Agent Based Individual Trafficguidance

Abstract

When working with traffic planning or guidance it is common practice to view the vehicles as a combined mass. From this models are employed to specify the vehicle supply and demand for each region. As the models are complex and the calculations are equally demanding the regions and the detail of the road network is aggregated. As a result the calculations reveal only what the mass of vehicles are doing and not what a single vehicle is doing.

This is the crucial difference to ABIT (Agent Based Individual Trafficguidance). ABIT is based on the fact that information on the destination of each vehicle can be obtained through cellular phone tracking or GPS systems. This information can then be used to provide individual traffic guidance as opposed to the mass information systems of today – dynamic roadsigns and trafficradio. The goal is to achieve better usage of road and time.

The main topic of the paper is the possibilities of using ABIT when disruptions occur (accidents, congestion, and roadwork). The discussion will be based on realistic case studies.

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1 Traffic Today

The objective of this document is to describe different situations in the ABIT (Agent Based Individual Traffic guidance) system. First I will cover the traffic situation of today. After this follows a section on disruptions. The two layers of ABIT are then described and a section on future perspectives conclude the paper.

The commuting of people is inevitable. Everyday we travel to fulfill demands of our daily life. We work, shop, workout, ferry our children and do many other things that require us to move from one place to another. Usually there is a choice of mode (car, bicycle, public transport) when travelling. In traffic modelling the populace is considered as homo economicus – the rational human. Given a choice the homo economicus will always choose the option providing the most convenience. This can be a very deterministic approach, but through advanced modeling and calibration it is possible to provide valuable insight and forecasts on the traffic of today and tomorrow.

The applications of models to traffic modeling and planning have lead to efficient, necessary and qualified decisions. These same models have repeatedly proven the most basic rule of mass car transport today. In 1952 Wardrop enunciated Wardrop's first principle: *"Under equilibrium conditions traffic arranges itself in a congested network in such a way that no individual trip maker can reduce his path cost by switching routes."* In traffic this is considered to be the Nash equilibrium. In other words, the percieved utility or cost of each route is the same under congestion. It is important to note that this does not mean that all routes take the same time, only that all routes are perceived equal *to the drivers*. Personal preferences may make one driver choose a more beautiful but longer route while another driver chooses a less time consuming route.

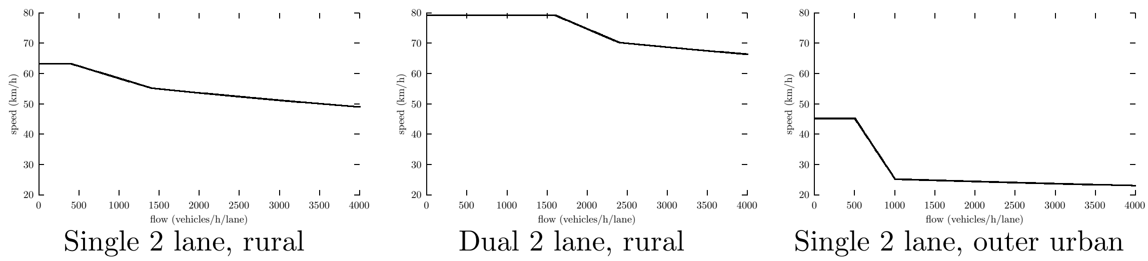
Wardrop's second principle is also interesting: *"Under social equilibrium conditions traffic should be arranged in congested networks in such a way that the average (or total) travel cost is minimised."* As [Ortúzar and Willumsen, 2001] illustrates by a simple example the social equilibrium is 0.5% better in the total travel time than the Nash equilibrium.

In a recent published article [Roughgarden and Tardos, 2002] on selfish routing the authors find that the Nash equilibrium can be far from the social equilibrium. In fact for complicated speed/flow relationships the ratio between the two equilibria is theoretically unbounded.

A most interesting statement also proposed in the article is: *... to match the performance of a centrally controlled network with selfish routing, simply double the capacity of every edge.*

In most cases drivers choose a route based on the time it takes to traverse it. This is also what most in-car navigation systems and route planning services do. In regular (non-congested) traffic nearly all cars follow the same route from origin to destination. As the traffic flow increases, the speed of a section decreases thus making the preferred route slower. Other routes become attractive and the traffic is diffused into the infrastructure. In this way autonomous vehicle traffic exhibits a form of self balanced sifting.

