimated to 12-18 months, depending on the complexity of the data collection and structuring.

8.3.3 Implementation of Solutions

The third phase is closely associated with the second phase as the possibilities in modelling and solving the model is closely related to the soft and hardware applications used when implementing the model. The development of the model and the collection of data needed are in this perspective not isolated from the third phase.

The possibilities when implementing the use of mathematical modelling as an OR tool are numerous. A further analysis of these possibilities is needed to define exactly what software and hardware suits the purpose best. In relationship to this it is noted that the ERP system contains modules with options in the fields of optimization. In the section below two other alternatives are outlined.

One possibility is using GAMS as modelling software in connection with a CPLEX solver. The use of GAMS is well proven both generally and in this case. CPLEX is not the only possible solver, but seems to be the most popular choice in other comparable cases. The use of GAMS has several setbacks though. When generating the input data for the model, GAMS and CPLEX need to be integrated with the data warehouse of the ERP system. The user-friendliness is lacking in GAMS as the user interface consisting of a standard text editor e.g. EMACS is very primitive.

Another possibility is to develop specialised software (i.e. in Java) for generating the input to a solver. This programme could be integrated with the existing data warehouse, and a more user-friendly user interface could be developed.

As stated above several other options exist, both regarding the modelling language and the choice of solver. An evaluation of these possibilities requires a closer analysis of the products available. This analysis is not the subject of this paper.

Resources: A project team of two people, with external assistance from it consultants.

Timeframe: It is estimated that the model, and the integration with the current ERP system, according to which option is chosen, can be carried out in approximately 12 month.

8.3.4 Maintenance of the System

The fourth phase of the implementation consists of operating and maintaining the system. From this point on, two tasks need to be performed continuously. The first task is the maintenance of input data, keeping this up to date at all times. The second task consists of maintaining the model, continually fitting the structure of the model to the realities i.e. new production sites are build existing sites closed etc.

Resources: One person (part time) at each modelled facility and one person part time in the SCO department. The involved personnel at the production facilities will be responsible for continually updating the data for the respective facilities in the model and the SCO department will be responsible for updating the model.

Timeframe: After 36 month a successful system implementation should be achieved. The maintenance will take place until the system is deactivated.

8.3.5 Implementation Conclusions

When assessing the costs and the potential benefits of carrying out an implementation of a supply network model, the project must be inspected stage by stage in order to identify if the project should continue. The costs evaluated in each stage should not include costs of earlier stages as these are seen as sunk costs and therefore not relevant for the decision at hand.

The accumulated costs must be compared to the benefits gained when considering the return on the investment. Finally the yearly costs of maintaining and running the system should be assessed compared with benefits. The estimated costs are shown in table 8.1 below (for details, see the enclosed public CD, file:*Costs of implementation.xls*).

	Stage 1	Stage 2	Stage 3	Stage 4	Accumulated costs
Costs	600.000	3.680.000	1.820.000	920.000	6.100.000

Table 8.1: Implementation costs for each implementation step. All costs are in DKK and no discount factor is consider.

At stage one the cost of defining the purpose and establishing the precise specifications has a cost of approximately 600.000 DKK within the first 6 months. The benefits obtainable in this prefatory phase are not tangible. There will definitely be benefits, gained from a closer study of the business processes and the necessary data, but these benefits are hardly quantifiable.

If the project is continued after the first stage, the costs for the following 18 month will be approximately 3.680.000 DKK. These costs cover further data collection, modelling of the network and the validation and verification of both elements. In this phase of the project the benefits become more tangible. Modelling the production and distribution and defining the need for data gives knowledge of the cost structures in the production and distribution system. This increased knowledge of the true costs of the products is valuable in either case.

If the project is still considered beneficial the project goes into the third implementation stage. The costs for this 12 month phase are approximately 1.820.000 DKK. The accumulated costs at the time of operation are 6.100.000 DKK. Comparing the accumulated investment with the potential advantages, a positive gain is indicated. However, to ensure this benefit a more detailed analysis is necessary.

The results from the evaluated scenarios are not scaleable and furthermore the investment costs of i.e. capacity expansions are not considered . Therefore a detailed cost benefit analysis and an evaluation of the return on investment (ROI) is not possible at the current stage. However, it does not seem unrealistically to obtain a positive ROI within the first years of operation. The ROI in the following periods of operation will subsequently rise considerably as only the maintenance costs have to be compensated.

The costs of implementation, compared to the potential value as a decision support system, indicate achievable benefits with relative small investments.

Chapter 9

Conclusion

In this report a model for a logistic intra-organisational supply network in an international context has been developed. The model has been used on a specific case in the production and distribution network at Novozymes A/S. The model has been developed through a four step approach, starting with a generalised model which has been adjusted to a case specific linear program. The model is based on a general structure with nodes representing production steps and arcs representing a flow of products in or across geographical regions. It is set up as an acyclic digraph.

The model deals with maximization of the net profit when considering two different flow types: A fiscal flow and a product flow. The problem considers production, distribution and storage costs, prices, transfer pricing, tax rates, exchange rates, import duties, export value added taxes, royalties and financial costs on a tactical-strategic decision level. Different scenarios are modelled in the software GAMS/CPLEX and based on these specific cases the following conclusions can be drawn:

The project shows that an improvement of the overall profit for the organisation is possible. In the scenarios a profit improvement of approximately 7% is achievable in the existing production system, and through investments in structural changes up to 14% profit increase is achievable. The scenarios do however not represent all aspects of the real life planning problem and the necessary investment costs have not been evaluated.

For use in the daily planning a lower profit increase must be expected, and a detailed analysis of the optimal solution is necessary to ensure feasibility with the non-economical aspects. Furthermore an investigation of more exact input data is necessary to achieve feasible plans. Finally the results are not scaleable, as the implemented products do not necessarily have the same characteristics as the rest of the portfolio.

The answer to the hypothesis is, that the use of optimisation in the planning process is only considered profitable as a decision support tool.

The scenarios show that the optimal solutions come from a trade-off between the different cost factors. Though i.e. distribution costs are increased the overall costs are reduced through tax reductions. The model emphasises movement of production to low taxation regions.

The influence of the key parameters; exchange rates, royalties and tax rates have been investigated. The results show that the relative profit improvement under changing exchange rates and royalties is rather constant; the absolute profit however is highly dependent on the levels. The factors with highest influence on the bottom line are the taxation levels. With a more equal taxation level across the regions the relative improvements obtainable are reduced.

Through the results it is seen that bottlenecks in the production primarily will occur in the first two phases in low taxation areas. Elimination of these bottlenecks will improve the gain in profit further.

The optimal solutions show that the use of cost minimisation in an international network is not sufficient. Though several articles describe minimisation models for global networks, the impact of regional taxation levels cannot be neglected. This emphasises the use of profit maximisation in an international network as essential. The use of domestic models in an international context is not adequate.

For network models of supply network two significant different flows are identified: A product flow modelling the production and distribution and a fiscal flow modelling the financial transactions. As the flow may be decoupled this is an important consideration when modelling supply network.

The developed model can be used for modelling general multi process production-distribution systems within an intra-organisational network in an international context.

Chapter 10

References

The following references are used in the report. The references are divided into different categories (A-F).

A The A-literature is OR articles focused on strategic of tactical optimisation of supply network. A review of the A-articles is presented in chapter 4.4.

B Other OR articles focusing on supply chain optimisation but where the perspective is considered less relevant for this thesis are categorised as B-literature.

C Articles with operations management and supply chain management aspects are in the C-category.

E E-literature represents case specific literature

F F-literature is more general literature on operations research and management.

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Appendix A

Notation

The following notation is used for sets and variables in the mathematical models.

$DistC_{REG}$	Variable. Distribution costs for region REG in local currency
$FinC$	Variable. Define the global financial costs for storage and distribution products.
$FinDistC_{REG,REG2,t}$	Variable. Define financial distribution cost for distribution from region REG to $REG2$ in time period t.
$FinStoC_{REG,t}$	Variable. Define financial storage cost in region REG for time period t
Npr_{REG}	Variable. The regional net profit in local currency for region REG
PR	Defines the set of all products Ű intermediate and final products.
$ProdC_{REG}$	Variable. Production costs for region REG in local currency
$PROREG$	Defines the set of regions where production takes place. A subset of REG.
$ProVik_{PRik}$	Auxiliary variable. Observes the average product value of a product in set $PRik$

$PRik$	Defines the set of intermediate products for the production phases i to k. A subset of PR.
REG	Defines the set of all regions where production, sales or storage takes place
$Regpr_{REG}$	Variable. Regional profit before tax in local currency for region REG
$Regprpos_{REG}$	Auxiliary variable for positive regional profit in region REG
$Regprneg_{REG}$	Auxiliary variable for negative regional profit in region REG
$RoyC_{REG}$	Variable. Royalty cost in local currency for region REG
$RoyTO_{REG}$	Variable. Turnover from royalties paid to region REG
$StoC_{REG}$	Variable. Storage cost for region REG in local currency
t	Defines the set of time periods.
TC_{REG}	Variable. Costs for region REG from intermediate transfer of products (acquirements) to region REG in local currency.
TO_{REG}	Variable. Turnover for region REG in local currency
TTO_{REG}	Variable. Turnover for region REG from intermediate transfer (sales) of products from region REG in local currency.
$xi_{PRik,REG,t}$	Flow variable, defining the amount of product $PRik$ in 1000 kg units in region REG for production phase i and time period t. Production of product $PRik$ in phase i
$xl_{PRkl,REG,REG2,t}$	Flow variable, defining the amount of flow from region REG to $REG2$ of product $PRkl$ in 1000 kg in phase l for time period t. Distribution flow of $PRkl$.
$yl_{PRkl,REG,t}$	Flow variable. Amount of product $PRkl$ in 1000kg stored in region REG in phase l from time period t to period $t + 1$ Storage flow of $PRkl$.
z	Variable. The objective function: The global net profit after tax

Appendix B

Abbreviations

The following abbreviations are used in the report.

APS	Advance Planning Systems
BoM	Bill of Materials
CLM	The Council of Logistics Management
CODP	Customer Order Decoupling Point
GAMS	General Algebraic Modeling System
ER	Exchange Rate
ERP	Enterprise Resource Planning
LP	Linear Program
MIP	Mixed Integer Program
MRP	Material Requirements Planning
MRPII	Manufacturing Resource Planning
OR	Operations Research
R&D	Research and Development
SC	Supply Chain
SCM	Supply Chain Management
TC	Transfer Cost
TO	Turnover
TP	Transfer Price
TTO	Transfer Turnover
VAT	Value Added Taxes

Appendix C

Program Structure

This section contains a description of individual files used in the GAMS program as well as the Excel spreadsheets used for analysis of the results. A description of the general relations and structures is presented in chapter 6, section 6.3.

Section C.1 contains a description of the Excel files. Section C.2 contains a description of the input and output files in the GAMS model.

C.1 Excel Analysis Tools (.xls)

The results from the scenarios are imported in Excel spreadsheets to compare results and evaluate key parameters in graphical form. The following is the content of the spread sheets that are used for all scenarios. For the individual scenarios some additional spread sheets are created to evaluate scenario specific conditions.

Figures on i.e. profit levels or capacity utilization etc. are generated in the sheets to provide an overview of the solution. Furthermore pivot-tables are used to enable comparison of the actual variable levels and marginal values. The mentioned files are created for each scenario presented in chapter 7.

xls. files	
Data alignment tool.xls	The tool is used to import the .out files from GAMS. The sheets with the names *Aligned* are *cleaned* versions of the .out-files without comments. The *Aligned*-sheets are used for input in the other excel-sheets.
Scenario analysis.xls	The results are imported from the data alignment tool. Key financial parameters are presented, i.e. profits, turnovers, costs, tax contributions etc. Capacity utilisation levels are evaluated.
Product flow.xls	The product specific flow is presented and compared. Capacity utilisation on product level is described.
Marginal values.xls	The marginal profit values on capacities and demands are presented.

C.2 GAMS File Content

The following input files (.inp, .cmd) are used in the GAMS programmes and the following output files (.out) are generated. For details on the programme relations see section 6.3.

The .inp files (data input files) contains tables, parameters and scalars with case specific input data.

price.inp	Average sales prices
transferprice.inp	Transfer prices
prodcost.inp	Production costs
distcost.inp	Distribution costs
storcost.inp	Storage costs
demand.inp	Regional demand
duty.inp	Import duties between regions
exvat.inp	Export value added taxes between regions
bom.inp	Bill of materials for all phases
capacity.inp	Capacity limits for production and storage
caputlfactor.inp	Capacity utilisation factors for products and phases
exrate.inp	Regional currency exchange rates
tax.inp	Regional tax rates
dfcost.inp	Interest rate for financial costs
dtime.inp	Distribution times between regions
stime.inp	Storage times
noflow.inp	Fixes not-allowed flows for product-region combinations
histplan.inp	Fixes flow to correspond to the historical production plan

The .cmd files (command files) contain display command and post-optimal parameter calculations.

caputlobs.cmd	Observes the total and relative capacity utilisation for the facilities i, k and m
caputlobsproduct.cmd	Observes the capacity utilisation level for each product in each facility
profitobs.cmd	Observes tax, VAT and duty payments Observes the marginal profit on demand
dispopt.cmd	Generates the output files (.out) for the optimal solution
dispreal.cmd	Generates the output files (.out) for the historical solution
disponscreen.cmd	Generates the display command for the variables in the output file (.lst)

The following .out files (output Files) with results are generated with GAMS and imported in Excel	
profit.out	Contains overall profit and regional profits
turnover.out	Contains turnover levels, transfer turnover and royalty turnovers
cost.out	Contains costs for production, distribution, storage, royalties and import
flowopt.out	Level of the flow variables x for the optimal solution
flowrea.out	Level of the flow variables x for the historical solution
storagerea.out	Level of the storage variables y for the optimal solution
storageopt.out	Level of the storage variables y for the historical solution
capacityutlopt.out	Capacity utilization level for production facilities. Relative and absolute, for the optimal solution
capacityutlrea.out	Capacity utilization level for production facilities Relative and absolute, for the historical solution
capacityusageopt.out	Capacity utilization level per product for all phases, optimal solution
capacityusagerea.out	Capacity utilization level per product for all phases, historical solution
marginallevels.out	Marginal levels on capacity constraints
demandmarginalopt.out	Marginal levels on demand, optimal solution
demandmarginalrea.out	Marginal levels on demand, historical solution
taxcontributions.out	Tax, import-exort duties and payments

Appendix D

Model size

The size of the model is estimated in this section. The estimations are based on analyses of the size of the individual sets defining in the individual equations (5.1 - 5.23). Only the linear non-integer equations are included in the estimation. The sets are as defined in section 5.1.6. The model size estimation is used in section 5.2.

The size function is implemented in the excel spreadsheet (modelsize.xls) on the CD. Here the problem size can be evaluated as a function of the size of the regional set, product set and number of time periods.

With REG_{max} defining the number of regions, PR_{max} defining the number of products and t_{max} defining the number of time periods, the number of variables is given by the following relations:

Flow Variables Xi: $REG_{max}PR_{max}t_{max}$
Xk: $REG_{max}PR_{max}t_{max}$
Xl: $REG^2_{max}PR_{max}t_{max}$
Xm: $REG_{max}PR_{max}t_{max}$
Xn: $REG^2_{max}PR_{max}t_{max}$
Xo: $REG_{max}PR_{max}t_{max}$
Xp: $REG^2_{max}PR_{max}t_{max}$
Xq: $REG_{max}PR_{max}t_{max}$
This corresponds to $t_{max}PR_{max}(5REG_{max} + 3REG^2_{max})$ flow variables.

