Estimation of Wind Speed in Connection to a Wind Turbine

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Abstract

The wind speed varies over the rotor plane of wind turbine making it impossible to determine from a single wind speed measurement taken by an anemometer. However, in this paper the wind speed on the rotor plane will be estimated by using a wind turbine as a wind measuring device. To realize the idea, a knowledge of the system characteristics is required, therefore the fundamental relations and principles of system dynamics will be presented. Several estimation methods such as Newton-Raphson method, Kalman filter method and extended Kalman filter method will be investigated in the paper.

Keywords

Wind turbine, Estimation of wind speed, Newton-Raphson method, Kalman filter, Extended Kalman filter.

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1 Introduction

Using a wind turbine for production of electrical energy requires a reliable operation, therefore a control system is usually considered. Some control methods require a knowledge of wind speed. For example, when we apply the gain scheduling method to a wind turbine [19] for compensating the variations in precess parameters and known nonlinear dynamics of the process, the wind speed, which will be taken as a scheduling variable, is the key role for determining the operating conditions of the precess. The wind speed is especially interested, because the nonlinearity of the precess, which comes from rotor dynamics, depends significantly on the wind speed. The fast varying of wind speed changes the working points rapidly.

But it is impossible to measure the effective wind speed on the rotor plane, because of the varying of wind speed on the plane. It is also impossible to predict the true wind speed passing through the rotor plane by measuring it by an anemometer, because the discrepancy between the measured wind speed and true speed on the rotor plane is considerably large. Even if it were possible, the presence of the turbine disturbs the wind speed and the measurement would need to be made at separate site where the wind speed correlated almost exactly with wind speed experienced by the turbine. However, if the wind speed passing the rotor plane can be estimated by using a wind turbine as a wind measuring device, the control system will not have to rely on the wind measurements made by an anemometer, and the estimated wind speed can be used instead of measured wind speed. For gain scheduling method the estimated wind speed can be used as a scheduling variable.

Several estimation methods such as Newton-Raphson method, Kalman filter method and extended Kalman filter method are investigated in this paper, a comparison will be made at last. Realizing these estimation methods requires some knowledges of the system characteristics, therefore the fundamental relations and principles of system dynamics will be presented first.

2 A Wind Turbine

2.1 A Brief Description

WINDANE 40 is taken as an example in this paper. It is a large scale horizontal axis wind power plant with rated power 750 KW. The plant has a three bladed rotor and an automatic yaw system to turn the rotor into the wind, the blade angle is adjusted by an automatic control system. The turbine is operated in nearly constant rotor speed of 34 RPM, and the rotor drives a four-pole generator by means of a gear box in the nacelle. The generator is normally connected to a large utility grid. Pitch control is employed to the turbine by applying PID control method. The system configuration is shown in Figure 1

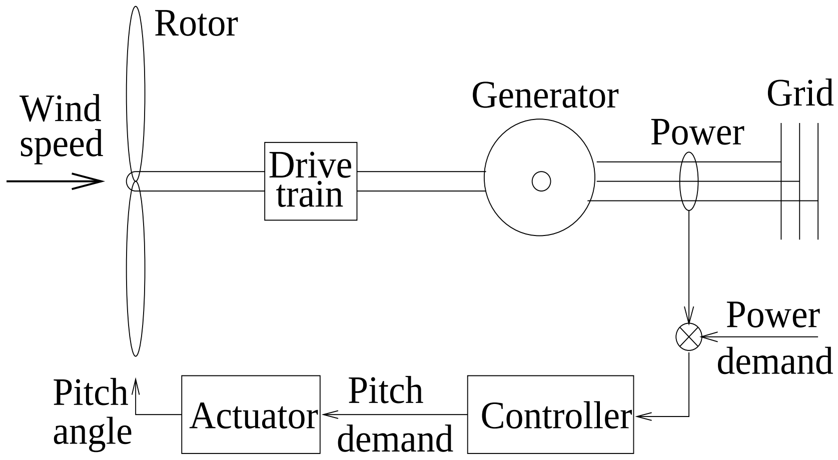

Figure 1: Wind turbine configuration

The wind energy is absorbed by rotor and transformed to mechanical power. Via a drive train the mechanical power is delivered to generator which yields a electrical power. The electrical power will normally be sent to the grid.

The wind turbine is normally operated between a lower and upper limited wind speed. The operation wind speed area of WINDANE 40 is 5 m/s - 25 m/s. The turbine can be started at wind speed of 5 m/s, and shut down at wind speed of 25 m/s. The minimum wind speed at 750 KW electrical power output is 15 m/s. Below this wind speed, the wind turbine will be adjusted to capture the energy from the wind as much as possible. Above this wind speed, the wind turbine will be regulated for power limitation at 750 KW until the turbine is shut down at maximum wind speed. We will restrict our attention to the high wind speed area, and pitch control will be used to the turbine for power limitation and overload prevention.