Interestingly a heterogeneous group of anonymous more or less selfish drivers almost fulfill



Single 2 lane, rural Dual 2 lane, rural Single 2 lane, outer urban

Figure 1: Speed/flow relationship for different road sections

Wardrop's first principle. Figure 1 shows traversal times for different types of road section given the flow into the section[1]. As the flow increases so does the traversal time. Here it is worthwhile to examine two routes at the same time.

Consider two routes, A and B, obeying the single 2 lane outer urban speed/flow relationship. At some point in time route A is 10% faster than B (flow 940 and 1000 respectively). Given the slope on figure 1, which at the onset of rush-hour is steep, only 3% of the cars (30) shall change route to obtain the Nash equilibrium.

For most networks there is as mentioned above a better social equilibrium, but this cannot be obtained by the action of any individual driver. Some cars must choose a worse route in order to make other cars get a sufficiently better route to achieve the social equilibrium. The situation can be described as a solution space with a local minimum (the Nash equilibrium) and a global minimum (the social equilibrium) which do not coincide. The problem is that the solution method, selfish routing, is incapable of sustaining the global minimum even if the flow distribution should occur. The problem described above is best reflected in the Prisoner's Dilemma, where no single prisoner can ensure himself a better outcome.

Returning to the models and the forecasts, today no country or major city can expect to see a decrease in the number of car-based commuters unless (more or less) drastic countermeasures are applied. This means that the length and intensity of congestion will become increasingly troublesome.

Congestion in it self is self balancing as described above. The problem is that it only balances properly under normal conditions. If the state and usability of the infrastructure diverges from general perception then the self balanced sifting can in worst case be replaced by a very severe jam or even complete traffic grid lock. At this point the driver becomes the most important part in the relative success in the progression of the traffic.

1.1 The Driver

As described earlier the mass of drivers tend to distribute themselves according to the Nash equilibrium. This happens primarily because each driver can be seen as an autonomous entity. At some point the driver will try another route and if perceived to be better then it will become the new preferred route. The term "preferred route" is used to reflect what is often encountered in traffic modeling: conservative drivers. Drivers stick to a specific route and if content with it they will seldom try other routes. This means that if the infrastructure changes then it will take some time before the equilibrium is reestablished. Trying a new route may be caused by several reasons, eg. curiosity, impatience, or advice. The driver may try something new for a change, become unsatisfied with the current route (to slow, long, boring, etc.), or the driver might have heard of a better route.

Above, this conservative attitude is presented almost as having a negative impact on the traffic, but this is not entirely so. If drivers are not conservative, the equilibrium would not become steady as too much of the flow would change path every day.

The behavior of drivers is important to mention. Although conservative attitude is good for the equilibrium, inconsiderate self righteousness is very bad for general flow. The steady state is still obtained by being conservative. The problem is that the often inconsiderate behavior might make the aggressor get a little bit (under congestion a very little bit) faster ahead, but at the expense of everybody else. The result is that the capacity of a given intersection is impeded by selfish or inconsiderate behavior.

The conservative attitude has besides the equilibrium conservation at least one other important impact. Recent research [Abdulhai and Look, 2003] shows that proposing new routes must be done with great care as it is shown to have impact on general safety. New routes mean that people do not know the local traffic conditions and evasion routes (bypassing congestion) are

[1] These are generated from the formulas given in [Ortúzar and Willumsen, 2001] page 326.

more complex[2].

1.2 The Communication

When driving to a predetermined destination we usually have a route or a very small subset of routes in mind; a preferred set of paths to the destination.

As mentioned above this subset of routes tends to be static. When altered, it is usually because we think or know that another route is better. The question is how can it be known if some route is better than another.

This can be done either by guessing, by becoming sufficiently unsatisfied with the current route or be told. Not so many years ago the latter was only performed primarily by the speaker on the traffic radio. This reaches many drivers, but it requires that the driver listen to the radio and it is the right station or the driver has a car radio that can switch to traffic announcements automatically.

Recent advancements in communication and surveillance have made it possible to add new and entirely different forms of communication to the "telling". Some of these are dynamic message signs (DMS)[3] and real time status messages available over fax, short message service (SMS) or World Wide Web. Se [MTO, 1999] or [DRD, 2004] for examples of the latter.

All of this information has increased safety as well as the utilization of the infrastructure.

