Optimal Design of Hierarchical Ring Networks

Tommy Thomadsen, *IMM, Technical University of Denmark, Kongens Lyngby, Denmark*
Thomas Stidsen, *IMM, Technical University of Denmark, Kongens Lyngby, Denmark*

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Introduction

Communication networks are often divided into subnetworks which are organized in a hierarchy containing more levels. The subnetworks may be of varying size and may be either ring networks or general mesh networks. We consider a two-level hierarchical network design problem where the subnetworks are rings, i.e. a hierarchical ring network design problem (HRNDP). The hierarchical ring network consists of local rings and a backbone ring interconnecting the local rings. An example of a hierarchical ring network is shown in figure 1.

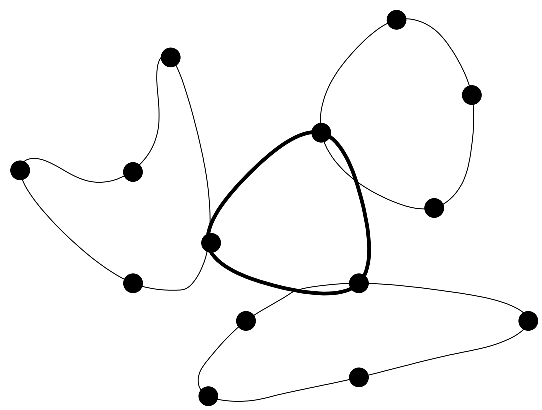

Figure 1: *An example of a hierarchical ring network.*

HRNDP optimizes total ring length (i.e. cable costs) and total capacity amounts (i.e. routing equipment costs) at the same time. Proestaki and Sinclair [4] describe a heuristic for a similar problem whereas we consider an optimal branch-and-price algorithm. The problem is generalizable to more levels, but this is not considered for the moment. We consider topological issues, so the problem does not depend on the particular type of routing equipment used. As the hierarchical ring network consists of interconnected rings, it can survive single-cable breakdowns and if dual homing is considered (i.e. all local rings have two nodes on the backbone ring), then the network can survive single-node breakdowns as well.

The pricing problem of the branch-and-price algorithm is a TSP like problem with a quadratic objective. We denote this problem the quadratic selective traveling salesman problem (QSTSP). QSTSP is interesting in itself since it seems to appear as a related problem or subproblem in other ring network problems, e.g. the p-cycle problem [1, 3, 5]. Gendreau, Labbe, and Laporte [2] study heuristics for a problem similar to QSTSP. We describe a branch-and-cut algorithm which finds provably optimal solutions to QSTSPs with 50 nodes in a few minutes.

In algorithms arising in the area of ring networks, it is common to initially generate a set of rings and choose the best subset among the initially generated set of rings. This has the drawback that potentially good rings are never considered. The branch-and-price algorithm uses information gained from the *current* best set of rings to generate new rings which improve the *overall* efficiency of the design. This is done by means of QSTSP. We believe that other ring problems (e.g. the p-cycle problem) can benefit from this idea.

The Problems

HRNDP is the problem of determining a set of node-disjoint rings (local rings) and a single backbone ring that interconnects all local-rings such that the cost of routing equipment and cables are minimized. In essence what is to be achieved is that nodes that communicate a lot should be on the same local ring and nodes far apart should be on different local rings. Finally the backbone ring is a shortest ring through one node in each local ring (as in figure 1) if single homing is required and two nodes if dual-homing is required.

If one considers a demand with endpoint nodes in different local rings, it will require traversal of (up to) three rings (two local rings and the backbone ring.) However a demand that has endpoint nodes on the same local ring requires traversal of the local ring only. Thus minimizing the routing equipment cost is effectively the same as maximizing the demand handled in local rings. We note that the demand in a local ring can be measured *locally* and hence is a suitable efficiency measure for local rings.

QSTSP is the problem of determining a local ring, i.e. to identify a subset of nodes and a ring on these nodes, such that the demand handled on the ring (weighted in a proper way reflecting the routing equipment costs) minus the cable cost is maximized. Also we add a reward for each node (penalty if negative) to the objective. This node-reward is used to control what nodes are most likely to be in the local ring generated. In essence it makes sure that the rings generated improve the *overall* network efficiency. Denote the weighted demand value by r_{ij}, the cable-cost by c_e and the node-reward by r_i. Let $y_i = 1$ if node i is in the ring, 0 otherwise and $x_e = 1$ if edge e is in the ring, 0 otherwise. The objective of QSTSP is then

$$\max \qquad \sum_{i,j \in V, i<j} r_{ij} \cdot y_i \cdot y_j - \sum_{e \in E} c_e \cdot x_e + \sum_{i \in V} r_i \cdot y_i \tag{1}$$

The demand handled locally on the ring is captured by the product $y_i \cdot y_j$ and hence the objective is quadratic. Note that without the node-reward, the objective would only capture *local* ring efficiency and as pointed out in [1], *overall* efficiency of the network is what is important.

The Algorithm

HRNDP is solved by maintaining a set of local rings, finding the most efficient subset of local rings, and using this information to iteratively generate local rings. When no more local rings that improve the overall efficiency exist, a backbone ring is designed which interconnects all rings. Pseudo-code for the algorithm is shown below.

LOCAL_RINGS = An initial set of local rings.
do
  Find the most efficient subset of LOCAL_RINGS covering all nodes. (*MASTER*)
  Generate local rings improving the solution and add to LOCAL_RINGS. (*PRICING*)
while local rings were generated
Find backbone ring interconnecting all local rings.

We have indicated the master-problem and the pricing-problem. The master problem is a set-partitioning problem (the local rings are the columns) which we solve optimally by linear programming. We use Ryan-Foster branching to obtain integer variables (this makes it branch-and-price). By considering the reduced costs of the local-rings, it can be shown that the pricing problem is in fact equivalent to the QSTSP problem where the dual variables related to each node are the node-rewards (r_i) mentioned above.

QSTSP is solved by branch-and-cut [6]. A straightforward linearization of the objective is used. Provable optimal solutions can be obtained in a few minutes for networks with up to 50 nodes and for networks with 30 nodes in 5 seconds.

Determining the backbone ring is a sort of generalized TSP problem, which, given the sizes of the networks considered is relatively easily solved.

Results

Preliminary results show that we can solve HRNDP with 20 nodes, all edges and full O-D demand matrix in less than 15 minutes. They also show that the main limitation of the perfomance is QSTSP (as expected), since it is called many times. In fact the solution time for the master-problem is negligible. Reducing the number of invocations of QSTSP, can be achieved by improving the initial set of local rings, by better control of the dual variables (i.e. stabilized column generation) and by using heuristics to generate local rings when possible.

We believe that HRNDP can be solved in reasonable time (since it is a design problem) and hope to increase the size of networks that can be considered. Specifically, this may be the case if one does not consider fully connected networks and sparse O-D demand matrices.

QSTSP is an interesting problem that seems to be generally applicable in ring network problems. It has not been widely considered and it seems likely that better solution algorithms can be developed.

Finally we point out that other ring problems can benefit from the ideas given in this paper. In particular the idea (which is built into the branch-and-price algorithm) of using the *current* best set of rings to generate new rings that improve the *overall* efficiency of the design.

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