Disruption Management in the Airline Industry – Concepts, Models and Methods

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15th April 2005

Abstract

The airline industry is notably one of the success stories with respect to the use of optimization based methods and tools in planning. Both in planning of the assignment of available aircraft to flights and in crew scheduling, these methods play a major role.

Plans are usually made several months prior to the actual day of operation. As a consequence, changes often occur in the period from the construction of the plan to the day of operation. Optimization tools play an important role also in handling these changes.

However, at the day of operation, no planning tool have been able to cope with the complexity of the re-planning given that the time span for proposing a solution is only a few minutes. Numerous suggestions for such subsystems have been put forward, but today no general tool is able to handle aircraft, crew, and passenger concurrently in a single system.

Currently, there is a gap between the reality faced in operations control and the decision support offered by the commercial it-systems targeting the recovery process. Though substantial achievements have been made with respect to solution methods, and hardware has become much more powerful, even the most advanced prototype systems for integrated recovery have severe limitations.

The current review accounts for the majority of subsystems mentioned in the literature in terms of the sub-problem addressed and the method used in each particular contribution. For each proposed system, also the computational experiments supporting the practical usability of the system is reported.

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1 Introduction

First we describe the basic planning processes of the larger modern airline companies, setting the scene for the key problem of disruption management shortly before or at the day-of-operation..

1.1 The planning process

Prior to the departure of any aircraft, a sequential planning approach normally takes place: First, the flight schedule is determined based on forecasts of passenger demand, available slots at the airports, and other relevant information. Thereafter, specific types of aircraft are assigned to the individual flights of the schedule providing anonymous rotations for flights in each fleet - this process is termed fleet assignment and aircraft routing. The different rotations must respect various types of constraints as e.g. maintenance and night curfews. In the following crewing phase, flight crew and cabin crew are assigned to all flights based on the schedule and the fleet assignment. The planning of flight and cabin crew is slightly different. For both crew groups individual flights are grouped to form pairings. Each pairing starts and ends at the same crew base. Note that these pairings are anonymous. Afterwards, pairings are grouped to form rosters for a given person. In

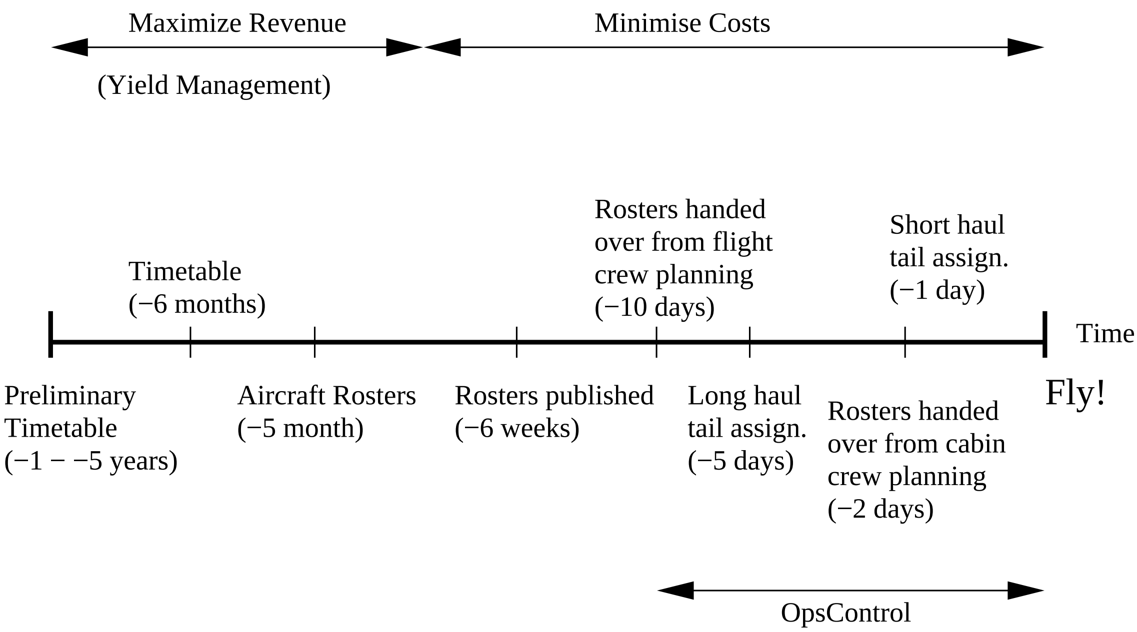

Figure 1: The time-line for the daily operation of an airline.

bidline rostering occasionally used for flight crew scheduling the pairings are grouped together to form anonymous rosters. The crew members then bid for these anonymous rosters, where usually senior crew members are favoured when assigning rosters to crew. Rosters are typically lines of work for 14 days or 1 month. Finally, physical aircraft from a given fleet are assigned to flights in the tail assignment process. The complete process is illustrated in Figure 1.

Constructing such a plan is in each case complicated as for aircraft maintenance rules have to be taken into account, the right capacity must be at the right place at the right time, and the characteristics of each individual airport have to be respected. For crew, there are regulations on flying time, off-time etc. based on international and national rules, but also regulations originating in agreements with unions, local to each airline.

After the planning phase follows the tracking phase, where changes in plans due to e.g. crew sickness, aircraft breakdowns, and changes in passenger forecasts are taken into account. This phase normally resides with the planning department.

The plans for aircraft assignments, crew assignments and maintenance of the flight schedule is handed over from the planning department to the operations control centre (OCC) a few days days ahead of the day of operation. The deadlines are different for different resources. Short-haul plans are usually handed over one day ahead of the operation date, while long-haul information is handed over three to five days before.

As the plan is handed over, it becomes the responsibility of OCC to maintain all resources so that the flight plan seen as an integrated entity is feasible. Events like crew sickness and late flight arrivals have to be handled. Furthermore, not only the immediately affected flights but also knock-on effects on other parts of the schedule can cause serious problems. The common practice in the industry of planning flight crew, cabin crew and aircraft separately reinforces the problem.

Generally, a disrupted situation (often just denoted a disruption) is a state during the execution of the current operation, where the deviation from the plan is sufficiently large to impose a substantial change. This is not a very precise definition, however, it captures the important point that a disruption is not necessarily the result of one particular event.

To produce recovery plans is a complex task since many resources (crew, aircraft, passengers, slots, catering, cargo etc.) have to be re-planned. When a disruption occurs on the day of operation, large airlines usually react by solving the problem in a sequential fashion with respect to the problem components: aircraft, crew, ground operations, and passengers. Infeasibilities regarding aircraft are first resolved, then crewing problems are addressed, ground problems like stands etc. are tackled, and finally the impact on passengers is evaluated. Sometimes, the process is iterated with all stakeholders until a feasible plan for recovery is found and can be implemented. In most airlines, the controllers performing the recovery have little IT-based decision support to help construct high-quality recovery options. Often, the controllers are content with producing only one viable plan of action, as it is a time consuming and complex task to build a recovery plan. Furthermore the controllers have little help in estimating the quality of the recovery action they are about to implement.

One generally available recovery option is cancellation of single flights or round trips between two destinations. From the resourcing perspective, cancellation is ideal - it requires no extra resources, it may even result in the creation of free resources, and little re-planning is required. However, from the passenger point-of-view it is the worst option, since a group of customers will not receive what they paid for. Indeed, determining the quality of a recovery option is difficult. The objective function is composed of several conflicting and non-quantifiable goals as e.g. minimizing the number of passenger delay minutes, returning to the plan as quickly as possible, and at the same time minimizing the cost of the recovery operation.

The current paper reviews the disruption management tools and recovery tools proposed in the OR literature. The terminology and general concepts regarding disruption management are assumed to be known, but are for convenience included in the Appendix. Part of the terminology was developed in the R&D project DESCARTES supported by the European Commission under the IST program in the 5th Framework programme.

Tools for planning, recovery and disruption management are in most cases based on a network representation describing how flights can be sequenced either in a rotation or in a crew pairing. To establish a common base for the presentation of the models and results in the succeeding sections, Section 2 presents the commonly used network representations and illustrates their use in modelling. Section 3 gives examples of prototypical papers on aircraft and crew planning, and Section 4 describes aircraft, crew, and integrated recovery as proposed in the literature. Section 5 briefly discusses robustness in relation to disruption management. Finally, Section 6 contains discussions of future prospects for disruption management systems in the airline industry. An appendix contains a description of the concepts and the terminology related to disruption management in the airline industry.

2 Network Models for Airline Optimization Problems

In this section we review standard network optimization models for airline planning problems. Though the details may vary, the networks used in these models are very similar. We first review the two basic networks used in planning, and then a network specifically designed to handle the recovery situation.

Based on the network descriptions, we then sketch prototypical models for fleet assignment and scheduling, for crew scheduling, and for disruption management. For simplicity, we consider a set of flight legs of a single fleet of aircraft in a given planning period.

2.1 Networks for Airline Optimization Models

The idea of the *connection network* or *time-space network* is to represent the possibilities for building rosters for aircraft (or crew). The network is an Activity-On-Node network

– the flights correspond to nodes in the network. It consists of a set of nodes, N, one for each flight leg. A flight leg is given by its origin, destination, departure time and date, and arrival time and date. The node i representing the flight leg l_i is connected by a directed edge (i, j) to the node j representing the flight leg l_j if it is feasible with respect to turn-around-times and airport to fly l_j immediately after l_i using the same aircraft. In addition, there are nodes indicating the position of each of the aircraft in the fleet both at the beginning and in the end of the planning horizon. These nodes are connected to those leg nodes which are feasible as first resp. last legs in the planning period. A path in the network now corresponds to a sequence of flights feasible as part of a rotation. Schedule information is not represented explicitly in the network, but used when building this. Maintenance restrictions are easily incorporated through the concept of a maintenance feasible path, which is a path providing sufficient extra time with the required intervals at a node corresponding to a station, where maintenance can take place. Note that the number of feasible paths may be very large - it grows exponentially with the planning time horizon.

The connection network resembles the networks seen in vehicle routing problems. The flights correspond to customers, the aircraft to vehicles, and the edges of the network describes which customers are feasible as successors of a given customer on a route.

In Table 1, a small sample of flights connecting Copenhagen (CPH), Oslo (OSL), Aarhus (AAR), and Warsaw (WAV) are given. Assume that the turn-around-time for an aircraft is 40 minutes in CPH and OSL and 20 minutes in AAR and WAV. The corresponding connection network is given in Figure 2.

Aircraft	**Flight**	**Origin**	**Destination**	**Departure**	**Arrival**	**Flight time**
AC 1	11	OSL	CPH	1410	1520	1:10
	12	CPH	AAR	1600	1640	0:40
	13	AAR	CPH	1730	1810	0:40
	14	CPH	OSL	1850	2000	1:10
AC 2	21	CPH	WAV	1430	1530	1:00
	22	WAV	CPH	1550	1650	1:00
	23	CPH	WAV	1730	1830	1:00
	24	WAV	CPH	1850	1950	1:00
AC 3	31	AAR	OSL	1500	1620	1:20
	32	OSL	AAR	1700	1820	1:20

Table 1: A sample schedule for Sample Air

One problem with the connection network is that it is difficult to view as a representation in time and space of the possible schedules. The basic idea of *time-line networks* is

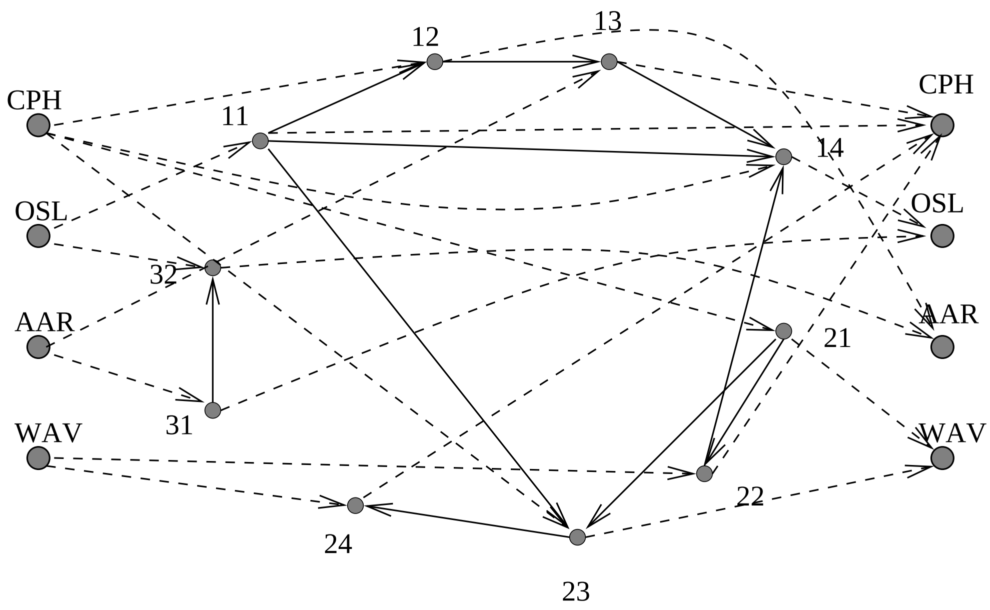

Figure 2: The sample schedule shown as a connection network. The feasible rotation for AC 1 shown in table 1 corresponds to the path OSL-11-12-13-14-OSL.

to represent the possible schedules in a natural way in a network. A time-line network has a node for each event, an event being an arrival or a departure of an aircraft at a particular station. Each station corresponds to a line to be thought of as the "time line" of that station, and all event-nodes for that station is located on the time line at the corresponding points in time. The length of the time line corresponds to the planning horizon. The edges of the network connect event-nodes corresponding to events that may follow each other in a sequence of events for one particular aircraft. Edges connecting nodes on the time lines for different stations correspond to flights feasible with respect to flying time, edges connecting nodes on the time line for a particular station correspond to grounded aircraft. Maintenance time is normally included in the flying time, so if maintenance is performed at the arriving station, the event time for the arrival is set to the true arrival time plus the maintenance time. In the same way as for the connection network, it is possible to describe a possible part of a rotation by a path in the network. However, time-line networks are Activity-on-Edge networks: Edges correspond to activities of an aircraft, and schedule information is represented explicitly by the event nodes of the path. The time-line network for Sample Air is shown in Figure 3.