2 The Future for Road Based Traffic

Over the recent years the amount of vehicles concurrently in transit has increased at an alarming rate. In Copenhagen, a capital with public transportation, a recent rapport [HUR, 2004] from The Greater Copenhagen Authority has shown that the average traveling speed on the highways during morning rush hour has dropped 15% to $37km/h$ in one year. As the critical mass of vehicles are approached on the highways so is the traffic in the center of the city. Traffic jams and grid locks are there to stay.

If we focus on the increasing vehicle traffic and leaving out alternatives two options are possible. Either increase the capacity of roads, which is immensely expensive, or change the usage of roads, which is equally complicated.

The latter is already attempted through the previously described communication media and through the use of dynamic intersectional control. The problem is that in most cases the individual driver has to decide solely by himself what to do in different traffic situations.

The idea of ABIT is to address this issue. Instead of forcing every driver to make his own choice unaided the system can propose a set of alternate routes. These can be based on far more information than can be communicated to the driver and thus work as decision support for route choice.

I have previously described how equilibrium was established between possible routes under normal conditions. Intelligent vehicle-wide routing will decrease the time to reach the equilibrium, thus reducing the overall transport cost.

The problem addressed in my PhD study is *what if it is not under "normal conditions"*. In theoretical terms, this means the system is disrupted or suffering from a disruption.

3 Disruptions

A disruption is a state where the expected best routes are significantly deteriorated.

[2] In the article the authors explicitly point out that the number of turning movements in non-preferred intersections is essential to the number of accidents under high load.

[3] Dynamic message signs are also called variable or changeable messages signs.

In other texts on traffic these states are simplified to incidents. Distinct occurences that are easy to identify and classify. A disruption is a much more general term for disturbances in the system.

The ABIT system is meant to cross the gap between dynamic traffic guidance and Operations Research (OR). The field within OR that is combined with traffic is *Disruption Management* (DM). DM researches in solution methods for resource efficient replanning when unforeseen events occur, see [Clausen et al., 2001] for further information.

Disruptions come in many shapes and sizes and a few of them can be:

- Accidents.
- Emergency vehicles.
- Demonstrations, parades or sport events.
- Congestion, jams and grid locks.
- Road work.
- The weather.

The main problem with disruptions is that they more or less immediately reduce the capacity of some part of the road network. If the load is below the new capacity nothing happens. The infrastructure can still accommodate the demand. The problem will occur if the load is above the new capacity. Depending of the degree of the overloading, queues will build up and the traversal time for routes affected by the queuing will increase, thus making them less attractive. This is exemplified in congestion where the significant presence of vehicles results in performance degredation.

Every disruption can be characterized in two ways. Firstly it can be described by its spatial representation – how large is the geographic area that is affected and if it is moving. Heavy snow is usually regional or global. An emergency vehicle is sectional and roaming as it moves around. A severe accident in an intersection may be multisectional or larger.

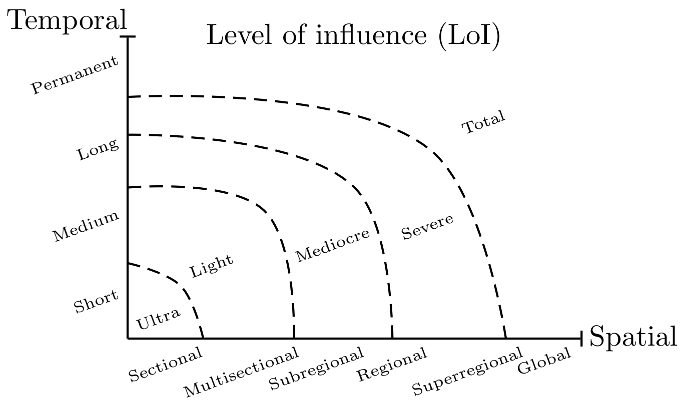
Secondly, it can be described by is temporal locality – the duration of the disruption. Minor accidents which are quickly alleviated exhibit short temporal locality, whereas major construction work is an example of long temporal locality. Similar to roaming spatial locality the term periodical temporal locality is used for repeating disruptions with a fixed schedule.