2.2 Model of Wind Turbine

The wind turbine model consists of several subsystems. The block scheme of the system is illustrated in terms of wind dynamics, rotor dynamics, drive train and generator dynamics, actuator dynamics. The dynamic behaviour of each subsystem will be described briefly in following sections. The block scheme of the wind turbine system is illustrated in Figure 2

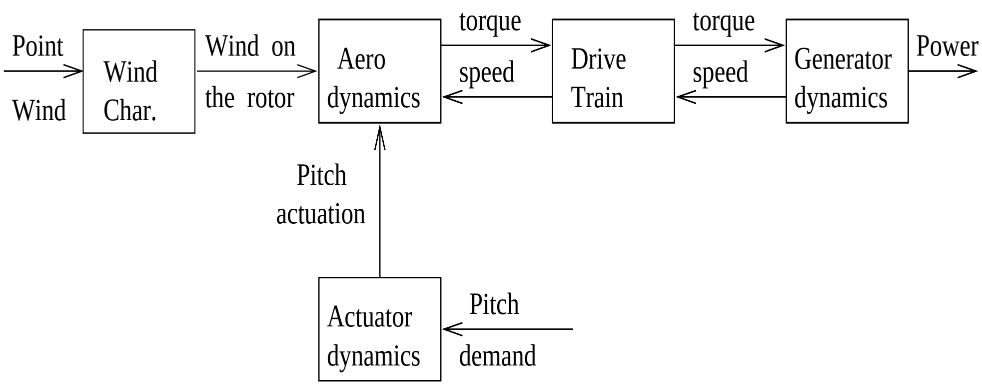

Figure 2: Block scheme of wind turbine

2.2.1 Wind Characteristics

According to Klingenberg [2], the frequency range of wind speed is divided into four regions

$$v_{wind} = v_{year} + v_{month} + v_{hour} + v_{turbulent} \tag{1}$$

each region is illustrated in Table 1.

Table 1: Frequency range of the wind speed

$v = v_{year} + v_{month} + v_{hour} + v_{turbulent}$		
Wind speed	Period	Frequency [Hz]
v_{year}	$\geq$ 2 months	0 - 1.9e-07
v_{month}	2 months - 10.5 hours	1.9e-07 - 2.7e-05
v_{hour}	10.5 hours - 20 min.	2.7e-05 - 8.3e-04
$v_{turbulent}$	20 min. - 0.1 sec.	8.3e-04 - 1e+01

The wind speed v on the rotor plane can be described as a slowly varying average wind speed v_m superimposed by a rapidly varying turbulent wind speed Δv.

$$v = v_m + \Delta v \tag{2}$$

The frequency range of average wind speed is below the hourly frequency range, which can be estimated by filtering the point wind speed measured by an anemometer. The turbulent wind speed will be modelled in the frequency range (8.3e-04 - 1e+01) Hz [6].

The turbulent wind speed on the rotor plane is modelled as a first order system driven by a white noise [2]

$$\Delta\dot{v} = -\frac{1}{\tau_v}\Delta v + \frac{K_v}{\tau_v}e \tag{3}$$

and

$$K_v = \sigma_p\sqrt{\frac{K_1}{v_m}} \tag{4}$$

$$\tau_v = \frac{1}{2\pi K_2 v_m} \tag{5}$$

where $e \in N(0,1)$ is a white noise, and $\sigma_p = 0.12v_m$.

2.2.2 Aerodynamics

The wind turbine is driven by the rotor torque T_r which is associated with the effective wind speed on the rotor plane v, the pitch angle of blades β and the rotor speed ω_r.

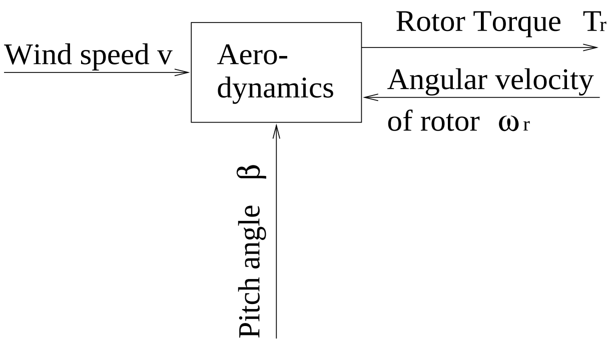

Figure 3: Aerodynamics

According to P.S. Andersen [11], the rotor power P_r is given as

$$P_r = \frac{1}{2}\pi\rho R^2 v^3 C_p(\lambda, \beta) \tag{6}$$

where $\frac{1}{2}\pi\rho R^2 v^3$ is theoretical part, and $C_p(\lambda, \beta)$ is efficiency part, and

ρ	air density $[kg/m^3]$
R	rotor radius [m]
C_p	power coefficient [-]
λ	tip speed ratio [-]

The tip speed ratio λ is defined as the ratio between the wind speed and blade-tip speed

$$\lambda = \frac{v}{\omega_r R} \tag{7}$$

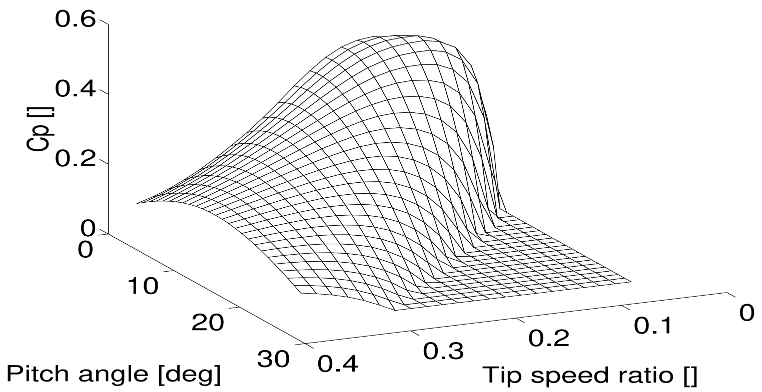

Figure 4: C_p is a function of λ and β (Negative values are replaced by zeros)

The power coefficient C_p is a nonlinear function of the tip speed ratio λ and pitch angle β which is illustrated as three-dimensional mesh surface in Figure 4. Data used in Figure 4 are calculated by Stig Øje.