To cope with disruptions, a third type of network called *time-band network* has been proposed. The basic idea is to represent the schedule in a time-line network fashion leaving out all arcs except those corresponding to the flights of the schedule. No ground

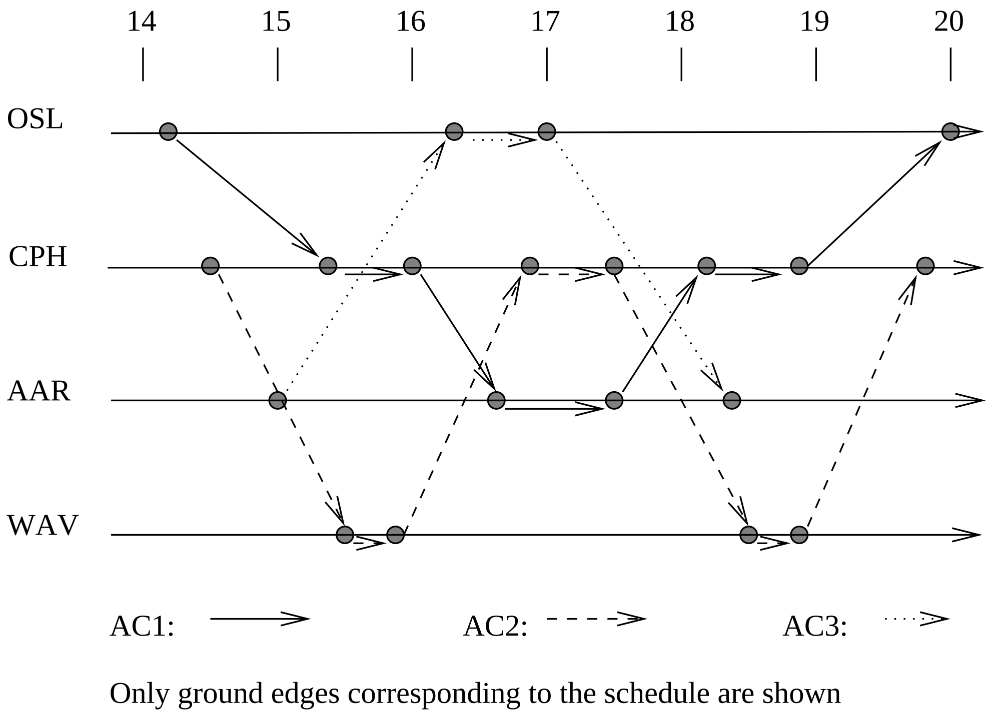

Figure 3: The sample schedule shown as a time-line network. The feasible rotation for AC 1 shown in table 1 corresponds to the AC1 path.

arcs are included - the arrival node i of a flight is simply joined with the departure node j of the next flight of the aircraft arriving in node i. The location of the node with respect to the time axis is that of i plus the turn-around time, i.e. the *availability time* of the aircraft, cf. Figure 4.

The time-band network is constructed in case of a disruption e.g. by an aircraft being out of service, that is, the network is not constructed a priori, but dynamically as a disruption occurs. Activities within discrete time intervals denoted time bands are aggregated at each station. The network has one node per station for each time band, called station-time nodes. In addition, there are station-sink nodes representing the end of the recovery period. The edges in the network are edges representing the scheduled flights. A scheduled A-to-B flight has an emanating edge for each A-time node, in which there is an aircraft available, and for which the flight can be flown within the recovery period. Each of these edges will end in the B-time node corresponding to the time where the aircraft becomes available at B. When drawn in a time-band figure, the edges appear as parallel edges from the A-station time line to the B-station time line. Finally, there are edges connecting each station-time node with the station-sink node for the relevant station. A re-schedule now corresponds to a flow in the network where edges of the original schedule carrying no flow correspond to canceled flights, and where re-timings correspond to flow in the "new" flight edges, indicating that flights are flown at a later time than scheduled. Each station-time node has a time label with the availability time of the first available aircraft in the corresponding time band.

The time-band network model for the sample schedule with AC 2 out of service from 14:00 until 21:00 due to maintenance and with time bands of 30 minutes is shown in Figure 5. The network is constructed in a stepwise fashion to avoid representing time-station nodes with no aircraft availability.

Two flows in this network, one starting from OSL, one starting from AAR, one ending in OSL, and one ending in AAR, determine a way to use the aircraft resources available. As an example, the path

OSL:1400-1429 → *CPH:1600-1629* → *WAV:1700-1729* → *CPH:1900-1929*
→ *OSL:2030-2059* → *OSL:sink*

represents a non-delayed flight 11, followed by a 1 hour delayed service to WAV on flight 21, then flight 22 (delayed 1 hour and 30 minutes) and then finally flight 14 delayed 10 minutes.

2.2 Optimization Models based on Networks

In the following, we describe three simplified prototype models from the literature illustrating the use of the networks just described in modelling airline optimization problems.

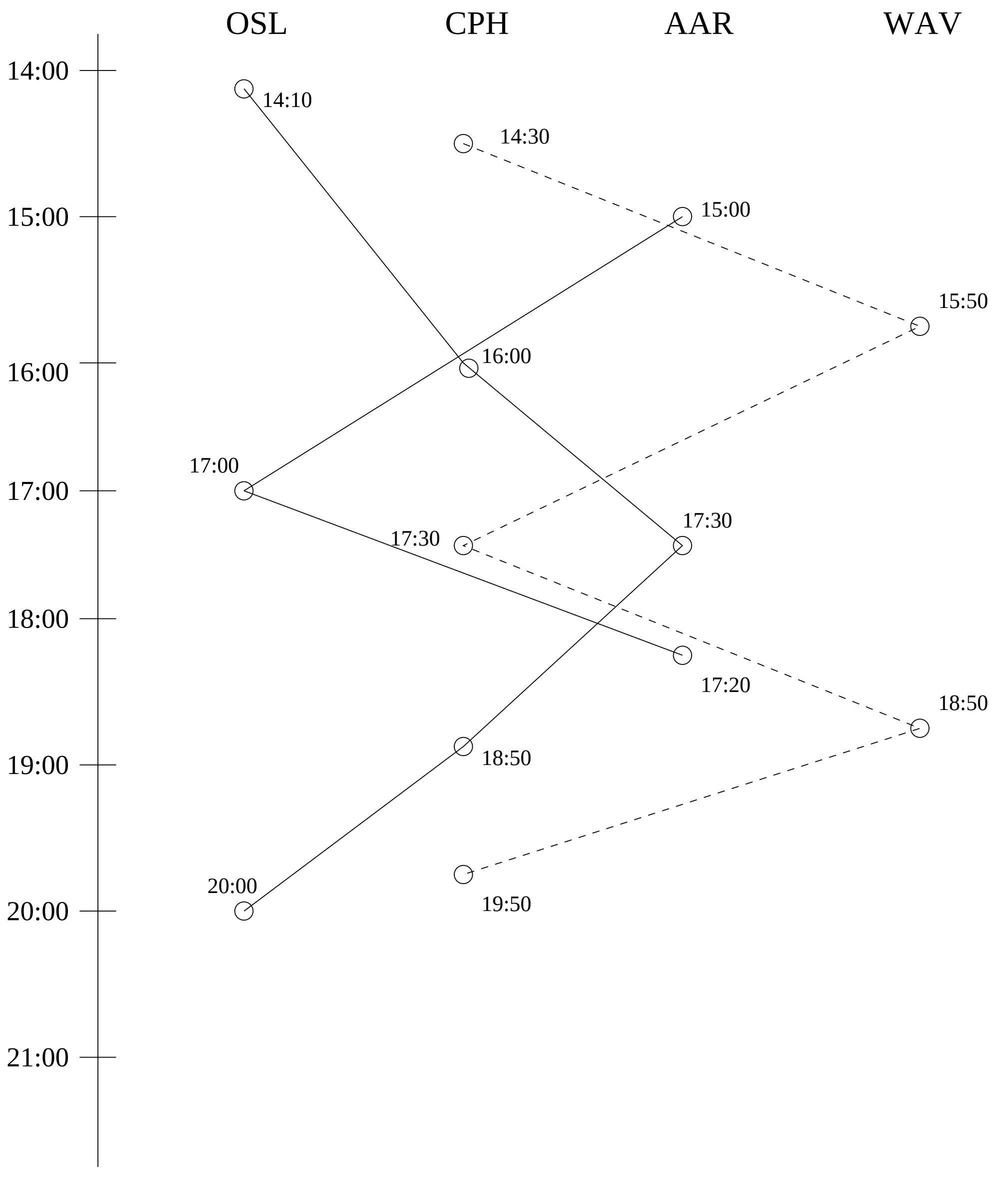

Figure 4: The sample schedule shown in a time-line network without ground edges.