Even though useful, the two characterizations are not sufficient. A stochastic function on the spatial and temporal locality, the level of influence (LoI), can here be introduced. The LoI of a disruption is the impact on the total system. It is generally a measure of how much that must be changed to achieve a new equilibrium. This function is the only measurement that actual assesses the influence on the system and not only the disruption itself. An example could be a sectional and medium accident under light traffic. This could be interpreted as light or ultra light as only local dispersion is needed to achieve a new equilibrium. On the other hand, the same accident during rush hour may exhibit severe LoI as the flow of diverted vehicles has nowhere to go. This usually causes long queues, which in some cases ultimately result in a grid lock.

Figure 2 on the following page is a visualization of the characterizations and the mean value of the LoI. The stochasticity of the function is introduced to cover examples as described above, where the same temporal and spatial locality can lead to different levels of influence.

The problem with a disruption is that it by definition is hard to predict. If it could be foreseen it would be considered advance knowledge. Knowledge that can be incorporated into the system in proper time and would potentially remove a disruption before it occurred.

A significant issue with a disruption is the driver's reaction to it. Will the driver stay on the current route or will another route be chosen? It is not an easy task to make the driver choose the best route.



The figure shows the mean value for the stochastic LoI function. The stochasticity is a result of the condition of the network and unforeseable consequences.

Figure 2: Classification guide

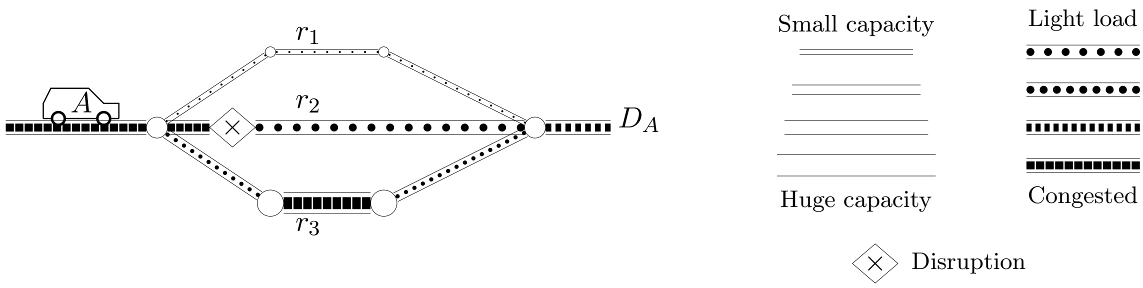


Figure 3: Example setup with one disruption

An abundance of information must be communicated to and analyzed by the driver if a qualified decision is to be made. This could become a significant safety hazard as cellular communication has proven to be. Here ABIT becomes interesting since it can act as decision support for the individual driver. There is much information available in the system, such as:

- The nature of the disruption
- Traveling speed for all roads in the network
- Optimal undisrupted route
- Optimal route given current conditions
- Impact of disruption over time
- Flow rates in the network over time

This is analyzed and only relevant information is presented to the driver. In the following sections I will cover the main ideas of ABIT in different situations.

3.1 Immediate Dispersion

The first and most obvious application is the immediate alleviation of a disrupted situation.

Consider a simple setup as in figure 3. Vehicle A has just encountered a queue on its way to destination D_A. In the setup there are three immediate alternatives: 1) stick to the original route; 2) choose the upper route; or 3) choose the lower route. For some reason the driver chooses the lower route thus running into another queue. What the driver did not know was

dir	current	normal
$\Uparrow$	10min	5min
$\Leftarrow$	7min	6min
$\Rightarrow$	11min	6min

a)

dir	current
$\Leftarrow$	7min
$\Uparrow$	+3min
$\Rightarrow$	+4min

b)

dir	current
$\Uparrow$	7min
$\Leftarrow$	+0min
$\Rightarrow$	+0min

c)

Figure 4: Examples of dashboard information

dir	current
$\Uparrow$	8min
$\Leftarrow$	9min
$\Rightarrow$	13min

Vehicle A
non-biased

dir	current
$\Leftarrow$	7min
$\Uparrow$	10min
$\Rightarrow$	13min

Vehicle B
biased to make a left turn

The two vehicles A and B are in the same queue heading for approximately the same destination.

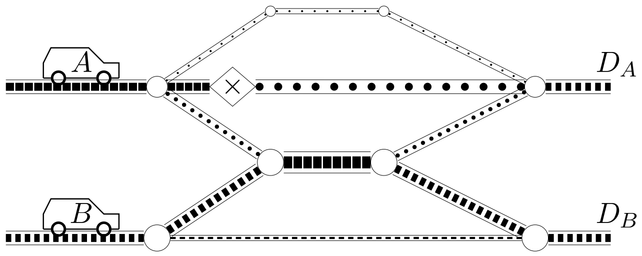
Figure 5: Example of selective biased dashboard information

that the traversal times for each route was 1) 10 minutes; 2) 7 minutes; and 3) 11 minutes. In this case the driver would have saved 4 minutes if information was available.