The relation between the rotor power P_r and rotor torque T_r is

$$T_r = \frac{P_r}{\omega_r} \tag{8}$$

which can be represented as a nonlinear function of wind velocity v, pitch angle β and rotor speed ω_r

$$T_r = f(\ v,\ \ \beta,\ \ \omega_r\ ) \tag{9}$$

and the linearized model is given by

$$T_r = T_{r,0} + \Delta T_r \tag{10}$$

and

$$\Delta T_r = \frac{\partial T_r}{\partial \beta}\Delta\beta + \frac{\partial T_r}{\partial v}\Delta v + \frac{\partial T_r}{\partial \omega_r}\Delta\omega_r \tag{11}$$

where

$$
\begin{aligned}
\Delta v &= v - v_m && (12)\\
\Delta \beta &= \beta - \beta_0 && (13)\\
\Delta \omega_r &= \omega_r - \omega_{r,0} && (14)
\end{aligned}
$$

v_m, β_0, $\omega_{r,0}$ and $T_{r,0}$ define a steady-state operating point. Δv, $\Delta\beta$ and $\Delta\omega_r$ are the small deviations from the steady-state operating point. $\partial T_r/\partial v$, $\partial T_r/\partial \beta$ and $\partial T_r/\partial \omega_r$ can be calculated from C_p curve according to operating point.

2.2.3 Drive train and Generator

The model involves a rotor with inertia J_r and an asynchronous generator with inertia J_g. The rotor torque T_r is delivered to the generator which yields a generator torque T_g through a gear box with gear ratio N_{gear}. The generator is connected to grid and the frequency of grid is ω_0. Since the dynamics of generator is faster than what is relevant for pitch control (about 10 Hz), the generator is modelled by its slip-torque curve.

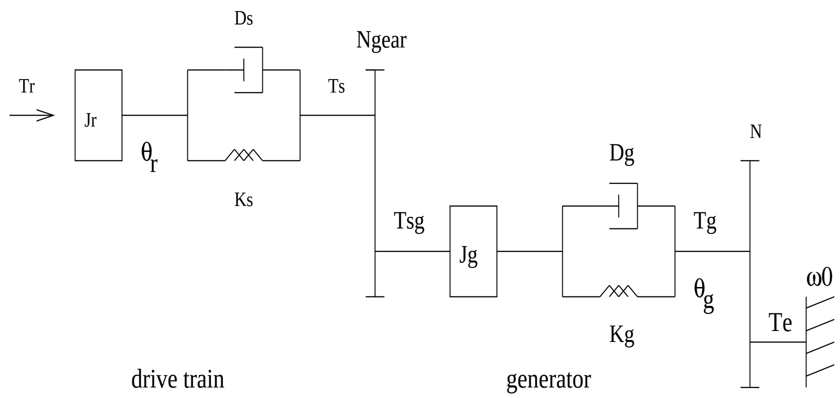

Figure 5: Drive train and generator

The equations of the motion of the drive train and generator are

$$
\begin{aligned}
J_r \dot{\omega}_r &= T_r - T_s && (15)\\
J_g \dot{\omega}_g &= \eta_{gear} \frac{T_s}{N_{gear}} - T_g && (16)
\end{aligned}
$$

and

$$
\begin{aligned}
T_s &= K_s \epsilon && (17)\\
T_g &= D_g \delta && (18)
\end{aligned}
$$

where ϵ and δ are defined by

$$\epsilon = \theta_r - \frac{\theta_g}{N_{gear}} \tag{19}$$

$$\delta = \omega_g - \frac{\omega_0}{N} \tag{20}$$

and

$$\omega_r = \dot{\theta}_r \tag{21}$$

$$\omega_g = \dot{\theta}_g \tag{22}$$

The output electrical torque T_e and electrical power P_e are

$$T_e = \eta \frac{T_g}{N} \tag{23}$$

$$P_e = T_e \omega_0 \tag{24}$$

ϵ	torsion angle of the drive train [rad]
δ	load angular velocity of the generator [rad/s]
θ_r	rotor shaft angle [rad]
θ_g	generator shaft angle [rad]
ω_g	angular velocity of generator [rad/sec]
T_s	mechanical torque [Nm]
K_s	spring coefficient of the drive train [Nm/rad]
D_g	damping coefficient of the generator [Nms]
N	number of pole pairs in generator
ω_0	grid frequency [rad/s]
η_{gear}	efficiency coefficient of gear box [-]
η	efficiency coefficient of generator [-]

2.2.4 Actuator

The actuator model describes the dynamic behaviour between the pitch demand from the pitch controller to the actuation of this demand.

$$\dot{\beta} = -\frac{1}{\tau_\beta}\beta + \frac{K_\beta}{\tau_\beta}\beta_{ref} \tag{25}$$

where β_{ref} is reference control input and β is actuator output (pitch angle).

