

Interconnected  
hydro-thermal systems

# Interconnected hydro-thermal systems

Models, methods, and applications

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## Preface

This dissertation is a partial fulfilment of the requirements to obtain the PhD degree at Informatics and Mathematical Modelling (IMM) at the Technical University of Denmark (DTU).

It is the result of an Industrial PhD study that has taken place in the period April 1999 through October 2002 with Elkraft System Ltd., the Transmission System Operator company in Eastern Denmark, as the industrial partner. The study has been financed by the Danish Academy of Technical Sciences (ATV) and Elkraft System.

The dissertation addresses different aspects of mathematical modelling for medium- to long-term analyses of hydro-thermal power systems. One of the main goals of the study has been the development of modelling tools for practical analyses and problem solving.

In addition, another theme of the dissertation is the discussion of the model developing process in general, when addressing a real-life problem. In particular conceptual model design and validation will be covered, as it is essential for the analysis and understanding of the problems and hence the ability to solve them efficiently.

The dissertation consists of eight research papers done during the project and a core paper giving the global insight and the background knowledge for the research work done in the accompanying papers as well as a summary of the results achieved.

Lyngby, October 14, 2002

Magnus Hindsberger



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A special thank goes to Associate Professor Andrew Philpott and the department of Engineering Science, University of Auckland for an excellent introduction to stochastic programming, fruitful scientific cooperation, and for making my stay in New Zealand a great experience.

I would also like to thank the participants in the Balmorel project for making this an interesting and fun project to participate in.

Also a special thank to the following persons for their helpful suggestions and support: Professor Saul I. Gass from Robert H. Smith school of Business, Associate Professor Stein-Erik Fleten from NTNU, Norway, Professor Jens Clausen from IMM/DTU, Associate Professor Lene Sørensen from CTI/DTU, Lia Leffland from ATV, and Bjarne Chr. Jensen, ATV appointed sponsor.

Finally, the support from family and friends has been invaluable, especially from my patient wife, Grete, and our lovely child, Simone.



## Summary

This dissertation addresses mathematical modelling applied to power system analysis within an international perspective. It consists of two parts: one of practical model development and one of theoretical model studies. The power systems to be analysed are more specifically those found in the Baltic Sea Region. They are characterised by having a mix of hydroelectric and thermal based production units, where the latter type includes the combined heat and power (CHP) plants that are widely used in e.g. Denmark and Finland. Focus is on the medium- to long-term perspective, i.e. within a time horizon of about 1 to 30 years.

A main topic in the dissertation is the Balmorel model. Apart from the actual model, analyses of how to represent different elements appropriately in the model are presented. Most emphasis is on the representation of time and the modelling of various production units. Also, it has been analysed how the Balmorel model can be used to create inputs related to transmissions and/or prices to a more detailed production scheduling model covering a subsystem of the one represented in the Balmorel model.

As an example of application of the Balmorel model, the dissertation presents results of an environmental policy analysis concerning the possible reduction of CO<sub>2</sub>, the promotion of renewable energy, and the costs associated with these aspects.

Another topic is stochastic programming. A multistage stochastic model has been formulated of the Nordic power system. This allows analyses to be performed where the uncertainty of the inflow to the hydro reservoirs is handled endogenously. In this model snow reservoirs have been added in addition to the hydro reservoirs. Using this new approach allows sampling based decomposition algorithms to be used, which have proved to be efficient in solving multistage stochastic programming problems.

For solving the stochastic model a new sampling based method was developed that performed as least as good as existing methods. Stopping criteria for use in this kind of algorithms are also addressed and a new one suggested, which ensures the quality of the solution with a user-specified probability.



## Resumé (in Danish)

Emnet for denne PhD afhandling er matematisk modellering af energisystemer med fokus på studier af el og kraftvarme i internationalt perspektiv. Mere specifikt er det elsystemet i Østersø-regionen eller delsystemer heraf, der analyseres. Kendetegnet for dette område er en kombination af vandkraft og termisk baseret elproduktion, hvor sidstnævnte inkluderer kraftvarme, altså samproduktion af el og fjernvarmevand. Denne type produktion udgør en stor del af produktionen i for eksempel Danmark og Finland. Fokus vil være på mellemlangt til langt sigt, dvs. med en tidshorisont fra omkring 1 år og op til 30 år.

Studiet, som ligger til grund for afhandlingen, har dels været orienteret mod praktisk modeludvikling og dels mod teoretiske model- og modelleringsstudier.

Hovedtemaet i afhandlingen er Balmorel modellen, som er resultatet af den praktiske modeludvikling. I afhandlingen præsenteres analyser af, hvorledes forskellige elementer af elsystemet bedst muligt kunne blive repræsenteret i modellen. Specielt har fokus været på repræsentationen af tid og af forskellige produktionsenheder. Ligeledes bliver det beskrevet, hvorledes Balmorel modellen kan bruges i samspil med mere detaljerede modeller, som til gengæld dækker et mindre område og/eller en kortere tidshorisont end Balmorel modellen.

Som eksempel på anvendelse af modellen præsenterer afhandlingen en analyse af mulighederne for CO<sub>2</sub> reduktion og øgning af produktionen fra vedvarende energikilder, samt på omkostningerne, der er forbundet med disse tiltag.

Et andet tema i afhandlingen er stokastisk programmering. Opbygningen af en model af det nordiske elsystem bliver beskrevet. Her er tilstrømningen til vandkraftværkerne stokastisk og delt op i et bidrag fra regn og et fra smeltevand. Dette muliggør anvendelsen af sampling baserede algoritmer, som har vist sig at være velegnede til løsning af denne type stokastiske problemer.

Til løsning af den stokastiske model præsenteres en ny sampling baseret algoritme, som i sammenligning med eksisterende algoritmer viser sig at være mindst ligeså hurtigt. Endelig bliver stop-kriterier for samling baserede algoritmer analyseret.

*To Simone*

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# 1 Introduction

The power sector has been one of the traditional areas in which Operations Research (OR) has been applied in practice. Numerous models and accompanying optimisation and simulation methods for decision support have been designed for applications ranging from short-term production planning to long-term transmission network expansion planning. With the liberalisation of the power sector taking place in many countries new problems arise and therefore new applications where OR could be useful can be added, for example; optimal bidding strategies for trading on power pools and tools for analysing market imperfections (see e.g. Read (1996) for a further discussion on this). Similarly, growing environmental concerns add issues of policy analysis related to emissions from the use of fossil fuels.

This PhD dissertation is centred on modelling of power systems and in particular the power systems found in the northern parts of Europe. These are characterised by having a mixture of production technologies where hydropower, nuclear power, and thermal power are the most important, each with its own possibilities and limitations. To this comes production on combined heat and power (CHP) plants and from wind turbines.

The dissertation can be divided into two main parts: methodological studies and practical model development.

A main theme of the methodological studies is the discussion of the model development process and in particular analyses to support model design decisions like for example the levels of detail in the models, though most aspects of the modelling process; conceptual model design, mathematical formulation, implementation, validation, and application will be addressed.

Another theme is the modelling of stochastic phenomena such as the future weather. Apart from formulating a stochastic model, methods for solving this particular type of models will be discussed.

On the practical side, a mathematical model that has been developed for empirical analyses is presented as well as a number of analyses made using it. This model, the Balmorel model, covers the power system in the Baltic Sea Region within a long-term time horizon.

In the rest of this chapter, the background, aims, and delimitation of the PhD project is given. This is followed by an overview of the rest of the dissertation and a reader's guide.

## 1.1 The changes in the energy sector

The background of the PhD project shall be found in the numerous changes that the Danish energy sector has experienced in the recent years. The first major change was the increasing focus on environmental questions expressed in the Bruntland report from 1987, which put the environment and sustainable development in focus.

This led to a new national energy plan, "Energy 2000—an action plan for sustainable development", which was presented by the Danish government in 1990 with the overall goal of reducing the CO<sub>2</sub> emissions by 20% in 2005. The result was a political agreement, see Energistyrelsen (2002-I), that promoted a conversion of district heating plants to CHP plants as well as an increased use of natural gas and renewable energy sources as a substitute for oil and coal.

In 1993 it was agreed to increase the use of biomass in the energy sector; see Energistyrelsen (2002-II). The goal was to reach an annual use of 1.4 million tons of biomass such as straw and wood by 2000.

*Table 1 - The targets of the Energy 21 plan*

<b>Actions/targets</b>	<b>1998 (statistics)</b>	<b>2005</b>	<b>2030</b>
Installed wind capacity	1470 MW	1500 MW	5500 MW
Used biomass for energy	64 PJ	85 PJ	150 PJ
CO <sub>2</sub> reduction compared with 1988	8 %	20 %	50 %

In 1996 the government presented their plan, Energy 21, for the development of the energy sector in the beginning of the next century, see Miljø- og energiministeriet (1996). Again the main elements for the electricity sector were the increased use of renewables (biomass and especially windpower) for electricity production and promotion of CHP to replace separate electricity and heat production. Some of the targets of this plan have been specified in Table 1.

In 1997 the international community met in Kyoto, Japan, to discuss the threatening global warming. It was agreed that industrialised countries should accept commitments

to reduce emissions of greenhouse gasses to 5.2% below 1990 levels by 2012. Some developed countries were allowed to increase emissions while others had to reduce. Denmark agreed to reduce the emission of CO<sub>2</sub> equivalents in average over the years 2008-2012 with 21% of the 1990 level, see European Commission (2001). As the Kyoto protocol treats a total of 6 greenhouse gasses and not just CO<sub>2</sub>, the reduction in CO<sub>2</sub> equivalents is a larger reduction than the 2005 target of Energy 21. The Kyoto meeting also opened for the discussion of flexible mechanisms as Tradable Emission Permits (TEP), which received the stamp of approval in the Marrakesh 2001 meeting.

After the oil crises in the seventies most Danish electricity production plants converted to coal. Due to the high CO<sub>2</sub> emission from coal combustion, these plants are now being converted to natural gas or closed down in order to get near the reduction agreed upon in Kyoto. This is supplemented by increased use of biomass, a large build-up of small-scale natural gas CHP plants, and the highest number of wind turbines per capita in the world. Thus environmental concerns have probably been the main reason for a large transformation of the electricity production system in the nineties, though the aspect of security of supply also is part of the rationale behind the transformation.

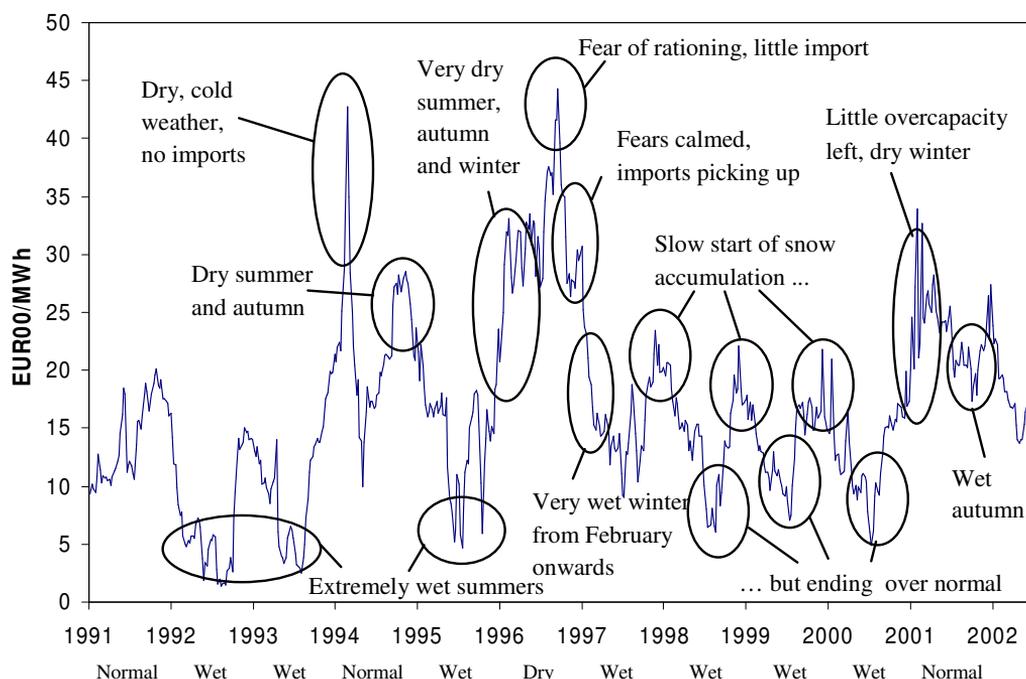


Figure 1 – Price development in Norway and the dependence on precipitation (original version by Norsk Hydro Energy)

The other big change in the electricity sector all over Europe in recent times is the liberalisation of the electricity markets. In Denmark this came with the electricity reform of 1999 implying e.g. that from January 2003 the market will be open for all consumers and that separate production companies and transmission system operators were established.

After the liberalisation started several power pools have opened in Europe. The first international power pool to open, Nord Pool, now covers the countries of Denmark, Finland, Norway, and Sweden. In general, the liberalisation has led to increased international trade during the 1990s.

The price of electricity in Denmark had typically been subject to only changes in fuel prices and taxation, but now it is also affected by the amount of water the hydro reservoirs in mainly Sweden and Norway receives. As illustrated in Figure 1 the impact on the price can be considerable. Figure 2 show why this may happen. Two supply curves  $P_1$  and  $P_2$  are shown where  $P_2$  describes the situation where the availability of hydropower is high due to a large reservoir content while  $P_1$  has less hydropower available for production. If the demand curve  $D$  intersects these two supply curves as in the figure, it can be seen that a relative small change in the availability of production capacity can result in a quite large change in price.

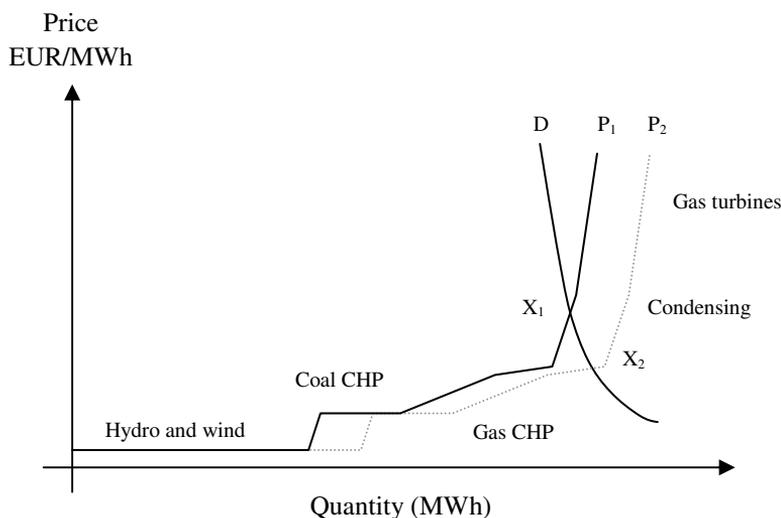


Figure 2 – Example of supply and demand curves for an electricity system

Since the liberalisation process started in the early nineties a consolidation has taken place. Fewer, but bigger, companies are left. This increases the risk, that they due to their size, can and will use market power, i.e. by acting strategically, they try to affect the market in order to increase their revenue. If the goal with the liberalisation was lower electricity prices, the use of market power may hinder this.

## 1.2 Challenges of the Danish electricity system

Due to the changes of the Danish electricity system mentioned above several physical and organisational challenges exist.

Firstly, a high proportion of fixed electricity production from CHP plants due to the demand for district heating exists. To this comes production from the increasing number of wind turbines. Can this production efficiently interact with the production in the hydro-dominated areas of Norway and Sweden? These areas have large hydro reservoirs where energy can be stored in case other sources produce the electricity needed to meet the demand. This may be hindered by market mechanisms and transmission bottlenecks however.

Secondly, the amount of water available for hydropower production each year varies considerably. In the very dry year 1996 the hydropower production in the Nordic countries equalled a little more than 150 TWh, while during the wet year 2000 it almost reached 250 TWh. As a comparison the annual Danish electricity consumption is roughly 34 TWh. This fluctuation may affect both electricity prices (as seen in Figure 1) and the security of supply.

Another challenge has been the liberalisation of the electricity market. How should this be organised in order to ensure an efficient market? This includes considerations on how to ensure the security of supply within a liberalised market.

To this comes the issue of environmental regulation. How can Denmark meet the national environmental goals as well as those of the international treaties that have been ratified? One scheme that has been used is promotion of renewable technologies. But how can this be done in liberalised markets? And are the actions chosen the best ones?

Finally, another important challenge covers the transmission system. In the Nordic electricity system presented in Section 2.1, the hydropower production is mostly found

in the northern regions while the electricity consumption is mainly in more populated regions in the south. In order to meet the peak demand a large transmission capacity is needed as the market otherwise will not work properly. Also, bottlenecks in the transmission system will make it easier for local dominant market actors to use market power to increase their revenues.

### **1.3 The aim of the PhD project**

The purpose of the PhD project was to participate in the development of a model for making power system analyses of the Baltic Sea Region. The main idea was that the study both should deal with practical model development and theoretical studies of power system modelling that the practical model development would benefit from.

On the practical side, the main objective of the PhD study has been the participation in the development of the Balmorel model. In short the Balmorel model is a long-term, multiregional model with an accompanying dataset describing the electricity and district heating system of the Baltic Sea Region. It is flexible in its requirements of the level of detail of data and is easy to expand and modify to comply with new aspects, which are sought analysed.