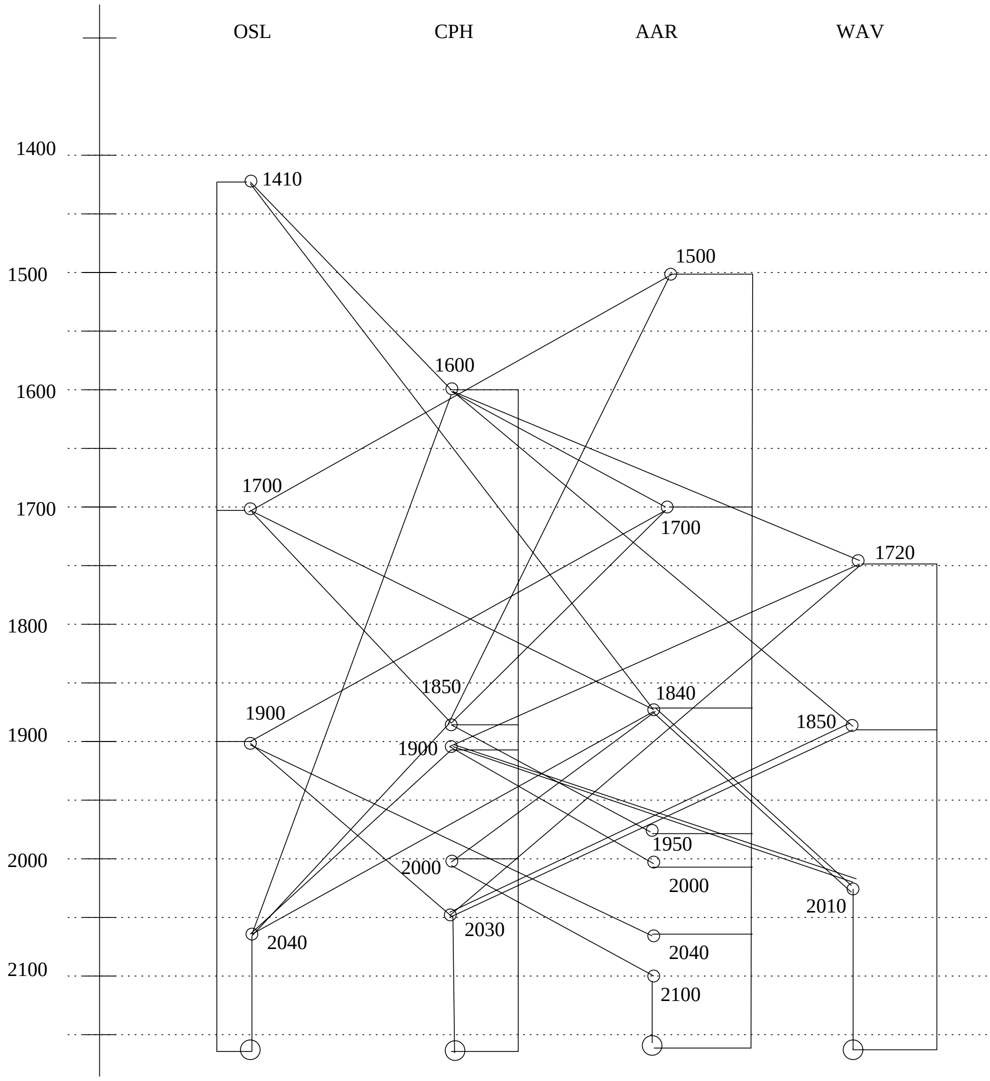

Figure 5: The Time-Band Network of the sample time schedule

2.2.1 Aircraft Fleeting and Routing with Connection Networks

The model described below can be found in "Benders Decomposition for Simultaneous Aircraft Routing and Crew Scheduling" [16] by Cordeau, Stojkoviĉ, Soumis, and Desrosiers. Assume that the fleeting problem has been solved, so a particular fleet has been assigned to each flight. For a given fleet consider the problem of assigning aircraft to flights over a fixed time horizon while respecting maintenance requirements. To keep the model simple we do not include considerations on the connectivity of the rotations, i.e. that the paths representing lines of work for the aircraft should form a connected network, if origin and destination nodes representing the same station are joined.

The set of available aircraft is called F, and for each aircraft $f \in F$, an origin o^f and a destination d^f relative to the planning horizon is given. The set of nodes $N^f = N \cup \{o^f, d^f\}$ consists of the flights, the origins and destinations. There are edges from each origin node to flights feasible as first flights for an aircraft located at the origin node, and edges into destination nodes from flights feasible as last flights with respect to the origin. Furthermore, the set Ω^f denotes the set of feasible paths between o^f and o^d in the network. If maintenance is to be taken into account, only maintenance feasible paths are considered. The relations between the flights and the paths are given by binary parameters a^i_ω taking the value 1 iff flight i is on path ω.

To determine which aircraft are to fly which flights, we define binary decision variables x_ω taking on the value 1 iff the flights on the path given by ω is flown by the aircraft determined by the origin node of the path. The constraints of the problem are that each flight must be in one of the selected paths, and that one path must be chosen for each aircraft. The routing problem now becomes:

$$\begin{array}{lll}
\text{minimize} & \sum_{f \in F} \sum_{\omega \in \Omega^f} c_\omega x_\omega & \\
\text{subject to} & \sum_{f \in F} \sum_{\omega \in \Omega^f} a^i_\omega x_\omega = 1 & i \in N \\
& \sum_{\omega \in \Omega^f} x_\omega = 1 & f \in F \\
& x_\omega \in \{0, 1\} & f \in F; \omega \in \Omega^f
\end{array}$$

Note the similarity with models for vehicle routing - the flights can be seen as customers and the aircraft as vehicles serving the customers. The connection network describes the possible routes of the vehicles in terms of feasible successor relations between customers. Hence, an immediate solution approach is Branch-and-Price, i.e. LP-based Branch-and-Bound combined with column generation, where each column represents a feasible path, cf. [9]. Further comments on the model and experimental results are given in Section 4.3.

2.2.2 Aircraft Fleeting and Routing with Time Line Networks

The model described in the following is a simplified version of a model appearing in “Flight String Models for Aircraft Fleeting and Routing” [7] by Barnhart, Boland, Clarke, Johnson, and Nemhauser. Consider the situation, in which the fleeting problem has not yet been solved, and let K be the set of fleets. One possibility is to use the time-line network model. Again, a path in the network is a representation of an aircraft rotation, here called a string. Assume now that maintenance requirements are taken into account: Feasible strings start at some maintenance station, end at a possibly different maintenance station, and has sufficient time for maintenance to be performed whenever necessary during the rotation. An augmented string is a string with the necessary maintenance time attached to the end of the string. The set of strings is denoted S, and for flight i, S_i^+, and S_i^- denote the set of augmented strings starting with the edge of flight i, resp. ending with i and maintenance.

The model has a binary decision variable x_s^k for each string $s \in S$ and for each fleet k, and parameters a_{is} describing the relationship between flight i and path s. The purpose of the model is to assign fleets to flights in a maintenance feasible fashion. As in the aircraft rotation model, a set of constraints ensuring that each flight is assigned to exactly one fleet is necessary:

$$\begin{aligned} \sum_{k \in K} \sum_{s \in S} a_{is} x_s^k &= 1 \quad i \in F \\ x_s \in \{0, 1\} & \qquad\quad s \in S \end{aligned}$$

In order to account for balance constraints in terms of number of aircraft at maintenance stations, given that strings start and end at maintenance stations and that not too many aircraft of a given fleet can be used simultaneously, count variables y_j^k are defined for each ground edge of the model including ground edges with maintenance. The value of y_j^k is the number of aircraft from fleet k on the ground at the station and time interval corresponding to the ground edge j.

Consider now a specific flight i and fleet k. If an augmented string s starting in i and using fleet k is chosen for the solution, then the number of aircraft from fleet k on the ground just before take-off of flight i must be one larger than the corresponding number just after take-off. This can be expressed as follows:

$$\sum_{s \in S_i^+} x_s^k - y^k_{(e_{i,d}^{-,k}, e_{i,d}^k)} + y^k_{(e_{i,d}^k, e_{i,d}^{+,k})} = 0 \quad i \in F, k \in K$$

Here, the indices e, i, d, and $-k$ resp. $+k$ of the ground variables indicate the last fleet-k event (e) at the relevant station before ("-") resp. after ("+") the departure (d) of flight i. Likewise, if i arrives at a maintenance station as the last station, we need a balance constraint reflecting this:

$$-\sum_{s \in S_i^-} x_s^k - y^k_{(e_{i,a}^{-,k}, e_{i,a}^k)} + y^k_{(e_{i,a}^k, e_{i,a}^{+,k})} = 0 \quad i \in F, k \in K$$

Finally, the complete model also contains constraints describing that not more than the available number of aircraft in fleet k is used simultaneously. These make use of a so-called count time, which is a point in time where all aircraft are counted, both the grounded ones and those in the air:

$$\sum_{s \in S_i} r_s^k x_s^k + \sum_{j \in G_k} p_j^k y_j^k \quad \leq N^k \quad k \in K$$

Here, r_s^k resp. p_j^k counts the number of times string s resp. ground arc j cross the count time, and N^k is the number of aircraft in fleet k.

2.2.3 Disruption Management with Time-Band Networks

The time-band network model described below is from “Optimizing aircraft routings in response to groundings and delays” [6] from 2001 by Bard, Yu, and Argüello. The model is as indicated previously used in connection with disruptions, for example when an aircraft becomes unavailable. Formally, the model becomes an integral minimum cost flow network model with constraints ensuring flow balance, and with indicator variables for cancellation of aircraft. The model has binary decision variables x_{ij}^k representing flow from station-time node i to station-time node j for flight k, and y_k representing the possible cancellation of flight k. One set of constraints of the model ensures that each flight is either canceled or flown:

$$\sum_{i \in P(k)} \sum_{j \in H(k,i)} x_{ij}^k + y_k \quad = 1 \quad k \in F$$

Here, $P(k)$ is the set of possible origin station-time nodes for flight k, and $H(k,i)$ the set of destination station-time nodes for flight k if starting at node i.

The flow balance at each station-time node i is modeled as follows:

$$\sum_{k \in G(i)} \sum_{j \in H(k,i)} x_{ij}^k + z_i - \sum_{k \in L(i)} \sum_{j \in M(k,i)} x_{ji}^k \quad = a_i \quad i \in I$$

Here, z_i is the number of aircraft from station-time node i routed directly to the corresponding station-sink node. $G(i)$ denotes the set of flights originating at node i, $L(i)$ the set flights terminating at i, $M(k,i)$ the set of origin station-time nodes for flight k ending in node i, and a_i the number of aircraft becoming available at node i at time 0. The corresponding flow balance constraint for a station-sink node j is as follows:

$$\sum_{k \in L(i)} \sum_{j \in M(k,i)} x_{ji}^k + \sum_{j \in Q(i)} z_j \quad = h_i \quad i \in J$$

The objective function in the model reflects the costs of delay and cancellation:

$$min \sum_{k \in F} \sum_{i \in P(k)} \sum_{j \in H(k,i)} d_{ij}^k x_{ij}^k + \sum_{k \in F} c_k y_k$$

The model can be solved to optimality using integer programming packages, or it can be handled using heuristics, if solution time is a critical factor.

2.3 Crew Scheduling

Crew Scheduling is the task of assigning a group of people to a set of tasks. Beside airlines similar crew scheduling problems appear in numerous transport contexts eg. in bus and rail transit, road and rail freight transport.

On passenger aircraft there are two groups of crew; flight crew flying the aircraft and cabin crew servicing the passengers. Each of the crew groups are further divided according to rank. Crew will typically get a plan of work for a two- or four-week period. The task of assigning crews to itineraries is generally a complex task. Therefore it is split into two stages: crew pairing and crew assignment (also known as crew rostering). The planning process usually takes place 2-6 weeks before the flights are operated.

In the crew pairing problem pairings are constructed. A pairing is a sequence of flights starting and ending at the base of the crew. The pairing is in this stage not assigned to a person, that is, it is a piece of work for an anonymous person. A pairing consists of flight legs where the crew member is working, and deadheads. Legs are grouped into duty periods (equal to a working day) which are separated by overnight stops. A schedule is sometimes referred to as a line of work (LoW).

In practice pairings for short and medium-haul problems may consist of up to 4 duty periods, while long-haul problems often result in longer duty periods. A legal pairing must satisfy a multitude of rules, partly governmental regulations, partly as collective agreements.

Based on the pairings the crew rostering problem or crew assignment problem assigns pairings to named persons. Here, the objective is to produce legal plans covering all pairings and in addition also incorporating vacation, training etc.

Basically, the crew pairing and the crew assignment models are Set Partitioning and Set Covering problems with one constraint for each task to be performed. In the crew pairing problem the task is a flight to be covered and in the crew assignment problem the task is a pairing/other work to be covered.

Crew pairing models are typically formulated as Set Partitioning problems. Here we want to find a minimum cost subset of the feasible pairings such that every flight is covered by exactly one selected pairing.

Let F be the set of flights to be covered and P the set of all feasible pairings. Decision variable y_p is equal to 1 iff pairing p is included in the solution, and 0 otherwise. The relation between pairing p and flight i is given by a_{ip}, which is 1 if p contains i and 0 otherwise. The cost of a pairing is denoted c_p and includes allowances, hotel and meal costs, ground transport costs and paid duty hours.

$$\begin{array}{llcll} \min & \sum_{p \in P} c_p y_p & & & \\ \text{subject to} & \sum_{p \in P} a_{ip} y_p & = & 1 & i \in F \\ & y_p \in \{0,1\} & & & p \in P \end{array}$$

The crew pairing problem is often solved in three phases: daily, weekly exceptions, and transition.

The daily problem is the problem most often discussed in the academic literature. The daily problem only considers flights that are flown at least 4 days a week. These flights are treated as if they were flown on a daily basis. The solutions are in the vast majority of cases only of academic importance e.g. for European and international traffic the solution is not directly applicable, mainly because of the substantial number of "irregularities" just before and during the weekend.

Therefore the weekly exceptions problem builds pairings by considering flights flown less that 4 days a week. Flights are associated to a specific day. At this phase special flights, charters etc. can be incorporated. The exceptions problem has been subject to less research than other areas of crew scheduling. One reference is [21].

Combining the solution of the daily problem with the weekly exceptions result in a solution that cover all flights in the weekly schedule exactly once.

Finally it is sometime necessary to solve the transition problem that appears when moving from one schedule to a new one. Here pairings for the small changeover period are constructed.