3 Estimation Methods

In this section three estimation methods will be expounded, which are Newton-Raphson method, Kalman filter method and extended Kalman filter method. Emphasis will be placed on the main principles. We will start from Newton-Raphson method.

3.1 Newton-Raphson Method

Newton-Raphson method is a kind of iterative technique for minimizing the cost function. it is chosen in this report, because it has a particularly fast convergence properties in most cases.

3.1.1 Principle of Newton-Raphson Method

The main idea of Newton-Raphson method is to estimate parameters by minimizing a cost function

$$\hat{\theta} = arg\underset{\theta}{min} J(\theta) \tag{26}$$

Suppose the minimum is locally unique and cost function has a positive definite Hessian matrix, we can approximate cost function $J(\theta)$ in the vicinity of a given start point θ_0 by the quadratic function

$$J(\theta) \simeq J_0 + g_0^T(\theta - \theta_0) + \frac{1}{2}(\theta - \theta_0)^T H_0 (\theta - \theta_0) \tag{27}$$

and the optimal solution will be found at stationary points

$$\nabla J(\hat{\theta}) = 0 \tag{28}$$

the estimator is given by

$$\hat{\theta} = \theta_0 - H_0^{-1} g_0 \tag{29}$$

The iteration scheme of searching algorithm is

$$\bar{\theta}_{n+1} = \bar{\theta}_n - H_n^{-1} g_n \tag{30}$$

$\bar{\theta}_n$ is the result of n iterations

$$\begin{aligned} g_n &= \nabla_\theta J(\theta)|_{\bar{\theta}_n} && (31) \\ H_n &= \nabla_\theta^2 J(\theta)|_{\bar{\theta}_n} && (32) \end{aligned}$$

and result of the iterative searching is

$$\hat{\theta} = \bar{\theta}_{n_{stop}} \tag{33}$$

The unbound minimizing problem is equivalent to the problem in finding the solution to

$$I(\hat{\theta}) = 0 \tag{34}$$

if exist

$$I(\theta) = \nabla_\theta J(\theta) \tag{35}$$

therefore g_n and H_n in the iteration form (30) can be rewritten as

$$g_n = I(\theta)|_{\bar{\theta}_n} \tag{36}$$
$$H_n = \nabla_\theta I(\theta)|_{\bar{\theta}_n} \tag{37}$$

3.1.2 Estimation of Wind Speed

The wind speed estimator is achieved by minimizing the cost function $J(t, v)$

$$\hat{v}_t = arg\min_v J(t, v) \tag{38}$$

and cost function is given by

$$J(t, v) = (P_{r,t} - f(v))^2 \tag{39}$$

where $P_{r,t}$ is a measurement of rotor power at time t, which is assumed that it is known, and $f(v)$ is

$$f(v) = \frac{1}{2}\pi\rho R^2 v^3 C_p(\lambda, \beta) \tag{40}$$

The problem is equivalent to finding the solution to

$$I(t, v) = P_{r,t} - \frac{1}{2}\pi\rho R^2 v^3 C_p(\lambda, \beta) = 0 \tag{41}$$

The iteration form of the estimator is

$$\bar{v}_{n+1} = \bar{v}_n - H_n^{-1} g_n \tag{42}$$

where $\bar{v}_n$ is the result of n iterations, H_n and g_n can be obtained from definition (36) and (37)

$$g_n = P_{r,t} - \frac{1}{2}\pi\rho R^2 \bar{v}_n^3 C_p(\bar{\lambda}_n, \beta) \tag{43}$$

$$H_n = \left[-\frac{3}{2}\pi\rho R^2 v^2 C_p(\lambda,\beta) - \frac{1}{2}\pi\rho R^2 v^3 \frac{\partial C_p}{\partial v} \right]_{\bar{v}_n} \tag{44}$$

where

$$\frac{\partial C_p}{\partial v} = \frac{\partial C_p}{\partial \lambda}\frac{\partial \lambda}{\partial v} = \frac{1}{\omega_r R}\frac{\partial C_p}{\partial \lambda} \tag{45}$$

$\partial C_p/\partial\lambda$ can be calculated from C_p curve, therefore H_n can be obtained

$$H_n = -\frac{3}{2}\pi\rho R^2 \bar{v}_n^2 C_p(\bar{\lambda}_n,\beta) - \frac{1}{2\omega_r}\pi\rho R \bar{v}_n^3 \frac{\partial C_p}{\partial \bar{\lambda}_n} \tag{46}$$

where

$$\bar{\lambda}_n = \frac{\bar{v}_n}{\omega_r R} \tag{47}$$

At sample time t, the measurement is $P_{r,t}$, the iteration will start from $\bar{v}_0 = \hat{v}_{t-1}$ and continue until

$$I(t, \bar{v}_{n_{stop}}) = (P_{r,t} - f(\bar{v}_{n_{stop}})) < c \tag{48}$$

where c is small value.