Storage Variables Yl: $REG_{max}PR_{max}t_{max}$
Yn: $REG_{max}PR_{max}t_{max}$
Yp: $REG_{max}PR_{max}t_{max}$
Yq: $REG_{max}PR_{max}t_{max}$
This corresponds to $4REG_{max}PR_{max}t_{max}$ storage variables.

Variables in the Objective Function Z: 1
Npr: REG_{max}
Regpr: REG_{max}
Regprpos: REG_{max}
Regprneg: REG_{max}
RoyC: REG_{max}
RoyTO: REG_{max}
TO: REG_{max}
TTO: REG_{max}
TC: REG_{max}
ProdC: REG_{max}
DistC: REG_{max}
StoC: REG_{max}

FinC: 1
FinDistC: $REG^2_{max}t_{max}$
FinStoC: $REG_{max}t_{max}$

This corresponds to $2+12REG_{max}+t_{max}REG_{max}*(1+REG_{max})$ variables in the objective function.

Variables to observe the product value ProVik: PR_{max}
ProVkl: PR_{max}
ProVmn: PR_{max}
ProVopq: PR_{max}
Which corresponds to $4PR_{max}$ auxiliary variables.

This leads to the total number of variables given by the function f:

$$\begin{aligned}
&f(PR_{max}, REG_{max}, t_{max}) \\
&= t_{max} PR_{max}(5REG_{max} + 3REG_{max}^2) \\
&+ 4 * REG_{max} * PR_{max} * t_{max} \\
&+ 2 + 12REG_{max} + t_{max} * REG_{max} * (1 + REG_{max}) \\
&+ 4PR_{max}
\end{aligned}$$

Appendix E

The Full Model

This appendix contains an overview of the mathematical formulation of the full model. Compared to the general model the full model is a relaxation of the integer constraints and a linearisation of the product values and financial costs. The difference between the general and the full model is presented in section 5.2

E.1 Formulation

$$maxz = \sum_{REG} Npr_{REG} ER_{REG} - FinC$$

$$\begin{aligned} Npr_{REG} = {} & Regprpos_{REG}(1 - Tax_{REG}) \\ & - Regprneg_{REG} \end{aligned}$$

$$Regpr_{REG} = Regprpos_{REG} - Regprneg_{REG}$$

$$\begin{aligned} Regpr_{REG} = {} & TO_{REG} + TTO_{REG} + RoyTO_{REG} \\ & - TC_{REG} - ProdC_{REG} - DistC_{REG} \\ & - StoC_{REG} - RoyC_{REG} \end{aligned}$$

$$TO_{REG} = \sum_{PRopq,t} xq_{PRopq,REG,t} Price_{PRopq,REG}$$

$$\begin{aligned} ProdC_{REG} = & \sum_{PRik,t} xi_{PRik,REG,t} PCi_{PRik,REG} \\ & + \sum_{PRkl,t} xk_{PRkl,REG,t} PCk_{PRkl,REG} \\ & + \sum_{PRmn,t} xm_{PRmn,REG,t} PCm_{PRmn,REG} \\ & + \sum_{PRopq,t} xo_{PRopq,REG,t} PCo_{PRopq,REG} \end{aligned}$$

$$\begin{aligned} StoC_{REG} = & \sum_{PRkl,t} yl_{PRkl,REG,t} SCl_{PRkl,REG} \\ & + \sum_{PRmn,t} yn_{PRmn,REG,t} SCn_{PRmn,REG} \\ & + \sum_{PRopq,t} yp_{PRopq,REG,t} SCp_{PRopq,REG} \\ & + \sum_{PRopq,t} yq_{PRopq,REG,t} SCq_{PRopq,REG} \end{aligned}$$

$$\begin{aligned} Distc_{REG} = & \sum_{PRkl,REG2,t} xl_{PRkl,REG2,REG,t} DCl_{PRkl,REG2,REG} \alpha \\ & + \sum_{PRmn,REG2,t} xn_{PRmn,REG2,REG,t} DCn_{PRmn,REG2,REG} \alpha \\ & + \sum_{PRopq,REG2,t} xp_{PRopq,REG2,REG,t} DCp_{PRopq,REG2,REG} \alpha \end{aligned}$$

$$\begin{aligned} TTO_{REG} = &\sum_{PRkl,REG2,t} xl_{PRkl,REG,REG2,t} TPl_{PRkl,REG,REG2} \gamma \\ &+ \sum_{PRmn,REG2,t} xn_{PRmn,REG,REG2,t} TPn_{PRmn,REG,REG2} \gamma \beta \\ &+ \sum_{PRopq,REG2,t} xp_{PRopq,REG,REG2,t} TPp_{PRopq,REG,REG2} \gamma \beta \end{aligned}$$

$$\begin{aligned} TC_{REG} = &\sum_{PRkl,REG2,t} xl_{PRkl,REG2,REG,t} TPl_{PRkl,REG2,REG} \zeta \beta \\ &+ \sum_{PRmn,REG2,t} xn_{PRmn,REG2,REG,t} TPn_{PRmn,REG2,REG} \zeta \\ &+ \sum_{PRopq,REG2,t} xp_{PRopq,REG2,REG,t} TPp_{PRopq,REG2,REG} \zeta \end{aligned}$$

$$RoyTO_{REG} = \sum_{REG2} (TO_{REG2} + TTO_{REG2}) Royal_{REG2,REG} \beta$$

$$RoyC_{REG} = \sum_{REG2} (TO_{REG} + TTO_{REG}) Royal_{REG,REG2}$$

$$FinC = \sum_{REG,t} (\sum_{REG2} FinDistC_{REG,REG2,t} + FinStoC_{REG,t})$$

$$
\begin{aligned}
&FinStoC_{REG,t} \\
&= \sum_{PRkl} \sum_{REG3} (\sum_{PRik} ProVik_{PRik} Prodmixk_{PRkl,PRik,REG3} \\
&+ \frac{PCk_{PRkl,REG3} ER_{REG3}}{card_{PROREG}}) yl_{PRkl,REG,t^{\kappa}} \\
&\\
&+ \sum_{PRmn} \sum_{REG3} (\sum_{PRkl} ProVkl_{PRkl} Prodmixm_{PRmn,PRkl,REG3} \\
&+ \frac{PCm_{PRmn,REG3} ER_{REG3}}{card_{PROREG}}) yn_{PRmn,REG,t^{\kappa}} \\
&\\
&+ \sum_{PRopq} \sum_{REG3} (\sum_{PRmn} ProVmn_{PRmn} Prodmixo_{PRopq,PRmn,REG3} \\
&+ \frac{PCo_{PRopq,REG3} ER_{REG3}}{card_{PROREG}}) yp_{PRopq,REG,t^{\kappa}} \\
&\\
&+ \sum_{PRopq} \sum_{REG3} (\sum_{PRmn} ProVmn_{PRmn} Prodmixo_{PRopq,PRmn,REG3} \\
&+ \frac{PCo_{PRopq,REG3} ER_{REG3}}{card_{PROREG}}) yq_{PRopq,REG,t^{\kappa}}
\end{aligned}
$$

$$
\begin{aligned}
&FinDistC_{REG,REG2,t}\\
&=\sum_{PRkl}\sum_{REG3}(\sum_{PRik}ProVik_{PRik}Prodmixk_{PRkl,PRik,REG3}\\
&+\frac{PCk_{PRkl,REG3}ER_{REG3}}{card_{PROREG}})xl_{PRkl,REG,REG2,t}\lambda\\
&\\
&+\sum_{PRmn}\sum_{REG3}(\sum_{PRkl}ProVkl_{PRkl}Prodmixm_{PRmn,PRkl,REG3}\\
&+\frac{PCm_{PRmn,REG3}ER_{REG3}}{card_{PROREG}})xn_{PRmn,REG,REG2,t}\lambda\\
&\\
&+\sum_{PRopq}\sum_{REG3}(\sum_{PRmn}ProVmn_{PRmn}Prodmixo_{PRopq,PRmn,REG3}\\
&+\frac{PCo_{PRopq,REG3}ER_{REG3}}{card_{PROREG}})xp_{PRopq,REG,REG2,t}\lambda
\end{aligned}
$$

with

$$ProVik_{PRik}=\sum_{PROREG}\frac{PCi_{PRik,PROREG}ER_{PROREG}}{card_{PROREG}}$$

$$
\begin{aligned}
ProVkl_{PRkl}=&\sum_{PROREG}(\sum_{PRik}ProVik_{PRik}\\
&\cdot Prodmixk_{PRkl,PRik,PROREG}\\
&+\frac{PCk_{PRkl,PROREG}ER_{PROREG}}{card_{PROREG}})
\end{aligned}
$$

$$
\begin{aligned}
ProVmn_{PRmn}=&\sum_{PROREG}(\sum_{PRkl}ProVkl_{PRkl}\\
&\cdot Prodmixm_{PRmn,PRkl,PROREG}\\
&+\frac{PCm_{PRmn,PROREG}ER_{PROREG}}{card_{PROREG}})
\end{aligned}
$$

$$
\begin{aligned}
ProVopq_{PRopq} = {} & \sum_{PROREG} (\sum_{PRmn} ProVmn_{PRmn} \\
& \cdot Prodmixo_{PRopq,PRmn,PROREG} \\
& + \frac{PCo_{PRopq,PROREG} ER_{PROREG}}{card_{PROREG}})
\end{aligned}
$$

s.t.

$$xq_{PRopq,REG,t} = Dem_{PRopq,REG,t}$$

$$
\begin{aligned}
\sum_{PRik} xi_{PRik,REG,t} &\leq Capi_{REG,t} \\
\sum_{PRkl} xk_{PRkl,REG,t} &\leq Capk_{REG,t} \\
\sum_{PRmn} xm_{PRmn,REG,t} &\leq Capm_{REG,t} \\
\sum_{PRopq} xo_{PRopq,REG,t} &\leq Capo_{REG,t}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{PRkl} yl_{PRkl,REG,t} &\leq Capl_{REG,t} \\
\sum_{PRmn} yn_{PRmn,REG,t} &\leq Capn_{REG,t} \\
\sum_{PRopq} yp_{PRopq,REG,t} &\leq Capp_{REG,t} \\
\sum_{PRopq} yq_{PRopq,REG,t} &\leq Capq_{REG,t}
\end{aligned}
$$

$$
\begin{aligned}
xi_{PRik,REG,t} &= \sum_{PRkl} xk_{PRkl,REG,t} Prodmixk_{PRkl,PRik} \\
\sum_{REG2} xl_{PRkl,REG2,REG,t} &= \sum_{PRmn} xm_{PRmn,REG,t} Prodmixm_{PRmn,PRkl} \\
\sum_{REG2} xn_{PRmn,REG2,REG,t} &= \sum_{PRopq} xo_{PRopq,REG,t} Prodmixo_{PRopq,PRmn}
\end{aligned}
$$

$$
\begin{aligned}
yl_{PRkl,REG,t} &= yl_{PRkl,REG,t--1} + xk_{PRkl,REG,t} \\
&\quad - \sum_{REG2} xl_{PRkl,REG,REG2,t} \\
yn_{PRmn,REG,t} &= yn_{PRmn,REG,t--1} + xm_{PRmn,REG,t} \\
&\quad - \sum_{REG2} xn_{PRmn,REG,REG2,t} \\
yp_{PRopq,REG,t} &= yp_{PRopq,REG,t--1} + xo_{PRopq,REG,t} \\
&\quad - \sum_{REG2} xp_{PRopq,REG,REG2,t} \\
yq_{PRopq,REG,t} &= yq_{PRopq,REG,t--1} \\
&\quad + \sum_{REG2} xp_{PRopq,REG2,REG,t} - xq_{PRopq,REG,t}
\end{aligned}
$$

$$
\begin{gathered}
xi_{PRik,REG,t} = p_{PR,REG,t} batchsize_{REG} \\
\sum_{PRik} p_{PRik,REG,t} \leq p^{max}_{REG,t} \\
p = 0, 1, ..., p^{max}
\end{gathered}
$$

$$
\begin{gathered}
\sum_{PRmn} xn_{PRmn,REG,REG2,t} = q^{real}_{REG,REG2,t} contsize \\
q^{int}_{REG,REG2,t} - 1 \leq q^{real}_{REG,REG2,t} \leq q^{int}_{REG,REG2,t} \\
DCn_{PRmn,REG,REG2,t} = q^{real}_{REG,REG2,t} contcost_{PRmn,REG,REG2.t}
\end{gathered}
$$

with $\alpha = \frac{ER_{REG="R2"}}{ER_{REG}}$
$\beta = \frac{ER_{REG2}}{ER_{REG}}$
$\gamma = (1 - ExVat_{REG,REG2})$
$\zeta = (1 + Dut_{REG2,REG})$
$\kappa = (1 + Dfcost)^{Stime} - 1$
$\lambda = (1 + Dfcost)^{Dtime_{REG,REG2}} - 1$

Appendix F

The Miniminitest Model

This appendix contains an overview of the implemented version of the minimini test model. The model has been used for validating and verifying the abilities of the model. The different features has been tested with the testdata shown in appendix H. This data was adapted approbiately according to the specific scenarios.