The essense of the decision is that it has to be made within a very short time. From A's perspective the decision is immediate, hence the term *immediate dispersion*. A driver in an ABIT equipped vehicle could be presented with the information on the dashboard as it is depicted in figure 4 a). Thus allowing the driver to easily make a qualified choice. In theory, the system could at every intersection indicate the benefit/loss for choosing a significantly different route as shown in 4 b), thus implicitly promoting dispersion even when the system is undisrupted. In an non-disrupted congested system the dash should look like 4 c) if the Nash equilibrium has been obtained.

Dispersion of a single vehicle is rather simple; find the new best route and propose that to the driver. Many people will choose accordingly. Others conclude that if the difference is 3 minutes, they will stick to the direct route. Finally, some may choose the right turn as they do not mind the extra time or simply just like the detour. Inspecting the speed/flow diagrams in figure 1 on page 2 shows that increasing the flow on a route most likely increases the traversal time of that route. If ABIT directs all vehicles to the upper route it too will become congested. Consequently, as the upper route has less capacity than the other two the impact can be much more severe than the 3 minutes saved in the single case.

Considering A's choice the crucial point is to divert exactly enough cars to make the upper and the direct route become equally time consuming. Given speed/flow graphs for the routes the equilibrium flows can be determined trivially or in not too complex cases it can be done rather quickly by microsimulation. The problem is to make the driver actually choose the right alternative. One approach to achieve the flow split operation could be to trick the drivers. Figure 5 shows a possibility of this selective biased decision support. This incurs one very significant problem – the credibility of the system. If the drivers do not trust the system then it will be of no use. Selective biasing must thus be used with great care and consideration.



Before disruption

dir	current
$\Leftarrow$	8min
$\Uparrow$	+2min

After disruption

dir	current
$\Uparrow$	11min
$\Leftarrow$	+1min

Figure 6: Extended setup with one disruption and dashboard information for vehicle B

It might be the best strategy to actually inform the driver that there are several different biased propositions, but the exact one is chosen for the current driver to promote a specific route choice.

Up to this point we have only considered immediate single route traffic and left interfering routes and traffic out of the considerations. If we broaden the scope and start considering just a little more of the world, then the situation becomes more complicated. Referring to figure 6, which is an extended version of figure 3 on page 6, consider the added vehicle B. As the system has all ready assessed some vehicles will be dispersed onto the shared section. This will increase the traversal time on it and consequently make the previously faster route slower than the straight alternative. Without ABIT, B would simply follow the expected fastest route and thus use an extra minute due to lack of information.

Both examples above only illustrate rather small savings, 3 minutes and 1 minute respectively. However consider the number of vehicles that could be involved. In rush hour thousands of vehicles will be on their way in the infrastructure. [TCC, 1999] states that the road system of Toronto accommodated no less than 5,2 million vehicles each day in 1996 (expected to become 8,1 million by 2021). If ABIT helps just 1% of these in saving 1 minute every day, then ~ 870 man hours would be saved every day. In 2021 the number of hours will be ~ 1350.

I believe that both the fraction of affected vehicles and the number of minutes saved is a pessimistic scenario and that they are in reality significantly higher. As the system is not yet developed or tested, the only indication of the possible impact is the effect of a currently widely deployed approach – DMS. As traffic safety is significantly increased the number of incidents are reduced and thus the overall performance of the infrastructure has risen.

So far, we have only considered the operation of immediate dispersion where ABIT acts as decision support to prevent the build up of back queuing. This is not the only intent of ABIT all though it is very important.

In [Lighthill and Whitham, 1955] the concept of kinematic waves are described in relation to vehicle traffic. From detailed formulations based on flows in floods the authors deduce similar behavior on roads. The essence is that at every point where there is a capacity reduction (e.g. disruption) or sudden surge in load (e.g. at intersections) waves of lower-than-average speed are generated. These consequence waves are then propagated through the traffic at different speeds. Some of the waves actually become shockwaves due to the capacity and load of the sections it traverses.