The estimate of wind speed at time t is

$$\hat{v}_t = \bar{v}_{n_{stop}} \tag{49}$$

3.2 Kalman Filter Method

Kalman filter is the best linear estimate in a least squares sense. It has been explained as follow.

3.2.1 Model for Estimation

In general case the wind turbine can be described by a nonlinear stochastic differential equation

$$\dot{x}(t) = f(x(t), u(t), t) + w(t) \tag{50}$$

with measurement model

$$y(t) = h(x(t), u(t), t) + e(t) \tag{51}$$

The vector f is a nonlinear function, $w(t)$ is zeros mean gaussian noise having spectral density matrix $R_1(t)$ and $e(t)$ is a white random sequence with zero mean and covariance matrix $R_2(t)$.

The nonlinear model can be split into two parts: one describing the nominal operating points and another describing the system dynamics. The nominal operating points determine the output when the system is at rest, and dynamic model of the system describes the deviations from the operating points when the system is excited. In the general nonlinear case, a very common approach to handle the state estimator is based on linearization of nonlinear functions around a nominal trajectory.

$$
\begin{aligned}
f(x(t),u(t),t) &= f(x_0(t),u_0(t),t) \\
&\quad + \left.\frac{\partial f}{\partial x}\right|_{x_0,u_0} x(t) + \left.\frac{\partial f}{\partial u}\right|_{x_0,u_0} u(t) + \cdots
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
h(x(t),u(t),t) &= h(x_0(t),u_0(t),t) \\
&\quad + \left.\frac{\partial h}{\partial x}\right|_{x_0,u_0} x(t) + \left.\frac{\partial h}{\partial u}\right|_{x_0,u_0} u(t) + \cdots
\end{aligned} \tag{53}
$$

where x_0 is the nominal operating point, u_0 is nominal input.

The linear dynamic model can be achieved

$$
\begin{aligned}
\dot{x}(t) &= A_c x(t) + B_c u(t) + w(t) \\
y(t) &= C_c x(t) + D_c u(t) + e(t)
\end{aligned} \tag{54}
$$

where

$$A_c = \left.\frac{\partial f}{\partial x}\right|_{x_0,u_0} \tag{55}$$

$$B_c = \left.\frac{\partial f}{\partial u}\right|_{x_0,u_0} \tag{56}$$

$$C_c = \left.\frac{\partial h}{\partial x}\right|_{x_0,u_0} \tag{57}$$

$$D_c = \left.\frac{\partial h}{\partial u}\right|_{x_0,u_0} \tag{58}$$

and assume that $D_c = 0$. Then the linear estimation algorithms can be applied on the model.

For wind turbine the state is chosen to be

$$x(t) = (\Delta\epsilon(t)\ \ \Delta\dot{\epsilon}(t)\ \ \Delta\delta(t)\ \ \Delta\beta(t)\ \ \Delta v(t))^T \tag{59}$$

and

$$u(t) = \Delta\beta_{ref}(t) \tag{60}$$

$$y(t) = \Delta P_e(t) \tag{61}$$

The output electrical power $P_e(t)$ is

$$P_e(t) = \Delta P_e(t) + P_{e,0} \tag{62}$$

$P_{e,0}$ is electrical power at stationary point, which is rated value of 750 KW.

Since the discrete-time Kalman filter is employed, a discrete-time model has to be achieved. Sampling time for the system is 0.1 second, the sampling frequency is twice of the cut off frequency of the system.

3.2.2 Kalman Estimator

An optimal estimator is sought to minimize the criterion $E\{(x_t - \hat{x}_t)^T(x_t - \hat{x}_t)\}$ for the discrete-time system

$$\begin{aligned} x_{t+1} &= A_d x_t + B_d u_t + w_t \\ y_t &= C_d x_t + e_t \end{aligned} \tag{63}$$

where w_t and e_t are mutually independent white noise sequence with zero mean and covariance

$$E\left\{\begin{pmatrix} w_t \\ e_t \end{pmatrix}\begin{pmatrix} w_t & e_t \end{pmatrix}\right\} = \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix} \tag{64}$$

the state estimator $\hat{x}_t$ is given by

$$\hat{x}_{t+1} = A_d\hat{x}_t + B_d u_t + K_t(y_t - C_d\hat{x}_t) \tag{65}$$

where Kalman gain is

$$\begin{aligned} K_t &= A_d\Sigma_t C_d^T(C_d\Sigma_t C_d^T + R_2)^{-1} \\ \Sigma_{t+1} &= A_d\Sigma_t A_d^T + R_1 \\ &\quad -A_d\Sigma_t C_d^T(C_d\Sigma_t C_d^T + R_2)^{-1}C_d\Sigma_t A_d^T \end{aligned} \tag{66}$$

Since the operating point is fixed here, the matrices A_d, B_d, C_d can be taken as constant matrices. Then K_t converges to a constant Kalman gain K as t goes to infinity.

$$\begin{aligned} K &= A_d\Sigma_\infty C_d^T(C_d\Sigma_\infty C_d^T + R_2)^{-1} \\ \Sigma_\infty &= A_d\Sigma_\infty A_d^T + R_1 \\ &\quad -A_d\Sigma_\infty C_d^T(C_d\Sigma_\infty C_d^T + R_2)^{-1}C_d\Sigma_\infty A_d^T \end{aligned} \tag{67}$$

The constant Kalman gain can be precalculated. The off-line calculation offers a computational advantage.