F.1 GAMS code

```
$Title Minimodel_time1 by the GAMS Sharks Inc. 2004
$eolcom//
```

```
option iterlim=999999999, reslim=300, optcr=0.0, solprint=ON, limrow=100, limcol=100;

Sets    REG             Main set regions          /R1, R2, R3/
        PROREG(REG)     Subset producing regions  /R1,R2,R3/
        PR              Products                  /P1*P63/
        PRik(PR)        Subset phase i            /P1*P3/
        PRkl(PR)        Subset phase k and l      /P4*P9/
        PRmn(PR)        Subset phase m and n      /P10*P27/
        PRopq(PR)       Subset phase o p and q    /P28*P63/
        t               Time periods              /1*4/         ;

ALIAS   (REG,REG2,REG3);



//input data are given in the data library

$include data/price.inp
$include data/transferprice.inp
$include data/prodcost.inp
$include data/distcost.inp
$include data/storcost.inp
$include data/demand.inp
$include data/duty.inp
$include data/exvat.inp
$include data/royalty.inp
```

```
$include data/bom.inp
$include data/capacity.inp
$include data/exrate.inp
$include data/tax.inp
$include data/dfcost.inp
$include data/dtime.inp
$include data/stime.inp

Free variables
        z               Netprofit after tax
        Npr(REG)        Regional netprofit in local currency
        Regpr(REG)      Regional profit before tax in local currency ;

Positive variables
        Regprpos(REG)   Positive regional profit in region REG
        Regprneg(REG)   Negative regional profit in region REG

        RoyC(REG)       Royalty cost in local currency
        RoyTO(REG)      Turnover from royalties

        xi(PRik,REG,t)  Production of product PRij in kg in region REG in phase i for
                        time period t
        xk(PRkl,REG,t)  Production of product PRkl in kg in region REG in phase k for
                        time period t
        xm(PRmn,REG,t)  Production of product PRmn in kg in region REG in phase m for
                        time period t
        xo(PRopq,REG,t) Production of product PRopq in kg in region REG in phase o for
```

```
                time period t

xl(PRkl,REG,REG2,t)     Distribution flow from region REG to REG2 of product PRkl
                        in kg in phase l for time period t
xn(PRmn,REG,REG2,t)     Distribution flow from region REG to REG2 of product PRmn
                        in kg in phase n for time period t
xp(PRopq,REG,REG2,t)    Distribution flow from region REG to REG2 of product PRopq
                        in kg in phase p for time period t

xq(PRopq,REG,t)         Sales flow in region REG of product PRopq in kg in phase q
                        for time period t

yl(PRkl,REG,t)  Storage of product PRkl in kg in region REG in phase l for
                time period t
yn(PRmn,REG,t)  Storage of product PRmn in kg in region REG in phase n for
                time period t
yp(PRopq,REG,t) Storage of product PRopq in kg in region REG in phase p for
                time period t
yq(PRopq,REG,t) Storage of product PRopq in kg in region REG in phase q for
                time period t

TO(REG)         Turnover for region REG in local currency
TTO(REG)        Turnover for region REG from intermediate transfer in local currency
TC(REG)         Costs for region REG from intermediate transfer in local currency
ProdC(REG)      Production costs for region REG in local currency
DistC(REG)      Distribution costs for region REG in local currency
```

```
        StoC(REG)       Storage cost for region REG in local currency

        FinC            Define financial cost for storage and distribution
        FinDistC(REG,REG2,t)  Define financial distribution cost for each time period
        FinStoC(REG,t)  Define financial storage cost in region REG for time period t
        ProVik(PRik)    Define product value of a product in set PRik
        ProVkl(PRkl)    Define product value of a product in set PRkl
        ProVmn(PRmn)    Define product value of a product in set PRmn
        ProVopq(PRopq)  Define product value of a product in set PRopq;


Equations
        Sumnetprofit            Annual enterprise netprofit (in dkk)
        Netprofit(REG)          Regional netprofit (in local currency)
        Posprofit(REG)          Regional profit auxillary equation ensuring linearity of
                                the tax function
        Profbftax(REG)          Regional profits before tax
        Royalcost(REG)          Royalty cost
        Royalincm(REG)          Royalty income
        Turnover(REG)           Regional turnover
        Transto(REG)            Regional turnover from internal transfer
        Transcost(REG)          Regional cost from internal procurement
        Prodcost(REG)           Regional production cost
        Distcost(REG)           Regional distribution cost
        Stocost(REG)            Regional storage cost

        FinCost                 Total financial cost for storage and distribution
```

```
FinDistCost(REG,REG2,t) Financial cost for distribution
FinStoCost(REG,t)       Financial cost for storage
ProdValik(PRik)         Average product value of product PRik in R1 currency
ProdValkl(PRkl)         Average product value of product PRkl in R1 currency
ProdValmn(PRmn)         Average product value of product PRmn in R1 currency
ProdValopq(PRopq)       Average product value of product PRopq in R1 currency

Cusdemand(PRopq,REG,t)  Customer demand satisfaction
PCapi(REG,t)            Capacity constraint for phase i
PCapk(REG,t)            Capacity constraint for phase k
SCapl(REG,t)            Capacity constraint for phase l
PCapm(REG,t)            Capacity constraint for phase m
SCapn(REG,t)            Capacity constraint for phase n
PCapo(REG,t)            Capacity constraint for phase o
SCapp(REG,t)            Capacity constraint for phase p
SCapq(REG,t)            Capacity constraint for phase q
Prodflowk(PRik,REG,t)   Flow conservation for production facilities at k for
                        time period t
Prodflowm(PRkl,REG,t)   Flow conservation for production facilities at m for
                        time period t
Prodflowo(PRmn,REG,t)   Flow conservation for production facilities at o for
                        time period t
Stoflowl(PRkl,REG,t)    Flow conservation for storage in facility l for
                        time period t
Stoflown(PRmn,REG,t)    Flow conservation for storage in facility n for
                        time period t
Stoflowp(PRopq,REG,t)   Flow conservation for storage in facility p for
```

```
                                time period t
        Stoflowq(PRopq,REG,t)   Flow conservation for storage in facility q for
                                time period t;

Sumnetprofit..          z =e=
                        sum(REG,Npr(REG)*ER(REG))-FinC;

Netprofit(REG)..        Npr(REG) =e=
                        Regprpos(REG)*(1-Tax(REG))-Regprneg(REG);

Posprofit(REG)..        Regpr(REG) =e=
                        Regprpos(REG)-Regprneg(REG);

Profbftax(REG)..        Regpr(REG) =e=
                        TO(REG)+TTO(REG)+RoyTO(REG)
                        -TC(REG)-ProdC(REG)-DistC(REG)-StoC(REG)-RoyC(REG);

Royalincm(REG)..        RoyTO(REG) =e=
                        sum(REG2, ((TO(REG2)+TTO(REG2))*Royal(REG2,REG)*ER(REG2)/ER(REG)));

Royalcost(REG)..        RoyC(REG) =e=
                        sum(REG2, ((TO(REG)+TTO(REG))*Royal(REG,REG2)));

Turnover(REG)..         TO(REG) =e=
                        sum((PRopq,t), xq(PRopq,REG,t)*Price(PRopq,REG));

Transto(REG)..          TTO(REG) =e=
```

```
                         sum((PRkl,REG2,t), xl(PRkl,REG,REG2,t) *TPl(PRkl,REG,REG2)
                                                                *(1-ExVat(REG,REG2)))
                        +sum((PRmn,REG2,t), xn(PRmn,REG,REG2,t) *TPn(PRmn,REG,REG2)
                                                                *(1-ExVat(REG,REG2)))
                        +sum((PRopq,REG2,t),xp(PRopq,REG,REG2,t)*TPp(PRopq,REG,REG2)
                                                                *(1-ExVat(REG,REG2)));

Transcost(REG)..        TC(REG) =e=
                         sum((PRkl,REG2,t), xl(PRkl,REG2,REG,t)   *TPl(PRkl,REG2,REG)
                                                *(1+Dut(REG2,REG))*ER(REG2)/ER(REG))
                        +sum((PRmn,REG2,t), xn(PRmn,REG2,REG,t)   *TPn(PRmn,REG2,REG)
                                                *(1+Dut(REG2,REG))*ER(REG2)/ER(REG))
                        +sum((PRopq,REG2,t), xp(PRopq,REG2,REG,t) *TPp(PRopq,REG2,REG)
                                                *(1+Dut(REG2,REG))*ER(REG2)/ER(REG));

Prodcost(REG)..         Prodc(REG) =e=
                         sum((PRik,t),  xi(PRik,REG,t)*PCi(PRik,REG))
                        +sum((PRkl,t),  xk(PRkl,REG,t)*PCk(PRkl,REG))
                        +sum((PRmn,t),  xm(PRmn,REG,t)*PCm(PRmn,REG))
                        +sum((PRopq,t), xo(PRopq,REG,t)*PCo(PRopq,REG));

Distcost(REG)..         Distc(REG) =e=
                         sum((PRkl,REG2,t), xl(PRkl,REG2,REG,t)  *DCl(PRkl,REG2,REG)
                                                                 *ER("R2")/ER(REG))
                        +sum((PRmn,REG2,t), xn(PRmn,REG2,REG,t)  *DCn(PRmn,REG2,REG)
                                                                 *ER("R2")/ER(REG))
                        +sum((PRopq,REG2,t), xp(PRopq,REG2,REG,t)*DCp(PRopq,REG2,REG)
```

```
                                                                 *ER("R2")/ER(REG));

Stocost(REG)..          StoC(REG) =e=
                         sum((PRkl,t), yl(PRkl,REG,t)*SCl(PRkl,REG))
                        +sum((PRmn,t), yn(PRmn,REG,t)*SCn(PRmn,REG))
                        +sum((PRopq,t), yp(PRopq,REG,t)*SCp(PRopq,REG))
                        +sum((PRopq,t), yq(PRopq,REG,t)*SCq(PRopq,REG));

FinCost..               FinC =e= sum((REG,t), sum(REG2,FinDistC(REG,REG2,t))+FinStoC(REG,t));

FinDistCost(REG,REG2,t)$(ord(REG)<>ord(REG2))..
                        FinDistC(REG,REG2,t) =e=
                        sum(PRkl,sum(REG3, sum(PRik, prodmixk(PRkl,PRik)*PCi(PRik,REG3)
                           *ER(REG3))+PCk(PRkl,REG3)*ER(REG3))/card(PROREG)
                           *xl(PRkl,REG,REG2,t)*((1+Dfcost)**Dtime(REG,REG2)-1))

                        +sum(PRmn,(sum(PRkl,(sum(REG3,sum(PRik,prodmixk(PRkl,PRik)
                            *PCi(PRik,REG3)*ER(REG3))+PCk(PRkl,REG3)*ER(REG3))
                            /card(PROREG))*prodmixm(PRmn,PRkl))
                            +sum(REG3,PCm(PRmn,REG3)*ER(REG3))/card(PROREG))
                            *xn(PRmn,REG,REG2,t)*((1+Dfcost)**Dtime(REG,REG2)-1))

                        +sum(PRopq,(sum(PRmn,(sum(PRkl,(sum(REG3,sum(PRik,prodmixk(PRkl,PRik)
                            *PCi(PRik,REG3)*ER(REG3))+PCk(PRkl,REG3)*ER(REG3))/card(PROREG))
                            *prodmixm(PRmn,PRkl))+sum(REG3,PCm(PRmn,REG3)*ER(REG3))/
                            card(PROREG))*prodmixo(PRopq,PRmn))
                            +sum(REG3, PCo(PRopq,REG3)*ER(REG3))/card(PROREG))
```

```
                    *xp(PRopq,REG,REG2,t)*((1+Dfcost)**Dtime(REG,REG2)-1));

FinStoCost(REG,t)..     FinStoC(REG,t) =e=

                         sum(PRkl,sum(REG3,sum(PRik, prodmixk(PRkl,PRik)*PCi(PRik,REG3)
                            *ER(REG3))+PCk(PRkl,REG3)*ER(REG3))/card(PROREG)
                            *yl(PRkl,REG,t)*((1+Dfcost)**Stime-1))

                        +sum(PRmn,(sum(PRkl,(sum(REG3,sum(PRik,prodmixk(PRkl,PRik)
                           *PCi(PRik,REG3)*ER(REG3))+PCk(PRkl,REG3)*ER(REG3))
                            /card(PROREG))*prodmixm(PRmn,PRkl))
                            +sum(REG3,PCm(PRmn,REG3)*ER(REG3))/card(PROREG))
                            *yn(PRmn,REG,t)*((1+Dfcost)**Stime-1))

                        +sum(PRopq,(sum(PRmn,(sum(PRkl,(sum(REG3,sum(PRik,prodmixk(PRkl,PRik)
                            *PCi(PRik,REG3)*ER(REG3))+PCk(PRkl,REG3)*ER(REG3))/card(PROREG))
                            *prodmixm(PRmn,PRkl))
                            +sum(REG3,PCm(PRmn,REG3)*ER(REG3))/card(PROREG))
                            *prodmixo(PRopq,PRmn))
                            +sum(REG3, PCo(PRopq,REG3)*ER(REG3))/card(PROREG))
                            *yp(PRopq,REG,t)*((1+Dfcost)**Stime-1))

                        +sum(PRopq,(sum(PRmn,(sum(PRkl,(sum(REG3,sum(PRik,prodmixk(PRkl,PRik)
                            *PCi(PRik,REG3)*ER(REG3))
                            +PCk(PRkl,REG3)*ER(REG3))/card(PROREG))*prodmixm(PRmn,PRkl))
                            +sum(REG3,PCm(PRmn,REG3)*ER(REG3))/card(PROREG))
```

```
                            *prodmixo(PRopq,PRmn))
                            +sum(REG3, PCo(PRopq,REG3)*ER(REG3))/card(PROREG))
                            *yq(PRopq,REG,t)*((1+Dfcost)**Stime-1));


ProdValik(PRik)..       ProVik(PRik) =e=
                        sum(REG, PCi(PRik,REG)*ER(REG))/card(PROREG);

ProdValkl(PRkl)..       ProVkl(PRkl) =e=
                        sum(REG3, sum(PRik, prodmixk(PRkl,PRik)*PCi(PRik,REG3)*ER(REG3))
                           +PCk(PRkl,REG3)*ER(REG3))/card(PROREG);

ProdValmn(PRmn)..       ProVmn(PRmn) =e=
                         sum(PRkl,(sum(REG3,sum(PRik,prodmixk(PRkl,PRik)*PCi(PRik,REG3)
                            *ER(REG3))+PCk(PRkl,REG3)*ER(REG3))
                            /card(PROREG))*prodmixm(PRmn,PRkl))
                            +sum(REG3,PCm(PRmn,REG3)*ER(REG3))/card(PROREG);

ProdValopq(PRopq)..     ProVopq(PRopq) =e=
                        sum(PRmn,(sum(PRkl,(sum(REG3,sum(PRik,prodmixk(PRkl,PRik)
                            *PCi(PRik,REG3)*ER(REG3))+PCk(PRkl,REG3)*ER(REG3))
                            /card(PROREG))*prodmixm(PRmn,PRkl))
                            +sum(REG3,PCm(PRmn,REG3)*ER(REG3))/card(PROREG))
                            *prodmixo(PRopq,PRmn))
                            +sum(REG3, PCo(PRopq,REG3)*ER(REG3))/card(PROREG);
```

```
//Constraints

Cusdemand(PRopq,REG,t)..
                        xq(PRopq,REG,t) =e= Dem(PRopq,REG,t);

PCapi(REG,t)..          sum(PRik,xi(PRik,REG,t)) =l= Capi(REG);

PCapk(REG,t)..          sum(PRkl,xk(PRkl,REG,t)) =l= Capk(REG);

SCapl(REG,t)..          sum(PRkl,yl(PRkl,REG,t)) =l= Capl(REG);

PCapm(REG,t)..          sum(PRmn,xm(PRmn,REG,t)) =l= Capm(REG);

SCapn(REG,t)..          sum(PRmn,yn(PRmn,REG,t)) =l= Capn(REG);

PCapo(REG,t)..          sum(PRopq,xo(PRopq,REG,t)) =l= Capo(REG);

SCapp(REG,t)..          sum(PRopq,yp(PRopq,REG,t)) =l= Capp(REG);

SCapq(REG,t)..          sum(PRopq,yq(PRopq,REG,t)) =l= Capq(REG);

Prodflowk(PRik,REG,t).. xi(PRik,REG,t) =e= sum(PRkl, (xk(PRkl,REG,t)
                                             *Prodmixk(PRkl,PRik)));

Prodflowm(PRkl,REG,t).. sum(REG2,xl(PRkl,REG2,REG,t)) =e= sum(PRmn,(xm(PRmn,REG,t)
```

```
                                                          *Prodmixm(PRmn,PRkl)));

Stoflowl(PRkl,REG,t)..  yl(PRkl,REG,t) =e= yl(PRkl,REG,t--1)+xk(PRkl,REG,t)
                                           -sum(REG2,xl(PRkl,REG,REG2,t));

Stoflown(PRmn,REG,t)..  yn(PRmn,REG,t) =e= yn(PRmn,REG,t--1)+xm(PRmn,REG,t)
                                           -sum(REG2,xn(PRmn,REG,REG2,t));

Prodflowo(PRmn,REG,t).. sum(REG2,xn(PRmn,REG2,REG,t)) =e= sum(PRopq, xo(PRopq,REG,t)
                                                              *Prodmixo(PRopq,PRmn));

Stoflowp(PRopq,REG,t).. yp(PRopq,REG,t) =e= yp(PRopq,REG,t--1)+xo(PRopq,REG,t)
                                            -sum(REG2,xp(PRopq,REG,REG2,t));

Stoflowq(PRopq,REG,t).. yq(PRopq,REG,t) =e= yq(PRopq,REG,t--1)+sum(REG2,xp(PRopq,REG2,REG,t))
                                            -xq(PRopq,REG,t);

Model NZMini /all/;
Solve NZMini using lp maximizing z;

Display
        " %%%%%%%%%%%%%%%%%%%%%%%%%%%%  PROFITS  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ",
        z.l, Npr.l, Regpr.l, Regprpos.l, Regprneg.l,
        " %%%%%%%%%%%%%%%%%%%%%%%%%%%%%  TURNOVERS  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ",
        TO.l, TTO.l, RoyTO.l,
        " %%%%%%%%%%%%%%%%%%%%%%%%%%%%%  COSTS  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ",
        ProdC.l, DistC.l, StoC.l, TC.l, RoyC.l,
```

```
" %%%%%%%%%%%%%%%%%%%%%%%%%%%%%  FLOW  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ",
xi.l, xk.l, xl.l, xm.l, xn.l, xo.l, xp.l, xq.l,
" %%%%%%%%%%%%%%%%%%%%%%%%%%%%%  STORAGE  %%%%%%%%%%%%%%%%%%%%%%%%%%%%% ",
yl.l, yn.l, yp.l, yq.l
" %%%%%%%%%%%%%%%%%%%%%%%%%%%%%  FINANCIAL  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%% ",
FinC.l, FinStoC.l, FinDistC.l, ProdValik.l, ProdValkl.l, ProdValmn.l, ProdValopq.l;
```