More specifically, the practical research work focused on the following tasks: model construction and implementation, data collection, model validation, as well as empirical analyses of actual problems with the model.

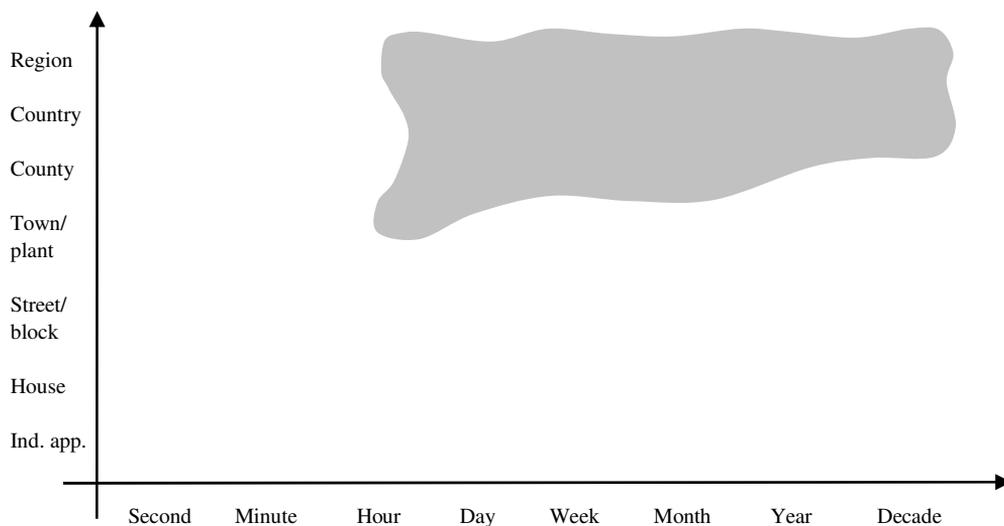
In relation to the theoretical part of the study, the main research has been in analysing how to model various aspects properly given the problem in focus. Also optimisation methods for solving stochastic programming problems have been analysed. These studies were undertaken in order to address modelling issues that arised during the Balmorel project.

### **1.4 Boundaries and delimitation of the model**

The initial delimitation in time and space of the model to be developed was as indicated in Figure 3.

It can be seen that focus of the model is generally on medium- to long-term issues, i.e. it should enable analyses within a 1-30 year time horizon. However, the model should also make it possible to carry out analyses with smaller time steps than a year due to the temporal variations of e.g. the demand of electricity.

Depending on the analysis to be made, the geographical scope could be the overall power system of the Baltic Sea Region or it could be more specifically oriented on the Nordic power system, the Danish power system, or even parts hereof.



*Figure 3 – Spatial and temporal delimitation of the project*

Hence, the model should represent elements at county, country, or regional level. This would allow a suitable representation of the overall transmission system. For a suitable modelling of CHP plants however, it may be necessary to look at heat demands for individual district heating networks found at town level. Going into more detail on the demand side, for example to model consumption for individual electrical apparatus or the heat demand for different types of houses, was not the intention.

Within this delimitation some of the aspects the model should be able to analyse were:

- International trade patterns
- Hydro-thermal interaction in the region
- The interaction between CHP plants and the rest of the power system
- Implication of changes in fuel costs, efficiency in energy transformation, and other parameters
- The impacts of the dominant stochastic phenomena
- Impacts of different national policies (taxes, emission quotas, etc.) on the environment and economy

These delimitations will be further elaborated in Chapter 2.

## 1.5 Overview of the dissertation

The work of the PhD study is documented in this dissertation as well as in the Balmorel main report; see Ravn et al. (2001-I). The Balmorel model, the result of the practical model development, is documented in the latter, while this dissertation mainly addresses the methodological studies carried out.

The dissertation consists of a core paper, of which this introduction is Chapter 1, as well as 8 research papers that have been written during the study.

Chapter 2 describes the fundamentals of hydro-thermal systems, i.e. power systems where both hydroelectric and thermally based production plants are found. Apart from the characteristics of the different production technologies the chapter also discusses transmission and stochasticity issues. Finally, descriptions of the power systems found within the Baltic Sea Region are included.

Chapter 3 gives a theoretical account of problem solving in general and especially of mathematical modelling for decision support. The mathematical modelling process is described and some modelling recommendations found in literature are presented. This leads to the choice of modelling guidelines to be used in the practical modelling work of the study.

Based on Chapters 2 and 3, Chapter 4 discusses the experiences obtained during the practical and theoretical modelling work done during the study. A main issue is the evaluation of the modelling guidelines used. Also, the theoretical analyses done as part of the study are motivated in the light of the overall modelling process.

Chapter 5 presents the general contributions of each of the accompanying research papers along with the conclusions of those.

Finally, in Chapter 6 the overall conclusions of the study are given and suggestions for further research are made.

An appendix has been included to the core paper, Appendix A, describing the present version of the Balmorel model (version 2.10, October 2002), supplementing the Balmorel main report in documenting the work done while participating in the development of this.

**Paper A** “Level of detail in modelling – An analysis of time scales in the Balmorel model” discusses the level of detail in mathematical models in general. Using an early version of the Balmorel model, computational analyses of using different time scales have been carried out. The results show that a rough division of time is reasonable for some analyses, while other times of analyses require a finer representation of time. The paper was presented at the workshop “Denmark in a North European liberalized electricity market”, in Copenhagen, November 1999, as well as the IAEE workshop on “Multiregion models, energy markets, and environmental policies”, in March 2000, in Helsinki, Finland.

**Paper B** “Bottom up modelling of an integrated power market with hydro reservoirs” is a similar analysis to that of Paper A but now the focus is on a particular plant in the system. It is shown that a fine representation of time is needed to analyse the behaviour of this type of plant and its impact on other parts of the system, while the overall picture is not similarly affected by changes in the time detail. This paper was published in the proceedings of the Second International Conference in “Simulation, Gaming, Training and Business Process Reengineering in Operations” in Riga, Latvia, September 2000.

In **Paper C** “Deterministic modelling of hydropower in hydro-thermal systems”, the suitability of a deterministic model in modelling larger hydro-thermal systems is analysed. This is done by comparing the results of models with both different time scales and number of restrictions with actual historical observations from the Nordic power system. It is concluded that for many types of results, e.g. system costs and expected annual average prices, a deterministic model can obtain fine results. However, when looking at the price development over the year, the estimates are of less quality.

**Paper D** “Multiresolution modeling of hydro-thermal systems” discusses the issue of how to combine models with different levels of detail both concerning time and geography. This is an important issue, as it often is desirable to analyse different parts with a different level of detail. The computational case uses the Balmorel model as the low-resolution model analysing the Nordic power system. The transmission patterns found in this are used as input to a high-resolution unit commitment model of the power system in eastern Denmark. It was also tried to use the price signals of the Balmorel model as input, but those results showed a less resemblance with historical observations than when transmission data was used. The paper was published in the proceedings of the IEEE conference “Power Industry – Computer Applications, PICA 2001” in Sydney, Australia, May 2001.

In **Paper E** “Co-existence of electricity, TEP, and TGC markets in the Baltic Sea Region”, an application of the Balmorel model is presented. In the paper the Balmorel model has been used for analysing the effects of partial overlapping markets of electricity, renewable electricity certificates, and tradable emission permits as few numerical analyses of such issues exist. The results show that depending on the targets set for tradable emission permits and renewable electricity certificates, the implications on the actual CO<sub>2</sub> reductions, the associated costs, and the possible revenues of companies within the system vary considerably. The paper appeared in *Energy Policy*, Volume 31, Issue 1, January 2003.

The present release of the Balmorel model (version 2.10, October 2002) is a deterministic model, i.e. it is unable to treat stochasticity endogenously. Analysing the effect of random realisations of data must be done exogenously before the model runs. Extending the model into a stochastic formulation would be relevant for answering many questions, e.g. for obtaining better estimations of the price developments within a year, cf. the description of Paper C.

To analyse how stochastics could be handled in such a model, a stochastic model has also been developed. This is basically a very simplified version of the Balmorel model that allows endogenous treatment of uncertainty, which is desired for certain analyses. The stochastic parameters included are used to represent the uncertain inflow that is received in the hydropower reservoirs.

In **Paper F** “Stochastic medium-term modelling of the Nordic power system”, the models itself and the modelling considerations done are presented. It was chosen to split the inflow into two parts: rain precipitation and snowmelt, which is a new approach to use. An advantage of this is that it allows sampling based methods to be used for solving the problem. Also, it includes knowledge that decision-makers have (the amount of snow in the mountains) in the model rather than having this included in the stochastic parameter. The model and some preliminary results were presented at the seminar “Investments and Risk management in a liberalised electricity market”, Copenhagen, Denmark, September 2001.

**Paper G** “ReSa: A method for solving multistage stochastic linear programs” presents a new algorithm that was developed for solving the stochastic model from Paper F. The algorithm is a sampling based algorithm, and in the paper, its performance is compared with those of existing similar algorithms. For the Paper F model the ReSa algorithm performed best. The work was presented at the conference “Stochastic Programming 2001”, Berlin, Germany, August 2001.

The analyses of Paper G revealed a problem with the stopping criterion used for the type of algorithms addressed in the paper.

In the last **Paper H** “Stopping criteria in sampling strategies for multistage SLP-problems”, a new stopping criterion for sampling algorithms is presented. Computational results using this and other different criteria have been included. The main issue is the trade-off between the quality of the solution and the computation time used to obtain it. The results show that the overall performance of the stopping criteria depends of the type of problem. The work was presented at the conference “Applied mathematical programming and modelling”, Varenna, Italy, June 2002.

## 1.6 Reader’s guide

The dissertation has been written for people in the power sector with an interest in the methodology and mathematics behind the models used. Especially they would benefit from discussions on the Balmorel project and on how to model various parts of power systems. Similarly, people in the research community working with energy planning models should find this dissertation interesting, both the issues on modelling of power systems and the presentation of the optimisation techniques.

It has been assumed that the reader will have a basic knowledge of OR. Readers without this knowledge will benefit from reading an introductory book such as Hillier and Lieberman (2001).

For full understanding of the stochastic programming parts, some basic understanding of this is required. For an introduction, Birge and Louveaux (1997) is recommended.

Also a basic knowledge of economical terms like supply and demand curves, assumptions behind perfect competition, and definitions of marginal and capital costs has been assumed. For an introduction to this, see e.g. Varian (1992).

Finally, knowledge of power systems is an advantage. The introduction in Chapter 2 should be sufficient for understanding most parts. Otherwise a classic reference is Wood and Wollenberg (1996). Note that in the dissertation the words power and electricity will be used interchangeably.

## 2 Hydro-thermal systems

This chapter will introduce the fundamentals of hydro-thermal systems. These are power systems where both hydropower plants with reservoirs and traditional thermally based power plants are found in larger scale. The combined power systems of the Nordic countries or the whole Baltic Sea Region are examples of hydro-thermal systems. Within these systems, combined heat and power (CHP) plants constitute a large proportion of the thermal units. Hence, the operation of CHP plants within hydro-thermal systems will also briefly be addressed.

In the next section the power systems of relevance to the dissertation will be introduced. This is followed in Section 2.2 by an overview of decision problems to be addressed with hydro-thermal models. Section 2.3 gives a description of the temporal characteristics of a hydro-thermal system. Section 2.4 introduces the characteristics of thermal dominated, hydro dominated, and mixed hydro-thermal systems. Sections 2.5 and 2.6 address the issues of transmission and stochasticity. Finally, in Section 2.7 conclusions related to the power systems studied throughout this dissertation are given.

### 2.1 The power systems in the region

After the liberalisation of the national power systems began, it has become increasingly important to look beyond the national borders when analysing power related issues of national interest. From a Danish point of view, it is most often relevant to look at the Nordic power system as a whole. Throughout this dissertation, the Nordic power system refers to the combined power system of Denmark, Finland, Norway, and Sweden. Iceland has been omitted, since it is not electrically connected to any of the other Nordic countries. In Table 2 some basic figures of the Nordic power system in 2001 have been given. An international power pool exists; see Nord Pool (2001), covering all four countries.

The Nordic region is interesting for several reasons. Firstly, it is a deregulated system with the power markets in all countries being fully liberalised or under liberalisation. Secondly, the mixture of production technologies is very varied as seen in Table 2. The current trend is that many thermal condensing units are being phased out while CHP plants and wind turbines are being built. Looking at individual countries Norway is hydro-dominated, while the Danish subsystem is considered thermal-dominated though more than 10% of the production now is from wind turbines. Finland and Sweden both have larger amounts of the different types of generation capacity. Thirdly, the

production from CHP plants is large compared with many other regions. Finally, the number and capacity of interconnections to other not included countries are (still) relatively small. Therefore the uncertainty of transmission to and from other countries is limited compared with the total system load.

*Table 2 – Capacities, production, and demand in 2001 in the Nordic countries (Iceland excluded) from Nordel (2002)*

	Denmark	Finland	Norway	Sweden	All
<b>Capacities (in MW)</b>					
Hydropower	11	2948	27571	16239	46769
Nuclear power	0	2640	0	9436	12076
Other thermal power	9983	11200	305	5753	27241
Windpower	2486	39	17	293	2835
Total capacity	12480	16827	27893	31721	88921
<b>Energy balance (in GWh)</b>					
Total production	36009	71645	121872	157803	386438
Total demand	35432	81604	125464	150512	393012
Net export	577	-9959	-4483	7291	-6574

The largest geographical area to be modelled as part of this study is the Baltic Sea Region; see Figure 4. It includes the Nordic power system described above in addition to the rest of the countries bordering the Baltic Sea; Russia, Estonia, Latvia, Lithuania, Poland, and Germany. The addition of these countries adds large district heating areas to the power system.

While the Nordic countries are quite similar in most other aspects than in how electricity is produced, the countries of the Baltic Sea Region differ considerably in terms of the economic situation and the political and institutional traditions. With respect to the longer-term development, this makes the region very interesting.

In the countries in the southeastern part of the region, the power plants are in general old. Thus, there is room for improvements in relation to existing older power plants, for example in terms of raising the thermal efficiency and in reducing emissions. The large district heating areas in these countries are currently to a large degree supplied from pure heat producing units. Conversion from pure heat to CHP units to improve the overall efficiency of the system, as seen in Denmark—see Section 1.1, is a development to be expected.

Finally, due to the differences in the supply system and between the economic levels of the countries in the region, trading with emission permits and Joint Implementation (JI) projects are very relevant issues within this area. JI projects allow one country—typical rich—to invest in the reduction of greenhouse gas emissions in another—typical poorer, and claim credit towards its own emission reduction targets given by the Kyoto protocol. Since 1998, the countries in the Baltic Sea Region have been working toward making the region a testing ground for Kyoto mechanisms such as JI; see e.g. BASREC (2002).



*Figure 4 – The countries in the Baltic Sea Region.*

## 2.2 Hydro-thermal decision problems

Decision problems in hydro-thermal systems are normally concerned with finding the optimal production levels of power plants for a system that includes both hydropower plants and thermal power plants. However, the geographical and temporal level of detail of these decisions can vary considerably, depending on the problem in mind. Figure 5 shows some decision problems sorted after the time horizon of the models needed to address them. It has been sketched that the level of detail for models doing short-term operation analyses usually are high while the level of uncertainty of data is little. For long-term analyses the opposite tends to be the case, as it will be shown.

Short-term (or operational) models tend to be very detailed in their description of the system and often include numerous restrictions and integer variables. But uncertainties are normally few and can mostly be well predicted. This includes the estimations of the power demand and the production from wind turbines.

In medium-term (tactical) models, results are in general sought with a lower time resolution than the operational models, which reduces the demand for detailed modelling. However, the uncertainty increases with the longer time scope as the estimates of the uncertain parameters becomes more unsure.

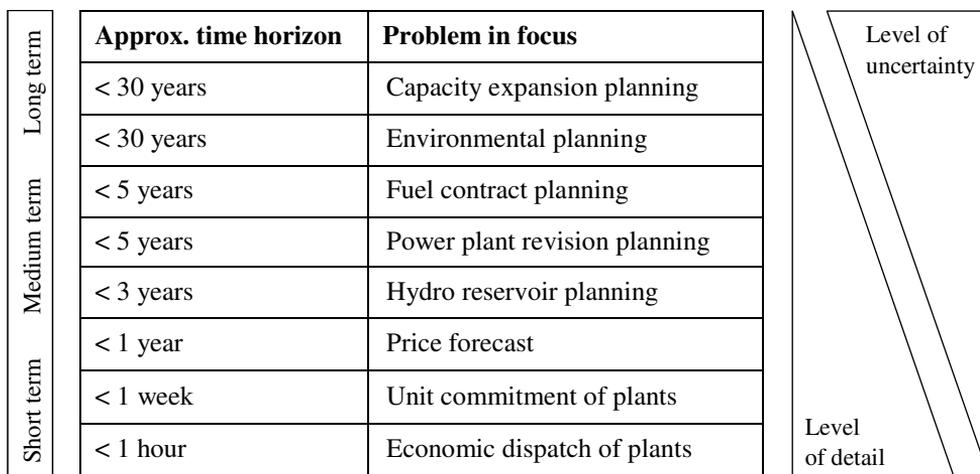


Figure 5 – Models for decision support and their time horizon

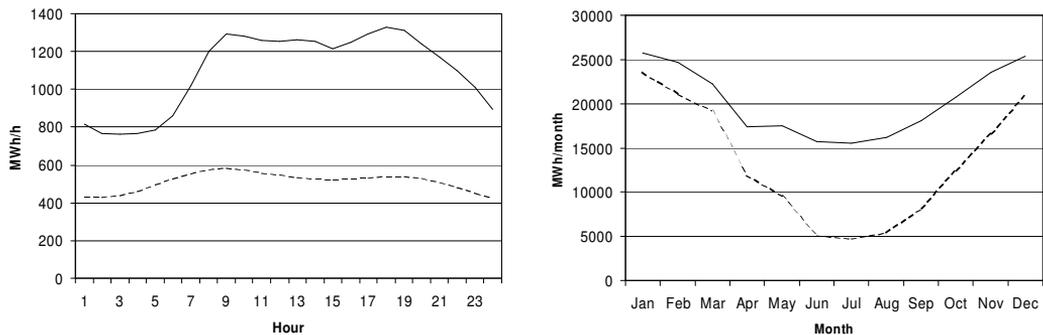
For the long-term (strategic) models, uncertainties are numerous while the need for a high level of detail is relatively small. Looking beyond 10 years ahead little is known

of the technical coefficients of new power plants, fuel prices, demands for energy, as well as the policies that regulate the energy markets. Thus operations of individual plants are not as interesting 30 years ahead as estimating the overall influence on the power system given various scenarios of future realisations of the uncertain parameters.