3.3 Extended Kalman Estimator

The problem of filtering and smoothing for nonlinear systems is considerably more difficult and admits a wide variety of solutions than does the linear estimation problem. In this paper the extended Kalman filter is considered as an optimal estimation algorithm for nonlinear systems.

3.3.1 Principle of Extended Kalman Filter

A better choice for linearization trajectory is to use the current estimate of the state. Linearizing the model about it at every sampling time and applying a Kalman filter to the resulting linearized model yields the algorithm known as *extended Kalman filter*.

Let discrete-time nonlinear system be given by

$$\begin{aligned} x_{t+1} &= f(x_t, u_t, t) + w_t \\ y_t &= h(x_t, u_t, t) + e_t \end{aligned} \tag{68}$$

The estimate of the state is

$$\hat{x}_{t+1} = f(\hat{x}_t, u_t, t) + K_t[y_t - h(\hat{x}_t, u_t, t)] \tag{69}$$

where Kalman gain can be obtained by

$$\begin{aligned} K_t &= A_d(\hat{x}_t)\Sigma_t C_d^T(\hat{x}_t)[C_d(\hat{x}_t)\Sigma_t C_d^T(\hat{x}_t) + R_2]^{-1} \\ \Sigma_{t+1} &= A_d(\hat{x}_t)\Sigma_t A_d^T(\hat{x}_t) + R_1 \\ &\quad - A_d(\hat{x}_t)\Sigma_t C_d^T(\hat{x}_t)[C_d(\hat{x}_t)\Sigma_t C_d^T(\hat{x}_t) + R_2]^{-1} C_d(\hat{x}_t)\Sigma_t A_d^T(\hat{x}_t) \end{aligned} \tag{70}$$

where the matrices $A_d(\hat{x}_t)$ and $C_d(\hat{x}_t)$ are the partial derivatives, which are calculated by linearizing the nonlinear model about current estimate of state x_t.

$$A_d(\hat{x}_t) = \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}_t} \tag{71}$$

$$C_d(\hat{x}_t) = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_t} \tag{72}$$

The extended Kalman filter is a kind of nonlinear minimum variance estimation algorithm. The Kalman gain K_t is actually a random variable depending upon the matrices $A_d(\hat{x}_t)$ and $C_d(\hat{x}_t)$.

3.3.2 Iterated Extended Kalman Estimator

The state estimator $\hat{x}_t$ can be improved by iterated extended Kalman filter. A very simple and natural way is to make the above recursion into two steps as a measurement update and a time update, and iterate the equations for the measurement update over a single sample period.

The time update is

$$\begin{aligned} \hat{x}_{t+1|t} &= f(\hat{x}_{t|t}, u_t, t) \\ \Sigma_{t+1|t} &= A_d(\hat{x}_{t|t-1})\Sigma_{t|t}A_d^T(\hat{x}_{t|t-1}) + R_1 \end{aligned} \tag{73}$$

and the measurement update is

$$\begin{aligned} \hat{x}_{t|t} &= \hat{x}_{t|t-1} + k_t[y_t - h(\hat{x}_{t|t-1}, u_t, t)] \\ k_t &= \Sigma_{t|t-1}C_d^T(\hat{x}_{t|t-1})[C_d(\hat{x}_{t|t-1})\Sigma_{t|t-1}C_d^T(\hat{x}_{t|t-1}) + R_2]^{-1} \\ \Sigma_{t|t} &= [I - k_tC_d(\hat{x}_{t|t-1})]\Sigma_{t|t-1} \end{aligned} \tag{74}$$

The iterated extended Kalman filter is obtained by replacing the equation (74) with

$$\xi_i = \hat{x}_{t|t-1} + k_t[y_t - h(\xi_{i-1}, u_t, t) - C_d(\xi_{i-1})(\hat{x}_{t|t-1} - \xi_{i-1})] \tag{75}$$

Kalman gain can be obtained by local iteration over a single sampling period

$$\begin{aligned} k_{t,i} &= \Sigma_{t|t-1}C_d^T(\xi_{i-1})[C_d(\xi_{i-1})\Sigma_{t|t-1}C_d^T(\xi_{i-1}) + R_2]^{-1} \\ \Sigma_{t|t,i+1} &= [I - k_{k,i}C_d(\xi_{i-1})]\Sigma_{t|t-1} \end{aligned} \tag{76}$$

$$i = 0, 1, 2, \cdots n$$

iteration starts from

$$\xi_0 = \hat{x}_{t|t-1} \tag{77}$$

3.3.3 Extended Kalman Filter for Wind Turbine

The nonlinearity in the system comes from rotor dynamics. The rotor torque T_r is a nonlinear function of wind speed v, the pitch angle β and rotor angular velocity ω_r. Since we have restricted our attention to the high wind speed area, and a PID regulator is applied to the system for the purpose of power limitation, we can assume that nominal operating point of the system is around the power output of 750 KW. The contour level of 750 KW is drawn in Figure 6. On the contour level of 750 KW, the nominal rotor angular velocity ω_r is fixed, and pitch angle β is a function of wind speed v. Therefore the nonlinearity of the model depends only on the wind speed. The nonlinear rotor model can be linearized by monitoring current estimate of wind speed at every sampling time.