Appendix G

The Final Model

This appendix contains an overview of the mathematical formulation of the final model (section G.1). And the implemented GAMS code (section G.2)

G.1 Formulation

$$maxz = \sum_{REG} Npr_{REG} ER_{REG} - FinC$$

$$\begin{aligned} Npr_{REG} &= Regprpos_{REG}(1 - Tax_{REG}) \\ &\quad - Regprneg_{REG} \end{aligned}$$

$$Regpr_{REG} = Regprpos_{REG} - Regprneg_{REG}$$

$$\textit{with } Regprpos, Regprneg \geq 0$$

$$\begin{aligned} Regpr_{REG} &= TO_{REG} + TTO_{REG} + RoyTO_{REG} \\ &\quad - TC_{REG} - ProdC_{REG} - DistC_{REG} \\ &\quad - StoC_{REG} - RoyC_{REG} \end{aligned}$$

$$TO_{REG} = \sum_{PRopq,t} xq_{PRopq,REG,t} Price_{PRopq,REG}$$

$$\begin{aligned}
ProdC_{REG} = & \sum_{PRik,t} xi_{PRik,REG,t} PCi_{PRik,REG} \\
& + \sum_{PRkl,t} xk_{PRkl,REG,t} PCk_{PRkl,REG} \\
& + \sum_{PRmn,t} xm_{PRmn,REG,t} PCm_{PRmn,REG} \\
& + \sum_{PRopq,t} xo_{PRopq,REG,t} PCo_{PRopq,REG}
\end{aligned}$$

$$\begin{aligned}
StoC_{REG} = & \sum_{PRkl,t} yl_{PRkl,REG,t} SCl_{PRkl,REG} \\
& + \sum_{PRmn,t} yn_{PRmn,REG,t} SCn_{PRmn,REG} \\
& + \sum_{PRopq,t} yp_{PRopq,REG,t} SCp_{PRopq,REG} \\
& + \sum_{PRopq,t} yq_{PRopq,REG,t} SCq_{PRopq,REG}
\end{aligned}$$

$$\begin{aligned}
Distc_{REG} = & \sum_{PRkl,REG2,t} xl_{PRkl,REG2,REG,t} DCl_{PRkl,REG2,REG} \alpha \\
& + \sum_{PRmn,REG2,t} xn_{PRmn,REG2,REG,t} DCn_{PRmn,REG2,REG} \alpha \\
& + \sum_{PRopq,REG2,t} xp_{PRopq,REG2,REG,t} DCp_{PRopq,REG2,REG} \alpha
\end{aligned}$$

$with\ \alpha = \frac{ER_{REG=''R2''}}{ER_{REG}}$

$$
\begin{aligned}
&TTO_{REG}\\
&= \sum_{PRkl,REG2,t} xl_{PRkl,REG,REG2,t} TPl_{PRkl,REG,REG2}\gamma\\
&+ \sum_{PRmn,REG2,t} xn_{PRmn,REG,REG2,t} TPn_{PRmn,REG,REG2}\gamma\beta\\
&+ \sum_{PRopq,REG2,t} xp_{PRopq,REG,REG2,t} TPp_{PRopq,REG,REG2}\gamma\beta\\
&\\
&+ \sum_{PRkl,t,ORD_{REG}=3} xl_{PRkl,REG,''R2'',t} TPl_{PRkl,REG,''R1''}\gamma_R 1\\
&+ \sum_{PRmn,t,ORD_{REG}=3} xn_{PRmn,REG,''R2'',t} TPn_{PRmn,REG,''R1''}\gamma_R 1\beta_{R1}\\
&+ \sum_{PRopq,t,ORD_{REG}=3} xp_{PRopq,REG,''R2'',t} TPp_{PRopq,REG,''R1''}\gamma_R 1\beta_{R1}\\
&\\
&+ \sum_{PRkl,t,ORD_{REG}=1} xl_{PRkl,''R3'',''R2'',t} TPl_{PRkl,REG,''R2''}\gamma_{R2}\\
&+ \sum_{PRmn,t,ORD_{REG}=1} xn_{PRmn,''R3'',''R2'',t} TPn_{PRmn,REG,''R2''}\gamma_{R2}\beta_{R2}\\
&+ \sum_{PRopq,t,ORD_{REG}=1} xp_{PRopq,''R3'',''R2'',t} TPp_{PRopq,REG,''R2''}\gamma_{R2}\beta_{R2}
\end{aligned}
$$

with $\beta = \frac{ER_{REG2}}{ER_{REG}}$,
$\beta_{R1} = \frac{ER_{''R1''}}{ER_{REG}}$,
$\beta_{R2} = \frac{ER_{''R2''}}{ER_{REG}}$,
and $\gamma = (1 - ExVat_{REG,REG2})$,
$\gamma_{R1} = (1 - ExVat_{REG,REG2=''R1''})$,
$\gamma_{R2} = (1 - ExVat_{REG,REG2=''R2''})$.

$$
\begin{aligned}
& TC_{REG} \\
& = \sum_{PRkl,REG2,t} xl_{PRkl,REG2,REG,t} TPl_{PRkl,REG2,REG} \zeta \beta \\
& + \sum_{PRmn,REG2,t} xn_{PRmn,REG2,REG,t} TPn_{PRmn,REG2,REG} \zeta \\
& + \sum_{PRopq,REG2,t} xp_{PRopq,REG2,REG,t} TPp_{PRopq,REG2,REG} \zeta \\
& \\
& + \sum_{PRkl,t,ORD_{REG}=1} xl_{PRkl,''R3'',''R2'',t} TPl_{PRkl,''R3'',REG} \zeta_{R3} \beta_{R3} \\
& + \sum_{PRmn,t,ORD_{REG}=1} xn_{PRmn,''R3'',''R2'',t} TPn_{PRmn,''R3'',REG} \zeta_{R3} \\
& + \sum_{PRopq,t,ORD_{REG}=1} xp_{PRopq,''R3'',''R2'',t} TPp_{PRopq,''R3'',REG} \zeta_{R3} \\
& \\
& + \sum_{PRkl,t,ORD_{REG}=2} xl_{PRkl,''R3'',REG,t} TPl_{PRkl,''R1'',REG} \zeta_{R1} \beta_{R1} \\
& + \sum_{PRmn,t,ORD_{REG}=2} xn_{PRmn,''R3'',REG,t} TPn_{PRmn,''R1'',REG} \zeta_{R1} \\
& + \sum_{PRopq,t,ORD_{REG}=2} xp_{PRopq,''R3'',REG,t} TPp_{PRopq,''R1'',REG} \zeta_{R1}
\end{aligned}
$$

with $\beta = \frac{ER_{REG2}}{ER_{REG}}$,
$\beta_{R1} = \frac{ER_{REG2=''R1''}}{ER_{REG}}$,
$\beta_{R3} = \frac{ER_{REG2=''R3''}}{ER_{REG}}$,

and $\zeta = 1 + Dut_{REG2,REG}$,
$\zeta_{R1} = 1 + Dut_{''R1'',REG}$,
$\zeta_{R3} = 1 + Dut_{''R3'',REG}$

$$RoyTO_{REG} = \sum_{REG2} (TO_{REG2} + TTO_{REG2}) Royal_{REG2,REG} \beta$$

$$RoyC_{REG} = \sum_{REG2} (TO_{REG} + TTO_{REG}) Royal_{REG,REG2}$$

$with\ \beta = \frac{ER_{REG2}}{ER_{REG}}$

$$FinC = \sum_{REG,t} (\sum_{REG2} FinDistC_{REG,REG2,t} + FinStoC_{REG,t})$$

$$\begin{aligned} FinStoC_{REG,t} = & \sum_{PRkl} ProVkl_{PRkl} yl_{PRkl,REG,t} \kappa \\ & + \sum_{PRmn} ProVmn_{PRmn} yn_{PRmn,REG,t} \kappa \\ & + \sum_{PRopq} ProVopq_{PRopq} yp_{PRopq,REG,t} \kappa \\ & + \sum_{PRopq} ProVopq_{PRopq} yq_{PRopq,REG,t} \kappa \end{aligned}$$

$$\begin{aligned} FinDistC_{REG,REG2,t} = & \sum_{PRkl} ProVkl_{PRkl} xl_{PRkl,REG,REG2,t} \lambda \\ & + \sum_{PRmn} ProVmn_{PRmn} xn_{PRmn,REG,REG2,t} \lambda \\ & + \sum_{PRopq} ProVopq_{PRopq} xp_{PRopq,REG,REG2,t} \lambda \end{aligned}$$

$with\ \kappa = (1 + Dfcost)^{Stime} - 1$
$and\ \lambda = (1 + Dfcost)^{Dtime_{REG,REG2}} - 1$

$$\begin{aligned} ProVkl_{PRkl,t} = & \frac{1}{card_{REG}} \sum_{REG} PCk_{PRkl,REG} ER_{REG} \\ & + \sum_{PRik^{\ast}} PCi_{PRik,REG} ER_{REG} Prodmixk_{PRkl,PRik,REG} \end{aligned}$$

$\forall_{REG=PROREG}$
$^{\ast}\forall_{PRik|Prodmixk_{PRkl,PRik,REG} \neq 0}$

$$\begin{aligned}
&ProVmn_{PRmn,t}\\
&= \frac{1}{card_{REG}} \sum_{REG} PCm_{PRmn,REG} ER_{REG}\\
&+ \sum_{PRkl^{\ast}} (PCk_{PRkl,REG} ER_{REG} Prodmixm_{PRmn,PRkl,REG}\\
&+ \sum_{PRik^{\ast 2}} PCi_{PRik,REG} ER_{REG} Prodmixk_{PRkl,PRik,REG})
\end{aligned}$$

$\forall_{REG=PROREG}$
$^{\ast}\forall_{PRkl|Prodmixm_{PRmn,PRkl,REG}\neq 0}$
$^{\ast 2}\forall_{PRik|Prodmixk_{PRkl,PRik,REG}\wedge Prodmixm_{PRmn,PRkl,REG}\neq 0}$

$$\begin{aligned}
&ProVopq_{PRopq,t}\\
&= \frac{1}{card_{REG}} \sum_{REG} PCo_{PRopq,REG} ER_{REG}\\
&+ \sum_{PRmn^{\ast}} (PCm_{PRmn,REG} ER_{REG}\\
&+ \sum_{PRkl^{\ast 2}} (PCk_{PRkl,REG} ER_{REG} Prodmixm_{PRmn,PRkl,REG}\\
&+ \sum_{PRik^{\ast 3}} PCi_{PRik,REG} ER_{REG} Prodmixk_{PRkl,PRik,REG}))
\end{aligned}$$

$\forall_{REG=PROREG}$
$^{\ast}\forall_{PRmn|Prodmixo_{PRopq,PRkl,REG}\neq 0}$
$^{\ast 2}\forall_{PRkl|Prodmixm_{PRmn,PRkl,REG}\wedge Prodmixo_{PRopq,PRmn,REG}\neq 0}$
$^{\ast 3}\forall_{PRik|Prodmixk_{PRkl,PRik,REG}\neq 0}$
$\wedge Prodmixm_{PRmn,PRkl,REG}\wedge Prodmixo_{PRopq,PRmn,REG}\neq 0$

$$xq_{PRopq,REG,t} = Dem_{PRopq,REG,t}$$

$$\sum_{PRik} xi_{PRik,REG,t} Cufi_{PRik,REG} \leq Capi_{REG,t}$$

$$\sum_{PRkl} xk_{PRkl,REG,t} Cufk_{PRkl,REG} \leq Capk_{REG,t}$$

$$\sum_{PRmn} xm_{PRmn,REG,t} Cufm_{PRmn,REG} \leq Capm_{REG,t}$$

$$\sum_{PRopq} xo_{PRopq,REG,t} Cufo_{PRopq,REG} \leq Capo_{REG,t}$$

$$\sum_{PRkl} yl_{PRkl,REG,t} Cufl_{PRkl,REG} \leq Capl_{REG,t}$$

$$\sum_{PRmn} yn_{PRmn,REG,t} Cufn_{PRmn,REG} \leq Capn_{REG,t}$$

$$\sum_{PRopq} yp_{PRopq,REG,t} Cufp_{PRopq,REG} \leq Capp_{REG,t}$$

$$\sum_{PRopq} yq_{PRopq,REG,t} Cufq_{PRopq,REG} \leq Capq_{REG,t}$$

$$xi_{PRik,REG,t} = \sum_{PRkl} xk_{PRkl,REG,t} Prodmixk_{PRkl,PRik,REG}$$

$$\sum_{REG2} xl_{PRkl,REG2,REG,t} = \sum_{PRmn} xm_{PRmn,REG,t} Prodmixm_{PRmn,PRkl,REG}$$

$$\sum_{REG2} xn_{PRmn,REG2,REG,t} = \sum_{PRopq} xo_{PRopq,REG,t} Prodmixo_{PRopq,PRmn,REG}$$

$$\begin{aligned}
yl_{PRkl,REG,t} &= yl_{PRkl,REG,t--1} + xk_{PRkl,REG,t} \\
&\quad - \sum_{REG2} xl_{PRkl,REG,REG2,t} \\
yn_{PRmn,REG,t} &= yn_{PRmn,REG,t--1} + xm_{PRmn,REG,t} \\
&\quad - \sum_{REG2} xn_{PRmn,REG,REG2,t} \\
yp_{PRopq,REG,t} &= yp_{PRopq,REG,t--1} + xo_{PRopq,REG,t} \\
&\quad - \sum_{REG2} xp_{PRopq,REG,REG2,t} \\
yq_{PRopq,REG,t} &= yq_{PRopq,REG,t--1} \\
&\quad + \sum_{REG2} xp_{PRopq,REG2,REG,t} - xq_{PRopq,REG,t}
\end{aligned}$$