In order to solve the pairing problem as stated above one possibility is to construct all legal pairings. The challenge is that the number of legal pairings can be extremely large, typically varying from 500,000 for a small airline to billions for large airlines.

Generation of pairings is done using one of the two network representations presented earlier: The flight network (mainly used for domestic and short-haul operations) and the duty time-line network (mainly appropriate for international and long-haul operations).

A legal pairing is represented by a path from the source to the sink in the network, where these are usually crew bases. Note that many paths from source to sink do not represent legal pairings. The network guarantees that connected flights match wrt. arrival and departure airport and that turn-around times are respected, but it does not prevent the path from violating rules like maximum flying hours etc.

In the other type of network, the duty period network, it is possible to build the duty period rules into the network, resulting in an extended set of arcs. In the duty period network nodes represent the departure and arrival of each duty period. Arcs in the network represent possible duty periods as well as legal connections between the duty periods. As in the flight network we complete the representation with a source and a sink.

In the duty period network numerous rules are satisfied by all source-sink paths, more than in the flight network. However, there will still be some rules that are not enforced by the network structure. These rules must be checked for each possible path in order to ensure legality.

Each flight path from source to sink that fulfills the constraints define a legal pairing. For smaller problems all legal pairings can be generated a priori. For larger problems, a limited a priori generation can be used as a heuristic. Here, only "good" pairings are generated. For example, instead of investigating every possible extension of the current path from a given node one may enforce that the crew has to leave on one of the four immediately succeeding connections. Optimality is not guaranteed, but a sound solution results. Recent approaches generate the pairings as they are needed in a column generation process. The problem of generating the pairings then becomes a variant of the shortest path problem.

Crew assignment problems are solved for each crew type (ie. captain, first officer etc.). The constraints of the crew assignment problem is:

- Each crew member should be assigned to exactly one work schedule. In case the airline is not required to use all crew members, a crew member might be assigned an empty schedule containing no work.

- Each pairing in the crew pairing solution is contained in the appropriate number of selected schedules (depending on how many crew members of each type are required).

Using the notation of [8], let K be the set of crew members of a given type and let P be the set of dated pairings to be covered. For each crew member k the set of feasible work schedules is denoted S^k. n_p is the minimum number of crew members needed to cover pairing p and γ_p^s is 1 in pairing p is included in schedule s and 0 otherwise. c_s^k is the cost of schedule s for crew k. Decision variables are x_s^k, which are 1 if schedule $s \in S^k$ is assigned to crew $k \in K$ and 0 otherwise. We can now formulate the crew assignment problem:

$$\begin{array}{llccl} \min & \sum_{k\in K}\sum_{s\in S^k} c_s^k x_s^k & & & \\ \text{subjec to} & \sum_{k\in K}\sum_{s\in S^k} \gamma_p^s x_s^k & \geq & n_p & p \in P \\ & \sum_{s\in S^k} x_s^k & = & 1 & k \in K \\ & x_s^k \in \{0,1\} & & & s \in S^k, k \in K \end{array}$$

Basically this problem is solved in the same way as the pairing problem. Either we a priori generate the rosters by constructing a path from source to sink in a network of pairings. This a priori phase can be done optimally or heuristically. Alternatively, we use a column generation approach and only generate the rosters as they are needed. The network representation is similar to the pairing problem, but instead of defining a path of flights as in the pairing problem the path consists of pairings.

3 Planning

To set the scene for the review of papers addressing recovery and disruption management, we review selected important papers regarding planning in the airline industry in this section. These papers discuss fleeting and routing, crew scheduling, and an integration of both of these.

3.1 Aircraft Fleeting and Routing

The issue addressed in “The aircraft rotation problem” [13] from 1997 by Clarke, Johnson, Nemhauser, and Zhu is to produce optimal rotations given the value of letting specific flights follow others (the so-called through value). The problem is initially described by a time-line network, where ground arcs have cost according to their through value. The model used in the solution procedure is, however, based on a connection network in order to allow an easy way of expressing that a solution must connect all flights into a tour, i.e. that broken rotations are not feasible. The problem is solved using Lagrangean relaxation where the relaxed constraints are subtour elimination constraints and constraints ensuring maintenance feasibility. The Lagrangean dual is solved with subgradient optimization. Test data for the method are from a major US carrier and consist of 11 instances ranging from 43 flights to 3818 flights. Two types of problems are solved: through value problems, in which the solutions may not be maintenance feasible, and rotation problems, which in case the through value solution is not feasible, produces a service feasible solution. For through value problems, the method gives provably optimal solutions for all instances in less than a minute. For rotation problems, the solutions lies within 5% of the solution of the through value problem. Here, the solution times are substantial, although less than an hour for the test instances.

The problem studied in “Flight String models for Aircraft Fleeting and Routing” [7] from 1998 by Barnhart, Boland, Clarke, Johnson, and Nemhauser is a combined fleeting and routing problem for aircraft. Models based on time-line networks as well as models based on connection networks are used. The solution technique is based on generating strings of flights respecting maintenance conditions. These strings are used in a Branch-and-Price framework, where the general model is time-line based, but where the column generation step is tailored and uses connection networks. The algorithm is tested on data provided by a long-haul airline with a planning horizon of a week, and with 1124 flights visiting 40 cities, and 9 fleets with 89 aircraft. The solutions are provably within 1 % of optimum and are found in appr. 5 hours. Also, a routing problem with connectivity constraints and through values is studied - this is modelled using connection networks and solved with a Branch-and-Cut-and-Price algorithm. The test data is 10 data sets from short-haul operations. All but one instance are solved to optimality, and the solution times range from a few seconds to 10 hours.

3.2 Crew Scheduling

Crew scheduling is generally acknowledged to be an extremely complicated task. In "Solving Airline Crew Scheduling Problems by Branch-and-Cut" [19] from 1993 by Hoffman and Padberg, the authors describe a Branch-and-Cut optimizer for solving both pure Set Partitioning Problems originating from crew scheduling and crew scheduling problems, which include other types of constraints specifying e.g. even distribution of time away from home base. The optimizer takes as input a very large set of columns each corresponding to a feasible crew rotation (roster). The resulting huge Set Partitioning problem is first reduced using simple, but efficient column and row reduction techniques. Then, an LP-based heuristic is applied in order to get a tight upper bound before starting the Branch-and-Cut module. The Branch-and-Cut module consists of an LP-based Branch-and-Bound combined with polyhedral cuts derived for the Set Packing polytope (e.g. clique cuts and odd-cycle cuts). The cuts are generated efficiently on-the-fly by specially tailored procedures. All components of the optimizer are described in detail, and extensive computational results are reported. These show that for many real-life problems, the combination of the tight upper bound found by the heuristic and the cut generation solves the problem without branching. In most cases (including those where branching appears), the solution time is small (less than 100 seconds) both for pure Set Partition problems and problems with base constraints. The authors nevertheless point out that a few of the test problems require much more computational effort - this is in line with the fact that the problem addressed is NP-hard.

3.3 Integrated Solutions

The complexity of integrated planning of resources such as crew and aircraft is orders of magnitude harder than separate planning of each resource. The topic is from an airline perspective extremely interesting, and the paper "Benders Decomposition for Simultaneous Aircraft Routing and Crew Scheduling" [16] from 2001 by Cordeau, Stojkoviȁ, Soumis, and Desrosiers is the first to address the issue. The research reported can be considered to be the first step to evaluate the technical difficulties and the potential benefits from a truly integrated planning system for crew and aircraft.

The basic idea is to construct two connection networks, one for the fleet, and one for the set of crew for the fleet. The underlying mathematical program is a mixed integer linear programming model, which contains constraints as those indicated in section 2 for each of the resources as well as constraints combining the resources - notably that crew does not change aircraft when connection time does not allow this. As noted in section 2, the decision variables of the model correspond to feasible paths for aircraft and feasible rosters for crew. Even for one resource, such a model presents a problem regarding solution. Therefore, the model is decomposed using Bender's decomposition. Each feasible aircraft assignment gives rise to a primal subproblem, which is the LP-relaxation of the

MILP problem for crew given the fixed aircraft assignment. This problem is solved using column generation, and the dual information is fed back into the Benders master problem, which is the LP-relaxation of the aircraft assignment problem with additional constraints. These are added incrementally as usual in the Bender's decomposition approach. The procedure alternates between solving master and subproblems until a specified stopping criterion is achieved. The integer version of Bender's master problem is solved with a heuristic, which implies that suboptimal solutions are found. Finally, an integer crew solution is found based on the solution to the primal subproblem.

The method is tested on the data of a weekly schedule supplied by a Canadian airline with 3205 short-haul legs. After fleet assignment, three fleets cover 2950 of these, and three problems hence result. For each of these, after determining the initial positions and the routing of aircraft, a crew scheduling problem is then solved to determine the crewing of the proposed routing. Each aircraft is assigned exactly one crew. Based on the initial positioning of the aircraft and crew, the integrated problem is then solved both using a direct approach without decomposition, and the described decomposition approach. Finally, the solutions obtained from the integrated planning approach are compared with those available from the traditional sequential planning performed. In general, the results are promising with savings between 5% and 10%. The solution times reported are though measured in hours not prohibitive given the planning horizon. The paper demonstrates the potential of integrated solution techniques, but it also implicitly highlights the complexity - the proposed solution corresponds to that crew are rostered in teams rather that individually.

The paper "Integrated Airline Planning" [35] by Sandhu and Klabjan formulates an integrated model covering the tactical planning of fleet assignment, aircraft routing, and crew pairing. The model is built on a time-line network representation of the problem, integrating de-facto standard models for fleeting and routing, and pairing. Two solution methods for this model are then compared with respect to efficiency and solution quality. The test problems used are problems from a major US carrier with a heavy hub-and-spoke structure, 5 crew bases, and eight hubs. The hardware used is a cluster of 27 900 MHz PCs. One solution method is based on Lagrangean relaxation and column generation, the other on integrality relaxation and Bender's decomposition.

Both solution approaches for the integrated model are computationally heavy: The running time for the most difficult problem is appr. 30 hours running all 27 processors in parallel. On the other hand, both approaches show a substantial benefit in terms of increased profit. When compared to the traditional stepwise planning approach, the yearly increase in profit for the largest problem in the test set is appr. 50 million USD. The paper documents the potential of integrated planning solutions, however, the computational resources required to reach the goal are non-standard.

4 Disruption Management

In this section we comment on most of the papers published on recovery and disruption management over the last 15 years. For each paper we comment on the problem addressed, the type of model used, the solution techniques discussed, and the computational experiences reported (including to some extent details on the computational equipment). The section is subdivided by resources (aircraft, crew, ...) and for each resource, the papers appear in chronological order).

In addition to the literature reviewed in this paper the conferences of the AGIFORS organisation often feature presentations within the area of airline disruption management. As contributions from these conferences at best are available in the form of presentation slides they are not considered in this paper. AGIFORS is the Airline Group of the International Federation of Operational Research Societies. Further information can be found at www.agifors.org.

The following subsections reviewing published papers are organized according to the resources (aircraft, crew, passengers etc.), which form the goal of the recovery procedures. However, a few papers with a more general approach deserve mentioning.

Rakshit et al. provide interesting insights to the potential savings of a decision support system in the paper entitled "System Operations Advisor: A Real-Time Decision Support System for Managing Airline Operations at United Airlines" [32]. The decision support system was implemented in 1992 and its impact convinced United Airlines of the need to develop other decision support systems for managing daily operations. The first version of the system were able to swap aircraft between flights and to propose re-timings. Later versions also considered cancellations (see the description of the work by Jarrah et al. [20]on page 22).

In "Irregular Airline Operations: A Review of the State-of-the-practice in Airline Operations Control Centers" from 1998 [15] Clarke provides an overview of the state-of-the-practice in Operations Control Centers (OCC) in the airline industry in the aftermath of irregular operations. The overview is based on field studies to several airlines. Clarke provides an extensive review of the literature within airline disruption management. Finally, Clarke propose a decision framework that addresses how airlines can re-assign aircraft to scheduled flights after a disruptive situation.

The paper "How Airlines and Airports Recover from Schedule Perturbations: A Survey" [18] by Filar et al. describes techniques that enhances utilization of airport capacities. In addition, methods that limit damage or provide recovery in disruptive situations are reviewed. The paper describes methods involving the Traffic Management, airport authorities and airlines (Operations Control).