## 2.3 Temporal characteristics of power systems

In the previous section some relevant time horizons were defined for different decision problems in power systems in general. This section will briefly introduce temporal aspects that are important to the operations of hydro-thermal power systems and thus should be considered within the overall time horizon of the analysis to be made.

Figure 6 sketches the diurnal and seasonal variations in the demand of power (bold line) of a typical Nordic country. Similarly, a typical demand for district heating (dotted line) has been depicted, as the power output of CHP plants is restricted by the amount of heat they must deliver. The figure illustrates how the variations in demand for power and district heat differ both when seen diurnally and seasonally. Power demand varies much during a day while the daily average change less over the year. For the heat demand the opposite is the case; smaller diurnal variations but large seasonal changes in demand.



*Figure 6 – Diurnal and seasonal variations of demand (electricity and heat)*

Besides from the consumption of electricity and district heating the production rate of some power plants may also be affected over time. Most obvious is the production from wind turbines that fluctuates highly. On average the pattern shows a larger production during winter than during summer. Also, the production during daytime is

in general higher than during night. The large daily fluctuations are indicated in the left graph of Figure 7. The right graph shows the average monthly wind energy contents for Denmark in the period 1979-2001 (index 100 = annual average) and hence the annual trend mentioned.

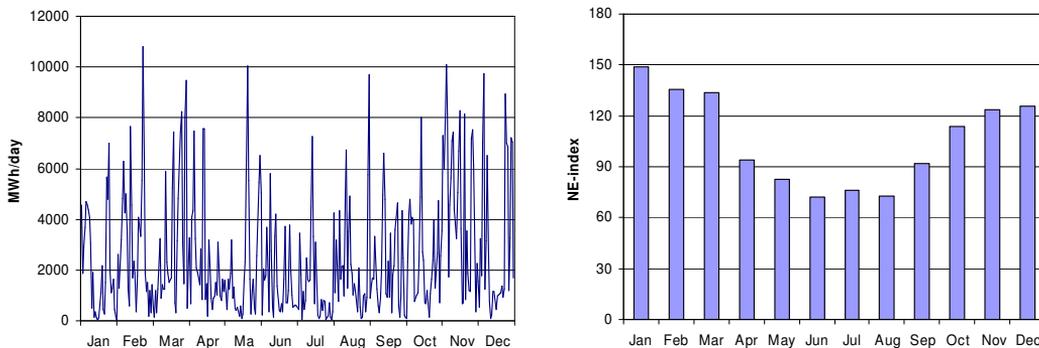


Figure 7 – Wind production in East Denmark 2001, data from Elkraft System (2002) and average monthly wind energy contents in Denmark in the period 1979-2001, data from Energi- og Miljødata (2002)

The production on hydropower plants is also affected over time. The left graph of Figure 8 shows the average monthly inflows (full line) to the Norwegian hydro reservoirs. It can be seen that the main inflow arrives late spring, early summer as the snow in the mountains melts. The winter inflow is limited, as the precipitation in these months is accumulated in the mountains as snow. The inflow varies from year to year as seen on the right graph of the figure, which shows the annual inflow sequences for the years 1990-2000 again for the Norwegian system.

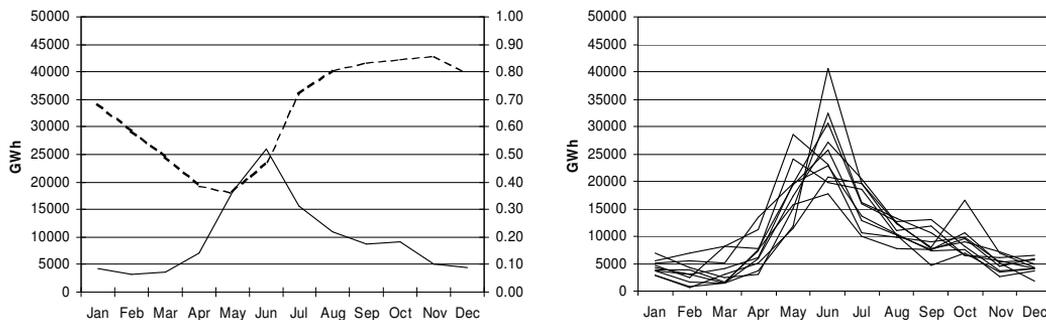


Figure 8 – Inflow and reservoir content of hydropower reservoirs (data from Nordel)

For the hydropower plants without any reservoirs (run-of-river plants) the production will at any time depend on the river flow. Some plants have a limited storage that allows them to store water for a few hours or days worth of production. These plants will partly be able to adjust their production to fit with the variations of the diurnal power demands. Other hydropower plants have larger reservoirs, which allow water to be stored for months or even years. The dotted line on the left graph of Figure 8 shows the average reservoir level (right axis) of the Norwegian hydropower plants as fractions of the total capacity. It can be seen that the storages are filled during summer and the water then gradually released till the next melting period comes.

## 2.4 Characteristics of hydro-thermal systems

In this section the thermal and hydropower characteristics will be introduced. The hydropower plants to be discussed are assumed to have larger hydro reservoirs.

In general, thermally based power is characterised by being decoupled in both time and space. Thus a decision to produce now will, with the exception of the very short-term view, not affect the ability to produce later. Similar, production on one unit does not in general affect the generation capacity of other units, though this may be the case when multiple natural gas fired power plants share the same gas pipeline, if the capacity of this line is insufficient for full production of all plants.

Production from hydropower plants with reservoirs is known to be coupled in time. Compared with thermal power, the capability of storing water and thus production for later is a major difference. Spatially, hydropower units may also be coupled as production on one plant may affect the production of other plants if these plants are located on the same river. Thus production on an upstream plant will release water to downstream plants and allow them to produce more.

The second main difference between hydro generation and thermal generation is the marginal cost of production. The cost of producing on hydropower plants is negligible as the water is freely received. Thermal power plants have a considerable fuel cost. Looking at a system of thermal power plants this gives a supply curve showing a high and increasing marginal cost, depending on the type of power plant and its fuel. These trends of the marginal costs were indicated back in Figure 2.

The last difference that will be addressed here is the uncertainty of future production. Most thermal power plants have fuel delivered from medium-term contracts. Thus little

uncertainty is seen on the costs of production on the short- to medium-term, while this can be considerable on the long-term scale. Similar can be said about the availability of the fuel (and thus the ability of future production). For hydropower systems, the inflows to the reservoirs are highly uncertain as shown later in Section 2.6. Hence, the possible future production is not known exactly.

When having both types of production, the energy availability risks are reduced as parts are affected by changes in price and availability of fuel only, while other parts are affected by weather only. Also, the whole system includes the fast regulating capacity of the hydropower plants, which is important in case of failures of other plants (forced outages) or in the transmission system in order to reduce the risk of a blackout.

The characteristics of thermal and hydropower technologies are summarised in Table 3 below.

*Table 3 – Characteristics of thermal and hydropower technologies*

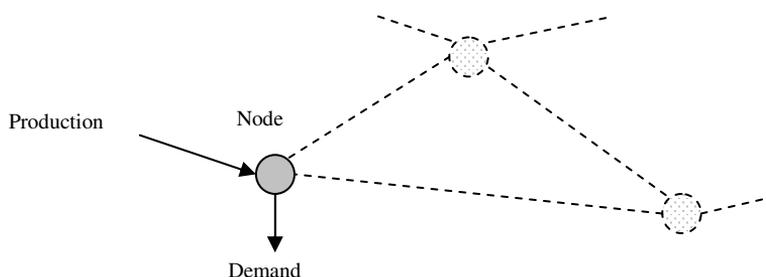
<b>Characteristics</b>	<b>Thermal</b>	<b>Hydropower</b>
General	<ul style="list-style-type: none"> <li>• High and growing marginal costs of production</li> <li>• No storage of electricity</li> <li>• Possible heat restrictions for CHP plants</li> </ul>	<ul style="list-style-type: none"> <li>• Low, non-growing marginal costs of production</li> <li>• Storage of electricity</li> </ul>
Short-term	<ul style="list-style-type: none"> <li>• Unit commitment important due to high startup costs and long startup times</li> <li>• Slow regulation capabilities</li> </ul>	<ul style="list-style-type: none"> <li>• Fast regulation capabilities</li> </ul>
Medium- to long-term	<ul style="list-style-type: none"> <li>• Decoupled in time</li> <li>• Little uncertainty on medium term</li> </ul>	<ul style="list-style-type: none"> <li>• Coupled in time</li> <li>• High uncertainty on medium term</li> </ul>

## 2.5 Transmission aspects

Hydro-thermal systems may cover larger areas where bottlenecks in the internal transmission network exist. This has been sketched in Figure 9. Production of electricity is available at a given part of the system (here seen as a node) and demand in that node must be fulfilled. It is possible to transmit power from a node with electricity surplus to a deficit node though with a minor loss. The transmission network from the node is sketched with the dotted lines. The distribution network existing at

each node is not modelled though a distribution loss may be given, so that the gross demand in the node corresponds to the net demand plus the distribution loss.

A hydro-dominated part of the system would from an isolated perspective have a low and rather constant price over the year, while a thermal dominated part would be expected to have a higher and more varied price level as indicated in Table 3. A transmission line between such two parts would level out the differences in price transmitting power from the low-price node to the high-price node. Whether the difference will wholly disappear depends on the production capacity in the interconnected parts, the hydro storage capacity, as well as the capacity of the connecting transmission line.



*Figure 9 – Transmission, production and demand*

CHP plants can produce both power and heat for district heating networks at the same time. Hence, each node may also have a district heat demand and a distribution loss associated. Production equal to this amount must take place at the node, since district heating is assumed to be unsuited for transmission due to high losses.

## 2.6 Stochasticity

For hydro-thermal systems much of the data concerned with describing the future is uncertain. For some of these data a distribution of possible values exists. Such data will also be denoted as stochastic data. Examples of the data that may be considered stochastic are the demand for electricity, the availability of the power plants, the production from wind turbines, and the inflow to the hydropower reservoirs.

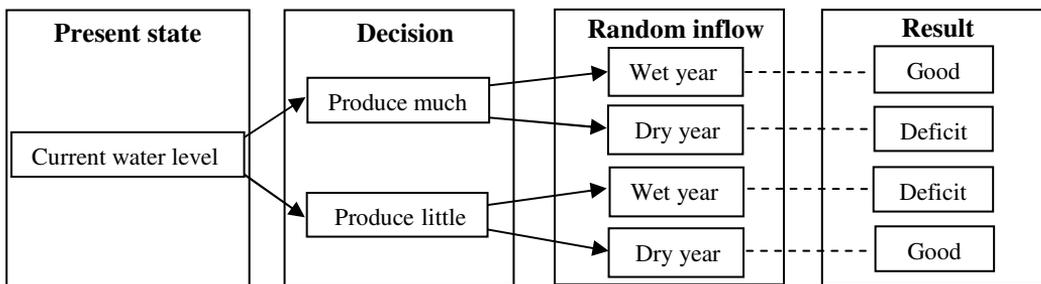


Figure 10 – A simplified stochastic hydro-thermal decision problem

The diagram in Figure 10 shows an example of the effects of stochastic parameters—in this case the inflow to the hydro reservoirs in a hydro-thermal system. In such a system power generator companies with hydropower storages have to decide how much water to use for production now and thus how much to save for later.

In the first case the company decides to use a lot of water for generation. If the inflow is high, it will still have plenty of water to cover production later on. If the inflow is low though, the production may have to be made on expensive reserve units. If the water on the other hand was saved and a lot of inflow is received spillage may occur. Since the company could have used this water for generation and thus reduced the fuel costs of thermal production this is a deficit result. But if the inflow is low the company may have a lot of water for production when everybody else is running short. The dependence on the price of the annual water inflow is shown in Figure 11.

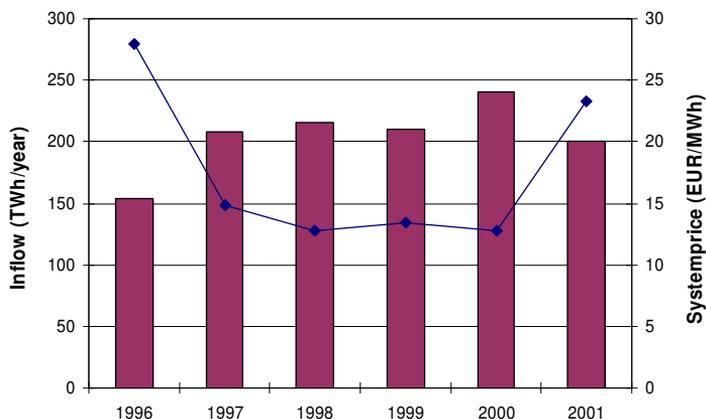


Figure 11 – The relationship between the observed inflow (bar – left axis) and the average spot price (line – right axis)

Within the delimitation of this dissertation the main stochastic parameter is the hydro inflow. Electricity demand and the availability of power plants can be well predicted if the main scope is on annual energies of production and demand and on larger groups of power plants. Production by wind turbines vary more, both in short term and in terms of energy produced each year (see Figure 7). However, the capacity and energy produced by windpower compared with hydropower in the region is small, cf. Table 2 (if limiting the geographical scope to Denmark, windpower becomes the major stochastic parameter). Also, for windpower the random outcomes affect the decisions of the actual hours only. The hydro inflow on the other hand can, due to the use of reservoirs (see Figure 8), affect production patterns and thus prices, fuel usages, and emissions in the longer-term perspective.

## 2.7 Summary

This chapter has introduced the fundamentals of hydro-thermal systems with focus on the relevant issues for medium- to long-term analyses within the hydro-thermal systems used in the case studies in the dissertation.

Given the discussions in the chapter, the model delimitation of Section 1.4 can now be further specified.

Some essential characteristics of the modelling tool that should be built are:

- representation of the long-term perspective
- representation of both seasonal and diurnal variations of relevant parameters
- representation of the main characteristics of plants found in the hydro-thermal system in view; i.e. hydropower, nuclear power, and other thermal power including CHP plants
- a geographical representation that enables the representation the transmission bottlenecks of international importance
- a stochastic representation of the inflow to the hydropower reservoirs
- representation of the implications on the environment
- representation of policy instruments

Similarly, some aspects have been considered as less important given the kind of questions asked and the implications it would have on the computation time. The excluded aspects include unit-commitment. Also, no stochastic representation of wind production, unit availability, and electricity demands should be made. Rather, average values should be used as discussed further in Chapter 4.

## 3 Dealing with problems

In the literature, many definitions of a problem exist. One such, Ritz et al. (1986), describes a problem as “a need that *must* be met”. This need could, among other things, be the need to understand the forces of nature (science), to alter the environment (technology), or to use scientific knowledge to alter the environment (engineering).

First of all, the definition indicates that somebody must find the gap between the situation now and the one desired to be important enough before it becomes a problem worth dealing with.

Also, according to this definition, a problem is not merely a question of decisions like “how to do?” or “what to do?”, but also may be one of understanding, i.e. “why does this happen?”. Finally, if more people are involved, a problem may be one of obtaining consensus “what can we agree on?”.

This chapter will in the first sections look at problem solving in general. However, as the main focus of the dissertation is on decision problems like those presented in Section 2.2, the focus will later on switch to this type of problems. A general definition of decision problems can be found in Ackoff (1981), who defines them as problems where alternative courses of action exist, which can have significant effects and there is doubt on which one to choose. The doubt arises due to the complexity of the problem.

In the next section, an introduction to problem complexity will be given. In Section 3.2 the overall problem solving process is described, followed in Section 3.3 by an overview of mathematical modelling, which in OR is an often-used tool for problem solving, especially in relation to decision problems. In Section 3.4 the evaluation of mathematical models is discussed, i.e. whether the models developed during the mathematical modelling process can be accepted for use in the problem solving process. This is followed by an introduction to the mathematical modelling recommendation that can be found in literature. Finally, in Section 3.6 the modelling guidelines that were followed as part of this project are presented.

### 3.1 Messes, problems, and complexity

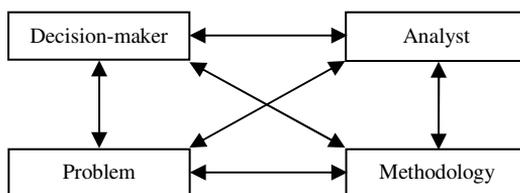
Problems can be divided into categories depending on how easy they can be solved. Focus in this dissertation, and hence the theoretical introduction in this chapter, is on problems that are hard to solve due to the complexity of the problem.