G.2 GAMS code

```
$Title Finalmodel by the GAMS Sharks Inc. 2005
$eolcom//


option iterlim=999999999, reslim=300, optcr=0.0, solprint=OFF, limrow=0, limcol=0;

Sets    REG             Main set regions             /R1,R2,R3,R10/
        PROREG(REG)     Subset producing regions     /R1,R2,R3/
        NONPROREG(REG)  Subset non producing regions /R10/
        PR              Products                     /P1*P20,P101*P120,P1001*P1020,
                                                      P10001*P10020/
        PRik(PR)        Subset phase i               /P1*P20/
        PRkl(PR)        Subset phase k and l         /P101*P120/
        PRmn(PR)        Subset phase m and n         /P1001*P1020/
        PRopq(PR)       Subset phase o p and q       /P10001*P10020/
        PRtgran(PRmn)   Tgran                        /P1001,P1003,P1005,P1009,P1016,P1017/
        t               Time periods                 /1*4/              ;

ALIAS   (REG,REG2,REG3);


//input data are given in the data library

$include ../data/price.inp
$include ../data/transferprice.inp
```

```
$include ../data/prodcost.inp
$include ../data/distcost.inp
$include ../data/storcost.inp
$include ../data/demand.inp
$include ../data/duty.inp
$include ../data/exvat.inp
$include ../data/royalty.inp
$include ../data/bom.inp
$include ../data/capacity.inp
$include ../data/caputlfactor.inp
$include ../data/exrate.inp
$include ../data/tax.inp
$include ../data/dfcost.inp
$include ../data/dtime.inp
$include ../data/stime.inp

Free variables
        z               Netprofit after tax
        Npr(REG)        Regional netprofit in local currency
        Regpr(REG)      Regional profit before tax in local currency ;

Positive variables
        Regprpos(REG)   Positive regional profit in region REG
        Regprneg(REG)   Negative regional profit in region REG

        RoyC(REG)       Royalty cost in local currency
        RoyTO(REG)      Turnover from royalties
```

```
xi(PRik,REG,t)  Production of product PRij in kg in region REG in phase i for
                time period t
xk(PRkl,REG,t)  Production of product PRkl in kg in region REG in phase k for
                time period t
xm(PRmn,REG,t)  Production of product PRmn in kg in region REG in phase m for
                time period t
xo(PRopq,REG,t) Production of product PRopq in kg in region REG in phase o for
                time period t

xl(PRkl,REG,REG2,t)     Distribution flow from region REG to REG2 of product PRkl
                        in kg in phase l for time period t
xn(PRmn,REG,REG2,t)     Distribution flow from region REG to REG2 of product PRmn
                        in kg in phase n for time period t
xp(PRopq,REG,REG2,t)    Distribution flow from region REG to REG2 of product PRopq
                        in kg in phase p for time period t

xq(PRopq,REG,t)         Sales flow in region REG of product PRopq in kg in phase q
                        for time period t

yl(PRkl,REG,t)  Storage of product PRkl in kg in region REG in phase l for time
                period t
yn(PRmn,REG,t)  Storage of product PRmn in kg in region REG in phase n for time
                period t
yp(PRopq,REG,t) Storage of product PRopq in kg in region REG in phase p for time
                period t
yq(PRopq,REG,t) Storage of product PRopq in kg in region REG in phase q for time
```

```
                        period t

        TO(REG)         Turnover for region REG in local currency
        TTO(REG)        Turnover for region REG from intermediate transfer in local currency
        TC(REG)         Costs for region REG from intermediate transfer in local currency
        ProdC(REG)      Production costs for region REG in local currency
        DistC(REG)      Distribution costs for region REG in local currency
        StoC(REG)       Storage cost for region REG in local currency

        FinC            Define financial cost for storage and distribution
        FinDistC(REG,REG2,t)  Define financial distribution cost for each time period
        FinStoC(REG,t)  Define financial storage cost in region REG for time period t
        ProVik(PRik)    Define product value of a product in set PRik
        ProVkl(PRkl)    Define product value of a product in set PRkl
        ProVmn(PRmn)    Define product value of a product in set PRmn
        ProVopq(PRopq)  Define product value of a product in set PRopq;


//no flow allowed from facilities i to p in the non producing regions
xi.fx(PRik,NONPROREG,t)=0;
xk.fx(PRkl,NONPROREG,t)=0;
xl.fx(PRkl,NONPROREG,REG,t)=0;
xm.fx(PRmn,NONPROREG,t)=0;
xn.fx(PRmn,NONPROREG,REG,t)=0;
xo.fx(PRopq,NONPROREG,t)=0;
xp.fx(PRopq,NONPROREG,REG,t)=0;
```

```
yl.fx(PRkl,NONPROREG,t)=0;
yn.fx(PRmn,NONPROREG,t)=0;
yp.fx(PRopq,NONPROREG,t)=0;


Equations
        Sumnetprofit            Annual enterprise netprofit (in dkk)
        Netprofit(REG)          Regional netprofit (in local currency)
        Posprofit(REG)          Regional profit auxillary equation ensuring linearity of
                                the tax function
        Profbftax(REG)          Regional profits before tax
        Royalcost(REG)          Royalty cost
        Royalincm(REG)          Royalty income
        Turnover(REG)           Regional turnover
        Transto(REG)            Regional turnover from internal transfer
        Transcost(REG)          Regional cost from internal procurement
        Prodcost(REG)           Regional production cost
        Distcost(REG)           Regional distribution cost
        Stocost(REG)            Regional storage cost

        FinCost                 Total financial cost for storage and distribution
        FinDistCost(REG,REG2,t) Financial cost for distribution
        FinStoCost(REG,t)       Financial cost for storage
        ProdValik(PRik)         Average product value of product PRik in R1 currency
        ProdValkl(PRkl)         Average product value of product PRkl in R1 currency
        ProdValmn(PRmn)         Average product value of product PRmn in R1 currency
        ProdValopq(PRopq)       Average product value of product PRopq in R1 currency
```

```
        Cusdemand(PRopq,REG,t)  Customer demand satisfaction
        PCapi(REG,t)            Capacity constraint for phase i at time t
        PCapk(REG,t)            Capacity constraint for phase k at time t
        SCapl(REG,t)            Capacity constraint for phase l at time t
        PCapm(REG,t)            Capacity constraint for phase m at time t
        SCapn(REG,t)            Capacity constraint for phase n at time t
        PCapo(REG,t)            Capacity constraint for phase o at time t
        SCapp(REG,t)            Capacity constraint for phase p at time t
        SCapq(REG,t)            Capacity constraint for phase q at time t
        Prodflowk(PRik,REG,t)   Flow conservation for production facilities at k for time
                                period t
        Prodflowm(PRkl,REG,t)   Flow conservation for production facilities at m for time
                                period t
        Prodflowo(PRmn,REG,t)   Flow conservation for production facilities at o for time
                                period t
     Stoflowl(PRkl,REG,t)    Flow conservation for storage in facility l for time period t
     Stoflown(PRmn,REG,t)    Flow conservation for storage in facility n for time period t
     Stoflowp(PRopq,REG,t)   Flow conservation for storage in facility p for time period t
     Stoflowq(PRopq,REG,t)   Flow conservation for storage in facility q for time period t;

//Objective function the sum of net profits in R1 currency minus the financial costs
Sumnetprofit..          z =e=
                        sum(REG,Npr(REG)*ER(REG))-FinC;

//The netprofit ie profit after tax for each region linearized
Netprofit(REG)..        Npr(REG) =e=
```

```
            Regprpos(REG)*(1-Tax(REG))-Regprneg(REG);

//Profits for each region split in two positive numbers representing positive and negative
  profit

Posprofit(REG)..        Regpr(REG) =e=
                                Regprpos(REG)-Regprneg(REG);

//The profit before tax for each region generated from all the auxiliary variables
Profbftax(REG)..        Regpr(REG) =e=
                                TO(REG)+TTO(REG)+RoyTO(REG)
                                -TC(REG)-ProdC(REG)-DistC(REG)-StoC(REG)-RoyC(REG);

//The royalty income recieved by each region to balance costs of research and development
Royalincm(REG)..        RoyTO(REG) =e=
                        sum(REG2, ((TO(REG2)+TTO(REG2))*Royal(REG2,REG)*ER(REG2)/ER(REG)));

//The royalty costs paid for each region to balance costs of research and development
Royalcost(REG)..        RoyC(REG) =e=
                        sum(REG2, ((TO(REG)+TTO(REG))*Royal(REG,REG2)));

//The turnover for each region
Turnover(REG)..         TO(REG) =e=
                        sum((PRopq,t), xq(PRopq,REG,t)*Price(PRopq,REG));

//Intercompany sales turnover
Transto(REG)..         TTO(REG) =e=
```

```
                       sum((PRkl,REG2,t), xl(PRkl,REG,REG2,t) *TPl(PRkl,REG,REG2)
                                                                *(1-ExVat(REG,REG2)))
                      +sum((PRmn,REG2,t), xn(PRmn,REG,REG2,t) *TPn(PRmn,REG,REG2)
                                                *(1-ExVat(REG,REG2))*ER(REG2)/ER(REG))
                      +sum((PRopq,REG2,t), xp(PRopq,REG,REG2,t)*TPp(PRopq,REG,REG2)
                                                *(1-ExVat(REG,REG2))*ER(REG2)/ER(REG))
//Additional transfer turnover R3 due to triangular trade between R3 R1 R2
                      +sum((PRkl,t)$(ORD(REG)=3), xl(PRkl,REG,"R2",t)*TPl(PRkl,REG,"R1")
                      *(1-ExVat(REG,"R1")))
                      +sum((PRmn,t)$(ORD(REG)=3), xn(PRmn,REG,"R2",t)*TPn(PRmn,REG,"R1")
                      *(1-ExVat(REG,"R1"))*ER("R1")/ER(REG))
                      +sum((PRopq,t)$(ORD(REG)=3), xp(PRopq,REG,"R2",t)*TPp(PRopq,REG,"R1")
                      *(1-ExVat(REG,"R1"))*ER("R1")/ER(REG))
//Additional transfer turnover R1 due to triangular trade between R3 R1 R2
                      +sum((PRkl,t)$(ORD(REG)=1), xl(PRkl,"R3","R2",t)*TPl(PRkl,REG,"R2")
                      *(1-ExVat(REG,"R2")))
                      +sum((PRmn,t)$(ORD(REG)=1), xn(PRmn,"R3","R2",t)*TPn(PRmn,REG,"R2")
                      *(1-ExVat(REG,"R2"))*ER("R2")/ER(REG))
                      +sum((PRopq,t)$(ORD(REG)=1), xp(PRopq,"R3","R2",t)*TPp(PRopq,REG,"R2")
                      *(1-ExVat(REG,"R2"))*ER("R2")/ER(REG));

//Intercompany purchasing costs
Transcost(REG)..      TC(REG) =e=
                       sum((PRkl,REG2,t), xl(PRkl,REG2,REG,t)     *TPl(PRkl,REG2,REG)
                                                *(1+Dut(REG2,REG))*ER(REG2)/ER(REG))
                      +sum((PRmn,REG2,t), xn(PRmn,REG2,REG,t)     *TPn(PRmn,REG2,REG)
                                                                  *(1+Dut(REG2,REG)))
```

```
                      +sum((PRopq,REG2,t), xp(PRopq,REG2,REG,t) *TPp(PRopq,REG2,REG)
                                                                  *(1+Dut(REG2,REG)))

//Additional transfer costs R1 due to triangular trade between R3 R1 R2
                      +sum((PRkl,t)$(ORD(REG)=1), xl(PRkl,"R3","R2",t)*TPl(PRkl,"R3",REG)
                                                        *(1+Dut("R3",REG))*ER("R3")/ER(REG))
                      +sum((PRmn,t)$(ORD(REG)=1), xn(PRmn,"R3","R2",t)*TPn(PRmn,"R3",REG)
                                                                        *(1+Dut("R3",REG)))
                      +sum((PRopq,t)$(ORD(REG)=1), xp(PRopq,"R3","R2",t)*TPp(PRopq,"R3",REG)
                                                                        *(1+Dut("R3",REG)))

//Additional transfer costs R2 due to triangular trade between R3 R1 R2
                      +sum((PRkl,t)$(ORD(REG)=2), xl(PRkl,"R3",REG,t)*TPl(PRkl,"R1",REG)
                                                        *(1+Dut("R1",REG))*ER("R1")/ER(REG))
                      +sum((PRmn,t)$(ORD(REG)=2), xn(PRmn,"R3",REG,t)*TPn(PRmn,"R1",REG)
                                                                        *(1+Dut("R1",REG)))
                      +sum((PRopq,t)$(ORD(REG)=2), xp(PRopq,"R3",REG,t)*TPp(PRopq,"R1",REG)
                                                                        *(1+Dut("R1",REG)));

//Sum of production costs
Prodcost(REG)..         Prodc(REG) =e=
                         sum((PRik,t),  xi(PRik,REG,t)*PCi(PRik,REG))
                        +sum((PRkl,t),  xk(PRkl,REG,t)*PCk(PRkl,REG))
                        +sum((PRmn,t),  xm(PRmn,REG,t)*PCm(PRmn,REG))
                        +sum((PRopq,t), xo(PRopq,REG,t)*PCo(PRopq,REG));

//Sum of the distribution costs
```

```
Distcost(REG)..         Distc(REG) =e=
                         sum((PRkl,REG2,t), xl(PRkl,REG2,REG,t) *DCl(PRkl,REG2,REG)
                                                                *ER("R2")/ER(REG))
                        +sum((PRmn,REG2,t), xn(PRmn,REG2,REG,t) *DCn(PRmn,REG2,REG)
                                                                *ER("R2")/ER(REG))
                        +sum((PRopq,REG2,t), xp(PRopq,REG2,REG,t)*DCp(PRopq,REG2,REG)
                                                                 *ER("R2")/ER(REG));

//Sum of the storage costs
Stocost(REG)..          StoC(REG) =e=
                         sum((PRkl,t), yl(PRkl,REG,t)*SCl(PRkl,REG))
                        +sum((PRmn,t), yn(PRmn,REG,t)*SCn(PRmn,REG))
                        +sum((PRopq,t), yp(PRopq,REG,t)*SCp(PRopq,REG))
                        +sum((PRopq,t), yq(PRopq,REG,t)*SCq(PRopq,REG));

//The sum of financial cost for distributing and storing goods, money flow out of the system
FinCost..               FinC =e= sum((REG,t), sum(REG2,FinDistC(REG,REG2,t))+FinStoC(REG,t));

//Financial cost for distributing goods
FinDistCost(REG,REG2,t)$(ord(REG)<>ord(REG2))..
                        FinDistC(REG,REG2,t) =e=
                        sum(PRkl,(sum(PROREG, sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0),
                            PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG))+
                            PCk(PRkl,PROREG)*ER(PROREG))/card(PROREG)
                            *xl(PRkl,REG,REG2,t)*((1+Dfcost)**Dtime(REG,REG2)-1)))

                        +sum(PRmn,(sum(PROREG, sum(PRkl$(Prodmixm(PRmn,PRkl,PROREG)<>0),
```

```
                            sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0
                                                $(Prodmixm(PRmn,PRkl,PROREG)<>0)),
                            PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG)
                            +PCk(PRkl,PROREG)*ER(PROREG))*Prodmixm(PRmn,PRkl,PROREG))
                            +PCm(PRmn,PROREG)*ER(PROREG))/card(PROREG)
                            *xn(PRmn,REG,REG2,t)*((1+Dfcost)**Dtime(REG,REG2)-1)))

                       +sum(PRopq,(sum(PROREG, sum(PRmn$(Prodmixo(PRopq,PRmn,PROREG)<>0),
                        sum(PRkl$(Prodmixm(PRmn,PRkl,PROREG)<>0
                                $(Prodmixo(PRopq,PRmn,PROREG)<>0)),
                        sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0
                        $(Prodmixm(PRmn,PRkl,PROREG)<>0$(Prodmixo(PRopq,PRmn,PROREG)<>0))),
                            PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG)
                            +PCk(PRkl,PROREG)*ER(PROREG))*Prodmixm(PRmn,PRkl,PROREG)
                            +PCm(PRmn,PROREG)*ER(PROREG))*Prodmixo(PRopq,PRmn,PROREG))
                            +PCo(PRopq,PROREG)*ER(PROREG))/card(PROREG)
                            *xp(PRopq,REG,REG2,t)*((1+Dfcost)**Dtime(REG,REG2)-1)));

//Financial cost for storing goods
FinStoCost(REG,t)..    FinStoC(REG,t) =e=

                         sum(PRkl,(sum(PROREG, sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0),
                            PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG))+
                            PCk(PRkl,PROREG)*ER(PROREG))/card(PROREG)
                            *yl(PRkl,REG,t)*((1+Dfcost)**Stime-1)))

                        +sum(PRmn,(sum(PROREG, sum(PRkl$(Prodmixm(PRmn,PRkl,PROREG)<>0),
```

```
    sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0
                $(Prodmixm(PRmn,PRkl,PROREG)<>0)),
    PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG)
    +PCk(PRkl,PROREG)*ER(PROREG))*Prodmixm(PRmn,PRkl,PROREG))
    +PCm(PRmn,PROREG)*ER(PROREG))/card(PROREG)
    *yn(PRmn,REG,t)*((1+Dfcost)**Stime-1)))

+sum(PRopq,(sum(PROREG, sum(PRmn$(Prodmixo(PRopq,PRmn,PROREG)<>0),
  sum(PRkl$(Prodmixm(PRmn,PRkl,PROREG)<>0
                $(Prodmixo(PRopq,PRmn,PROREG)<>0)),
  sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0
  $(Prodmixm(PRmn,PRkl,PROREG)<>0$(Prodmixo(PRopq,PRmn,PROREG)<>0))),
    PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG)
    +PCk(PRkl,PROREG)*ER(PROREG))*Prodmixm(PRmn,PRkl,PROREG)
    +PCm(PRmn,PROREG)*ER(PROREG))*Prodmixo(PRopq,PRmn,PROREG))
    +PCo(PRopq,PROREG)*ER(PROREG))/card(PROREG)
    *yp(PRopq,REG,t)*((1+Dfcost)**Stime-1)))

+sum(PRopq,(sum(PROREG, sum(PRmn$(Prodmixo(PRopq,PRmn,PROREG)<>0),
 sum(PRkl$(Prodmixm(PRmn,PRkl,PROREG)<>0
              $(Prodmixo(PRopq,PRmn,PROREG)<>0)),
 sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0
 $(Prodmixm(PRmn,PRkl,PROREG)<>0$(Prodmixo(PRopq,PRmn,PROREG)<>0))),
  PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG)
  +PCk(PRkl,PROREG)*ER(PROREG))*Prodmixm(PRmn,PRkl,PROREG)
  +PCm(PRmn,PROREG)*ER(PROREG))*Prodmixo(PRopq,PRmn,PROREG))
  +PCo(PRopq,PROREG)*ER(PROREG))/card(PROREG)
```

```
                          *yq(PRopq,REG,t)*((1+Dfcost)**Stime-1)));

//Product values generated according to production costs used to evaluate the financial costs
ProdValik(PRik)..       ProVik(PRik) =e=
                        sum(PROREG, PCi(PRik,PROREG)*ER(PROREG))/card(PROREG);

ProdValkl(PRkl)..       ProVkl(PRkl) =e=
                        sum(PROREG,
                        sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0),
                        PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG))
                                                +PCk(PRkl,PROREG)*ER(PROREG))/card(PROREG);

ProdValmn(PRmn)..       ProVmn(PRmn) =e=
                        sum(PROREG, sum(PRkl$(Prodmixm(PRmn,PRkl,PROREG)<>0),
                            sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0$
                                        (Prodmixm(PRmn,PRkl,PROREG)<>0)),
                            PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG)
                            +PCk(PRkl,PROREG)*ER(PROREG))*Prodmixm(PRmn,PRkl,PROREG))
                                     +PCm(PRmn,PROREG)*ER(PROREG))/card(PROREG);

ProdValopq(PRopq)..     ProVopq(PRopq) =e=
                        sum(PROREG, sum(PRmn$(Prodmixo(PRopq,PRmn,PROREG)<>0),
                         sum(PRkl$(Prodmixm(PRmn,PRkl,PROREG)<>0$
                                                  (Prodmixo(PRopq,PRmn,PROREG)<>0)),
                         sum(PRik$(Prodmixk(PRkl,PRik,PROREG)<>0$
                         (Prodmixm(PRmn,PRkl,PROREG)<>0$(Prodmixo(PRopq,PRmn,PROREG)<>0))),
                         PCi(PRik,PROREG)*ER(PROREG)*Prodmixk(PRkl,PRik,PROREG)
```

```
                         +PCk(PRkl,PROREG)*ER(PROREG))*Prodmixm(PRmn,PRkl,PROREG)
                         +PCm(PRmn,PROREG)*ER(PROREG))*Prodmixo(PRopq,PRmn,PROREG))
                         +PCo(PRopq,PROREG)*ER(PROREG))/card(PROREG);



//Constraints

//Fullfillment of customer demand
Cusdemand(PRopq,REG,t)..        xq(PRopq,REG,t) =e= Dem(PRopq,REG,t);

//Capacity bounds
PCapi(REG,t)..          sum(PRik,xi(PRik,REG,t)*Cufi(PRik,REG)) =l= Capi(REG,t);

PCapk(REG,t)..          sum(PRkl,xk(PRkl,REG,t)*Cufk(PRkl,REG)) =l= Capk(REG,t);

SCapl(REG,t)..          sum(PRkl,yl(PRkl,REG,t)*Cufl(PRkl,REG)) =l= Capl(REG,t);

PCapm(REG,t)..          sum(PRtgran,xm(PRtgran,REG,t)*Cufm(PRtgran,REG)) =l= Capm(REG,t);

SCapn(REG,t)..          sum(PRmn,yn(PRmn,REG,t)*Cufn(PRmn,REG)) =l= Capn(REG,t);

PCapo(REG,t)..          sum(PRopq,xo(PRopq,REG,t)*Cufo(PRopq,REG)) =l= Capo(REG,t);

SCapp(REG,t)..          sum(PRopq,yp(PRopq,REG,t)*Cufp(PRopq,REG)) =l= Capp(REG,t);

SCapq(REG,t)..          sum(PRopq,yq(PRopq,REG,t)*Cufq(PRopq,REG)) =l= Capq(REG,t);
```

```
//Flowconstraints
Prodflowk(PRik,REG,t).. xi(PRik,REG,t) =e= sum(PRkl, (xk(PRkl,REG,t)
*Prodmixk(PRkl,PRik,REG)));

Prodflowm(PRkl,REG,t).. sum(REG2,xl(PRkl,REG2,REG,t)) =e= sum(PRmn,(xm(PRmn,REG,t)
                                                                *Prodmixm(PRmn,PRkl,REG)));

Stoflowl(PRkl,REG,t)..  yl(PRkl,REG,t) =e= yl(PRkl,REG,t--1)+xk(PRkl,REG,t)
                                                     -sum(REG2,xl(PRkl,REG,REG2,t));

Stoflown(PRmn,REG,t)..  yn(PRmn,REG,t) =e= yn(PRmn,REG,t--1)+xm(PRmn,REG,t)
                                                     -sum(REG2,xn(PRmn,REG,REG2,t));

Prodflowo(PRmn,REG,t).. sum(REG2,xn(PRmn,REG2,REG,t)) =e= sum(PRopq,
                                             xo(PRopq,REG,t)*Prodmixo(PRopq,PRmn,REG));

//Stoflowp(PRopq,REG,t)..       yp(PRopq,REG,t) =e= yp(PRopq,REG,t-1)+xo(PRopq,REG,t)
                                                        - sum(REG2,xp(PRopq,REG,REG2,t));
//Stoflowq(PRopq,REG,t)..       yq(PRopq,REG,t) =e= yq(PRopq,REG,t-1)
                                                        +sum(REG2,xp(PRopq,REG2,REG,t))
                                                        -xq(PRopq,REG,t);

Stoflowp(PRopq,REG,t).. yp(PRopq,REG,t) =e= yp(PRopq,REG,t--1)+xo(PRopq,REG,t)
                                            -sum(REG2,xp(PRopq,REG,REG2,t));
Stoflowq(PRopq,REG,t).. yq(PRopq,REG,t) =e= yq(PRopq,REG,t--1)+sum(REG2,xp(PRopq,REG2,REG,t))
                                            -xq(PRopq,REG,t);
```

```
Model NZOpt "Optimal Linearized Model" /all/
      NZHigh "Linear Solution is rounded to nearest higher integer " /all/
      NZReal "Historical Real Life Solution" /all/;


//Define output files
file results   /profit.out/,
     turnovers /turnover.out/,
     costs     /cost.out/,
     flowopt   /flowopt.out/,
     flowrea   /flowrea.out/,
     storagerea /storagerea.out/,
     storageopt /storageopt.out/,
     capacityrea /capacityutlrea.out/,
     capacityopt /capacityutlopt.out/,
     capusageopt /capacityusageopt.out/,
     capusagerea /capacityusagerea.out/,
     marginal /marginallevels.out/,
     profitmarginopt /demandmarginalopt.out/,
     profitmarginrea /demandmarginalrea.out/,
     taxation /taxcontributions.out/;


//Solves the linearized model to optimality
$include ../scenariodata/noflow.inp
```

```
Solve NZOpt using lp maximizing z;
$include ../displaycommand/caputlobs.cmd
$include ../displaycommand/profitobs.cmd
$include ../displaycommand/caputlobsproduct.cmd
$include ../displaycommand/dispopt.cmd
$include ../displaycommand/disponscreen.cmd


//ceil function is used to find nearest higher integer value
//xi.fx(PRik,REG,t) = ceil(xi.l(PRik,REG,t));
//Solve NZHigh using lp maximizing z;


//flow variables are fixed to simulate the real life solution
//$include ../scenariodata/histplan.inp
//$include ../scenariodata/noflow.inp
//xl.fx(PRkl,REG,REG2,t)$(ord(REG)<>ord(REG2)) = histplanl(PRkl,REG,REG2,t);
//xp.fx(PRopq,REG,REG2,t) = histplanp(PRopq,REG,REG2,t);
//Solve NZReal using lp maximizing z;
//$include ../displaycommand/caputlobs.cmd
//$include ../displaycommand/profitobs.cmd
//$include ../displaycommand/caputlobsproduct.cmd
//$include ../displaycommand/dispreal.cmd
//$include ../displaycommand/disponscreen.cmd
```