A shockwave is characterized as a situation where the vehicles rather suddenly comes to an almost complete halt and then slowly regains speed. According to the article the only way to make a wave fade is by controlling the speed or flow of vehicles just before the wave. The speed can be controlled by asking the drivers to slow down in advance and the flow can be decreased by diverting traffic before the wave.

Even though the immediate dispersion has been inaugurated the disruption is not necessarily dealt with. The simple extension above showed the effect on nearby vehicles. If the disruption

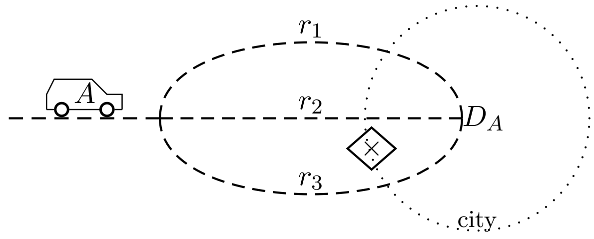


Figure 7: Individual alleviation example

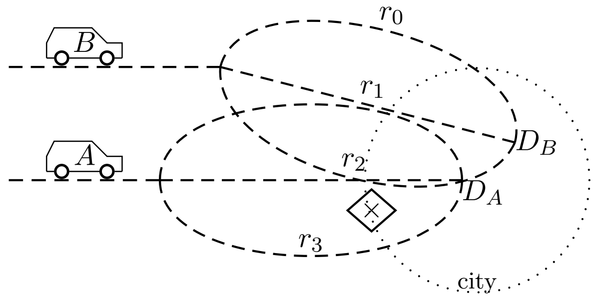


Figure 8: Extended individual alleviation example

has medium LoI or even higher, the number of vehicles and routes affected increase dramatically. At this point immediate information is not sufficient and both theory and practice become increasingly complex as we turn to individual alleviation.

3.2 Individual Alleviation

In the previous section immediate dispersion dealt with the local, both spatial and temporal, impact of a disruption. However, individual alleviation is the overall measure to minimize the impact of a disruption. The idea is to divert traffic in a much larger scope than with immediate dispersion.

Consider figure 7, vehicle A is heading for the destination D_A. As the disruption $\times$ happened earlier it has all ready been assessed to be of severe impact. Dispersion of traffic is thus impeding routes r_2 and r_3. At present route r_1 is best, but as it takes (according to the system) 15 minutes to reach the impeded area the disruption may be alleviated at that time. If that was the case, then both r_2 and r_3 will be faster than r_1. This little scenario depicts a very onerous complication in individual alleviation, the concept of time and propagation. I expect that this combined with the nature of disruption is the crucial success criterion in ABIT. Extensive modeling and forecasting must be applied to get anywhere near a viable approach.

If the disruption is severe it is unlikely that the situation returns to normal within the relatively short time span of 15 minutes. The expected best route for vehicle A is then r_1 in this situation.

If we extend the scenario with vehicle B as in figure 8 we again see that simply diverting A might cause secondary disruptions (jams due to insufficient capacity), if B continues onto the direct route. Depending on the prediction of the flow over time for the individual sections of the route, B may arrive sooner by continuing along route r_0.

As with immediate dispersion we bend the routes to accommodate the new flow pattern. Figure 9 depicts different bending patterns according to a disruption levels of influence.

The problem with redistribution of flow, both immediate dispersion and individual alleviation, is that the disrupted infrastructure might not be able to accommodate the flow. In theory this means infinite queuing, in practice it yields jams and in extreme cases grid locks where the traveling speed is close to zero.

At present, we have only considered dissipating disruptions. It is important to realize the system's potential when experiencing imminent disruptions. An imminent disruption is a dis-

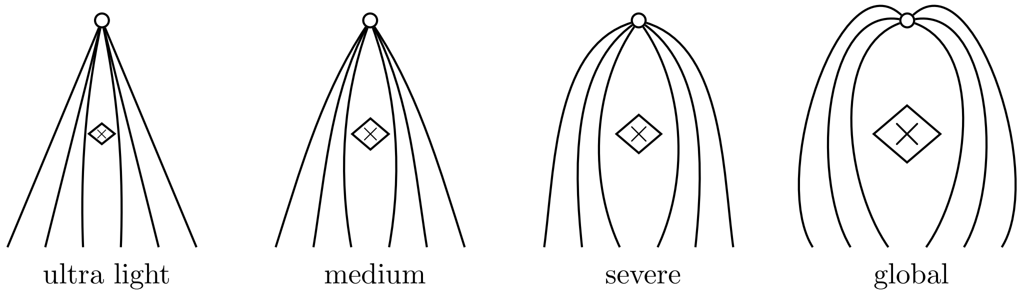


Figure 9: Level of influence (LoI) bending patterns

dir	status	current
$\Uparrow$	minor accident	10min
$\Leftarrow$		7min
$\Rightarrow$	heavy traffic	11min