In Figure 6, if we assume that the nominal operating point for the system is at point 0 defined by v_m, β_0, and $\omega_{r,0}$ and working point for rotor is around point 1 defined by v_1, β_1 and $\omega_{r,1}$, which is determined by current estimate of wind speed. The linearized rotor model at nominal operating point 0 is given by equations (10) - (14). If we attempt to apply the extended Kalman filter for estimation of wind speed, the linearization of rotor model about point 1 should be carried out

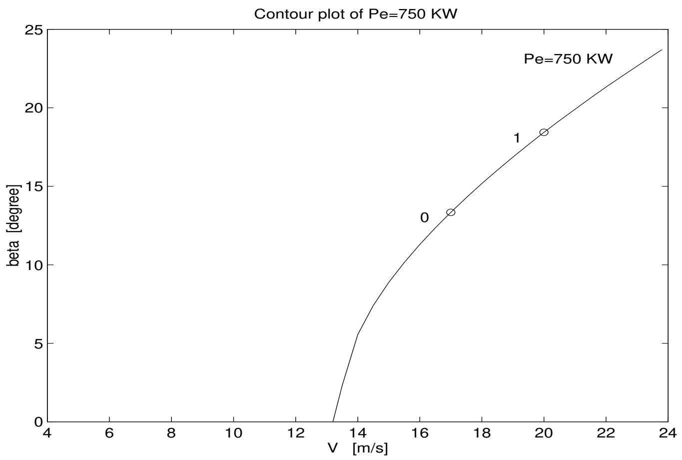

Figure 6: Contour plot of output electrical power $P_e = 750\ KW$

$$T_r = T_{r,1} + \left.\frac{\partial T_r}{\partial v}\right|_{v_1} \Delta v' + \left.\frac{\partial T_r}{\partial \beta}\right|_{v_1} \Delta \beta' + \left.\frac{\partial T_r}{\partial \omega_r}\right|_{v_1} \Delta \omega_r' \tag{78}$$

where

$$\begin{aligned} \Delta v' &= v - v_1 && (79) \\ \Delta \beta' &= \beta - \beta_1 && (80) \\ \Delta \omega_r' &= \omega_r - \omega_{r,1} && (81) \end{aligned}$$

It can be easily proved that the stationary rotor torque and rotor angular velocity on the contour level of 750 KW are constant, i.e., $T_{r,1} = T_{r,0}$ and $\omega_{r,1} = \omega_{r,0}$. Then we have

$$\begin{aligned} \Delta v' &= v - v_1 + v_m - v_m && (82) \\ &= \Delta v - \Delta v_1 \end{aligned}$$

$$\Delta\beta' = \Delta\beta - \Delta\beta_1 \tag{83}$$
$$\Delta\omega'_r = \Delta\omega_r \tag{84}$$

therefore small deviation of rotor torque ΔT_r is

$$\begin{aligned}\Delta T_r &= \left.\frac{\partial T_r}{\partial v}\right|_{v_1} \Delta v' + \left.\frac{\partial T_r}{\partial \beta}\right|_{v_1} \Delta\beta' + \left.\frac{\partial T_r}{\partial \omega_r}\right|_{v_1} \Delta\omega'_r \\ &= \left.\frac{\partial T_r}{\partial v}\right|_{v_1} \Delta v + \left.\frac{\partial T_r}{\partial \beta}\right|_{v_1} \Delta\beta + \left.\frac{\partial T_r}{\partial \omega_r}\right|_{v_1} \Delta\omega_r - \Delta T_r(1)\end{aligned} \tag{85}$$

where

$$\Delta T_r(1) = \left.\frac{\partial T_r}{\partial v}\right|_{v_1} \Delta v_1 + \left.\frac{\partial T_r}{\partial \beta}\right|_{v_1} \Delta\beta_1 \tag{86}$$

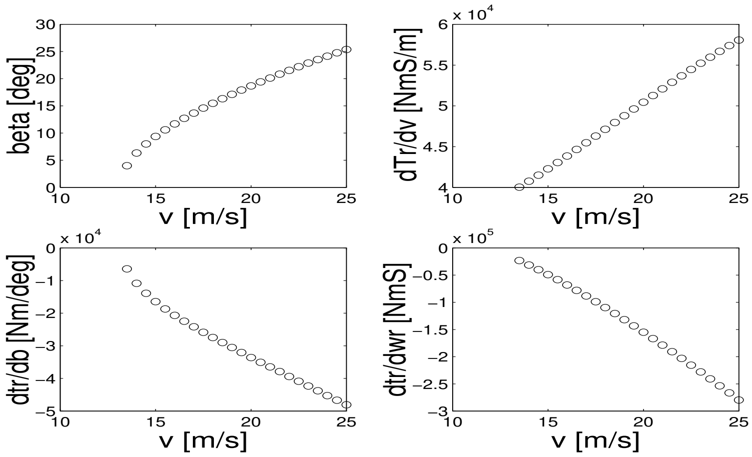

Figure 7: Working point of rotor model ($P_e = 750\,KW$)

Parameters of working points for the rotor is searched by estimate of wind speed $\hat{v}_t$ at sampling time t, the scheduling table for working point is illustrated in Figure 7.