Appendix H

Validation and Verification

This section contains the details regarding the test scenarios presented in section 5.4. In the scenarios a formalized verification of the *miniminitest-model* is performed.

The following scenarios are modelled and presented below:

1. The simple case, where production takes place in individual regions only.
2. Transfer pricing and the effects of cross regional flow.
3. International taxation and the non-linearities of tax functions.
4. Import duties.
5. Export value added taxes.
6. Royalties.
7. Converging and diverging material flows with BoM constraints.
8. Storage of products while considering multiple time periods.
9. Financial cost of storing and distributing.

Initially the input data are defined in section H.1. The implemented GAMS tables are presented in section H.2. None of the data reflect reality but are constructed for the specific cases. All GAMS input and output files for each scenario are given on the enclosed CD in the folder *miniminitest model GAMS*.

H.1 Validation Scenarios

Input Data for Validation

The initial data values are given in local currency pr. unit product for each region; except distribution costs which are defined in $R2$ currency. The capacities are given as the total capacity for each facility and are defined in unit the product is measured, depending on the process phase. Taxes, import duties, export value added tax and royalties are defined in percentage and are set to zero to start with. Distribution time and storage time is given in months. Factors that are not possible in real life are not allowed (NA). The initial data values are the same for all time periods and all regions when considering the exchange rate and are given in the tables H.1 to H.6.

Three producing regions $R1$-$R3$ and one sales region $R4$ where no production occurs are modelled. The products divided into the intermediate concentrates $P1$-$P2$ and the final blends $P3$-$P6$.

The tables H.1 and H.2 describes the input data for each region

- *Dem* defines the demand for a product $P1$-$P6$. $P1$ and $P2$ represent concentrates that are not sold to any end customers.
- *Price* defines the sales prices in the individual regions. Prices are given in the regional currencies pr. product unit.
- TPx defines the transfer price for each product in the currency of the sending region. The transfer price is defined for each phase x where transfer is possible and is given pr. unit of the product.
- PCx defines the production costs for each production phase x. The costs are given in local currencies pr. unit of the outgoing product. A cost is only defined where production is possible.
- DCx defines the distribution costs in phase x for each unit of a product. The costs are given in $R2$ currency and is only valid where cross regional flow occurs. Distribution within a region does not have a cost in the model.
- SCx defines the storage costs for a product unit in the storage facility in phase x. The storage costs are given in local currencies and are defined for a time period of one month. Only the direct variable storage costs are included in the parameter.

In the tables H.3 and H.4 the following input data are given:

		Dem	$Price$	TPj	TPl	PCi	PCk
$R1$	P1	NA	NA	18	NA	6	NA
	P2	NA	NA	18	NA	6	NA
	P3	10	36	NA	18	NA	6
	P4	10	36	NA	18	NA	6
	P5	10	36	NA	18	NA	6
	P6	10	36	NA	18	NA	6
$R2$	P1	NA	NA	3	NA	1	NA
	P2	NA	NA	3	NA	1	NA
	P3	10	6	NA	3	NA	1
	P4	10	6	NA	3	NA	1
	P5	10	6	NA	3	NA	1
	P6	10	6	NA	3	NA	1
$R4$	P1	NA	NA	NA	NA	NA	NA
	P2	NA	NA	NA	NA	NA	NA
	P3	10	662,98	NA	NA	NA	NA
	P4	10	662,98	NA	NA	NA	NA
	P5	10	662,98	NA	NA	NA	NA
	P6	10	662,98	NA	NA	NA	NA

Table H.1: Data input for period 1-4

	PR	DCj	DCl	SCj	SCl	SCm
$R1$	P1	0,1	NA	6	NA	NA
	P2	0,1	NA	6	NA	NA
	P3	NA	0,1	NA	6	6
	P4	NA	0,1	NA	6	6
	P5	NA	0,1	NA	6	6
	P6	NA	0,1	NA	6	6
$R2$	P1	0,1	NA	1	NA	NA
	P2	0,1	NA	1	NA	NA
	P3	NA	0,1	NA	1	1
	P4	NA	0,1	NA	1	1
	P5	NA	0,1	NA	1	1
	P6	NA	0,1	NA	1	1
$R4$	P1	NA	NA	NA	NA	NA
	P2	NA	NA	NA	NA	NA
	P3	NA	0,1	NA	NA	110,5
	P4	NA	0,1	NA	NA	110,5
	P5	NA	0,1	NA	NA	110,5
	P6	NA	0,1	NA	NA	110,5

Table H.2: Data input for period 1-4

	$Capi$	$Capj$	$Capk$	$Capl$	$Capm$
$R1$	100	100	100	200	200
$R2$	100	100	100	200	200
$R3$	NA	NA	NA	NA	200

Table H.3: Data input - capacities - for period 1-4

	ER	TAX	$Duty$	$ExVat$	Roy
$R1$	1	0	0	0	0
$R2$	6	0	0	0	0
$R3$	0,0543	0	0	0	0

Table H.4: Financial data input for period 1-4

- $Capx$ defines the total capacity for a facility in phase x for one time period (3 months). The capacities are given in the units of the outgoing products and consider both production capacities (phase I and k) and storage capacities (phase j, l and m)
- ER defines the exchange rate for local currency with respect to the $R1$ currency.
- TAX defines the tax rate for each region. The tax rate is given in percent of the positive profit.
- $Duty$ defines the import duties in percentage of the transfer price for each region.
- $ExVat$ defines the export value added taxes in percentage of the transfer price for each region.
- Roy defines the royalties paid on sales for each region and are defined in percentage.

The factors for the financial costs are given in table H.5. The following factors are given.

- $Dtime$ defines the distribution time between regions. The columns define the sending region and the rows the receiving region. All times are given in months and define the distribution time from door to door. It is not possible to distribute any products from $R4$ as no production occurs in this region.
- Dfcost defines the interest rate pr. month for tied up capital when storing and distributing products.
- Stime defines the average storage time for products within a region.