In Ackoff (1981) a system of interacting problems (whether decision, understanding, or consensus) is denoted a *mess*. A mess is not solved but managed e.g. by identification of individual problems within the mess that can be solved independently. The number of interrelations is one kind of complexity that may make it difficult to solve problems, as the process of identification itself may be hard.

In Daellenbach (2001) this is denoted human/social complexity as this is associated with the human perception of the mess and the interrelations between the different problem stakeholders, i.e. the humans who are part of the problem solving process.

The OR literature has traditionally focused on problems, models, and methods for solving those, but all these only exists within a social context. Every problem will belong to a human or a group of humans who want this problem to be solved. The problem is going to be analysed by humans, and eventually humans are to decide how to act (if needed). All these people are stakeholders in the problem.

Vidal (1997) describes decision-making as a social process with the elements shown in Figure 12.



*Figure 12 – Elements of the decision-making process*

It introduces stakeholders in the roles of decision-makers, who will ultimately decide what actions to be taken, and analysts, or experts, who are to analyse the problem for the decision-makers. The social interactions between these as indicated by the arrows add human/social complexity to the problem as discussed in Vidal (1997) and Borges (1998). However, even more stakeholders may need to be considered as e.g. Ulrich (1983) also identified the clients and the affected people as relevant groups.

Another kind of complexity identified in Daellenbach (2001) is the technical complexity that is associated with the physical, mathematical, and computational nature of individual problems.

Parts of the technical complexity may be due to the size and structure of the problem that makes it hard to solve. Examples of this are certain combinatorial optimisation problems where the computation time of all known methods for solution grows exponentially with the size of the problem. The technical complexity can also be due to the uncertainty of describing the system, e.g. of data estimation.

Other types of uncertainty increase the human/social complexity instead. Examples are those related to defining the problem and, if several stakeholders are found, to create a consensus on what the problem is and in the perception of the system, in which the problem exists. In Sørensen (1994) and Friend and Hickling (1987) different kinds of uncertainty are discussed in more detail.

The problems addressed in this dissertation include both technical and human/social complexity. Both types are found in relation to the Balmorel model. No specific problems to be solved are defined for this model, rather it is designed as both a means of discussion where problems are identified and agreed on (human/social complexity), and as a model for analysing and solving those problems (technical complexity). Technical complexity is also found in the stochastic model, as this model includes up to 130 million scenarios for the future inflow to the hydro reservoirs. Finding the optimal strategy for the overall operation of the hydropower system here is a complex decision problem due to the problem size.

### 3.2 The problem solving process

OR is dealing with complex decision problems. Hence, in the literature many suggestions on how such problems should be dealt with in a structured way can be found. Figure 13 shows a schematic view of the problem solving inspired by the works of Drucker (1966), Ackoff (1978), and Garvin (1993).

<b>Phase:</b>	<b>Activity:</b>
1. Identify problem	} Recognition/structuring
2. Analyse problem	
3. Set up alternatives	} Decision
4. Evaluate and choose	
5. Implement solution	} Acting
6. Control	

*Figure 13 – The problem solving process*

Firstly, the problem must be identified. This is followed by an analysis of what causes the problem and an overview of the available resources (human, machinery, time, etc.). Within the resource limits different alternative solutions are then suggested and evaluated against one or more criteria for selection of the best action. Finally, the chosen alternative is implemented in practice and the outcome evaluated to see if further actions are needed.

This is a linearised model of how to solve problems. However, this kind of programmed approaches has led to the belief by many practitioners that meticulously following the steps of the process is the best way to obtain the correct solution. This is by Coppola (1997) called one of the four horsemen of problem solving. Rather, Coppola says, the process should be seen delinearised, where skipping steps or stepping back from time to time should be seen as a necessity. This allows the problem to be redefined, new information on the effects of decisions is constantly gathered, and this information is valued and put into use rather than being discarded; see Duncan et al. (1995). This has been sketched in Figure 14 for the structuring and decisioning stages (phases 1-4 of Figure 13), as these will be the ones focused on in the following.

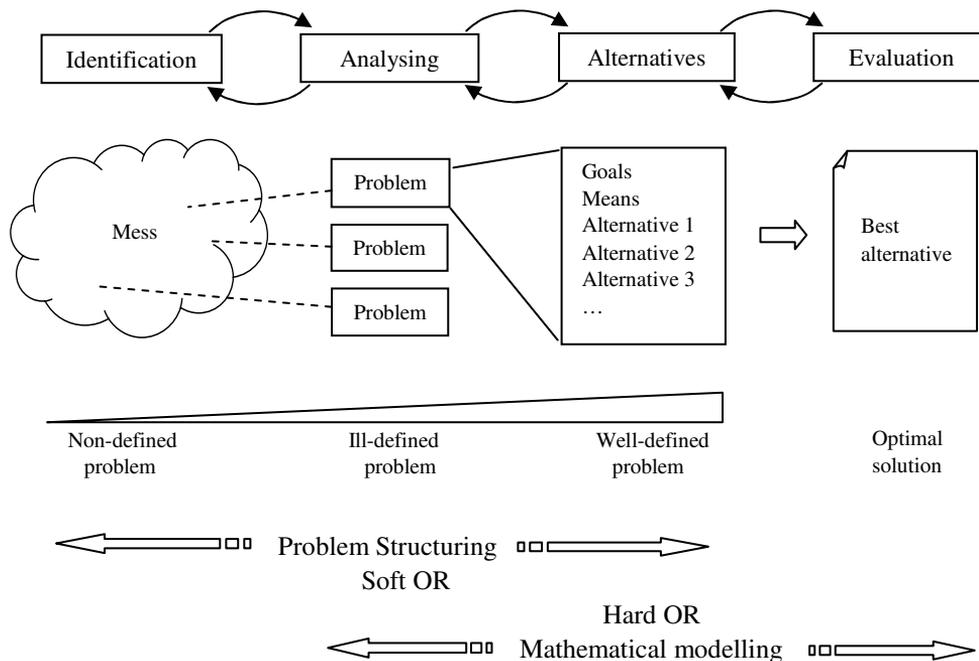
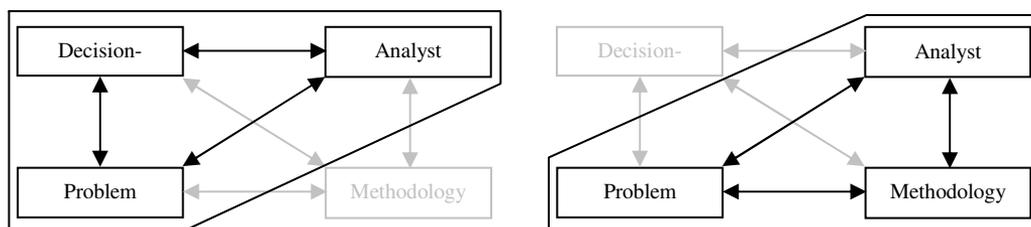


Figure 14 – Delinearised problem solving process (parts of)

In the problems structuring stage, i.e. phases 1 through 3, participation of the different problem stakeholders is essential. A useful approach during these phases is using Problem Structuring Methods as presented by Rosenhead (1989). These methods, also known as Soft OR, are efficient for analysing the problem and creating consensus between the decision-maker, the analyst, and any other stakeholders, of the definition and the understanding of the problem, as well as the objectives to be pursued. The goal is to reduce the human/social complexity of the problem.

In OR, the problem analysis (phase 2) of decision problems typically leads to the formulation of a mathematical model, which is used for decision support (phases 3 and 4). Here, the analyst will try to solve the defined problem using mathematical modelling, or Hard OR, as this is a useful tool for dealing with problems with a high technical complexity. As a precondition, however, the problem has to be defined, or structured, in a way so it can be formulated as a mathematical program. Figure 15 shows the main interactions between the elements of the decision-making process that was sketched in Figure 12.



*Figure 15 – Problem structuring (left) and mathematical modelling (right) as part of the decision-making process*

From the figure it can be seen that the principal human-human interaction is part of the problem structuring part of the process. However human interaction during the mathematical modelling process may still be needed as the analysts may need advice or approval during the process of modelling.

The rest of this chapter will discuss mathematical modelling in more detail, as using this for solving decision problems is as mentioned the primary focus of this dissertation.

### 3.3 The mathematical modelling process

Mathematical modelling within OR has been seen as the traditional way to analyse many of the complex, but well-structured, problems faced by industry or governmental institutions. This section will introduce the mathematical modelling process.

Figure 16 sketches the basic steps in modelling a specific system. Again, focus is on the problem to be analysed. In the following, this is assumed to be a decision problem. However, in general the problem may also be for the decision-maker to improve the understanding of a process (e.g. to determine the relationship between parameters in a production process) or in case of several participants, to achieve a consensus on data and the behaviour of the system in view. Building a mathematical model will help with solving these problems even though the model is not used.

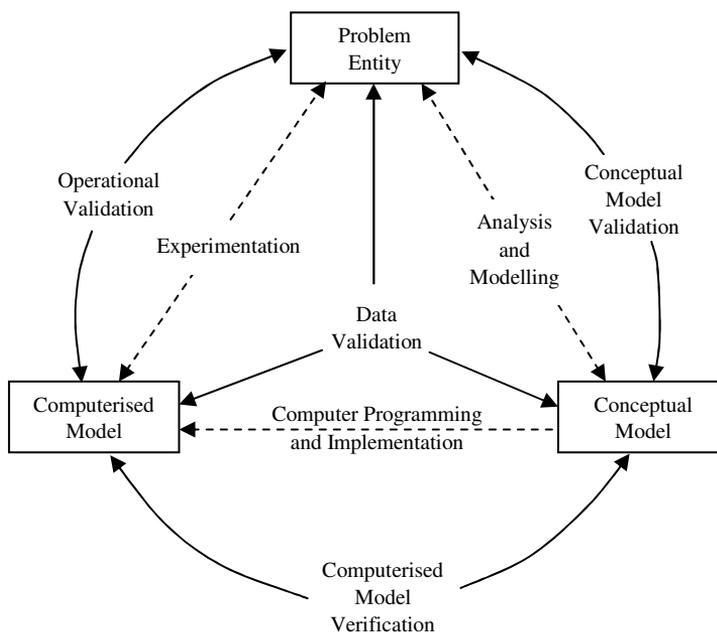


Figure 16 – Simplified version of the modelling process; from Sargent (1999)

On the figure the dotted lines show the actions directed in building the model while the bold lines show the actions needed to ensure that the model built is valid. The double arrows of indicate that the process of modelling is iterative or delinearised, just as the process of problem solving (see discussion around Figures 13 and 14).

Below a further elaboration of the modelling phases is given.

**The analysis and modelling phase** leads to a *conceptual model*. During this phase the system in focus is defined. A system is a limited part of the real world that will be analysed. Outside the system is the environment, which might influence the system, but cannot be controlled. Building a conceptual model requires the definition of parameters (dependent and independent), their possible values, as well as the numerical relationships between those. In addition, for optimisation models, a criteria function must be defined. Overall, this phase should lead to the formulation of a mathematical model of the problem.

Taking the Balmorel model as an example, this phase has numerous important decisions that must be made. This includes the overall delimitation of the type of questions to ask as e.g. illustrated in Section 1.4, the choice of model type, and how best to represent the different model elements. An example of this is the choice of time resolution and between deterministic and stochastic programming. Chapter 4 describes these examples in more detail.

**The computer programming and implementation phase** is the transfer of the mathematical model into a computerised model. The model and solution method can be programmed from scratch but often some modelling/simulation languages are used that permit a near mathematical formulation of problems and efficient solution by accompanying optimisation or simulation software.

However, sometimes the analyst must implement methods for solution, as no existing ones fit with the model of the problem. Some model types are easily solved while others cannot—even with the technology and knowledge of today—be solved to optimality. Instead the method implemented must be able to find good, but not necessary optimal solutions to the models.

In relation to the Balmorel model, it was chosen to use the algebraic modelling language GAMS, see Brooke, Kendrick, and Meeraus (1989), and using commercial solvers to solve the linear programming problem formulated in this, as very efficient solvers for linear programming problems are available. This was to assure that focus could be on the model formulation and not on implementing the model or solution methods. See further the discussion in Section 4.3.

For the stochastic model that was built, a solution algorithm was developed as no existing code for efficient solution existed.

Finally, **the experimentation phase** addresses the computations made using the computerised model. Looking at problems in general, this can be computations to verify whether a hypothesis holds or simulating the consequences of taking different actions. For decision-problems, the results of the model run should indicate which decisions to take given the problem in view.

Many experiments have been made with the Balmorel model during the development stage. Some of the experiments made have been for analysing a specific problem. Typically of these is the one presented in Paper E. But mainly experiments have been made to test the validity of the model and to assist in the decision-making during the analysis and modelling phase. This is part of the important issue during model development known as model *Validation* and *Verification* (V&V). As seen in Figure 16 the V&V is part of all the phases of the modelling process.

In order to trust the model results, it has to be determined whether the underlying assumptions and mechanics of the model are sound and the data used is acceptably accurate. Validation of a model ensures this. Verification on the other hand is to check that the formulated mathematical model and the dataset to be used are implemented and ran correctly on the computer. In general, verification is testing whether the model is implemented as intended while validation is whether the intended model is the right model for the problem.

The various V&V steps in the process of modelling can be presented as follows.

**The data validation** concerns whether data is available for use, reliable, consistent, and up-to-date. Relevant issues to consider include how the data is measured and what assumptions that have been made. Also, the validation should ensure that the transformation of data from the ones collected to the ones required by the model has been done correctly.

**The conceptual model validation** is addressing whether the choices during the analysis and modelling phase are sound. The design choices may be backed by earlier experiences. If no such are available, the choices should be based on theoretical considerations or empirical analyses, to make sure that the assumptions, the defined model elements, and their relations are sound compared with the answers to be found.

**The computerised model verification** concerns whether the conceptual model has been correctly transformed into a computer program. This includes the implementation of any method, whether optimisation or simulation, used for analysing it.

**The operational validation** is dealing with the results of the whole model. Basically, it is tested whether the overall model results behave like the real world does. In general, this can be analysed by either historical or model comparison. In the historical comparison, input similar to those found in the real world should result in a behaviour reasonably close to the one observed in reality. Model comparison may be done if there is no real world data to compare with but also to see the performance of the computerised model developed in comparison with similar existing models.

V&V is actions performed by the analyst during the model development. However, it is not enough that the model developer trusts the results, as the decision-maker, who is to use its results for assistance when taking decisions, should share this belief. This leads to the discussion of model evaluation in the next section, where the acceptance of the model as seen by the decision-maker is addressed. This also includes an overview of literature relevant for both V&V and model evaluation.

### 3.4 Evaluation of models

In order to be used by decision-makers, mathematical models must be credible, useful, and feasible within the resources of the organisation. Models that fit all these criteria are given a model *accreditation*. The decision-maker or an independent third part gives the accreditation after a model *evaluation* or *assessment*; see e.g. Gass (1983) and Landry and Oral (1993).

*Table 4 – Criteria for model evaluation*

<b>Criteria</b>	<b>Comments</b>
Credibility: <ul style="list-style-type: none"> <li>• Accuracy</li> <li>• Robustness</li> </ul>	The model output portrays the real world The interval of input data that gives accurate output
Usability: <ul style="list-style-type: none"> <li>• Effectiveness</li> <li>• Computation speed</li> <li>• Transparency</li> <li>• Adaptive</li> </ul>	The model output helps solving the actual problem Time used for generation output for given input User understanding of model mechanics Easy handling of input/output and making modifications
Feasibility: <ul style="list-style-type: none"> <li>• Cost</li> <li>• Time</li> <li>• Data availability</li> <li>• Knowledge</li> </ul>	Cost of development, acquiring data, and maintenance Time used for development and maintenance Data available with the quality needed, also in the future Expertise in running model and interpreting results

Much of the work done during the model evaluation corresponds to the V&V done by the analyst. For an accreditation, however, more than just V&V is necessary. Overall, the acceptance of a model can be evaluated against such different criteria as the credibility, usability, and feasibility mentioned earlier. In Table 4 a more comprehensive list of some possible criteria is presented inspired by the works of Sørensen (1994) and Willemain (1995).

Note that different stakeholders in the problem solving process may weight the criteria differently. Also, some of the listed criteria are interrelated, e.g. a high accuracy may result in increased effectiveness of the model, which however may make it more costly in resources for data gathering now and in the future.

**Credibility** relates to the model user's confidence in the model, see Sargent (1999). Credibility can be increased through V&V. Though V&V is important it is also costly in resources as indicated in Figure 17. So the level of V&V should depend on how critical the quality of the model results is.

Model accuracy is important in order to obtain credibility. The model must represent the real world "sufficiently" well. Robustness is the model's ability to produce acceptable results though input is greatly varied. Some types of models have no problems with this. But in other types of models, large changes of the model input may result in output of too low quality. Sensitivity analyses are useful in analysing the robustness of models.