In the paper "Airline Disruption Management: Perspectives, Experiences and Outlook" [22] from 2004 Kohl et al. provide a general introduction to airline disruption management including a description of the planning processes in the airline industry. The present (almost manual) mode of dealing with disruption and recovery is presented, followed by a review of existing litterature on developments within automated optimized recovery. Furthermore, the paper reports on the experiences obtained during the large-scale research and developement project DESCARTES, supported by EU, on airline disruption management. Among the results of the project were a first prototype of a multiple resource decision support system.

4.1 Aircraft Recovery

Teodorovic and Guberinic were among the first to study the aircraft recovery problem in "Optimal Dispatching Strategy on an Airline Network after a Schedule Pertubation" [43] from 1984. Here, one or more aircraft are unavailable and the objective is to minimize the total passenger delays by reassigning and retiming the flights. The problem is solved separately for each fleet. The model is based on a type of connection network, which consists of two types of nodes. The first type represents the flights to be flown whereas the other represents operational aircraft. The model is solved by finding the shortest Hamiltonian path in the network which is solved using a Branch-and-Bound algorithm. The authors present a very simple example with only 8 flights.

"Model for Operational Daily Airline Scheduling" [44] from 1990 by Teodorovic and Stojković extends the previous work described above to consider also airport curfews. Nodes in the network represent flights to be flown and are grouped in stages. Each stage represents a flight number in the chain of flights to be made by the first aircraft considered. An initial node with arcs to all nodes in stage 1 is added. Stage 1 contains only nodes representing the flights starting from airports with available aircraft. The following stages contains flights to be flown later. An arc in the network indicate that the two flights can be operated in succession. The cost of an arc is the total time loss of passengers on the i'th departure after the $(i - 1)$'st takeoff. The solution method is greedy: First, a shortest path (schedule) for the first aircraft is generated. Nodes used in this shortest path are removed and the shortest path method is invoked again to generate the schedule for the next plane, and so forth. The method is tested on a small example of 14 aircraft and 80 flights. By extracting sub-problems hereof a test-set of 13 instances is generated and tested. Running times on a PC/XT are in the range of 5 to 180 seconds. The quality of the solutions is not discussed.

In "Model to Reduce Airline Schedule Disturbances" [45] from 1995, Teodorovic and Stojković further extend their model to include also crew considerations. The model proposed still solves the problem individually for each aircraft type. Their approach is based on two objectives where the first priority objective is to maximize the total number of flights flown and the second objective is to minimize the total passenger time loss on flights that are not canceled. The proposed framework schedules crew before aircraft.

The authors also conducted experiments with the reverse order of scheduling. However, computation time increases as this problem is much larger and scheduling of the aircraft requires constant checks on crew feasibility. The question on whether to first design the crew or the aircraft rotations is similar to the “cluster-first or route-first” decision in vehicle routing. Crew rotations are scheduled using either the “first-in-first out” (FIFO) principle or a sequential approach based on dynamic programming (DP). When linking legs into routes by the FIFO principle, every leg arriving at an airport is linked to the first leg departing from the same airport. A chain is completed when the crew member is out of hours. The DP approach constructs a connection network with flight legs as nodes. An edge from leg i to leg j indicates the feasibility of crew flying j immediately after i. The cost of each edge represents the corresponding ground time for the crew. Now, the shortest path containing the maximum number of legs is found. This is the rotation for the first crew. The flights covered by this rotation is then removed from the network, and the process is repeated for the next crew group rostered together. A connection network with the rotations for crew as “legs” is the generated. An edge represents that a particular tail is able to fly the two rotations involved in sequence. The length of the edge is defined to be the number of flights in the latter of the rotations constituting the edge. A longest path in this network is identified, corresponding to a line of work for an aircraft including as many flights as possible. If two paths are equally long the path with the smallest passenger delay is chosen. Thereafter, the nodes and corresponding edges are removed from the network, and the process is repeated in a greedy fashion.

Teodorovic and Stojković propose an algorithm that describes how the checks of the technical maintenance requirements are handled. If infeasibilities are found the dispatcher first tries to reshuffle the aircraft rotations. If this does not work the dispatcher changes one of the parameters (for instance cancel or re-time a flight). The proposed method is tested on 240 different randomly generated numerical examples. The largest examples consist of 80 legs. 4-5 disturbances are generated at random for each of the 240 instances. The method performs at least as good as “naive solutions” (simply canceling disturbed flights) in almost all of the cases. The tests were run on a 16 MHz 286 PC. Running times for the FIFO approach was 2 seconds and for the DP approach 140 seconds for the biggest instances with 80 legs.

The paper “A Decision Support Framework for Airline Flight Cancellations and Delays” [20] from 1993 by Jarrah, Yu, Krishnamurthy, and Rakshit discusses the two major techniques for solving the aircraft recovery problem: cancellation and re-timing. A time-line network is used to model the problem data and three methods ared discussed: The successive shortest path method for cancellations, and two models based on the same type of network and allowing cancellations resp. re-timings. Also, the possibility of swapping aircraft is taken into account, where swaps can be with spare aircraft or with overnight layovers. The time-line network has two types of nodes per station - flight nodes and aircraft nodes. These are used to model the aircraft-to-flight assignments. Both models are minimum cost flow models. The models are tested on a network with three airports each having considerable air traffic. The re-timing model can typically save part of the delay minutes and produce a substantially better solution with respect to cost. Both minor and major disruptions are tested in the test scenarios. The results from the cancellation model are not as easy to interpret. The three test scenarios here are based on United Airlines' B737 fleet and a regional subdivision of the United States. In both cases, the running time of the models are counted in seconds on a DEC workstation - short enough to allow for real-time use.

In "Decision Support for Airline System Operations Control and Irregular Operations" [30] from 1996 Mathaisel describes the business process as well as the IT challenges faced with the desig and implementation of a decision support system for airline disruption management. Furthermore, the paper discusses how a simple network flow problem can be used for modelling the aircraft recovery problem. First, the non-disruptive network is constructed. Here, all planned aircraft routings are represented by setting the upper and lower bounds of the binary flow variables to "1". The network is then altered in order to describe the disruptive situation. The author discusses several types of disruptions that must be taken into account when designing the algorithm that alters the network. These include ground delays, inflight delays, cancellations, station closure and diversions. The altered network is an expanded version of the non-disruptive network usually consisting of a larger number of arcs and in some cases also additional nodes. The lower bounds of the flow variables are reset to "0" and the resulting problem is solved by the Out-of-Kilter algorithm. The model is capable of using cancellation as well as retiming. However, the paper does not discuss multiple types of aircraft, crew considerations, or solution time.

"Swapping Applications in a Daily Airline Fleet Assignment" [42] from 1996 by Talluri investigates the challenge of changing equipment type while maintaining feasibility of the schedule. The problem is to change the AC type on a specific flight at a minimum cost. As this process is done at operations control after the original planing phase computation speed is an important issue, and a solution must be found within 2 minutes. Solutions are categorized wrt. the number of overnight swaps needed. Being able to make the change without affecting the overnight position of an AC is desirable mainly due to maintenance. An algorithm with polynomial running time that finds a possible swap contained in the same day is presented. If no such swap exists, the algorithm returns this negative answer. Furthermore an algorithm allowing at most k overnight changes also with polynomial running time is presented. Both algorithms are based on the connection network. The solutions delivered by the algorithms are valid wrt. turn around rules, fleet size, and assignment of each flight, whereas maintenance and crew considerations are not checked. Testing is very limited and only documented by a single instance. For a connection network of two equipment types, 700 arcs and 200 nodes ten swapping solutions was found within 4 seconds on an IBM RS560.

“A GRASP for Aircraft Routing in Response to Groundings and Delays” [4] from 1997 by Argüello, Bard, and Yu describes a heuristic approach for the reconstruction of aircraft routes when one or several aircraft are grounded. The heuristic is based on randomized neighborhood search. An initial solution to the problem consists of aircraft routes and cancellation routes (sequences of flights operated by an individual aircraft, and sequences of canceled flights, which could be operated by an individual aircraft). In each step of the solution process, all pairs of two routes (of which at least one must be an aircraft route) from the current solution are investigated. For each such pair, all sets of feasible re-routes covering the flights from the two routes are constructed respecting flight coverage and aircraft balance at stations. Each set of feasible re-routes is assigned a score reflecting the cancellation cost and delay cost of the route set. A limited number of these are stored in a restricted candidate list. The selection is based either on quality relative to the current solution or on absolute quality. Finally, a random member of the candidate list is chosen as the starting solution for the next step of the algorithm. Each run is allowed 2 CPU seconds, and 5 independent runs per instance is performed. The quality of the solution is established through a comparison with a lower bound found using the LP-relaxation of a time-band formulation of the recovery problem. The method is tested on B757 fleet data from Continental Airlines with 16 aircraft and 42 flights. The recovery period is set to one day. All instances grounding from 1 to 5 out of the 16 aircraft at the beginning of the day are investigated. The results obtained by the proposed method are clearly superior to just canceling the flights serviced by the grounded aircraft. In more than 70% of the instances, the GRASP solution is within 5% of optimality.

In the working paper "The Airline Schedule Recovery Problem" from 1997 by Clarke [14], an approach to the aircraft recovery problem is presented that in many ways corresponds to the classical fleet assignment approach. Here, the time-space or connection network used in many airline-related solution methods is called a "Schedule Map". The network is used to generate legal paths throughout the time horizon. Flights can be retimed by incorporating delay arcs into the model. These arcs are incorporated before running the solution algorithm. An integer programming model with binary variables for using a path and for canceling of flights is presented. The model provides primitive extensions for crew, slots and gates. The paths are generated using an algorithm for the constrained shortest path problem. The objective is a sum of direct costs of reassignment, revenue spill costs and operating revenues. Experimental results for 3 different solution procedures (ranging from a simple greedy approach to a complex column generation approach) are presented for test sets from a major US domestic carrier. Tests are run on a Sun Sparc 20 workstation. The case studies have multiple aircraft types, 35-177 aircrafts, 180-612 flights and 15 or 37 airports. The results suggest that it is possible to reschedule flights in the aftermath of irregularities, although no running times are reported so it is hard to see if it remains feasible in a dynamic on-line environment.

The papers “Airline Scheduling for the Temporary Closure of Airports” [49] and “Multifleet routing and multistop flight scheduling for schedule perturbation” [50] from 1997 by Yan and Lin, resp. Yan and Tu are based on the same underlying model and can both be seen as preliminary investigation of methods for aircraft recovery. The topic of [49] is recovery when an airport is temporarily completely closed, whereas [50] addresses the particular situation of temporary shortage of one aircraft. The underlying model is a time-line network, in which flights are represented by edges from origin to destination. Furthermore, the network has position arcs corresponding to potential ferrying of an aircraft. The possibility of retiming an aircraft is modeled by introducing several arcs per flight and imposing a constraint indicating that at most one of these can be in the solution. Finally, the possibility of modifying a one-stop flight from i over j to k in a non-stop $i - k$ flight possibly supplemented with $i - j$ or $j - k$ flights is introduced. Maintenance considerations are not taken into account.

In [49], the models resulting from adding any combination of these possibilities to the basic time-line network are investigated. The solution methods are network flow methods, and if side constraints are present, these are combined with Lagrangean relaxation and Lagrangean heuristics. Tests are performed on data from China Airlines (Taiwan) with 39 flights to be served by 17 aircraft. The experiments on this small data set show a major advantage using all three proposed network modifications, and the running times reported are short (49 seconds at worst on an HP735).

[50] considers the situation, in which one aircraft becomes non-operational, but considers several fleets of aircraft. The network described above is modified with a supply node added at the point in time and space, where the absent aircraft is recovered. One such network is built for each fleet in question. If an aircraft type C can substitute another type B, the network for type C contains edges corresponding to the flight flown with type B (since a C-aircraft might fly such a flight). Hereby, swapping between fleets are made possible. To allow for re-timing, edges “parallel” with the flight edges are included with specific time intervals. The model becomes an integer multi-commodity flow model, which is solved using a combination of Lagrangean relaxation and network simplex, and a Lagrangean heuristic. Results are provided again based on data from China Airlines with 24 stations, 273 flights and 3 types of aircraft. Several types of recovery strategies are tested including limited re-timing, positioning, and the modification of multistop flights. 10 scenarios with all combinations of strategies are again tested, and convergence to within 1% of optimality is reported within 30 minutes computing time (HP735) for most scenarios.

The two papers “A Decision Support Framework for Handling Schedule Perturbation" and "A Decision Support Framework for Multi-Fleet Routing and Multi-Stop Flight Scheduling" [51, 52] from 1996 by Yan and Yang resp. Yan and Young describe the same models as the papers [49, 50]. Furthermore, much of the text in the papers is identical. Also, the case study described originates from the same data set. [51] has a more detailed experimental section than [49], but the essential conclusions are the same. Regarding [52], the differences to [50] are more profound. Though the modeling framework and the solution methods suggested are identical, the proposed strategies for solving the perturbation problem are slightly different. Again, the experimental section is more elaborate, but the main conclusions remain the same.