$Dtime$				$Dfcost$	$Stime$
	$R1$	$R2$	$R4$		
$R1$	0	1	2	0.0000	3
$R2$	1	0	1		
$R4$	NA	NA	NA		

Table H.5: Data input for period 1-4 (verification input)

BOM	$P1$	$P2$
$P3$	1	0
$P4$	1	0
$P5$	0	1
$P6$	0	1

Table H.6: Bill of materials at production facility k (verification input)

The storage time is given in months.

The data set used for the verification and validation consists of four final products ($P3$-$P6$) which are sold to the costumers. The final products are made from the intermediate products $P1$ and $P2$. The mix of $P1$ and $P2$ to produce $P3$-$P6$ are given in the BOMs (Table H.6). The row describes the input products ($P1$-$P2$) given in litres (l) and the column describes the output product $P4$-$P6$ given in kg. Conversions from liquid to solid products as well as non-balancing masses in production are thereby ensured by the BOM. To begin with no blending or separation of products is considered and the bill of material only defines one-to-one flow between intermediate and finished products.

Case 1: Production in separate regions

The initial values assume that production costs, storage costs and capacities are equal in both regions. Furthermore no taxes, import duties, export VAT and royalties are considered. Transfer prices are given as the sum of production and storage costs at the primary storage phase. With this case the basic structure of the model i.e. the equations generating the net profit after tax (z), turnover, transfer turnover, production costs, distribution costs, and transfer costs can be verified.

Given the abovementioned values it is expected that each production region will fulfil own demands and thereby avoid additional costs from cross regional product transfer. For $R4$ a transfer from either $R1$ or $R2$ should occur in order to fulfil demand.

Through this case it is possible to evaluate if flow conservation and BOM constraints when assuming one-to-one flow at the nodes are fulfilled, eq. 5.53 - 5.54. Furthermore, it is simple to see if the customer demand is fulfilled as required 5.50 and if the right key numbers are calculated 5.33-5.44. Transfer should only occur between a primary stock in a producing region and the secondary stock in $R4$. Furthermore, no use of storage capacity is expected as the capacities within each period ensure that the periodic demand can be fulfilled.

The output shows that the model generates both the objective equation 5.33 and the sub equations 5.34 - 5.44 as expected. Turnovers and transfer turnovers are generated in each region and the corresponding production costs, distribution costs and transfer costs are deducted. The resulting profits are summed to the objective value z. The correct prices, transfer prices, production costs, and distributions costs are used in the equations. The equations fulfilling demand 5.50 and all the constraints regarding the capacities and the product flows, 5.51 - 5.54, are set up correctly.

The test model showed that under these equal conditions the optimal solution was generated as expected. The simplicity of the model ensures that this result can be checked manually, and thereby verifying the models ability under these conditions. Since no production in R4 is possible import occurs (in form of a cross regional flow). With financial costs, all products would be shipped from $R2$ and not from $R1$ due to the different shipment times and thereby different financial costs however in the case with 0% interest, both solutions are equal. In this solution all products for the $R4$ market are produced in $R1$.

Through the verification it is proved that the correct economical values - corresponding to the product flows - are generated. It can be seen that global net profit and regional turnovers and costs balance as expected. Furthermore, all conversions between the regional currencies are performed as expected. The corresponding key account numbers in local currencies summed for all four periods are given in table H.7.

	$R1$	$R2$	$R4$
TO	5760	960	106076,8
TTO	2880	-	-
RoyTO	-	-	-
ProdC	3840	320	-
DistC	-	-	1767,956
StoC	-	-	-
TC	-	-	53038,674
RoyC	-	-	-
NPr	4800	640	51270,17
RegPr	4800	640	51270,17
RegPrPos	4800	640	51270,17
RegPrNeg	-	-	-
FinC	-		
Z	11423,97		

Table H.7: Results verification case 1

Case 2: Cross regional flow

To evaluate the effects of transfer pricing and cross regional flow the model is forced to perform distribution between the two producing regions. This cross regional distribution inflicts extra distribution costs to the entire objective function and adds an additional turnover for the sending region as well as an additional cost to the receiving region. The cross regional distribution is motivated through a lowered production cost in one region. As long as the cost savings from cheaper production is higher that the additional distribution costs, the model is expected to perform a transfer distribution of products.

The following changes are implemented for all time periods: Production costs for all products in $R1$ are reduced from six to one $R1$ currency pr. unit while production costs in $R2$ are kept at the initial level (see table H.8). This should force the model to allocate more production to $R1$ where production costs are lower. Since production costs for both phases are lower the flow is expected to take place between the primary and secondary storage in phase l.

Running the model enables to check the equations handling transfer costs 5.42 and turnovers 5.41 as well as the distribution costs 5.40 more thor-

		PCi	PCk
R1	P1	1	NA
	P2	1	NA
	P3	NA	1
	P4	NA	1
	P5	NA	1
	P6	NA	1

Table H.8: Changes in input for verification case 2

oughly as flow is initiated on the arcs. Also it is possible to check if the flow constrains considering more regions are respected: 5.53 for production node j and 5.54 for the storage nodes. These equations are all set up correctly.

The cross regional flow of the final products $P3$-$P6$ between $R1$ and $R2$ occurs in phase l as expected. Not all production is allocated to $R1$ due to capacity constraints on the $R1$ production facilities and buffer storages. This shows that the capacity constraints (5.51 and 5.52) are respected as well.

The corresponding economical flow shows a balance between the transfer turnover for $R1$ and the equivalent transfer cost for $R2$ and $R4$. Furthermore the distribution costs are as expected with the given product flow and cost factors. The results are given in table H.9. When the $R1$ capacity is not constrained all production takes place in $R1$ and the products are afterwards distributed to $R2$ and $R4$. This has been modelled, but the results are not presented here.

Case 3: Tax rates

The linearization of the tax function is evaluated by checking if the model charges taxes on a negative profit, eq. 5.34, 5.35 and 5.36. This is done by ensuring a loss in one region and a concurrent positive net profit in another region.

To ensure the loss sales prices in $R2$ are reduced from 6 R2 currency to 3 R2 currency for each product and are halved for $R4$ from 662,98 R4 currency to 300 R4 currency while the prices in $R1$ are retained. Raising the taxes in all regions to 30% makes it possible to evaluate if the model charge taxes

	$R1$	$R2$	$R4$
TO	5760	950	106076,8
TTO	4320	-	-
RoyTO	-	-	-
ProdC	800	160	-
DistC	-	8	1767,956
StoC	-	-	-
TC	-	240	53038,674
RoyC	-	-	-
NPr	9280	552	51270,17
RegPr	9280	552	51270,17
RegPrPos	9280	552	51270,17
RegPrNeg	-	-	-
FinC	-		
Z	15375,97		

Table H.9: Results verification case 2

only on the region with positive profit, and if the use of auxiliary variables is as expected.

Since the turnover does not cover the cost of intermediate products a loss is ensured in region $R4$. Furthermore the production costs in $R2$ are not covered from sales in own region as the total production costs are 2 R2 currency where the sales price is 1 R2 currency. (see table H.10 for details)

The generated equations model the tax function as desired. When the tax rates are set to 30% in all regions this is seen to be implemented correctly as a factor 0,7 in the generated net profit equations.

The model tries to avoid losses in the region $R2$ by generating cross regional transfer from $R2$ to $R4$. Since the transfer prices for products are higher than the production costs this is the only profit contribution to the $R2$ region. With the test case used the profit in the $R2$ region is zero. This is due to the fact that the transfer turnover to $R4$ exactly covers the losses in own region. Lowering the $R4$ demand forces a loss in the $R2$ while raising the demand ensures a profit.

The economic results are a positive profit before tax in $R1$ and a negative profit before tax in $R4$. The results show that taxes are only charged in $R1$ where the net profit after tax is 30% lower than the profit before tax.

		Price	*TAX*
R1	P3	36	0,3
	P4	36	
	P5	36	
	P6	36	
R2	P3	1	0,3
	P4	1	
	P5	1	
	P6	1	
R4	P3	300	0,3
	P4	300	
	P5	300	
	P6	300	

Table H.10: Changes in input for verification case 3

The results are presented in table H.11.

Case 4: Import duty

The impact of duties between regions is tested by defining an import duty for $R4$. The import duty for $R4$ should result in a $R4$ transfer cost [eq. 5.42] that does not balance with the corresponding transfer turnover [eq. 5.41] from the sending region. The $R4$ duty is defined to 10% while the rest of the data are kept on the initial level (table H.12).

The resulting flow is as in case 1. All products for the $R4$ market are produced in $R1$. However the corresponding economical flow has changed (as it can be seen in table H.13). The transfer cost in $R4$ are 3168 R1 currency and the transfer turnover in $R1$ are 2880 R1 currency. The difference is 10% corresponding to the import duty. The difference also influences the enterprise net profit before tax which is lowered due to the higher cost.

Case 5: Export VAT

Where import duties influence the costs of transfer for the buying region the export value added taxes influences the turnover for the supplying region.

	R1	R2	R4
TO	5760	160	48000
TTO	-	480	-
RoyTO	-	-	-
ProdC	1920	640	-
DistC	-	-	1767,956
StoC	-	-	-
TC	-	-	53038,674
RoyC	-	-	-
NPr	2688	0	-6806,63
RegPr	3840	0	-6806,63
RegPrPos	3840	0	-
RegPrNeg	-	-	6806,63
FinC	-		
Z	2318,4		

Table H.11: Results verification case 3

	Duty
R1	0
R2	0
R4	0,1

Table H.12: Changes in input for verification case 4

	R1	R2	R4
TO	5760	960	106076,8
TTO	2880	-	-
RoyTO	-	-	-
ProdC	3840	320	-
DistC	-	-	1767,956
StoC	-	-	-
TC	-	-	58342,541
RoyC	-	-	-
NPr	4800	640	45966,303
RegPr	4800	640	45966,303
RegPrPos	4800	640	45966,303
RegPrNeg	-	-	-
FinC	-		
Z	11135,97		

Table H.13: Results verification case 4

	VAT
R1	0,05
R2	0,1
R4	0

Table H.14: Changes in input for verification case 5

Due to the similarity of the formulation of the equations the same test approach as in section H.1 is used for this evaluation. The changes in input are presented in table H.14.

The model is run with export VAT defined for both $R1$ and $R2$. With equal production, storage and distribution costs, the production will be allocated to the region with lowest VAT. If the VAT is only defined for the $R1$ region all production will be allocated to $R2$. With equal VAT in both regions the production could be allocated to either one of the regions due to similar costs. The result is an export production from $R1$. The financial impact is a lower transfer turnover in the $R1$ region compared to the corresponding transfer cost in $R4$. The values are presented in table H.15.

	R1	R2	R4
TO	5760	960	106076,8
TTO	2736	-	-
RoyTO	-	-	-
ProdC	3840	320	-
DistC	-	-	1767,956
StoC	-	-	-
TC	-	-	53038,674
RoyC	-	-	-
NPr	4656	640	51270,17
RegPr	4656	640	51270,17
RegPrPos	4656	640	51270,17
RegPrNeg	-	-	-
FinC	-		
Z	11279,97		

Table H.15: Results verification case 5

	Roy
R1	0
R2	0,40
R4	0

Table H.16: Changes in input for verification case 6

Case 6: Royalties

Royalties are an economical flow between regions - even when no product flow takes place between those regions. Therefore the equations handling the royalties are validated. In this case, the royalty turnover for one region should correspond to the royalty cost for the other region. The royalties are paid on all sales of products and semi-finished produces given by the turnover and transfer turnover for the region. The only changes to the original input are the royalties defined in table H.16.

First it is verified that the correct equations are generated from the model (equation 5.43 and 5.44). The output shows that the equations for the royalty income are set to zero for $R2$ and $R4$ while for $R1$ it is set up containing the correct nonzero coefficient. The royalty costs also function

	R1	R2	R4
TO	5760	960	106076,8
TTO	2880	-	-
RoyTO	2304	-	-
ProdC	3840	320	-
DistC	-	-	1767,956
StoC	-	-	-
TC	-	-	53038,674
RoyC	-	384	-
NPr	7104	256	51270,17
RegPr	7104	256	51270,17
RegPrPos	7104	256	51270,17
RegPrNeg	-	-	-
FinC	-		
Z	11423,97		

Table H.17: Results verification case 6

as desired as only the equation for $R2$ is nonzero.

The royalties paid to $R1$ are changed to be 40% for all products and intermediate products sold in $R2$. The model shows that a royalty cost occurs in $R2$ (384 R2 currency) with a corresponding turnover in $R1$ (2304 R1 currency). The product flow is as in case 1. The corresponding economical values are presented in table H.17. As expected the objective value is the same as in case 1. Since no tax differences occur the result is only a movement of the regional profit but with the same overall result.

Case 7: BOM and product flow

In the initial data set a one-to-one product flow is considered assuming that 1L of product $P1$ corresponds to 1 kg $P3$. However in the real life some products are aggregated - i.e. through blending - while other products are disaggregated. The initial case handles the de-aggregation where $P1$ is transformed into $P3$ and $P4$. However to ensure that the BOM and product flow constraints are not violated while aggregating the products the BOM data are changed. In the new data input the final enzyme $P3$ is a blend of the products $P1$ and $P2$. The relation is 1 L $P1$ and ¡ L $P2$ to

BOM	P1	P2
P3	1	0,5
P4	1	0
P5	0	1
P6	0	1

Table H.18: Bill of materials at production facility k (Verification case 7)

	Capi	Capj	Capk	Capl	Capm
R1	100	100	40	200	200
R2	80	100	100	200	200
R4	NA	NA	NA	NA	200

Table H.19: Capacities for verification case 7

1 kg $P3$. This should result in a higher need for product $P2$ to fulfil the given customer demand.

In the real life production it is possible to import semi finished products from other facilities in other regions. In the model this is handled through the cross regional flow in phase j. To ensure, that the model handles the mixture of products from different regions as intended, the production capacities are reduced to ensure a product flow between regions. The capacity for the production facility i in the region $R2$ is reduced to 80 L pr time period and the production capacity for facility k in R1 is reduced to 40 kg pr. time period. Thereby a flow between $R1$ and $R2$ in the phase j is unavoidable when the customer demand has to be fulfilled.

From the output it appears that the constraints containing the BOM (prodmixjk) in the GAMS code are set up and the correct data are implemented according to the model equation 5.53. Thus it is ensured that one product number going into a production facility, from all the different sending regions is equal to outgoing products which need the ingoing product number as an ingredient.

The result of the case is that the product flow constraints are respected and the products are mixed as intended. The flow results in a 25% raise in the $P2$-production for each facility in phase i. The reduced capacities force higher allocations to $R1$ in the phase i. The concentrate from $R1$ is afterwards distributed to $R2$ for further processing in phase k. This ensures

	R1	R2	R4
TO	5760	960	106076,8
TTO	720	480	-
RoyTO	-	-	-
ProdC	2280	640	-
DistC	-	4	1767,956
StoC	-	-	-
TC	-	120	53038,674
RoyC	-	-	-
NPr	4200	676	51270,17
RegPr	4200	676	51270,17
RegPrPos	4200	676	51270,17
RegPrNeg	-	-	-
FinC	-		
Z	11039,97		

Table H.20: Results verification case 7

that the output products in phase k in region $R2$ are a mixture of products from $R1$ and $R2$ and shows that the model handles the mixture of products between products and regions as intended.

The following financial result is a lower profit, due to higher production costs (table H.20).

Case 8: storage constraints

In the abovementioned cases fulfilment of demand via production in the same time period has been possible. To test if the storage constraints are respected a test of the multi period problem is necessary. In the following scenario the constraints - storage flow 5.54, storage capacity 5.52 and storage cost 5.39 - are tested.