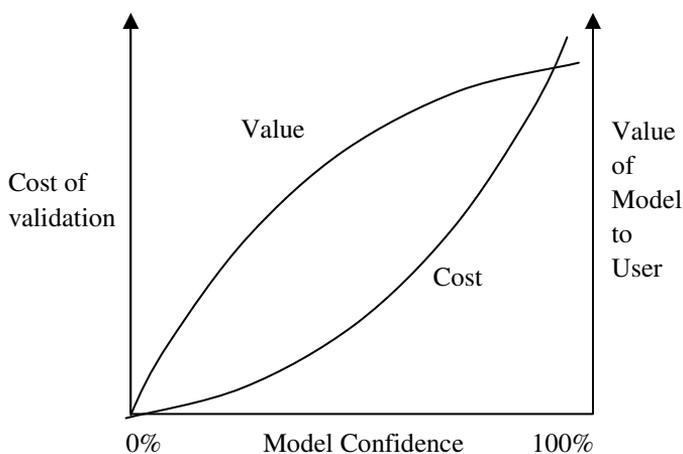


Figure 17 – Cost vs. user value of model; from Sargent (1999)

**Usability** of the model is something different. It describes how effectively the model contributes in solving the problem in hand. A model can be very accurate but not really answering the questions the decision maker wants. This is especially the case when a model is used for other analyses than the ones it was originally built for. Criteria as computation speed, transparency, and how easy the model is to use and update are also important.

Finally, the **feasibility** addresses whether the decision-maker (or client) has the resources needed to purchase and operate the model. Common issues are costs and the time it takes to get a usable model. However, some models may require data of a quality that is not available, maybe because data of this quality does not exist, or because it is considered confidential by those having it. Finally, the organisation may not have the expertise in running that type of model and, more importantly, to interpret the results correctly.

For more information in the field of validation, verification and accreditation (VV&A) some literature (though little compared with the importance) on these topics exists. One of the pioneers is Gass; see e.g. Gass (1977), Gass (1979), and Gass (1980). Much of his work is related to applications in the energy sector. More general works of VV&A is Miser and Quade (1988), which addresses modelling and validation within the scope of system analysis, as well as the special issue of European Journal of Operational Research about validation, see Landry and Oral (1993). Also, several papers have been published addressing VV&A of simulation models, see Balci (1997), Robinson (1997), and Sargent (1999) for some recent examples.

Few recent papers documenting the evaluation of actual models exist. Some of those that exist are Wood (1986), where experiences with third-party validation for energy models are presented and Sørensen (1994), which concerns the validation of policy planning models related to the analysis of acid rain.

The next section will introduce some modelling recommendations, which can be found in literature seen in the light of model evaluation.

### 3.5 Modelling recommendations in literature

Though an important part of OR, relatively little work on mathematical modelling has been published. Instead, literature (and teaching) tends to focus more on different model types and how to solve those; see Powell (1995). Some of the earliest work that

has been done on modelling is Little (1970) from where the following list of desired model properties has been obtained:

1. **Simple** – simplicity of a model may be the best argument for getting it accepted when decision-maker is not a mathematician. (usability, feasibility)
2. **Robust** – changes in parameters should lead to none or only few changes in the model, in order to keep it sufficient “valid”. (credibility)
3. **Easy to control** – the model user should know what input to use to obtain any kind of output. This is obtained by clearly defined and documented model mechanics. (usability)
4. **Complete** – should include essential elements in model and ignore those that do not affect the issue in great degree. (credibility, usability)
5. **Adaptive** – it should be easy for the user to update input and to some degree also the model structure. (usability)
6. **Easy to communicate with** – it should be easy for the user and/or analyst to make changes in input data and get the new output quickly. The computation speed is essential as it makes it easy to see the effects of different scenarios, performing sensitivity analyses on parameters, etc. (usability)

The parentheses after each point refer to which of the model evaluation criteria from Section 3.4 that are addressed by the property.

It can be seen that both credibility and usability are taken into account, but little is mentioned that relates to the feasibility of the model. In Daellenbach (1997) it is argued that the work of Little does not fully grasp the needs for model acceptance. In his paper the list of properties is extended to put more emphasis on usability and feasibility by adding properties saying that:

7. **Model appropriateness** – the model should be appropriate in relation to the situation studied.
8. **Output appropriateness** – model output should be relevant and with no needs for extensive translation/transformation of it to be useful.

Here, the first point (in the work by Daellenbach given the number 6, as he only listed 5 of the 6 properties given by Little) states, that the model should have a proper relation between credibility, usability, and feasibility, while the last point further addresses the usefulness of the model.

Also, it is argued that credibility is not only a property of the model, but also of the modelling process as whole. It is the credibility of the model seen by the decision-maker that matters, and this depends on the ability of the developer to establish the ‘real’ credibility of the model in the mind of the decision-maker. Thus, this also becomes related to the communicative skills of the model developer.

In Pidd (1999) a “rough guide to modelling” is given based on the author’s experiences. Instead of being focused on model properties like the work of Little and Daellenbach, this is a guide in modelling. The six basic principles of modelling presented by Pidd are:

1. **Model simple; think complicated** – basically this principle calls for adequate simple, but well-thought-out models.
2. **Be parsimonious; start small and add** – build a simple model and add refinements until it fits its intended purpose.
3. **Divide and conquer; avoid megamodels** – this principle calls for decomposition of complex problems into smaller managerial pieces, which can be solved easier independently.
4. **Use metaphors, analogies, and similarities** – look at similar systems or associate with earlier work to get inspiration, basically; think creatively
5. **Do not fall in love with data** – Lots of data exists and much time can be spent analysing this. However, data should be collected not because it is available, but because of the model requirements.
6. **Modelling may feel like muddling through** – this principle is an attack on the typical linearised process, go from step 1 to step 2 to step 3 etc. Rather a delinearised process should be used where new discoveries are taken into account by moving steps backward as discussed in relation to Figure 13.

Actual implementation and use of a mathematical model require that it matches the evaluation criteria set up by the decision-maker. Hence, the model design should always be guided by these. Following the modelling recommendations given in this section will help, though they do not say anything about the weighting of the different criteria. Also, one must recognise that the recommendations are ideals defined by one or more people. People’s perception of ideals may differ. And finally, as all problems differ, some recommendations may be wrong in some settings.

In the next section the major modelling principles to use during this project will be introduced.

### 3.6 Modelling guidelines used in this project

In the next chapter, the mathematical modelling done as part of this project will be discussed being based on the aspects on problem solving and mathematical modelling that have been introduced in this chapter. For the modelling, it was chosen to follow the three guidelines given below. They are based on the recommendations from the literature, but picked specifically to deal with the challenges of the Balmorel project, which are presented in Section 4.2.

**Incremental modelling** – this guideline is basically the *start small and add* principle. This ensures a high level of control over the model, as it is easier to validate each small addition than a major model built without any validation during its development. Also, following this principle ensures that a prototype model quickly is built, which can be used for generating preliminary results. These may be useful for decision support in some instances, though with little credibility. However, it may show that some alternative ways of acting are infeasible or too costly at an early stage if the indications are strong enough.

**Use modelling tools** – this guideline calls for the use of algebraic modelling languages such as GAMS, see Brooke, Kendrick, and Meeraus (1989), AMPL, see Fourer, Gay, and Kernighan (1993), and AIMMS, see Bisschop and Roelofs (2002), as well as spreadsheets as tools in the modelling process. The use of modelling languages allows focus to be on modelling issues and not implementation. Also, it should ensure flexibility, as models formulated in modelling languages compared with those implemented in traditional programming languages in general more easily can be adapted for other uses than those they originally were intended for. Spreadsheets are useful for efficient storing and transforming large amounts of data, which will be needed for managing a large real-life model.

**Appropriate representation** – this principle is that all design choices should be guided by the given evaluation criteria, i.e. the credibility, usability, and feasibility criteria given by the decision-makers (see Section 3.4). Thus, contact with the decision-makers and other relevant stakeholders should be prioritised, to ensure clearly defined evaluation criteria that all (if several) can agree on. In the model design, the choice of the model type is especially important as this has major implications on what that is possible in terms of types of analyses and expected computation time. Luckily, much work has been published about the use of different model types and the analyses made with those. So these choices can often be based on earlier experiences made. The actual representation of the model elements and their relationships are more difficult,

as the analysis to be made seldom is exactly as one already made. When it is not clear what choices that should be made, analyses should be done for guiding the decisions as they may show which representation (or even model type) to use taking the trade-offs between different criteria into account.

The modelling guidelines above will be elaborated further in Section 4.3 along with the discussion of the experiences with using them.

## 4 Experiences of the project

This chapter will, based on the concepts introduced in Chapter 3, discuss the experiences in problem solving and in particular in mathematical modelling that have been obtained during the PhD project.

Section 4.1 will give an overall view of the topics touched upon during the study. As it will be seen, the main focus has been on mathematical modelling. Section 4.2 will introduce the modelling challenges of the Balmorel project. This is followed in Section 4.3 by a discussion of the Balmorel modelling process in relation to the recommendations of Section 3.6. Some case studies related to this are presented in Sections 4.4 and 4.5. The handling of data is discussed in Section 4.6 while in Section 4.7, the capabilities of the Balmorel model are described. Finally, in Section 4.8 the overall conclusions are given.

### 4.1 Problem solving and modelling in the project

This section gives an overview of which parts of the problem solving process that have been mostly dealt with. Table 5 shows a list of the main topics of the problem solving and mathematical modelling processes, which were introduced in Chapter 3. To the table a column has been added for each research paper and for the Balmorel project as whole, and the number of “plusses” (see definition below the table) is used to represent to which extent a particular issue has been dealt with in each of the particular papers and in the Balmorel project overall.

The most evident point of the table is that compared with the various papers, the Balmorel project covers most of the topics that were presented in Chapter 3. The obvious explanation of this is that while the papers focus on specific issues, the Balmorel modelling project has been a real practical problem solving process and therefore all aspects had to be covered.

Also obvious is the lack of “plusses” in the rows of the implementation and control phases of the problem solving process section. This is because this is the work of an analyst and hence, the issue on how to implement chosen decisions has not been dealt with. However, tools like the Balmorel model can be used for follow-up analyses to make sure that the implementation works as intended. Therefore, the Balmorel project has been given a single plus in the control phase row.

*Table 5 – Aspects of problem solving and mathematical modelling covered in the Papers A-H and the Balmorel project*

	A	B	C	D	E	F	G	H	Balmorel*
<b>Problem solving process</b>									
Problem definition	+	+	+	+	+	+++			+++
Problem analysis	+++	+++	+++	+++	+	+++	+++	+++	+++
Alternatives					+++				+++
Evaluation and choice					+++				+++
Implementation									
Control									+
<b>Stakeholders</b>									
Decision-maker									+++
Analyst	+++	+++	+++	+++	+++	+++	+++	+++	+++
Others									++
<b>Mathematical modelling</b>									
Analysis and modelling	+++	+++	+++	+++	+	+++			+++
Model implementation	+	+	+	+	+	+	+++	+++	+++
Experiments	+	+	+	+	+++	+			+++
<b>Validation and verification</b>									
Data validation	+	+	+	+		+			+++
Conceptual model validation	+++	+++	+++	+++		++			+++
Implementation verification	+	+	+	+		+	++	++	+++
Operational validation	+	+	+	+		+	+	+	+++
<b>Evaluation of models**</b>									
Credibility	+++	+++	+++	+++		++		++	+++
Usability	+	+	+				++	++	+++
Feasibility	+								+++

+++ main topic

++ partly dealt with

+ related to some parts

\* This column is not only representing the mathematical model, but rather the Balmorel project as whole, i.e. a framework for communication, cooperation, and analyses in the region.

\*\* This is not evaluation of the decision-maker or third part. Rather, it is how the analyses and the thoughts behind in each of the papers may relate to the criteria of the evaluation process. Hence, it is actions of the analyst done to improve the acceptance of the model.

This also explains the missing marks for stakeholders other than the analyst, again with the Balmorel project as the exception, which has continuously been in touch with decision-makers and other potential users. As stated in Section 3.1 a major issue of the Balmorel project, apart from actual model development, was to facilitate cooperation

between states and organisations in the region, as lack of this and mutual understanding of the regional power system, the mechanics (technical and economical) behind, and the future challenges/problems were some of the problems to be addressed by the project. Hence, the project also deals with many of the “softer” themes related to the human/social complexity.

Looking at the papers, the main focus has been on problem analysis and as part hereof, mathematical modelling.

As a further note on model evaluation, the rows generally show that especially the credibility and usability of the models have been thought of during the study. Feasibility is mostly addressed in the sense that a more simple and easy-to-use model in general would be more feasible than otherwise. The experiences so far show that the model indeed is considered to be feasible for different organisations to use for analyses.

Overall, it can be seen that most papers deal with mathematical modelling and especially the analysis and modelling phase, i.e. the construction and validation of the conceptual model and the gathering and validation of data. Hence, focus in the following sections will be on these issues using the Balmorel project as case.

## **4.2 Challenges of the Balmorel model development**

As previously mentioned a main goal of the model was that it should act as a means of communication between decision makers in the Baltic Sea Region as previous studies like the Baltic 21-Energy (1998) and Baltic Ring Study (1998) reports showed that the lack of overview of the energy system seen in a liberalised, international perspective and the roles of the involved parties were problems to be dealt with.

Hence, as objectives the Balmorel modelling project was to result in:

- a set of relevant data for the region that could be agreed on
- a mathematical model based on a common understanding of the mechanisms of the energy system, i.e. the relationship between model parameters as well as the interpretation of the results (costs, prices, emission, etc.)
- identification of relevant questions for discussion and further analysis

To facilitate the use of the model as a means of communication, the following goals of the project were set:

- the model should be transparent and fully documented
- the data should be public and fully documented
- the model should be easily accessible for new potential users
- the model should be flexible so that identified problems could be properly addressed

In order to achieve these goals, firstly it was chosen to implement the model in a modelling language, as this would make the model implementation more transparent and easy to modify cf. the discussion in Section 3.6. Secondly, the model should be fully documented in terms of assumptions, data description, and the model mechanisms and limitations. Also, the dataset should be based on public sources and be well documented to facilitate the discussion of the power system in the region.

Finally, the model with dataset and documentation was to be available freely from the Internet as an open-source software. This would allow all interested parties to comment on the model and the data and allow those who had the modelling language installed and an adequate solver, to download and run the model. However, doing this properly was a major challenge. How should it be organised? How often should the model and the documentation be updated? Who should do this? The answers have not fully been found, nor will they be discussed in this dissertation. Instead, focus will be on the challenges that the Balmorel project objectives created for the model design.

The development of any model must be focused on solving the problem that initiated the project. Here the Balmorel project is special in the sense that no specific problem to be analysed was defined. Rather the model should be able to answer different questions addressing the power and CHP sector restructuring in the Baltic Sea Region on the longer term. Having one single model for many types of analyses has some advantages, such as:

- There is only one model to maintain
- There is only one dataset to update
- There is only one model to become skilled at

However, it is also a major challenge, as the model evaluation criteria as introduced in Section 3.4 relate to a specific problem. Hence, a simple model for one analysis may be a complex model for another. So the challenge lies in making a model that will be appropriate in its credibility, usability, and feasibility for many different analyses with little or no modifications.

To deal with this, it was decided to develop a base model, which could answer many general questions within the delimitation given in Section 2.7 without modifications. Similarly, the dataset should be suitable for dealing with these questions and overall, the model and its data should have a high degree of validity so that these questions could be answered without additional validation and verification of the model. In the Balmorel project, the overall validity of the base model has been sought achieved using the incremental modelling and appropriate representation guidelines introduced in Section 3.6.

Most important, the model had to be flexible, so that the level of resolution found in the dataset could be adjusted to fit with the answers sought in different analyses and so that the model structure itself could be changed to answer questions that lie outside the capabilities of the base model. The full model and data documentation and the implementing of the model in a modelling language, as already discussed, are the main steps taken to make many types of model modifications possible with little programming effort. This is discussed further in the next section along with the other experiences with model design and analyses, which has been gained during the study in relation to the guidelines introduced in Section 3.6.

### **4.3 Modelling and the Balmorel project**

In this section the modelling process of the Balmorel project will be discussed with focus on the guidelines introduced in Section 3.6. These guidelines are partly linked, as the incremental approach and the modelling tools both are used in order to efficiently build and handle a large model. Also, the use of these two guidelines is one way to assure an appropriate model representation, which the last guideline addresses.

#### **Incremental modelling:**

The Balmorel project started in 1999 with the development of a simple prototype model. In mathematical terms it was simple, being a linear programming model that did dispatch of electricity and heat in a market under the assumption of perfect competition. It was a static model, as the optimisation was done for a single time period only, and it was deterministic, as all data was assumed known.

In the following months, the model evolved with each improvement being small and added one-by-one as each addition was verified to work out as intended. In this way multiple regions with transmission constraints in between were added, as was a finer representation of time with dynamics of hydropower (within the year) and possible

investments (from year to year). Also issues as environmental policies were included over time.

The analyses presented in the Papers A though C documents some of these additions. Also, they are presentations of results from the prototype that has been presented at various conferences and seminars throughout the region since 1999—the first year of the project. This has led to valuable feedback from other researchers and potential users of the model from early stages and is one of the big advantages of using the incremental approach apart from the easiness of making and validating each small addition.

However, there are not only advantages with this approach. As the model evolves over time, more and more information about the problem arises as tests are run (this is an advantage) but it may show that the model misses some essential elements that the current structure or model type do not allow, making a complete rewrite necessary.

So using the incremental approach does not allow the model developer to avoid careful planning before starting with the simple model, as this may reveal some of the issues that may become important to include in the future. In this way the model implementation can be made to allow this element of change to be incorporated in the easiest possible way.