The discrete-time linear state-space model at sampling time t can be described by

$$\begin{aligned} x_{t+1} &= A_d(\hat{v}_t)x_t + B_d(\hat{v}_t)u_t + H_d(\hat{v}_t)d_t + w_t \\ y_t &= C_d(\hat{v}_t)x_t + e_t \end{aligned} \tag{87}$$

where

$$d_t = \Delta T_r(k) \tag{88}$$

where k is the operating point, searched by $\hat{v}_t$. Then the extended Kalman estimator can be implemented on the model of the plant.

4 Simulations

Simulations for three estimation methods will be carried out. The advantage and disadvantage will be concluded.

4.1 Newton-Raphson Method

Three kind of situations will be investigated, which are

1. If rotor power P_r is known.
2. $P_r = P_e$.
3. $P_r = P_e/\zeta$.

- If rotor power P_r is known
  First of all, we will assume that the rotor power P_r is measurable. The estimate of wind speed is plotted in Figure **??**. The variance of estimation error is
  $$V\{e\} = 4.0372 \cdot 10^{-12} \tag{89}$$
  where $e = v - \hat{v}$, and mean value of estimation error is $5.1702 \cdot 10^{-9}$. The estimation result is perfect in this case, and the estimation error is only numerical error.

- $P_r = P_e$
  Since the rotor power P_r is unmeasurable in practice, we will relax from the unrealistic attempt and assume that P_r is equal to electrical power P_e which is measurable. The estimate of wind speed $\hat{v}$ is shown in Figure **??**. The variance of estimation error is
  $$V\{e\} = 0.1449 \tag{90}$$
  and mean value of estimation error is 0.4939. This bias comes from energy loss in the system, which can be seen in Figure **??**, where the rotor power P_r and the electrical power P_e are plotted. The figure shows that the dynamics of rotor power P_r is faster than the dynamics of electrical power P_e, and there exists energy loss from P_r to P_e. The efficiency coefficient is
  $$\zeta = \eta_{gear} \cdot \eta \tag{91}$$

- $P_r = P_e/\zeta$
  Now we shall take into account the energy loss in the system, and use P_e/ζ instead of P_r. The simulation is shown in Figure **??** - **??**. The variance of estimation error is
$$V\{e\} = 0.1507 \tag{92}$$
  and mean value of estimation error is 0.0736, which is improved significantly.

  The advantage of Newton-Raphson method is

  1. It is not necessary to know the whole system model.
  2. It is not necessary to know the mean wind speed.

  The disadvantage of Newton-Raphson method is

  1. $P_r = P_e/\zeta$ gives estimation error.

4.2 Kalman estimator

Figure **??** gives the actual wind speed and estimate of wind speed by applying Kalman filter method, Figure **??** gives the estimation error. The variance of estimation error is
$$V\{e\} = 0.1468 \tag{93}$$
and mean value of estimation error is 0.1660.

4.3 Extended Kalman estimator

Figure **??** gives the actual wind speed and estimate of wind speed by applying the extended Kalman filter, Figure **??** gives the estimation error. The variance of estimation error is
$$V\{e\} = 0.0635 \tag{94}$$
and mean value of estimation error is 0.0236

The advantage of Kalman filter and extended Kalman filter method is

1. Estimation is more accurate than Newton-Raphson method.

The disadvantage of Kalman filter and extended Kalman filter method is

1. It requires a knowledge of whole system model.
2. It requires a knowledge of mean wind speed.

5 Conclusions

Table 2 gives variances and mean values of the estimation errors from three kind of methods. All data are obtained from 200 second simulations.

	Newton ($P_r = P_e/\zeta$)	Kalman	Extend Kalman
$V\{e\}$	0.1507	0.1468	0.0635
mean(e)	0.0736	0.1660	0.0236

Table 2: Variance $V\{e\}$ and mean value mean(e) of the estimation error $e = v - \hat{v}$

Newton-Raphson method with the assumption $P_r = P_e/\zeta$ gives a small mean value of estimation error, but it gives the largest variance in three methods, the reason is the rotor power P_r is not measurable and the assumption $P_r = P_e/\zeta$ gives the estimation error.

Kalman filter method gives both large variance and mean value of estimation error, because the linear model for Kalman filter design is fixed at nominal operating point. It can be clearly seen from long period simulation, the estimation error increases when the wind speed deviates from the nominal wind speed. The reason is that the difference between the nominal model and actual system increases when the difference between the actual wind speed and nominal wind speed increases. The Kalman filter algorithm yields less filtering accuracy than the extended Kalman filter algorithm, but it offers computational advantage that Kalman gain can be calculated off-line.

Extended Kalman filter gives the best performance, which has the smallest variance and mean value of estimation error in three methods. The extended Kalman filter method has an advantage that simple Kalman filter method has not, it is that the linear model for Kalman filter design can be calibrated for operating points by monitoring the estimate of wind speed, therefore the nonlinearity of the rotor dynamics can be considered by estimator, and the estimation error will not increase when the wind speed deviates from the nominal wind speed.

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