To ensure the use of storage facilities between the time periods demand for time period two in $R4$ has been raised and the production capacity in phase i in $R1$ has been reduced. This ensures that production within the time period is not enough to fulfil the entire demand and the use of storage is forced.

		t=1	t=2	t=3	t= 4
R4	P1	NA	NA	NA	NA
	P2	NA	NA	NA	NA
	P3	10	50	10	10
	P4	10	50	10	10
	P5	10	40	10	10
	P6	10	40	10	10

Table H.21: Demand pr time period for verification case 8

	Capi	Capj	Capk	Capl	Capm
R1	80	100	100	200	200
R2	100	100	100	200	200
R4	NA	NA	NA	NA	200

Table H.22: Capacities for verification case 8

The equations for the storage flow are set up according to the desired structure. There is a slight difference when setting up the equation for the last storage phase, as this phase can receive goods from different regions but can only send goods to its own region. This function the other way around than in all previous storage phases where goods can only be received from own region but sent to different regions.

The demand in phase i in time period t=2 is higher than the total production capacity in the period. Therefore it is seen that 20 kg of product $P5$ is stored in $R1$ while 50 kg $P4$ and 10 kg $P6$ are stored in $R2$ from time period 1 to time period 2. The total demand for time period 1 and 2 is 380 kg and the total production capacity is 360 kg. This means, that to fulfil demand an initial storage of 20 kg is necessary in phase 0. This means 20 kg of $P3$ is stored from time period 4 to time period 1. Since it is assumed that periods are cyclic the final stock level in period 4 corresponds to the starting level in time period t=0. Therefore it can be seen, that the storage flow constraints 5.54 are respected.

The storage of goods results in a storage cost from stored products in the regions $R1$ and $R2$.

	R1	R2	R4
TO	5760	960	198894
TTO	2880	420	-
RoyTO	-	-	-
ProdC	3840	600	-
DistC	-	-	3314,917
StoC	120	80	-
TC	-	-	99447,514
RoyC	-	-	-
NPr	4680	700	96131,569
RegPr	4680	700	96131,569
RegPrPos	4680	700	96131,569
RegPrNeg	-	-	-
FinC	-		
Z	14099,944		

Table H.23: Results verification case 8

Case 9: Financial cost

So far the direct operating costs of producing, storing and distributing have been evaluated. In the following the financial costs on storage and distribution are evaluated. To ensure financial costs a multiple time period problem with storage and cross regional distribution is necessary.

In the model, the following inputs are used: For storage between time periods, an average storage time of 3 months is assumed. The financial storage cost are the interests on storing the product for three months with an interest rate of 0,95% pr. month (12% p.a.), this affects the equations 5.45, 5.46 and 5.47. All costs are based on the average product values given by the equations 5.48 and 5.49.

When distributing the financial costs depend on the distribution time. For the test case it is assumed that the distribution time between $R1$ and $R2$ is one month; from $R1$ to$R4$ 2 months and from $R2$ to $R4$ 1 month. With an interest rate of 0,95% pr month this results in different cost on the different arcs.

Due to the fact that there are a financial cost on both storage and distribution the total financial costs consists of two inputs.

Dtime					
	R1	R2	R4	Dfcost	Stime
R1	0	1	2	0.0095	3
R2	1	0	1	0.0095	3
R4	NA	NA	NA	0.0095	3

Table H.24: Input data for verification case 9

		t=1	t=2	t=3	t= 4
R4	P1	NA	NA	NA	NA
	P2	NA	NA	NA	NA
	P3	10	50	10	10
	P4	10	50	10	10
	P5	10	40	10	10
	P6	10	40	10	10

Table H.25: Demand pr time period for verification case 9

The demands and capacities from case 8 are used as they ensure a use of the storage facilities in $R1$ and $R2$ and a financial cost from distribution to $R4$.

Due to the different financial costs the product flow is not the same as in case 8. The higher cost on the arc from $R1$ to $R4$ compared to the arc between $R2$ and $R4$ results in a higher production allocation to $R2$. Products are stored in $R2$ in time period 1 and in $R1$ in period 1 and 4.

Furthermore a transfer of products from $R2$ and $R1$ to $R4$ takes place. These are the arcs where corresponding financial costs should occur. Though the input without financial costs is similar to the input in case 8 the optimal flow with financial cost differs slightly. The result is a higher allocation of products to the $R2$ region from where the distribution time and thereby

	Capi	Capj	Capk	Capl	Capm
R1	80	100	100	200	200
R2	100	100	100	200	200
R4	NA	NA	NA	NA	200

Table H.26: Capacities for verification case 9

	Value in R1 currency
P1	6
P2	6
P3	12
P4	12
P5	12
P6	12

Table H.27: Product values for verification case 9

	t=1	t=2	t=3	t=4
R1	3,453	-	-	3,453
R2	20,716	-	-	-

Table H.28: Financial storage costs for verification case 9

the financial distribution costs to $R4$ are lower. Products are distributed from $R2$ whenever possible, however due to capacity constraints some distribution from $R1$ occurs.

Another effect of the financial costs is that storage of products take place earlier in the supply chain. This is due to the fact, that products have a lower value and the financial costs of storage will be lower for a given time period.

With the product flow and the given interests the financial storage costs in $R1$ currencies are

The financial distribution costs in R1 currency are

The financial costs are not a part of the regional costs but only occur in the overall profit as an overall financial cost. The result is are presented in table H.30.

	t=1	t=2	t=3	t=4
R1	9,163	13,745	-	-
R2	-	13,68	4,56	4,56

Table H.29: Financial distribution costs for verification case 9

	R1	R2	R4
TO	5760	960	198894
TTO	1800	600	-
RoyTO	-	-	-
ProdC	3120	720	-
DistC	-	-	3314,917
StoC	240	60	-
TC	-	-	99447,514
RoyC	-	-	-
NPr	4200	780	96131,569
RegPr	4200	780	96131,569
RegPrPos	4200	780	96131,569
RegPrNeg	-	-	-
FinC	73,329		
Z	14026,615		

Table H.30: Results verification case 9

Conclusion on the validation

The behaviour of the equations corresponds satisfactory to the realities being modelled. With the nine cases the equations 5.33 - 5.54 are verified individually and it is shown that the model performs the intended calculations. The verification was made on the *miniminitestmodel*.

H.2 Input GAMS Data

This section shows the basic data input used for verifying the functions in the code. The funtions are tested individually by changing the data input for each of the scenarios. E.g. the programs ability to calculate duties are tested by applying positive values in this data, and checking if the models calculations are correct. The strucutre of the tables and parameters corresponds to the structure used in the final model.

```
Bill of Material input with rows for products showing
input needed from previous production phase:

Table   Prodmixjk(PRk,PRi) Bill of materials at production
        facility k
                P1      P2
        P3      1       0
        P4      1       0
        P5      0       1
        P6      0       1       ;


Capacity bounds for each facility given in units of product
flow in the given phase:

Parameter       Capi(REG) Overall capacity in kg for
                facilities in region REG in phase i
        /       R1      100
                R2      100
                R10     0       /;

Parameter       Capj(REG) Overall capacity in kg for
                facilities in region REG in phase j
        /       R1      100
                R2      100
                R10     0       /;

Parameter       Capk(REG) Overall capacity in kg for
                facilities in region REG in phase k
        /       R1      100
```

```
                R2      100
                R10     0       /;

Parameter       Capl(REG) Overall capacity in kg for
                facilities in region REG in phase l
        /       R1      200
                R2      200
                R10     0       /;

Parameter       Capm(REG) Overall capacity in kg for
                facilities in region REG in phase m
        /       R1      200
                R2      200
                R10     200     /;

Demands stated for each end product for each region in
each time period:

Table   Dem(PRk,REG,t) Demand in kg for product PRk in
        region REG for timeperiod t
                R1.1    R2.1    R10.1
        P3      10      10      10
        P4      10      10      10
        P5      10      10      10
        P6      10      10      10

        +       R1.2    R2.2    R10.2
        P3      10      10      10
        P4      10      10      10
        P5      10      10      10
        P6      10      10      10

        +       R1.3    R2.3    R10.3
        P3      10      10      10
        P4      10      10      10
        P5      10      10      10
        P6      10      10      10

        +       R1.4    R2.4    R10.4
```

```
        P3      10      10      10
        P4      10      10      10
        P5      10      10      10
        P6      10      10      10      ;

The dfcost is the interest rate used when calculating the
cost of investing in goods stored or distributes

Scalar  Dfcost Interest rate for distribution or storage
        /0.0000/ ;

Distribution costs in each phase for each product between
all possible combinations of regions:

Table   DCj(PRi,REG,REG2) Distribution cost pr kg in R2
        currency for product PRi between REG and REG2 in phase j
                R1.R2   R2.R1
        P1      0.1     0.1
        P2      0.1     0.1     ;

Table   DCl(PRk,REG,REG2) Distribution cost pr kg in R2
        currency for product PRk between REG and REG2 in phase l
                R1.R2   R2.R1   R1.R10  R2.R10
        P3      0.1     0.1     0.1     0.1
        P4      0.1     0.1     0.1     0.1
        P5      0.1     0.1     0.1     0.1
        P6      0.1     0.1     0.1     0.1     ;

The time factor when distributing between regions. Used for
calculating the financial costs of distribution.

Table   Dtime (REG,REG2)  Distribution time in month between
        supplier REG and Buyer REG2
                R1      R2      R10
        R1      0       1       2
        R2      1       0       1       ;

The duty between all possible combinations of regions:
Table Dut(REG,REG2) Duty supplier(Vertical), buyer(Horisontal)
```

```
                R1      R2      R10
        R1      0.00    0.00    0.00
        R2      0.00    0.00    0.00    ;

Exchange rate for all the regions used in the verification:
Parameter       Er(REG) Exchange rate for region REG R1
                        currency used as index 1
        /       R1      1.0000
                R2      6.0000
                R10     0.0543  /;

The export value added taxes between all possible com-
binations of regions:
Table   ExVat(REG,REG2) supplier(Vertical),buyer(Horisontal)
                R1      R2      R10
        R1      0.00    0.00    0.00
        R2      0.00    0.00    0.00    ;

The prices stated for all the products in local currencies:
Table   Price(PRk,REG)  Price in local currency pr product
                        pr kg PRk in region REG
                R1      R2      R10
        P3      36      6       662.98
        P4      36      6       662.98
        P5      36      6       662.98
        P6      36      6       662.98  ;

Production costs stated for all products in all phases for
all regions in local currencies:

Table   PCi(PRi,REG) Production cost pr kg for product PRi in
                     region REG in phase i
                R1      R2
        P1      6       1
        P2      6       1       ;

Table   PCk(PRk,REG) Production cost pr kg for product PRk in
                     region REG in phase k
                R1      R2
```

```
        P3      6       1
        P4      6       1
        P5      6       1
        P6      6       1       ;
```

The level of royalty payments between all possible combinations of regions:

```
Table   Royal(REG,REG2) Royalties from sale in region REG
        (supplier vertical) to region REG2 in percentage
        of sales price(buyer horisontal)
                R1      R2      R10
        R1      0       0       0
        R2      0       0       0
        R10     0       0       0       ;
```

The average storage time when storing is needed has been estimated to 3 month:

```
Scalar Stime Estimated average storage time between time
periods in
months /3/ ;
```

The cost of storing stated in local currencies for each product and period:

```
Table   SCj(PRi,REG)    Storage cost pr kg for product PRi in
                        region REG in phase j
                R1      R2
        P1      6       1
        P2      6       1       ;

Table   SCl(PRk,REG)    Storage cost pr kg for product PRk in
                        region REG in phase l
                R1      R2
        P3      6       1
        P4      6       1
        P5      6       1
        P6      6       1       ;

Table   SCm(PRk,REG)    Storage cost pr kg for product PRk in
                        region REG in phase m
```

```
                R1      R2      R10
        P3      6       1       110.50
        P4      6       1       110.50
        P5      6       1       110.50
        P6      6       1       110.50 ;

The tax rates for the different reions:
Parameter       Tax(REG) Tax rate for region REG
        /       R1      0.0
                R2      0.0
                R10     0.0     /;

Transferprices used for internal transfer of goods in the
company stated for each product, each phase and between all
regional combinations:
Table   TPj(PRi,REG,REG2) Transferprices in sending regions
currency pr kg product PRi from region REG to REG2 in phase j
                R1.R2   R2.R1
        P1      18      3
        P2      18      3       ;

Table   TPl(PRk,REG,REG2) Transferprices in sending regions
currency pr kg product PRk from region REG to REG2 in phase l
                R1.R2   R2.R1   R1.R10  R2.R10
        P3      18      3       18      3
        P4      18      3       18      3
        P5      18      3       18      3
        P6      18      3       18      3       ;
```

Appendix I

CD contents - public CD

On the enclosed CD, please find the following files:

report.pdf
PDF-version of the final report.

report.ps
Postscript version of the final report.

Number of variables.xls
Excel file where the size of the model can be evaluated. The implemented function is a function of the size of the regional set, the product set and the number of time periods. The implemented function is described in appendix D.

Costs of Implementation.xls
The Excel file contains the cost estimates for the four step implementation plan, described in section 8.3.

Folder: Miniminitestmodel GAMS
Contains the input and output data (.lst) from the verification and validation scenarios 1-8 (see appendix H for details). Furthermore the implemented GAMS program with the miniminitestmodel is enclosed.

Folder: Finalmodel GAMS
Contains the implemented GAMS program (finalmodel.gms) and the implemented display commands (.cmd). (see appendix G for details). The

implemented data and analysis are only available on the company specific CD.

Appendix J

CD contents - the company specific CD

On the enclosed CD with case specific information please find the following files. The following are the contents of the files (fi:) and sub-folders (FO:):

Folder: Final model GAMS
Contains the implemented data in GAMS. In the subfolders are the results from the scenario analysis (as described in chapter 7) as well as the implemented final model in GAMS (The model is described in chapter 6, the program structure is described in section 6.3). The following subfolders and files are on the CD:

fi: *Finalmodel.gms:* The implemented GAMS program.
FO: *Scenario 1-8:* The output files (.out and .lst) for the individual scenarios.*
FO: *Displaycommand:* The commands to generate output data.
FO: *Data:* The implemented input data (.inp) for the model.
FO: *Scenariodata:* Case specific data, the historical production plan and not-allowed flows.

*Please notice: the Excel files refer to a specific data import path for automatic data update. When an error occurs, press *cancel import*.

Folder: scenarios output
Contains the tools used for scenario analysis. A subfolder (Sc1-Sc8) is

generated for each scenario described in chapter 7. The general contents of the subfolders is described in appendix C.1.

Folder: Implemented DATA
Contains the analysis of the master data. In the Excel files the input data for implementation is generated.

fi: *VNR PR ID:* Definition of the relations between the product numbers and product IDs.
fi: *Bom and capacity usage for implementation.xls:* Analysis of the bills of materials and the capacity utilisation levels. The bill of materials (prodmix), capacity utilisation factors (caputl), production costs (PC) and transfer prices (TP) are generated for each product.
fi: *Capacity utilisation:* The capacity limits are created based on analysis of the capacity utilisation for the historical solution.
fi: *Recovery line evaluation:* Identification of the individual recovery lines for each country and product. Used to find the factors for capacity utilisation and capacity limits for each product.
fi: *sales data for implementation:* Analysis of sales data. Used to generate demand (Dem), average prices (Price), transfer prices (Price) for each product. Historical production flow is generated.
fi: *stor, dist and div data for implementation:* Generation of storage costs (SC), distribution costs (DC) for each product.

Folder: master DATA
Contains the master data used for analysis of the products (see the implemented data folder)

fi: *Bom data for all products:* Contains the bills of materials.
fi: *Capacity data for fermentation and recovery for all products:* Contains the capacity utilisation rules for the individual production lines and products.
fi: *sales data for all products:* Contains the historical sales data.