An example is the time structure in the Balmorel model. Making it right was difficult and time consuming, but when it got to work, no further corrections of this have been needed due to later additions, as those in general had been thought of, as the time structure was made. This structure, where annual energies and a profile of the variations over the year are given for each time dependent parameter, has made it easy to change the level of resolution as it was done in the analyses in the Papers A-C.

In general, the Balmorel model evolved with small increments though version 1 (a description of an early version can be found in Paper A) though with some exceptions. In particular, district heating was included in the very first prototype model rather than being an extension to an even simpler power-only model. However, the inclusion and validation was rather straightforward as the mechanisms already had been thought through in the conceptual modelling phase.

The step to Balmorel version 2 however was a major rewrite of many parts (a description of version 2 can be found in Appendix A).

The major addition was going from inelastic to elastic demand for electricity and heat. This modification had been planned from the beginning, so all elements of the model prior to version 2 were made with this in mind, so that no major obstacles for implementing the elastic demand in the model were made. Though the elasticities normally are described as nonlinear relations between price and demand, the model was kept formulated as a linear program as a reasonable linear program formulation could be made of the elastic demands and linear programming was to be preferred for the sake of computation speed and to ease later additions as shown below.

Also, in version 2 more data was added, as many parameters were to be dependent on time and geography. Finally, the notation and structure of the model files were updated to make it consistent and more logical as the many small increments had left the code ill-structured and the naming convention of the model elements was inconsistent.

While the major revision from version 1 to version 2 went without major problems it was also simple in the sense that the general model type, the deterministic linear programming formulation, was the same. However, some interesting topics to analyse need other formulations.

One such issue is analysing unit commitment, which in a medium-term perspective may be desirable for larger power plants, such as nuclear. It requires integer variables, but working with integer variables in an otherwise linear model is easier than if the model formulation had been changed to include non-linear relationships. So by having kept the linear programming model type, this extension becomes easier.

Also extending the existing linear model into a stochastic linear program is rather straightforward though new solution methods may be needed depending on the size of the problem (see Papers F-H).

Finally, another possible field to apply the model in is for the analysis of market power. This is in general difficult to formulate as an optimisation problem. However, addressing market power is still possible within GAMS, the modelling language used for implementation. Common techniques to use are Supply Function Equilibria, see e.g. Halseth (1999), and Cournot game theory, which may be formulated as a mixed complementary problem; see e.g. Rivier, Ventosa, and Ramos (2000). While the formulation of the model may be difficult and the computation times considerable long, it may still be possible to reuse much of the general structure and data. To which degree however, is currently not known, but also here a linear formulation is expected to be the most efficient.

In conclusion, the incremental design has basically worked as intended. All additions have been fairly easy to control and validate. It should be recognised that major revisions still may be needed as the experiences with the model show some new elements must be included that requires changes in the general structure of the existing model. However, the linear programming formulation should ensure considerable flexibility on modelling and possibility of solving larger model than otherwise.

**Modelling tools:**

This guideline calls for the use of modelling tools such as algebraic modelling languages and spreadsheets where one of the former, GAMS, was used for easing the model development and enhancing the flexibility of model and the latter to ease the handling of the large amount of data that the model would need.

During the 1970s it was recognised that the computerised algorithms for solving large mathematical programs were used little in applications as it was very time consuming to make the data preparation, data transformation, and output generation procedures on the computer. In this light GAMS and other modelling languages were developed; see Brooke, Kendrick, and Meeraus (1989).

For this project GAMS was chosen as implementation language, though the main characteristics of modelling languages in general are the same. In Brooke, Kendrick, and Meeraus (1989) and Fourer, Gay, and Kernighan (1993) examples of the main characteristics are given. In general the modelling languages:

- Provide a high-level language for the compact representation of large models
- Allow changes to be made in model specification simply and safely
- Allow unambiguous statements of algebraic relationships
- Permit model descriptions that are independent of solution algorithms

In these languages large models can be formulated easily. Also, usually the modelling languages are bundled with one or more solvers, which are software that can solve mathematical models. The solvers interface directly with the modelling languages, which handle the data transformation needed to communicate with them. Hence, much more time can be spent on formulating and modifying models than implementing the model and solution algorithms on the computer.

Also, the use of modelling languages enhance the usability of the model, i.e. whether the model is easy to use, modify, and control, which are important criteria for model acceptance.

Finally, a model implemented in a modelling language is due to the algebraic notation in general much easier to understand than if a traditional programming language had been used, especially for people with a background in mathematical programming. This ensures a high model transparency.

As a drawback, the computation times when using modelling languages, when compared with implementations in e.g. C++, in general are somewhat longer. Also, interfacing with other program parts, e.g. a user interface is difficult in GAMS, as it must be done using file exchange. Direct transfer of data using the computer memory is currently not possible.

But in general, the experiences with using GAMS are positive, as it has been possible to handle and easily modify a large-scale optimisation model giving the flexibility that was desired. The model proved to be too large to be efficiently solved by the BDMLP solver, which was part of the GAMS package. Using the CPLEX solver removed these problems.

In the project, spreadsheets were to be used for handling the data for the model. The advantages of spreadsheets over text-files as GAMS uses for input is that columns and rows of data can easily be created and moved around, transformations of various kinds can be done, and comparison of data is easy, for example by visual aids such as graphs. Also, the data can be presented better due to the use of graphs and different colours and text fonts.

These advantages have proved to be useful during the Balmorel project, where the Microsoft Excel spreadsheet was used. However, in general, the data was copied into text-files manually for use in the model. One might expect that this could be done automatically using macros or VBA scripts, which also have been used to some degree. But it was found that while the model still was under development, this reduced the flexibility, as changes in the model structure and design in GAMS often required extensive changes in the Excel spreadsheets. Generally, Excel was used for preparing data for the new structure, but the macros for creating text-files for GAMS were not updated regularly.

For models that are intended for one type of analysis, a spreadsheet-based user interface for handling data and output generation such as of graphs, should generally be regarded as a good idea. For the Balmorel model however, one will often need to do model modifications before the model analysis is made. Here, the improved usability obtained by an automated link between Excel and GAMS will be a trade-off with the

flexibility given by using GAMS as the model modifications may require changes in the Excel-files, that for many users are impossible or time consuming to make. Hence, much of the effort in making the interface may be wasted.

### **Appropriate representation:**

A main issue in modelling is to find the appropriate representation of the various model elements, so that important aspects are included while a high transparency is retained and computation power is not wasted on unimportant issues.

The representation is given by the assumptions and design choices of the conceptual model. The assumptions and design choices may be based on experiences and logical thinking. However, sometimes it is not clear what to choose. Then the decision can be guided by empirical analyses as those presented in some of the papers, e.g.:

- Representation of time (Paper A, B, and C)
- Representation of pumped storage hydropower (Paper B)
- Representation of hydropower with reservoirs (Paper C)
- Representation of thermal production units (Paper D)
- Representation of stochastic parameters (Paper F)

Choices made during the model design will rarely solely support the fulfilment of all evaluation criteria. Rather, it will in general support the fulfilment of some, while others are weakened. For example, a wish for more completeness will in general make models less simple and usable (as models grow and computations take longer). The choice made depends on the trade-offs and the importances of the evaluation criteria.

Also an issue is the model simplicity, which is usually considered as an objective in order to improve the model transparency, and hence the future acceptance of the model as seen by the decision-makers. However, sometimes the simplicity of the model may have to be defended by the modeller. If the decision-maker is a technician himself with knowledge of the influences on the system, then the issue for the analyst may be to explain that it is the right simplifications that have been made in order to reduce costs, computation time, or other decision factors.

The resulting model as described in Appendix A appears large in terms of the number of parameters that is to be specified in the dataset. However, the level of detail found in the dataset is not needed for all analyses. A less detailed representation can easily be used instead. The basic idea is to allow the simplest model to be run given the question to be answered. In the next section this is exemplified using the time resolution as case.

## 4.4 Case study: model simplicity and time resolution

In this section the background for the analyses made concerning the resolution of time in the Balmorel model will be further discussed.

As the main time step, the Balmorel model uses years, as most data are available with yearly updates. However, in Section 2.3 there was given a rationale for a subdivision of the year due to the temporal characteristics found in a hydro-thermal system.

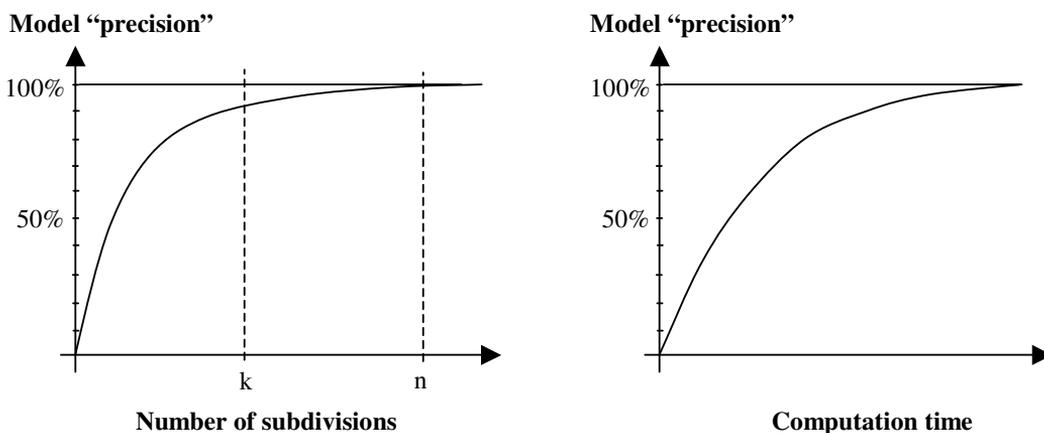


Figure 18 – Relationships between precision, detail, and computation time

The left graph of Figure 18 sketches a relationship between the level of detail and the model precision. Here, a number of  $k$  subdivisions of the year is shown to give a certain result (indicated as approximately 90% “precision”) though the quality can be improved by choosing a higher number. There will however always be a trade-off with the problems of getting data at a higher level of resolution and, as shown in the right graph of Figure 18, with the computation time. In general, one should expect that the last 10% of precision would be more than or at least as time consuming to obtain as the first 10%.

Subdividing the time in the model into  $n$ , where  $n > k$ , segments will give results that cannot be improved by a finer subdivision of time. This is not to be considered at the ultimate true answer (in some cases it might be), rather the best possible result given the restrictions of the modelling, e.g. linear programming, other aspects of model structure, estimation of demand and other uncertain parameters, etc.

Papers A-C present some analyses made with the Balmorel showing that depending on the question asked, very different finenesses of the time scale are required in order to obtain reasonable results. How many subdivisions that are needed and how they are to be defined is not intuitive. Analyses must be made in order to see how well different subdivisions of time works for different problems.

Based on the papers mentioned, Table 6 indicates how many subdivisions that are needed to answer different questions with reasonable accuracy. Refer to the specific papers for the exact numbers.

*Table 6 – Different questions ordered by the number of subperiods needed to analyse them (see the papers for more information)*

<b>Type of question</b>	<b>Paper</b>	<b>Subperiods</b>
Overall costs in system	A	Few ↓ Many
Net annual import/exports	A	
Main investment patterns	A	
Monthly hydro storage management	C	
Monthly price estimation	C	
Specific plant usage	B	

As a note to the specific plant usage, the amount of subdivisions of the year depends a lot on the specific plant. Nuclear power plants and other typical base load units requires very few time steps while peak load units and diurnal storages (as in Paper B) require a much finer representation of time.

Also note that the numbers of subperiods given in the papers are for the Balmorel model. Using the conclusions for other models looking at other systems or at another level of detail may lead to erroneous results. So the numbers given are not to be considered as absolute numbers for all models seeking those answers. For a linear programming model though, that model systems with a similar production system, similar variations over time, and with a similar description of geography as the models used in this project, the values should be close to what to expect.

Returning to the current version 2.10 of the Balmorel model, this has 12 seasonal and 12 diurnal subdivisions for a total of 144 subdivisions of the year. As the papers indicate, this is sufficient for many types of questions regarding medium- to long-term issues. However, it may be inappropriately high for answering some questions when comparing the extra gain in quality with the extra computation time needed. So the

model should allow less resolution to be selected and be able to perform the analyses without the need for modification of the data files. One way of achieving this is to have several datasets, each with different resolutions. The ultimate goal however must be to have one dataset, that can be used to generate model data at any level of resolution up to the one defined for the main dataset.

Currently, this has not been fully implemented in the Balmorel model. However, all necessary data for implementing any choice between 1 (no subdivision at all) and 144 time segments of the year (i.e. the full subdivision as in the dataset) is present. Moreover, the problems of including more time segments than 144 per year are only related to data acquisition and computation time issues. The logic of handling any number of time segments is already present and supported by GAMS and its solvers.

## **4.5 Case study: dealing with stochasticity**

This section will present another case study. Here, the issue of handling stochastic parameter values as briefly was introduced in Section 2.6 will be discussed. The related uncertainty can be addressed in several ways. This section will describe how it has been handled in the models made during this PhD study for three different parameters.

Firstly, for the production from wind turbines the expected values of production have been used. I.e. average seasonal values like the ones shown in Figure 7 are used and similarly, average values for the diurnal time steps are used. Hence, each year will have an average wind energy production, which is desirable for many analyses. However, it may also be a drawback, as years with low wind production require more production from non-wind technologies, which is not reflected by this type of modelling. Using average values for production will lead to an underestimation of prices, as the backup units needed in hours with little or no wind production often are thermal units with high marginal costs.

When looking at shorter periods than a year using average values becomes even more disadvantageous, as it no longer just is an energy issue, but also starts to become a capacity issue. Hence, the investments needed in new capacity may be underestimated both in terms of the real needs and what the model would estimate as needed, if the uncertainty was handled another way.

Still, it has been chosen to use the expected values as the production on wind turbines is still limited compared with the overall system, but with the growth in the installed

capacity seen the recent years, this may change the conclusion. Also, if the focus is on a subsystem with a relative high share of windpower (for instance like Denmark cf. Table 2) a better representation of the uncertainty related to this type of production may be needed.

Secondly, the power plant outages have been dealt with by capacity reduction, i.e. a unit that is only able to produce 85% of a year in average due to both planned and unplanned outages has its capacity reduced with 15%. Much of the arguments for and against this approach are the same as for the use of average values for the windpower production above. In Jørgensen and Ravn (1997) it was shown that this approach was both simple and effective when looking at annual energies.

Lastly, the inflow to the hydropower reservoirs will be discussed. This was in Section 2.6 found to be the most important stochastic parameter within the scope of the Balmorel model. In the current Balmorel model version 2.10 (October 2002) the hydro inflows are handled as the windpower production and availability of the power plants: by the use of average values. I.e. for each month the inflows to the reservoirs correspond to the monthly average observed historically for that country.

The examples above are simple ways of representing uncertainty in the models. The data is analysed exogenously in this case to find the average values of production, availability, and inflow. Stochastic programming is another way of representing uncertainty on important parameters, but compared with the approaches above, it is done endogenously in the model.

A deterministic program gives the optimal solution for a given future. If the future situation varies from this, the solution may end up inferior to others. By using stochastic parameters e.g. for the inflow to a hydro-thermal system, the model will be able to determine the optimal strategy given the uncertainty of the inflow.

Below, some numerical results based on the work from the Papers C and F are presented. These papers discuss respectively a deterministic and a stochastic model of the Nordic hydro-thermal system and the results obtained using them. Both models are based on equations found in the Balmorel model version 1 though somewhat modified from the official version.

On Figure 19 some monthly price estimates for the two models are shown. The left graph shows the average monthly prices as predicted by the deterministic model for three different inflow scenarios. The right graph shows the simulation results for

similar inflow scenarios using the stochastic model. The annual average prices predicted by these models are roughly at the same level for the different scenarios. Also, those annual prices are close to those observed historically.

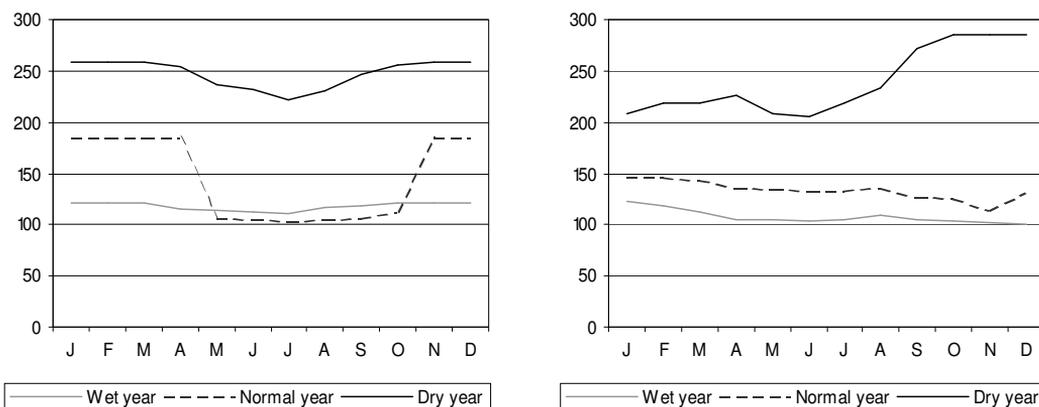


Figure 19 – Spot price estimates for Sweden in DKK/MWh by the deterministic (left) and stochastic (right) models

Looking at monthly prices, in Figure 1 the historical price development for Norway was shown. It can be seen that normally the price during summer is lower than the price during winter with an average difference of 6-7 EUR00/MWh  $\sim$  50 DKK/MWh. This is because the summer months have the most inflow as the snow melts in the mountains. So the availability of the hydropower production is high and at the same time, the electricity consumption is low. However, for some years, the opposite may be the case, as happened in 1996, which was an extremely dry year. However, when using the 1996 inflow profile the deterministic model of Paper C still predicts lower prices during the summer as illustrated in Figure 19 while the stochastic model show an increasing price over the year as the shortage gets worse. So in this sense, the stochastic model is more precise.