"Real-time Decision Support for Integration of Airline Flight Cancellations and Delays Part I (resp. II): Mathematical Formulation (Resp. Algorithms and Computational Experiments)" [11] and [12] from 1997 by Cao and Kanafani make use of the same type of time line network as [20] discussed above, however, the model presented allows for a solution combining delays and cancellations. The model derived is a special type of quadratic 0-1 integer program, the quadratic term in the objective function stemming from the cost incurred by interdependent changes in aircraft-to-flight assignments. The authors present a tailored Linear Programming Approximation algorithm for the problem, which finds a locally optimal solution. By subdividing their solution space and running the algorithm on each subdivision, they enhance the probability of identifying the global optimal solution to the problem. The algorithm is tested on a set of randomly generated scenarios with 20-50 airports, 30-150 aircraft, 5-12 surplus aircraft, 65-504 flights, and appr. 25% delayed aircraft. The running times range from 26 seconds to 869 seconds (VAX-6420). The scenarios tested have a high number of stand-by aircraft and hence seem quite unrealistic, and the quality of the solutions is difficult to assess. Furthermore, the work by [25], in which a reproduction of the results have been tried without success, suggests that the description of the model is not complete.

"Airline Schedule Perturbation Problem: Landing and Takeoff with Nonsplitable Resource for the Ground Delay Program", "Airline Schedule Perturbation Problem: Ground Delay Program with Splitable resources", and "On the Airline Schedule Perturbation Problem Caused by the Ground Delay Program" [28, 27, 29] by Lou and Yu all address the problem of schedule perturbation resulting from the Ground Delay Program of the Federal Aviation Authorities. The last two papers are, though the publishers are different, identical. Each operating airline of an airport has at the beginning of each day a number of assigned slots for landing and take-off. The slots exactly match the activities of the airline. The arrival slots for the airlines may, however, be changed due to e.g. deteriorated weather conditions. In that case, slots may be moved in time or possibly canceled. The problem for each airline is now to determine the assignments of flights to available slots to minimize inconvenience and knock-on effects. Hence, delays are not directly connected to a specific flight leg but to the fact that the landing slots for each airline are moved in time. The problem is therefore to reschedule the flights. In [28] from 1998, the subproblem in which aircraft and all crew are scheduled together is considered. A number of different objectives are discussed. For all of these, the problem is an assignment problem and very efficient solution methods are readily available. The paper is theoretical and methodological in nature. No implementations or experiments are reported. Considering schedules, in which aircraft and crew are scheduled separately, the problem becomes more complex. If the objective is to minimize the maximum delay of out-flights, which is crucial because these delays are the ones giving rise to down-line knock-on effects, the general problem is strongly NP-hard. In [27, 29], also from 1998, a special version is shown to be polynomial, and a heuristic is developed for the general version. The paper presents the result for a real-life problem from American Airlines with 71 incoming flights. A large improvement in the number of aircraft delayed more than 15 minutes compared to the original schedule can be achieved by the rescheduling of flights using the proposed methods. The solution time is appr. 15 seconds.

The master thesis “Disruption Management in the Airline Industry” [25] from 2001 by Løve and Sørensen takes as starting point the model of [11, 12] described above. The results are, however, discouraging, and therefore, alternative methods for employing cancellations and retimings in response to disruptions are investigated. The methods are local search based heuristics. The results show that steepest descent local search is almost as good as other local search based heuristics in terms of quality, and much faster with a running time of few seconds even for large problem instances. Also, the work shows that solutions are sensitive to the costs assigned to the different recovery strategies, such that structurally different solutions can be obtained by varying the relative importance of the costs. The data used in [25] are randomly generated. However, in the DESCARTES project [22] a feasibility study on real data from British Airways with 80 aircraft, 44 airports, and 340 flights has been carried out showing the same general tendency. The Dedicated Aircraft Recovery module of the DESCARTES project is based on the methods described in the project. A summary of the findings can be found in [26].

The two papers “Balancing user preferences for aircraft recovery during irregular operations” and “Multiple fleet aircraft schedule recovery following hub closures” [46, 47] by Thengvall, Yu, and Bard from 2000 resp. 2001 are both based on the classical timeline network as described by [49, 50], which in addition to flight arcs and ground arcs has protection arcs and through-flight arcs. The two latter types of arcs makes it possible in the evaluation of a proposed recovery schedule to prioritize the deviation from the original schedule by giving special emphasis to flying all legs in a flight with several stops with the same aircraft. In [46] the model is used to produce recovery schedules for single fleet recovery in case of minor disruptions as e.g. unavailability of a limited number of aircraft. Crewing and maintenance are not taken into consideration. The model is an integer single-commodity network flow problem with side constraints, which is solved by standard optimization software. As a supplement, a heuristic to construct an integral solution from a fractional LP-solution is implemented. The approach is tested on real-life data from Continental Airlines (B757 schedule with 16 aircraft and 13 stations, and B737-100 with 27 aircraft and 30 stations). Results indicate that the approach clearly allows for construction of different recovery schedules corresponding to changes in priorities between delay minute costs, cancellation costs, and cost of deviation from original schedule. Computing times are sufficiently small to allow for real-time use.

"Balancing User Preferences for Aircraft Schedule Recovery during Irregular Operations" [46] from 2000 by Thengvall, Bard, and Yu addresses the situation of complete closure of a hub, i.e. a situation causing a major disruption for a larger airline. The model described above is extended to allow for multiple fleets. Each fleet may have subfleets organized hierarchically, and substitution within subfleets are allowed accordingly. Three models are built: Two so-called preference models, of which one is a pure network model and one is a generalized network model, and a model based on time bands as introduced in [5]. The preference models are both based on time-line networks as described previously for each subfleet. The models are all MIP-models. Their relative performances are initially investigated leading to the choice of the first preference model for detailed investigation. Detailed experiments are performed leading to the conclusion that the model produces reschedules of quite high quality in computing times less that 30 minutes for the largest problem with a hub closure of 10 hours and a recovery period of 24 hours for a schedule with 3 hubs and 2921 flights between 149 stations.

In “Optimizing aircraft routings in response to groundings and delays” [6] from 2001, Bard, Yu and Argüello introduces the time-band network described in Section 2. The resulting model is an integral minimum cost flow model with additional constraints ensuring that a flight is either canceled or flown by a unique aircraft. The authors develop a solution methodology with an initialization step, in which the flight schedule is input and the time bands decided followed by the generation of the time band network. Finally, the mathematical formulation of the integer programming problem is derived.

This problem is then relaxed by ignoring the integrality constraints and solved to optimality. Based on the LP-solution, an integer-valued solution is derived, which is finally turned into a schedule. The cost is calculated and compared to the lower bound provided by the LP-relaxation. The approach is tested on a Continental Airlines B737-100 fleet schedule with 162 flights covering 30 stations and serviced by 27 aircraft. 427 test cases are reported: 27, in which one aircraft is grounded, and 100 cases for each case of two, three, four and five aircraft grounded. The time bands are varied from 5 minutes to 30 minutes, allowing also variations between hub stations and spoke stations. Using the lower bounds derived and the actual cost of the solutions, the quality of the solutions can be assessed, and also the solution time is reported. The results depend on the time band resolution, but are generally encouraging with respect to quality. Regarding solution time, these are highest for fine resolutions, the maximum being 750 seconds on Sun Sparc 10.

The thesis “The Flight Perturbation Problem - operational aircraft rescheduling” [3] of Andersson considers the aircraft recovery problem. Based on the same model three different solution methods are developed. The model used is based on the connection network resulting in a multi-commodity flow formulation. An aircraft is represented by two nodes: an “aircraft source node” (where and when is the aircraft at the start of the recovery period) and a corresponding “flight sink node”. It is assumed that the aircraft must pick up its original schedule after the recovery period. The three solution methods are a Lagrangian relaxation-based heuristic, a method based on Dantzig-Wolfe decomposition, and finally a heuristic based on tabu search. Computational results are based on data from a domestic carrier from Sweden operating 5 fleets with a total of 30 aircraft. Instances consist of 98–215 flights and 19–32 airports. Results are only reported for the last two heuristics as the performance of the first is described as being be clearly inferior. While the Dantzig-Wolfe-based method is good for the smaller instances, larger ones have to be solved using tabu search. Solution times range from 10 seconds to 1100 seconds for the Dantzig-Wolfe based method. The running time of the tabu search approach remains below 10 seconds and the method consistently delivers competitive solutions.

The paper “Rerouting Aircraft for Airline Recovery” [33] from 2003 by Rosenberger, Johnson and Nemhauser considers the case of aircraft recovery. The proposed model addresses each aircraft type as a single problem. The model principally follows an approach traditionally used in planning problems, namely a Set Partitioning master problem and a route generating procedure. The objective is to minimize the cost of cancellation and retimings, and it is the responsibility of the controllers to define the parameters accordingly. In order to solve the master problem in due time, a heuristic is used to select only a subset of aircraft to be involved in the Set Partition problem. The heuristic determines for each disrupted aircraft a number of other aircraft with routes allowing a swap with the disrupted aircraft. The legs of these routes are those included in the route generation procedure. This approach results in running times between 6 and 16 seconds for 3 real-size problem instances. The paper contains an impressive testing using SimAir [34] simulating 500 days of operations for the three fleets ranging in size from 32 to 96 aircraft servicing 139-407 flights.

Authors	Model	Functionality: Canx	Retim3	Fleets	Data	Dimensions: AC	Fleets	Flights	Solution time	Objectives
Teodorovic, Guberinic [43]	CN	No	Yes	No	G	3	1	8	NA	Delay minutes
Teodorovic, Stojković [44]	CN	Yes	Yes	No	G	14	1	80	180	canx and delay minutes
Teodorovic, Stojković [45]	CN	Yes	Yes	No	G	NA	1	80	140	canx and delay minutes
Jarrah, Yu, Krishnamurthy, Rakshit [20]	TLN	Yes	Yes	No	RL	NA	9	NA	0-30	delay, swap and ferrying
Mathaisel [30]	TLN	Yes	Yes	No	NA	NA	NA	NA	NA	“disruption effects”
Talluri [42]	CN	No	No	Yes	G	NA	NA	NA	10	swaps when changing AC type
Argüello, Yu, Bard [4]	–	Yes	Yes	Yes	RL	16	1	42	2	route cost and cancellation cost
Clarke [14]	CN	Yes	Yes	Yes	RL	177	4	612	NA	costs minus revenues
Yan, Lin [49]	TLN	Yes	Yes	No	RL	17	1	39	49	costs minus revenue
Yan, Tu [50]	TLN	Yes	Yes	yes	RL	273	3	3	1800	costs minus revenue
Cao, Kanafani [11, 12]	TLN	Yes	Yes	No	G	162	1	504	869	revenue minus costs
Lou, Yu [27, 29]	NA	No	Yes	NA	RL	NA	NA	71	15	number of delayed flights
Lou, Yu [28]	NA	No	Yes	NA	RL	NA	NA	71	15	delayed flights
Løve, Sørensen [25]	TL	Yes	Yes	No	RL	80	1	340	6	revenue minus costs
Thengvall, Bard, Yu [46]	TLN	Yes	Yes	No	RL	27	1	162	6	revenue minus cost
Thengvall, Yu, Bard [47]	TLN	Yes	Yes	Yes	RL	332	12	2921	1490	revenue minus cost
Bard, Yu, Argüello [6]	TBN	Yes	Yes	No	RL	27	1	162	750	delay and canx
Andersson [3]	CN	Yes	Yes	Yes	RL	30	5	215	10-1100[a]	revenue
Rosenberger, Johnson, Nemhauser [33]	NA	Yes	Yes	No	G	96	1	407	16	cost of canx and retimings

Table 2: Model is either connection network (CN), time line network (TLN), time band network (TBN). Data is either generated (G) or real-life (RL) instances. Solution times are in seconds. Yan, Yang [51] is not mentioned as it is identical to [49]. Fleets indicate whether multiple fleets can be dealth with concurrently.