You are advised to continue straight ahead as the left route is expected to become congested. Keep left – right lane ahead is closed in next intersection.

dir	status	current
$\Leftarrow$		7min
$\Uparrow$	minor accident	10min
$\Rightarrow$	heavy traffic	11min

Distance to next intersection: 400m
Time to next intersection: 2min
Intersection layout $\leftarrow \,||\, \Uparrow \,|\, \otimes \,||\, \rightarrow \,|\, \rightarrow$

Figure 10: Examples of enhanced dashboard information

ruption that is bound to appear and is known in advance. An example is roadwork. A specific section or intersection has reduced capacity as new tarmac must be applied. Outside rush hour this is no problem as the load is below the reduced capacity. Flow forecasting will indicate that if traffic is distributed as usual it will result in a disruption. The intent of ABIT in this case is to reduce the disruption by diverting traffic even before the problem becomes critical. A side effect of this is that the consequence waves are reduced.

3.3 Disruption Avoidance

Imminent disruptions is the first step to actual disruption avoidance, which can be interpreted as a proactive remedy to avoid disruptions. Based on accident prediction models, flow forecasting and capacity assessment it might be possible to reduce the accident risk, which may be a way to reduce the number of accidents and their severity.

Combined with LIWAS described in [Hansen, 2004] it may prove even more efficient to increase both safety and utilization of the infrastructure. Figure 10 shows some examples of enhanced information to the driver.

4 Further possibilities

Once the system is operational the abundance of information gathered can be put to even further uses. Except for the extreme surveillance possibilities, which are inherent to the system, several other more user-oriented applications are possible.

A very simple yet very important usage is the in-vehicle warning signal on inbound emergency vehicles. In these cases every minute counts and thus appropriate warning of involved vehicles and extended intersectional control may decrease the transit time for the emergency vehicle.

Furthermore, the light controlled intersections can become increasingly intelligent if they are not only programmed to perform a specific schedule. They can also be controlled according to the flow information in the system, thus increasing the vehicular throughput in specific directions to alleviate a disruption or increase overall flow.

The information can also be used for the public transport as road based vehicles can be forecasted with much better precision than today. GIS is getting increasingly implemented in public transport thus allowing for arrival time prediction. However, this is based on aggregate modelling and not on exact information as possible with ABIT.

A more commercially oriented application could be parking lot assignment. The driver selects which part of town or specific lot and the system provides route and reserves a lot for the car.

5 Conclusion

Even though the potential in ABIT is encouraging, there are some important issues that must be considered.

Gathering the real time status information is today insufficiently precise, as per section status must be available. [Roughgarden and Tardos, 2002] calculates the impact of imprecise or out-of-date data in the decision process. The conclusion of their research in relation to ABIT is that slightly out-of-date information is sufficient. The system will not need to know the exact state of entire network all the time. It might therefore be sufficient to use only a fraction of the vehicles to gather real time status of the infrastructure.

Immediate dispersion is, when compared to individual alleviation, theoretically simple. In practice the question of the necessary fraction of ABIT-enabled vehicles is crucial. Is it sufficient to have 10% of the vehicles ABIT-enabled to make a difference or is the critical fraction even higher?

The Nash equilibrium example showed that only a fraction of vehicles is necesarry to reobtain equilibrium in slightly pertubed situations. I believe that the greater the fraction of ABIT-enabled vehicles the higher LoI of disruptions can be alleviated.

The technical details of communication and server structure as well as in-vehicle implementation is also an unexplored field. Ongoing work on the ex-hoc infrastructure in [Hansen, 2004] might yield valuable insights to this problem.

Given the lack of enforcible incentive (such as the London congestion charge) for the individual driver, I expect that ABIT will only be capable of guiding to the Nash equilibrium and not the social equilibrium. ABIT is inherently depending on the autonomous driver and can therefore only try to make a difference. It is up to the drivers to actually make the difference.

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