The main drawback with using stochastic programming is the computation time. While the deterministic model can be solved in 30 seconds, the stochastic model takes up to two hours to solve as illustrated in the papers. But also the need for more data, and the fact that the model results are harder to interpret must be counted in.

In conclusion, it would be desirable if the Balmorel model was extended into a stochastic program. There is however a limitation to this recommendation. A stochastic model such as the one presented in Paper F would be too ambitious for many analyses;

especially those related to the general capacity expansion under environmental constraints in the next decades. Rather, the model from Paper F was primarily built for over a single year to analyse the price developments in detail.

In most cases a model with for example 6 seasons and 2 possible realisations of inflow per season, would give many of the benefits of a more complex stochastic program with relative few drawbacks. Most importantly, it will only be few times larger than the current Balmorel model (the stochastic formulation includes  $2^5 = 32$  seasons vs. 12 in the current deterministic model). This would allow reasonably fast computations compared with the model in Paper F and the results would show the capacity needed to deal with a dry year in the region as well as indicate the possible price span during that year depending on the different realisations of the inflow. Also no special solution algorithm as those presented in Paper G is needed. A normal linear programming solver, as those used for solving the Balmorel model, is all that is required.

However, such a model would start with the same reservoir level every year and thus the same price for the first season. This is reasonable for the current year, but for future years modelled, the price span for the first seasons will be too narrow. Also, the price seen over the years will be less volatile than seen historically due to the longer seasons (cf. the implication on the results illustrated in Paper C).

## 4.6 Data acquisition

As mentioned in Section 4.2, a major goal of the Balmorel project was the construction of a database of the regional power system with consistent data and with a level of resolution as indicated in the Sections 1.4 and 2.7. This section presents some of the experiences with the data collection. First some general issues will be addressed followed by a more detailed discussion of the particular types of data needed.

In general, collecting data for such a large region where the traditions for collecting statistical data differed considerably was a major challenge. For the countries in northwestern Europe, the traditions are strong and most data needed was accessible in the quality wanted. This was not the case for the former east-block countries where some types of data, for instance in relation to district heating, were mostly missing.

As with the representation of different aspects in models (see discussion around appropriate representation in Section 4.3), data, where the quality is higher than actually needed (similar to the situation in Figure 18), may be needed to justify the model for potential users. This was one of the possible pitfalls that the model could

expect to encounter. Hence, a lot of time was spent on data collecting and validating to ensure that few would reject the model due to incorrect data values for various parameters.

A related issue here is data uncertainty. Here, the issue is not that some parameters are uncertain due to their stochastic nature but rather that only estimates of the actual values can be obtained. Model results may be highly sensitive in relation to the data input. Thus, if an estimate of a parameter is adjusted with 1%, the effects on various results may be much higher. Sensitivity analyses should thus be made for all main parameters in order to see if this is the case.

Finally, the set of data to be produced needed to be one that potential users of all the represented countries generally could agree on. To succeed in this, model and data-advisors in the former east-block countries were added to the project. They have helped by collecting national data that was not otherwise available. Some of this did not exist, and had to rely on the advisors' best estimates. However, with the help of the local advisors for data collecting, data validation, and overall model validation, the model got a dataset, that to a higher degree than otherwise possible, should satisfy potential users throughout the region.

Looking at the data required by the model, it can be divided into the following groups:

- Technological data
- Fuel data
- Temporal data
- Geographical data
- National data

The data is in general described in the Balmorel reports; see Ravn et al. (2001-I) and Ravn et al. (2001-II). Below some additional comments have been made.

**Technological data:** This represents data that describes the production system. The liberalisation has reduced the amount of unclassified information for instance relating to efficiencies or costs of production. In the future, more and more of this information will have to rely on estimates. To ensure consistency between different technologies in terms of costs, efficiencies, and other parameters a spreadsheet was developed as described in Chapter 5 of Ravn et al. (2001-I). This enabled factoring in the extra costs for example of having a de-sulphuring unit at a power plant so that these costs proportionally would be the same for all technologies. Especially for designing the expected future technologies, this proved very helpful.

**Fuel data:** This group of data relates to the emission factors of different fuels and the price developments of those. Some of the fuels are sold internationally with a unified price in all countries. These include coal and oil. Other fuels are national which differ in availability and price between the countries. Examples of these are oil shale in Estonia and peat in e.g. Finland and Sweden.

**Temporal data:** The temporal data is basically profiles for all parameters that have a over-the-year time-dependence, whether this is seasonal, diurnal, or both. The most important of these parameters were introduced in Section 2.3.

**Geographical data:** This type of information relates to the geographical structure of the power system. Most important is the transmission network and the installed capacities of different production technologies in the geographical entities in the start year and the expected rate of decommissioning. Such data is usually available from the national transmission system operators (TSO), as they need this data to ensure a reliable operation of the transmission system. So this type of data will continue to be available at the required level of detail.

**National data:** This data group includes estimates of electricity and district heating demands, the price elasticities of those, taxes and emission quotas, as well as the annuity factor. The elasticities and how to find them is discussed in more detail in Grohnheit and Klavs (2000) and in the Balmorel appendices, Ravn et al. (2001-II).

The annuity factor is used to describe the annual costs of an investment, i.e. annual instalments of a loan taken to make the investment. Assuming a discount rate of 10% and a payback time of 10 years, will give an annuity factor in that country of approximate 0.16. This high discount rate and rather low expected economic lifetime of the investment (10 years) implies a high competitive market. Having a 5% discount rate and a 20-year payback time instead reduces the annuity to 0.08, i.e. the annual costs of the loan are halved. However, such conditions are in general only available in regulated markets where the value of the investments in the future is better known.

In conclusion regarding the data collection work, the dataset currently (October 2002) available for the Balmorel model version 2.10 is reasonable in terms of accuracy and consistency. As well as the actual numbers, a documentation of the data has been made. This describes the assumptions, any transformations done, and where to find the data sources in order to make future updates easier. Making the documentation proved to be more time consuming than expected.

As for possible improvements, much information related to CHP and district heating in the former East block countries still rely on guesswork, but this should improve as traditions are created in those countries for collecting that type of data. Also, the estimation of the price elasticities of electricity in the countries must remain uncertain as in general only little experiences with price vs. demand behaviour of electricity have been obtained in the various countries up till now. Again, this should improve in the future. Finally, sensitivity analyses should be made of more parameters than it has been done till now to check how important the quality of the estimates are for different parameters, such as those related to elasticities.

## **4.7 Using the Balmorel model**

This section will elaborate more on what to do when the model is used, what the model can do, and finally, what it can be modified to do.

Use of the Balmorel model is driven by the need for modelling tools that arise when problems are identified. If the problem identification and analysis show that the problem is suitable for being analysed by a mathematical model, and that the scope of the problem faced lies within the delimitation of the Balmorel model, the researcher may chose to use this model for the analysis.

A general delimitation of the working area of the Balmorel model version 2.10 was given in the Sections 1.4 and 2.7. More specifically, within this delimitation the model has been designed for addressing questions related to monthly/annual energies (production, consumption, hydro storage, and transmission), capacities of yearly investments in different production technologies and transmission lines, and environmental issues that relates to annual emissions for different technologies and the annual use of different fuels.

A major model assumption here is the existence of perfect competition. Also, uncertainties in the model are to be handled exogenously such as by the use of average values or scenario analysis. For a further discussion the general capabilities of the model, see “The Balmorel Model Structure” in Ravn et al. (2001-II).

The problem analysis may show that the answers sought may be given by the available version of the Balmorel model, which then can be used without modifications. Most often however, modifications will be necessary. If this is the case, the modelling cycle shown in Figure 16 (Chapter 3) is to be used.

Hence,

- Based on the problem description, a conceptual model of the modifications needed must be made and transformed into a mathematical model.
- The available version of the Balmorel model is modified with the extensions of the mathematical model
- The updated model is used for analysing the problem. It must be decided if further modifications are required to get a suitable representation of the model

As expressed in Chapter 3, V&V of the modifications (i.e. conceptual model validation, computerised model verification, data validation, and operational validation) must be performed to ensure the model validity and also the model and model output appropriateness, as it should be assured that the questions are answered with a proper trade-off between quality, the computation time, and other resources needed, including the requirements of further data collection.

An example of a specific analysis that required modifications of the Balmorel model is presented in Paper E. This paper discusses the conceptual model development, the computer implementation, and presents the results of the analysis.

Not all modifications are easily made even with the flexibility of GAMS taken into account. The list below is ordered by how hard different types of modifications are (easiest first):

1. Changing existing data
2. Adding more information of the kind already available
3. Reducing the level of resolution
4. Adding new restrictions, new data, and/or new output routines
5. Changing of model type or changing the basic model structure

Ad. 1 – Changing existing data is the most easy modification. However, one must be sure that the new values entered are valid (i.e. efficiencies are between 0 and 1, consumptions are positive, etc.). The model itself does some checks for data validity.

Ad. 2 – Adding more information to tables already found in the model is almost as easy as updating the existing data assuming that the data to be entered has been found and are consistent with the existing (see previous section).

Ad. 3 – Reducing the resolution may many times be simple, but some problems may arise. For geography, exclusion of various countries changes the overall transmission network and excludes the import/export of the deselected countries with countries

outside the model. So while the reduction is easy to do, it may cause unwanted changes to the results. Reducing the resolution of time may be as simple, but often new profiles for demands, availability, etc. must be added. Finally, reducing the number of fuels or production technologies is difficult as they are interlinked (one cannot remove a fuel without removing the technologies using it) and the capacities of the existing power plants must be updated to represent the new system.

Ad. 4 – Adding new model restrictions, data, and output routines requires more work. Even with relatively small and simple changes, validation of the modification itself and of its effects on the overall model must be made, while the previously mentioned changes in general should keep the model valid. An example of a modification belonging to this group is the already mentioned one presented in Paper E. Other examples are:

- Adding modelling of pumped storage power plants as presented in Paper B
- Adding more constraints on nuclear production, as it was found necessary for modelling the Lithuanian energy system; see Elkraft System et al. (2002)

Ad. 5 – If one chooses to change the model type, for instance to introduce integer variables, or some basic model properties like the objective function, some major work is required. In general, the whole conceptual model must be validated again. Also, as the interpretation of the variables and the dual variables of the restrictions may be completely different and cause many of the output routines to be unusable. Still, a wide range of such “hard” modifications is possible within a modelling language as GAMS. Examples of such changes are:

- Changing the model from a deterministic linear program formulation to one using stochastic linear programming. It has already been discussed that the implications on the computation time is large, other solution procedures may be needed, and requires work on developing a scenario tree suitable for the problem.
- Adding integer variables, e.g. to include unit commitment decisions changes the interpretation of some results and again has large effects on the computation time.
- Changing the model, so it can use Cournot game theory for analyses of use of market power, the price of power must be part of the model formulation and not, as now, a result. Using a Mixed Complementary Problem formulation allows this, but requires many changes in the model formulation and the output procedures.

In conclusion, this section has shown that the Balmorel model has many possible uses. However, it must be reckoned that models developed from scratch for a specific purpose may in relation to many of the model evaluation criteria, as those sketched in Section 3.4, perform better than the Balmorel model. Such models will on the other hand require time to develop, gather data, and get used to—time that may be considerably longer than the time needed for modifying the Balmorel model where one already may have the expertise running the model and interpreting the results. In addition, much of the data and the general relationships between the various elements of the power system that would be needed, are already present in the Balmorel model.

## 4.8 Conclusions

This chapter has presented the experiences obtained during the Balmorel project. A main theme was the discussion of the modelling guidelines which resulted in the following conclusions.

**Incremental modelling** – This approach was useful for the Balmorel project as it ensured a high level of control during the modelling and much valuable feedback already at early stages. The project also showed that careful thoughts still are needed before the implementation. Especially, one must consider the possible future needs, so that the model implementation does not hinder the addition of these unnecessarily.

**Modelling tools** – Also the use and choice of modelling tools proved advantageously during the Balmorel project and can hence be recommended in similar cases, i.e. cases where few analyses are done with same settings and the model is not used regularly. In other cases, such as where few changes to the model are expected in the future and where the model is used daily, hourly, or more, coding in traditional languages can result in a higher-speed and more user-friendly application than using a modelling language could allow.

**Appropriate modelling** – Choices to be made during the model design are complex decision problems themselves, i.e. where there is doubt on which alternative to choose. Which choice to make depends on the problem and the evaluation criteria of the decision-makers. In settings where no earlier experience exists, numerical analyses should guide the decisions-making. This also implies that the conclusions of the analyses in this dissertation can be used as guidelines for taking decisions in similar settings. For instance, many of the conclusions of the Balmorel analyses could be reused when the stochastic model was developed as the overall settings (e.g. the time scale and linear relationships) very much equalled the ones from the experiments.

## 5 Contributions of the papers

In this chapter the contributions of each of the research papers are introduced and commented and the conclusions of them given. The papers with similar themes have been grouped together.

### 5.1 Papers A-C

These three papers all discuss the level of detail in modelling. Paper A starts with a general discussion of this and the main conclusion is that a high level of detail apart from the quality aspect of the results may have a lot of drawbacks, for example in costs and maintainability.

All papers include some computational experiments done during the development of the Balmorel model related to finding the appropriate level of detail compared with the answers sought. A major contribution of the papers is that they address an issue seldom discussed in the OR literature and add computational results to support the claims.

Most of the experiments concern the modelling of time. In Paper A the effect on the results was analysed when the main time step, the year, was divided into further subperiods. One major conclusion was that the answer to the question of how to divide a year into subperiods is non-trivial, i.e. it cannot be answered beforehand. Another conclusion was that the gain in accuracy in the results decreases as the level of detail grows.

The work presented in Paper B is a continuation of that from Paper A. Again the focus is on the modelling of time, but this time the geographical scope is national rather than multinational. Apart from the issue of time resolution as in Paper A this paper also addresses how to model time by discussing how profiles of electricity and heat demands could be approximated by either a duration curve or chronological time structure. Specifically, it examines which time structure that is appropriate for a model in order to reflect the operation of the pumped storage power plant in Lithuania.

The last paper of this group, Paper C, addresses how to model hydropower in a deterministic, medium-term model. Computations with different numbers of restrictions on the management of the hydropower reservoirs and with different timescales have been carried out. The results presented in the paper show that for results like total system costs, division of production among technologies, and net

annual import/export a relative rough level of detail will do well. The annual level of inflow will influence much more on the results.

When using 6-12 seasonal subperiods, it appears that a non-stochastic model can obtain good annual price estimates in general. However, when analysing the price developments within the year, the results were less good. For example, the larger hydropower regions in Norway and Sweden had little or no differences in the spot-price estimates during the year, as the hydro storages are large enough to level out the production. In other regions, like in Denmark, Finland, and the southern parts of Sweden, the price was lower during the summer than during winter as normally observed, though with less variance than observed historically. For scenarios with little inflow the model still predicted lower prices during summer, which is contrary to what that was observed historically in 1996, the latest year with less than normal inflow.

Overall, the number of subdivision needed to get reasonably good results of the use of hydropower over the year is lower than seen in Paper B. This is because focus no longer is on a specific plant, which is small when compared with the overall system, but rather on a large part of the system, as hydropower generates approximate 50% of the electricity in the region.

## 5.2 Paper D

The main contribution of the paper is the discussion of combining different models to better represent a larger geographical area and/or time horizon while keeping a high level of detail on the subsystem in focus.

Firstly, the paper addresses how to approximate highly detailed data for use in less-detailed models. Here, focus is on the modelling of CHP units (while modelling of hydropower was in focus in Paper C).

Secondly, the paper shows how results from a wide-looking low-resolution model can be used as input parameters in a more delimited model with higher resolution. As computational case a Balmorel based model of the Nordic countries were used to give transmission conditions to a unit commitment model of the power system in eastern Denmark. These signals were either in quantities or prices for import/export. The latter type of signal was inferior to the first one with respect to correspondence between the model results.

Apart from hydro-thermal systems as presented in the paper, an example of where multiresolution modelling may become desirable is for analysing the effects of large-scale windpower in longer-term models. The relevant resolution of time to use could be in the order of one hour, as the wind may vary considerably during a day. But having hourly time steps for the entire model may for data collecting and computational reasons be impractical. Analysing in detail a single month or two of the year that the model may cover, could give a good approximation of what to expect in the other months.

### **5.3 Paper E**

The reduction of the CO<sub>2</sub> emission and promotion of renewable energy are two increasingly important issues in the industrialized countries. This paper analyses how these goals can be pursued in a liberalised power market by looking at the application of two policy instruments; tradable emission permits (TEP) and tradable green certificates (TGC) to the electricity sector in an international context. The presentation of the model and numerical results for the ongoing discussion are the main contributions of this paper.

Both an abstract model formulation suitable for defining and analysing basic functionalities of the policy instruments and a model suitable for adding numerical results to the discussion are presented. The latter model is basically a modified version of the Balmorel model, so that it follows the description of the policy instruments given in the abstract model formulation. This includes also an extension of the dataset with respect to the modelling of renewable energy technologies and the potentials of installing those in each region.