[a] The tabu search has a max of 10 seconds, whereas the Dantzig-Wolfe algorithm goes as high as 1100

4.2 Crew Recovery

“The Operational Airline Crew scheduling Problem” [41] from 1998 by Stojkoviĉ, Soumis, and Desrosiers formulates the crew recovery problem as an integer non-linear multicommodity flow problem. The idea is that in the disrupted period, the duties of the crew are dissolved in order to make replanning feasible. The master problem is a Set Partition type model which is solved by Branch-and-Bound with LP-relaxation and column generation. The subproblems are shortest path problems based on a time line network, in which duties are represented as edges between the start node and the end node of the duty. A separate graph is constructed for each crew member. The model and method is tested on data from a major U.S. carrier, and only cockpit personnel for one fleet positioned at the carrier's base is considered. The disruptions considered consist of three delayed aircraft, and one indisposed crew member away from base. Scenarios with one or two crew members per aircraft are tested, and both one day horizons and seven days horizons are tested. The results are encouraging, showing that the column generation approach though varying considerably with respect to computing times is feasible for smaller problems.

Crew management as a Crew Pairing Repair (CPR) problem is treated by Wei et al. in “Optimization Model and Algorithm for Crew Management During Airline Irregular Operations” and "A Decision Support Framework for Crew Management During Airline Irregular Operations” [48, 37] from 1997/98. The two papers are, though the publishers are different, identical. Here the underlying assumption is that the flight schedule has been fixed and thus is given. The challenge is now to repair the pairings that are broken by the modified schedule. The objective is to return the entire system to the original schedule as soon as possible and in the cheapest way. The problem is solved based on a space-time network, which is considered for a certain time window. Start of the window is the current time and the end of the window is the end of the recovery period by which the resources should have returned to their original schedule.

Each airport is described by two columns of nodes. The first column represents crew that has arrived to the airport from another flight or that are signing in here. They are placed according to when they are ready. Flight nodes in the second column represent the departure of flights. Reserve nodes (placed in the first column) represent the availability of stand-by crew. Return nodes force the crew to return to their original schedule at the end of the window. There are four types of arcs in the network: flight arcs represent the flight from one airport to another, stand-by arcs emanates from stand-by crew nodes to those flights at the same airport that can be served by standby crew, scheduled arcs emanate from crew nodes to the originally scheduled flight nodes, and finally return arcs represent the returning of crew to their original schedule.

The cost of the arcs reflect preferences or penalties. It is assumed that each crew member can be associated with only one fleet type. Furthermore several crew members are grouped and rostered together, i.e. they have the same roster. The network basicly defines the feasible pairings. A generalised Set Covering problem with the constraints of covering all flights is then solved. Solution time is restricted to only a few minutes and the operations controller needs multiple good solutions. The solution method is a depth-first Branch-and-Bound algorithm. At each node, the problem is defined by the set of (still) uncovered flights and a list of pairings that are modified. When the set of uncovered flights is empty the corresponding Branch-and-Bound node represents a feasible solution to the CPR problem. At each non-leaf node in the search tree a flight is selected among the set of uncovered flights. A candidate crew list is built and the best crew member is chosen. The change pairing of the crew member must satisfy:

- it must be possible to return the crew member to the return node,
- the pairing must be legal, and
- the new pairing should be as close to the original one as possible.

If the two first items cannot be satisfied the node is fathomed. A further pruning feature is that a node is fathomed if the number of modified pairings is larger than that of the solution found so far.

The stopping criterion of the algorithm is that a predetermined time limit has been reached or a required number of solutions has been produced. Computational experiments are based on data from an unknown source. The largest instance comprises 6 airports, 51 flights in a two-day period, and 18 pairings. This rather small instance is the basis of 8 scenarios with a different number of delays and cancellations. The running times range from a fraction of a second to 6 seconds producing from 1 to 3 solutions (3 solutions being one of the stopping criteria).

In “Airline Crew Recovery” [24] from 2000 Lettovsky et al. presents a method for recovering crew in the case of disruptions. Preprocessing techniques are used to extract a subset of the schedule for rescheduling. Among the techniques are that only selected pairings (a restricted set of crews) are broken up, and not necessarily into single flight legs but consecutive flights with no swapping opportunities. A fast crew-pairing generator then constructs feasible continuations of partially flown crew trips. Deadheads can be given a priori. The crew recovery problem is then formulated and solved as a generalised Set Covering problem using 3 different branching strategies and incorporating variable fixing. Delaying flights is also incorporated but it is not clear how.

A test of the method on a schedule of 1296 flight legs from a major U.S. carrier is presented. The legs are covered by 177 pairings. Three scenarios of irregular operations are set up: 1) is a small-size maintenance-related disruption, 2) represents a weather disruption implying decreased landing capacity at an airport and finally 3) presents a major disruption having three airports hit by a snowstorm. The first two scenarios required no branching. Solution times range from 1 to 115 seconds. For the longest solution time

4642 new pairings were generated. In scenario 1 no flight had to be canceled whereas scenario 2 and 3 resulted in up to 21 cancellations. It should be noted that “crew members” are aggregated into groups of people that cannot be split for the length of the pairing. This situation is seldomly found among European airlines.

"Airline Crew Scheduling - From Planning to Operations" by Medard and Sawhney [31] discusses the crew recovery problem. The authors stress that the problem is structurally different from the crew pairing and rostering problems because contrary to the planning phase these two subproblems have to be solved at the same time in the recovery phase. This means that both rules on the pairing and the rostering level have to be respected. Thus, they note that the recovery challenge is to merge the pairing characteristics into a rostering problem which is modeled at the flight level. Medard and Sawhney consider the so-called rostering time window decomposition technique and refer to the fixed part of the roster before (and after) the disruption as the carry in (and the carry out) for the crew member in question. Within the recovery window, flights are de-assigned from the disrupted crew as well as from a group of other crew, referred to as helper crew. It should be noted that some crew member might have days off or training duties within the time window, these cannot be changed. Also, newly scheduled flights may be added to the pool of de-assigned flights. Medard and Sawhney propose an optimization model which is the flight-based equivalent to the original pairing-based rostering model, where the de-assigned flights replace the pairings. The optimization model is formulated as a Set Covering model which is solved using column generation. The columns are generated by finding shortest paths either by the use of a Depth First Search strategy (DFS) or by a reduced cost column generator (COLGEN). Medard and Sawhney test their methods on small to medium sized scenarios ranging from 14 to 885 planned crew members with up to 77 illegal rosters. The computation times range from 12 to 840 seconds on a 1 GHz PC. The results are encouraging however for some of the more complicated scenarios the time limit of a few minutes is not respected and the authors conclude that the column generation schemes must be refined. This can for instance be obtained by applying more crew specific information in the generation scheme.

“A Proactive Crew Recovery Decision Support Tool for Commercial Airlines during Irregular Operations” by Abdelghany et al. [1] addresses the problem of flight crew recovery for an airline with a hub-and-spoke network structure. The paper discusses the disruption management problem in detail and subdivides the recovery problems into four categories: Misplacement problems, rest problems, duty problems, and unassigned problems. Based on detailed information regarding the current plan and pool of problems, the recovery problem is then solve in steps. In the solution method, delaying, using stranded crew, swapping, deadheading, and using standby crew are used as means of recovery. The proposed model is an assignment model with side constraints, which takes into account timings and bounds on regarding the use of different means (as e.g. use of undisrupted crew members in the recovery solution). Due to the stepwise approach, the proposed solution is suboptimal. Computational results are reported for a situation from a US carrier with 18 problems. The solution involves 121 crew members and is found in less than 2 minutes using CPLEX to solve the mathematical model. From the information given it is hard to judge the practical applicability of the proposed method.

4.3 Integrated Recovery

A framework for integrated airline recovery is presented in Lettovsky's Ph.D. thesis "Airline Operations Recovery: An Optimization Approach" [23] from 1997. This is the first presentation of a truly integrated approach in the literature, although only parts of it is implemented.

The thesis presents a linear mixed-integer mathematical problem that maximizes total profit to the airline while capturing availability of the three most important resources: aircraft, crew and passengers. The formulation has three parts corresponding to each of the resources, that is, crew assignment, aircraft routing and passenger flow. In a decomposition scheme these three parts are "controlled" by a master problem denoted the Schedule Recovery Model.

The Schedule Recovery Model (SRM) determines a plan for cancellations and delays that satisfy some of the constraints. Now the three other problems can be solved separately. For crew and aircraft we have a Crew Recovery Model (CRM) and an Aircraft Recovery Model (ARM). The Passenger Flow Model (PFM) will find new minimum cost itineraries for disrupted passengers.

The solution algorithm is derived by applying Benders' decomposition to a mixed-integer linear programming model of the problem. SRM determines a plan for cancellation, delays and equipment assignment considering landing restrictions. Then for each equipment type f we solve ARM_f and for each crew group c we solve CRM_c returning Benders feasibility or optimality cuts to the SRM. Finally PFM evaluates the passenger flow. In this way the built-in hierarchy of the framework to a large extent resembles the present manual process at many airlines. Lettovsky points out that as the model can become large and complex to solve it is important to keep the recovery period as small as possible. As in other approaches, all assignments of duties outside the recovery period are fixed and only tasks within the recovery period can be changed. It is also noted that multiple solutions can be generated. By only adding Benders optimality cuts from the PFM the framework will produce the most "passenger friendly" solution, still adding feasibility cuts from the ARM_f and CRM_c.

Rescheduling aircraft and pilots for one day is the topic in "An Optimization Model for the Simultaneous Operational Flight and Pilot Scheduling Problem" [38, 40] by Stojkoviȅ and Soumis from 2000/2001. The disruption addressed is either that of disrupted crew schedules or that of delayed incoming aircraft. The problem is "to modify planned

Authors	Model	Functionality			Data	Dimensions		Solution	Objectives
		Canx	Retiming	Indv. Rostering		Crew	Flights	time	
Stojković, Soumis, Desrosiers [41]	TLN	No	Yes	Yes	RL	NA	210	1200	pairing costs
Wei, Yu, Song [48, 37]	STN	No	Yes	No	NA	18	51	6	Return to schedule cheapest way
Lettovsky, Johnson, Nemhauser [24]	TLN	Yes	(Yes)	No	RL	38	1296	115	revised pairing cost
Medard, Sawhney [31]	TLN	NA	Yes	Yes	NA	885	NA	840	Illegal crew, uncovered flights, and affected crew
Abdelgahny et al. [1]	NA	No	Yes	Yes	RL	121	NA	2	Deadhead, standby, and swap

Table 3: TLN Time Line Network. STN Space Time Network. RL Real-life. Solution times are in seconds

duties for a set of available pilots to cover a set of flights by delaying (if necessary) some of them". Some flights have fixed departure times, some others have more flexible times in terms of a flight specific time window. Stand-by pilots are available at some stations. The paper uses a connection network with explicit representation of each pilot, and the model hence becomes an integer non-linear multicommodity flow model with additional constraints. The problem is solved using column generation with a master problem and a subproblem per pilot. The solution may include the use of stand-by pilots. The model and solution method has been tested on three problems the largest of which has 59 pilots, 79 aircraft, and 190 flights of which 52 are originally delayed. The solution strategy of combining the modification of the aircraft schedule and the crew schedule using the proposed model is compared with a "simulated" traditional manual solution procedure, where first aircraft and then crew are dealt with. The results are encouraging, both in terms of quality and in terms of computing times.

The working paper "The Operational Flight and Multi-Crew Scheduling Problem" [39] again by Stojkoviĉ and Soumis from 2000 builds on the model derived in [38, 40], but extends this to work with multiple crew members. This makes the situation addressed more realistic. The extension is achieved by using a number of copies of each flight corresponding to the number of crew required. A set of constraints ensuring that the departure time for all copies of each flight is added to the model. The solution process is similar to that described in [38, 40]. Three different models are tested: One corresponding to that from the previous work with strict flight covering constraints, one in which there is a linear cost for missing crew members, and one with a cost for each flight with missing crew. In the solution process, artificial crew members are used to ensure feasible solutions. Results are reported for four test scenarios each originating in the closure of a domestic hub airport. It is demonstrated that using both the second and the third model, substantial improvement compared to the initial situation can be obtained. However, the solution times for large problems are prohibitive in an on-line situation (more than an hour).

The paper "Flight Operations Recovery: New Approaches Considering Passenger Recovery" by Bratu and Barnhart [10] presents two models that considers aircraft and crew recovery and through the objective function focuses on passenger recovery. These are based on the flight schedule network. Retiming is incorporated by representing a flight f by several arcs, one for each possible departure time. The same technique is used in eg. [3, 46]. While crew is incorporated into the models they do not consider how to recover disrupted crews.

In the Passenger Delay Model (PDM) model delay costs are modelled more exactly by explicitly modelling disruptions, recovery options and delays costs, whereas in the Disrupted Passenger Metric (DPM) model delay costs are only approximate. Based on the single instance for which both methods are tested the execution time for PDM is roughly a factor 25 higher than for DPM. The models are solved using OPL Studio. To test the models an OCC simulator is developed. Data are provided for the domestic operations of a major US airline. It involves 302 aircraft divided into 4 fleets, 74 airports and 3 hubs. Furthermore 83869 passengers on 9925 different passenger itineraries per day are used. 3 different scenarios with different levels of disruption is presented. Execution times ranges from 201 to 5042 seconds. Due to its excessive execution times the PDM is considered unfit for operational use. For all scenarios the DPM generate solutions with noticeable reductions in passenger delays and disruptions.

5 Disruption Management by Robustness

An interesting research topic closely related to disruption management is robust planning. The central idea is to incorporate the possibility to absorb disruptions and remain feasible (or at least facilitate an easy recovery) into the schedule.

In the masters thesis of Y. Ageeva entitled "Approaches to Incorporating Robustness into Airline Scheduling" [2] an approach for producing robust aircraft assignments is developed. The approach is an extension of the classical flight string model [7]. The measurement of robustness is based on identifying and building a schedule where strings meet each other as often as possible in order to create opportunities to change the schedule. Two definitions are central. Two sequences of flights meet at points P_1 and P_2 within δ on departure, if the departure airport at the two points are identical and the departure at the points are within δ in time, in other words, the strings meet at the two points P_1 and P_2 for at least δ minutes. Overlaps are used in the notion of robustness by the following definition: An overlap within time δT occurs at a point P_1 if two sequences meet on departure or arrival. A sequence is now called robust within time δT if there exists an overlap within time δT at some point P'. In general, a way to increase robustness of an airline schedule is to provide ways for more aircraft routes to intersect at different points, so that aircraft can switch strings if needed. Specifically, the goal is to minimize $P - A$, where P is the number of potential overlaps and A is the actual number of overlaps. Ageeva incorporates the robustness measure in the flight string model now maximizing profit and $P - A$. The model is implemented using ILOG's OPL optimization language. Basicly a column generation scheme is used to generate an optimal LP-solution thereafter solving the resulting the LP-model as an IP-model. Furthermore, constraints are added in order to generate all possible optimal solutions. Experiments are carried out on 4 instances with the number of flights ranging from 14 to 37. Larger problems were too complex and time consuming to solve. No computing times are reported. For each subset a number of alternative optimal solutions are compared based on robustness. In some cases the model provides an increase in robustness of up to 35% as compared to the original string model. Furthermore, the optimal cost of the final solution was preserved.

“Constructing Robust Crew Schedules with Bicriteria Optimization” [17] from 2002 by Ehrgott and Ryan describes the construction of robust crew schedules as a measure of avoiding disruptions. The basic observation is that knock-on effects from disruptions are often caused by short ground times between consecutive flight legs in a roster requiring change of aircraft. Rosters chosen to minimize costs often contain short ground times and are therefore vulnerable to disruptions. Therefore, a method to generate rosters with longer ground times and fewer changes of aircraft will have a potential for generating a robust crew schedule. Technically, the robustness of a roster is measured by the expected amount of delay based on historical data. The delay is only incurred if a change of aircraft occurs after the delayed leg. The problem now becomes a bi-criterion multi-objective generalized Set Partition problem. The paper describes the development of a constraint-based solution method, which is then applied to data from a domestic New Zealand carrier. The results show that at a small cost, robustness can be built into the generated rosters. The extra cost originates in an increase in the number of two days rosters. The rosters in general display larger median ground times. Also, experiments aiming directly at minimizing the number of aircraft changes are reported. These again show more two days rosters, but surprisingly also smaller median ground times due to that some very large waiting times between consecutive flights are eliminated by the two days rosters.

Shebalov and Klabjan addresses the robustness in crew pairing from another point of view in “Robust Airline Crew Pairing” [36], where the idea is to construct pairings, which allows for “move-up crew”, in the DESCARTES project called crew re-linking. When a crew c_1 member comes in late, it may be possible to let another crew member c_2 take over the line of work of c_1 and let c_1 fly the remaining part of c_2’s line of work, i.e. to perform a relinking of the linked schedules of c_1 and c_2. The hypothesis of the paper is that plans with many possibilities for crew move-up are less sensitive to disruptions.

Maximizing the number of possible move-ups is achieved by including the number of move-up crew in the objective function of the pairing problem. The traditional Set Partition model for the crew pairing problem is extended with constraints describing the feasibility of potential move-ups, e.g. that the two pairings in question has the same number of days remaining.

The model is solved using a combination of Lagrangean relaxation and column generation. The approach is then tested on three real-life instances with varying number of crew bases and hubs. The computational environment is a PC with a Pentium III 333 Mhz processor and 512 MB memory using CPLEX 7.5. The paper contains a detailed evaluation of the effect of different parameters on the solution as well as a study of the robustness of the generated pairings compared with the original pairings. The robustness is measured by number of values as e.g. deadheads, number of uncovered legs, and operational flight-time-credit (which measures the operational crew cost). The comparison is also given in terms of annual savings, showing considerable savings with one disruption per day, and less saving in case of fewer disruptions. The main conclusion is that “there is a fine line where the trade-off (between robustness and crew cost) is beneficial and robust solutions produce significant annual savings”.

6 Conclusion and Further Research

The field of disruption management in the airline industry has been increasingly active over the last decade, and in the last years also commercial tools for disruption management have become available.

The requirements for a tool as seen from the airline companies are, however, still substantially different from the services offered by commercial tools, and from the performance seen in all the prototype tools proposed in the literature. The fact that virtually all papers published address single reaource systems (aircraft, crew or passenger recovery) is indicative of this fact. Although development in computational speed indicate that during the next decade a number of performance infeasibilities will be resolved, the substantial gap between the ideal integrated recovery tool and the prototype tools proposed by software companies and research institutions will most likely not be closed in the near future.

It is worth mentioning that despite the large number of papers on the topic, the underlying graph models are more or less identical, and the resulting mathematical programs are in most cases multicommodity flow problems with side constraints.

A large number of subjects for further research within the field of disruption management are readily available. We mention just a few of these here: Quality versus computing time for both dedicated and integrated recovery methods, disruption management versus robustness, and disruption management and robustness for other transportation industries as e.g. the railway industry. Therefore we expect that disruption management will be a very active research area in the future, both in the context of transportation, and more generally in connection with logistics as e.g. supply chain management optimization.

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Appendix

Planning and Disruption Management - Concepts and Terminology

The airline industry is known for its extensive use of acronyms, abbreviations and specialised jargon. In this section we introduce the most essential concepts and terminology related to the planning processes on the day of operations.

Furthermore, we provide the reader with an introduction to more general papers dealing with planning problems on the day of operations within the airline industry.

Concepts and terminology

Terms and concepts used in planning and operations:

Timetable	The official set of flights to be flown by the airline described by the departure and arrival times for each of the flights in the programme.
Schedule	A set of plans listing all tasks to be completed in order to cover the flight programme. The term is also used to describe the present situation: The operations are said to “run according to the schedule”.
Planning	The process of planning the tasks to be carried out before the plans are published. The exact timings of the planning phase differs across airlines and the resourced being planned. The planning phase for the flight and the cabin crew members usually ends 4-6 weeks before the day of operations, when the crew planning departments publish the crew rosters for the coming 4 weeks period.
Tracking	The process of monitoring and maintaining the plans between the time of publication and the day, when plans are handed over to operations control. Again, the exact timings differ across airlines and the resources being planned for. Usually, the plans are handed over to operations control 24 hours before the day of operations for short-haul flights and three to five days before for long-haul flights.
Operations Control	The process of managing all resources (i.e. aircraft, crew, passengers, cargo, terminals, catering etc.) on the day of operations.

General airline industry terms:

Base	An airport central to the airline. It is usually the base for both crew and aircraft. In a hub-and-spoke network the base corresponds to a hub.
Out station	An airport that is not a base airport for the airline in question. In a hub-and-spoke network the out stations correspond to the spokes.
Curfew	Special restrictions for an airport regarding conditions for operating aircraft. A curfew may e.g. define a certain time interval for which specific aircraft types cannot operate in that airport. One example is the *night jet ban* which means that landings and take-offs are forbidden during a specified time interval.
Long-haul	A long distance flight, typically used for intercontinental flights.
Short-haul	A short to medium distance flight, typically used for national and transcontinental flights.
Through connection	Two flight legs to be flow by the same aircraft, usually for reasons of convenience for passengers.

Concepts and terminology specific to crew:

Duty	A set of subsequent flights for a crew member. For some airlines a duty is defined as the set of flights spanning one day.
Pairing	A set of subsequent duties starting and ending at the crew members home base. Some airlines also refers to a pairing as a trip, indicating the round-trip for the crew.
Roster	A set of subsequent pairings starting and ending at the crew members home base. In addition to activities as flying flight legs, the roster also holds off-duty days, leave, training etc.
Dead-heading	Re-positioning of crew. The crew members fly as passengers in order to be available at another airport. Dead-heading crew is costly as the crew takes up seats and is paid to fly as passengers.
Night stop	a Crew member's duty ending at an outstation. The crew member will be away from the base during the night. Night stopping is expensive as this incurs costs for hotel accommodation and allowances. On the other hand, night stopping is necessary for an airline to be able to offer early flights from outstations to the base.

Concepts and terminology specific to aircraft:

Fleet	All aircraft of a specific type used by an airline. An airline is said to be a single-fleet airline when the airline only operates one specific type of aircraft. For example, Ryan Air is a single-fleet airline since they only operate Boeing 737 aircraft. Multi-fleet airlines, on the other hand, operate several types of aircraft. Each fleet of aircraft may be divided into subfleets. For example, an airline may decide to operate part of their 737 fleet without business class seats to increase the passenger capacity.
Fleet assignment	The process of making the initial assignment of each flight to a particular aircraft fleet.
Aircraft rotation	The route or the schedule of a particular physical aircraft. Identical to aircraft routing.
Tail number	The unique identification of a particular aircraft in a fleet, also called the aircraft registration.
Tail swapping	The process of moving flights planned for a particular aircraft registration to a different registration of the same aircraft type.
Ferrying	Re-positioning of aircraft, i.e flying without passengers. Ferrying is extremely costly and is used very rarely.

Concepts and terminology common to both crew and aircraft:

Turn-around-time	The minimum required time in the schedule from the arrival of one flight leg to the departure of the subsequent flight leg. The turn-around-time for an aircraft is usually used for refueling, loading and unloading the baggage, cleaning, reloading catering etc. Each aircraft type has a specific minimum turn-around-time. Larger aircraft usually have longer turn-around-times as it takes longer time to refuel and clean a larger aircraft. The minimum turn-around-time for crew is a union-agreed time period which might depend on the airport, whether or not the subsequent leg is an international flight and the time of day.
Open flight	Also referred to as an uncovered flight. A flight leg which has not been assigned the required number of cabin or flight crew members or alternatively a flight leg that has not been assigned a specific aircraft tail. It is the responsibility of the Operations Controllers to ensure to that all open flights are covered at the time of take-off.
Standby	Crew or aircraft not assigned to a particular flight leg. Standby aircraft and/or crew are said to be a free resources, which can be allocated to uncovered flight legs.

Disruption management definitions:

Disruption	An event or a series of events that renders the planned schedules for aircraft, crew, etc. infeasible. A complete list of disruptions is not possible as a disruptions often is caused by a combination of events going wrong at the worst point in time. Simple disruptions include e.g. a late incoming aircraft due to technical problems before take off at the preceding departure airport, crew calling in sick, technical problems with an aircraft, and bad weather implying a reduced number of operations at the airports.
Delay	The situation when a flight is coming in late due to unforeseen circumstances.

Tools of recovery within disruption management:

Retiming	The change of the departure time of a flight to later point in time, usually due to a delay or another disruption.
Fleet swapping	The process of moving flights planned for a particular aircraft registration to a different registration on a different aircraft type (compared with tail swapping).
Re-linking	Splitting a crew member's original itinerary due to a disrupted service, assigning an alternative sequence of legs to operate.

Terminology for Disruption management systems:

Dedicated recovery system	A system producing feasible options for a specific resource. For example, a dedicated flight crew recovery system resolves disruptions by looking solely at the flight crew resource and ignoring aircraft, cabin crew, passengers etc.
Integrated recovery system	a system producing options that are feasible across the resources in question. An integrated recovery system must have access to all information for the resources affected in order to be able to produce a set of options that are feasible for these.