Simulation results for the countries in the Baltic Sea Region are given for 30 scenarios that all differ in the targets set for the TEP and TGC markets. A major element of the analysis is that the markets are partially overlapping in the sense that some countries are part of both the TEP and TGC markets, while others only are part of one of them (or none).

It is shown that depending on the goals (using either TEP or TGC) different results will be obtained. In general, the introduction of TEP and TGC markets will imply a restructuring of the electricity sector, for instance by a significant increase in windpower capacities for certain combinations of the targets. Also, depending on the specific combination of targets, the revenue by having different technologies will be

changed as some combinations favour the companies with hydropower capacity while others favour those with thermal capacity.

This will have to be counterbalanced by access to production technologies that have fast regulation properties and/or that may maintain voltage stability. However, the price signals of TGCs (and to some extent also TEPs) that will enhance windpower investments will simultaneously hamper investments in technologies that are a precondition for extensive use of windpower technologies like gas turbines. Also, an immediate consequence is increased pressure on transmission lines.

Finally, the simulation results show whether a country that is included in a particular market will benefit from this or not.

## 5.4 Paper F

Deterministic models can give fine and meaningful results in many cases. However, there are limitations for instance in relation to analysing the development of prices over a year as it was shown in Paper C.

The main contribution of Paper F is the presentation of a stochastic programming model, which was developed to deal with such analyses. It uses a two-storage formulation of the inflow/reservoir system. Apart from the water in the hydro reservoirs available for immediately production it includes a reservoir for snow stored in the mountains that eventually will melt and be transferred to the hydro-reservoir. This is in many ways more realistic and also makes the inflow, which is defined by stochastic parameters, serially independent. This allows the model to be solved by the efficient sampling based methods presented in Paper G.

Computational results are given for a model of the Nordic hydro-thermal system, and the results look promising. Compared with modelling the system without the snow-reservoir, the time for computation is not considerably higher while the results, when the snow-reservoir is included, look more accurate when compared with history.

While hydropower has been the main reason for applying stochastic programming to hydro-thermal systems, the issue of applying stochastic programming to windpower will arise as more and more capacity of this is installed. As the stochasticity is interesting on hourly basis or less for windpower, the techniques used for hydropower cannot be assumed to be usable for windpower applications without modifications, as

the relevant time horizon for hydropower is considerably longer. The use of multiresolution modelling as presented in Paper D will be very relevant in relation to include the stochasticity of both types of production.

## 5.5 Papers G-H

Unlike the previous papers, the papers in this group are related to the mathematical methods used for solving models.

For solving the stochastic hydro-thermal model built (Paper F), a new solution method, Reduced Sampling (ReSa), has been developed. This is based on the nested Benders decomposition technique and uses sampling to reduce the computation time. The main contribution of Paper G is the introduction of this method. Using the model from Paper F as computational case the performance of the ReSa algorithm is compared with those of existing algorithms like SDDP and Abridged Nested Decomposition.

This method will always perform better or at least as well as the SDDP method as it with some parameter settings will function exactly the same way. In most cases however, ReSa should be clearly superior to SDDP, as it was for the stochastic model presented in this dissertation.

When compared with Abridged Nested Decomposition (AND), no clear conclusions on which one is fastest can be made, as their performances were quite similar.

Finally, the issue of stopping criteria for sampling based algorithms as ReSa and SDDP has been analysed. As the traditional criterion referred has many drawbacks, a new criterion is presented in Paper H that ensures a given degree of convergence has been reached with certain probability. In Paper H the performance of the ReSa algorithm when using different stopping criteria is analysed empirically with the stochastic model from Paper F as case. Looking at the trade-off between the computation time and the quality of the solution, it can be seen that the new stopping criterion in many cases perform well, though a longer computation time in general must be expected in order to get the specified quality assurance.

## 6 Conclusions and further research

This PhD study has dealt with mathematical modelling of the power systems found in the Baltic Sea Region. It has included elements of both practical model development and theoretical model and modelling studies.

### **Practical model development:**

The main contribution here is the participation in the development of a functioning model, the Balmorel model, which is now fully documented and in use several places. This work included design considerations (based on the theoretical studies), implementation, data collection, validation and verification, as well as using the model for actual policy analysis.

As an overall conclusion of this project, it can be said that a flexible and useful tool for several kind of analyses now exists. The design has been made with further modifications in mind such as those mentioned in Section 6.1.

### **Theoretical studies:**

The theoretical part of the study has included aspects of power system modelling, stochastic programming (modelling and optimisation), and the process of mathematical modelling for problem solving.

In relation to power system modelling, the main contribution has been the experimentations to find the appropriate modelling for different purposes. This includes analyses of the level of detail needed, different model representations of hydropower, and in relation to the choice between deterministic and stochastic modelling.

Within the field of stochastic programming the main contributions of the study are the presentation of a new two-part modelling of the hydro inflow, the development of a new sampling-based algorithm for solving multistage stochastic linear programming models, as well as the proposal of a new stopping criterion to be used for this type of algorithms. All three contributions in this field look promising in terms of possible applications.

In general the conclusions of the theoretical work mentioned above are presented in the Papers A-H. But also the modelling process as a tool for problem solving has been studied. The experiences using the Balmorel project as a case study were discussed in

Chapter 4. In general, the modelling guidelines that were used proved to be effective in the Balmorel project and can be recommended for modelling projects with similar challenges as those given in Section 4.2.

Overall, this project has been very a fruitful learning experience for the author. It has given valuable insight into state-of-the-art knowledge at the universities and at the same time showed the requirements of those, who are to use the modelling tools for real.

More specifically, the project has improved the author's knowledge of mathematical modelling in general, and modelling of power systems in particular, including stochastic programming.

Finally, it has also broadened the view of the author, showing that mathematical modelling is not a stand-alone tool, but must be seen as a part of a larger problem solving process where many non-mathematical skills are needed.

## **6.1 Further research and development**

This section will focus on the issues that, at the conclusion of the study, remain as areas, where more research and development would be desirable as a continuation of the practical modelling work. Suggestions for further research in relation to the theoretical part are made in the individual papers.

Starting with the relatively simple, the modelling analyses presented in this dissertation show that the appropriate level of resolution of e.g. time depends on the problem to be solved. However, only the issue of time has been analysed. Analyses of the resolution of other dimensions such as geography, the production technologies, and the representation of demand, should be made to find the appropriate levels required for different types of problems. This would also improve the overall validity of the model.

More demanding will be the extension of the Balmorel model for making it possible to address other problems, though still within the delimitation given in Section 2.7. As mentioned in Section 4.3, analysis of market power could be one such extension, as tools for such analyses are frequently demanded as result of the power market liberalisation.

Another addition that should be worked on is the extension of the model into a stochastic program taking the uncertainty of hydropower production into account. This was discussed in Section 4.5.

Also, the model could be extended into modelling natural gas as a third energy commodity in addition to electricity and district heating. The future investments in natural gas transmission pipelines will to a high degree be related to the use of this for producing electricity and heat. Having one single model for analysing the future development of the energy system would be desirable for many analyses.

Finally, some more general challenges were indicated in Sections 4.2 and 4.3: “How is an open-source model maintained?” and “How can the Balmorel model be more user friendly without removing the flexibility of GAMS?”. The amount of future users of the model may very well depend much on how well these challenges are handled.

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## Appendix A – The Balmorel model

This appendix will present the present version of the Balmorel model (currently version 2.10, as per October 2002).

The Balmorel model is a partial equilibrium model. The equilibrium refers to that price-elastic demands for electricity and heat can be specified. It can also be used as a pure non-elastic model, i.e. with a fixed demand regardless of the prices of electricity and heat. The model time scope is from 1995 to 2030. The model optimises the production of electricity and heat, the transmission of electricity, and simulates investment decisions concerning building of new production and transmission capacity, if it a year becomes economically profitable or necessary in order to meet the demand. For use in the investment decisions the model includes a large technology catalogue with data for current and expected future production technologies.

Mathematically, the model is formulated as a linear programming model within the modelling language GAMS, see Brooke, Kendrick, and Meeraus (1989).

Figure 1 shows a schematic overview of the model as it looks in version 2.10. Given the parameters and included restrictions the optimal values of the decision variables are found by optimisation with the objective to maximise the consumers and producers surplus.

### A.1 Geography and time

Geographically the model elements are defined on system level, country level, region level, or area level. The overall system is divided into countries where each country may be split into one or more regions that again may be split into one or more areas.

With respect to geography, all technology data is system wide, national policy data is defined per country, and all parameters and variables concerning transmission, natural resources, hydro reservoirs, and electricity demand are given by region. Finally, data concerning the production and the capacity of electricity and heat is defined on area basis.

Many parameters are given for each year in the time horizon, which is the main optimisation period of the model. However, the year can be split into subperiods to describe seasonal and diurnal variations of parameters, which may affect the operation of the system. The dynamics between the different time steps are discussed below.

**BALMOREL VER. 2.10 OVERVIEW**

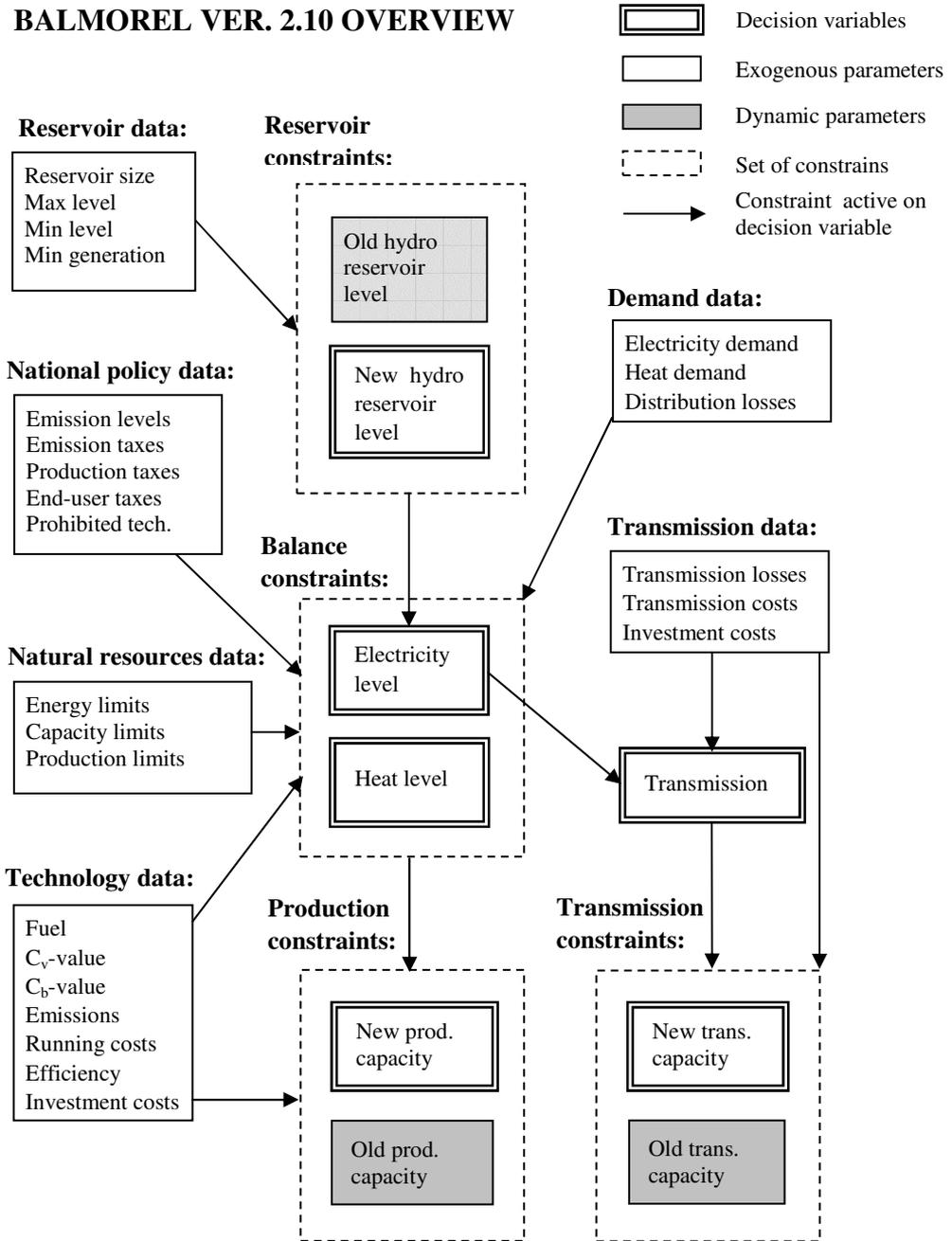


Figure 1 – Sketch of the Balmorel model version 2.10

## A.2 Model dynamics

Within the year, the hydro reservoir level is dynamic, as this is updated from season to season with perfect foresight of the parameters that might affect the operation of the reservoirs. Similarly, short-term heat and electrical storages (e.g. a hydro pumped storage as the one presented in Paper B) link the time segments that are making up the diurnal variation.

Compared with the dynamic implementation of the hydropower and short-term storages, the model is quasi-dynamic in its linkage of the years. This means that the model is solved sequentially for each year within the time scope with no information about the future, but is dynamic, in the sense that capacities of production technologies and transmission lines are adjusted and carried on from year to year.

Compared with a perfect foresight implementation, the quasi-dynamic model may choose to invest in a technology, which is made unprofitable in the following year, e.g. due to changes in demand, taxes, or emission quotas. So the model is not robust for such changes though decision makers would often know such changes some years in advance. Perfect foresight models will optimise the whole time horizon in one step and thus have full knowledge of all changes in demand and policies of the future. In many cases though, this would not be realistic for a 30-year time horizon.

As the current model that is solved year-by-year requires considerable computer memory to be solved efficiently, it has been chosen to keep the quasi-dynamic implementation, as a full dynamic implementation would not have many advantages over this approach, but would be time consuming to solve on most computers at the time being.

## A.3 Decision variables

The types of decision variables are illustrated on Figure 1. While the levels of electricity, heat, and transmission are found for each subperiod of the year (i.e. both the seasonal and diurnal entities), the hydro reservoir levels are found for each seasonal entity. Finally, the new production and transmission capacities built are defined for years only.

Apart from the optimal values of the decision variables, the model delivers other results. For instance, the shadow prices belonging to the balance constraints for electricity and heat can be interpreted as the expected market prices for those

commodities during the specific time periods. Also, given the optimal values of the decision variables, results like emissions and costs of production can be found.

## **A.4 Further references**

Note that the model will be continuously modified in the future. For an up-to-date model and accompanying documentation, the reader should consult the Balmorel project website [www.balmorel.com](http://www.balmorel.com).

**PAPER A**

**LEVEL OF DETAIL IN MODELLING  
AN ANALYSIS OF TIME SCALES  
IN THE BALMOREL MODEL**

Revised version of the paper “Bottom up modelling of an integrated power market” presented at the conference: “Multi-region models, energy markets and environmental policies”, Helsinki, Finland, March 9-10, 2000.

“Everything should be made as simple as possible, but not simpler”

- *Albert Einstein*

# Level of detail in modelling—an analysis of time scales in the Balmorel model

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**Abstract:** This paper will discuss the level of detail in modelling. The main conclusion is that a high level of detail apart from the quality aspect of the results may have a lot of drawbacks. The paper includes computational experiments done during the development of the Balmorel model. Here it was analysed the effect on the results when the main time step, the year, was divided into further subperiods. It has been concluded that the gain in accuracy in the results decreases as the level of detail grows. Also, it is clear that the answer to the question of how to divide a year into subperiods is non-trivial, i.e. it cannot be answered beforehand. Rather, the answer should be based on analyses as the one presented here.

**Keywords:** Modelling, time representation, power market, Baltic Sea Region.

## 1 Introduction

The present paper presents some reflections on bottom up modelling of an integrated power market. The scope of the modelling is the Baltic Sea Region, where in a number of studies, in particular Baltic 21 – Energy (1998) and Baltic Ring (1998), it has been expressed that a model for the analysis of the hydro-thermal system in the countries around the Baltic Sea is desirable. Of special interest is the interaction between hydropower found in the northern part of the region and combined heat and power (CHP) plants mainly found to the south. The model should be a long-term model capable of providing assistance for policy analyses.

The work presented here is carried out as a part of the Balmorel project where the objective is to develop a model, including the relevant data set, to be used as analysis tool in the region. This project is carried out in co-operation between research institutions around the Baltic Sea.

The Balmorel model is at the present state (version 1.00, March 2000) a demand driven model, though it is the ultimate goal to produce a partial equilibrium model. The model time scope is from 1995 to 2030. The model optimises the production of electricity and CHP heat and simulates investment decisions concerning building of new capacity of different technologies, if it a year becomes economically profitable or necessary for reasons of capacity.

In the present paper we describe the modelling, and present preliminary examples of analysis carried out using the model.

The paper is organised as follows. First we discuss the issue of the level of detail in modelling. In Section 3 we give an overview of the Balmorel model describing it in the present state of development. In Section 4 we use the model to illustrate, in the form of a case study, the effects of varying the fineness of the time representation. Finally, Section 5 presents some general conclusions and some perspectives for the further work with the model.

## **2 Some considerations on model details**

The design of a model involves the determination of how much detail it should contain. The obvious temptation in any model work is to include too much detail, from the belief that omission of detail implies less accuracy, and therefore a ‘not so good’ model. This is based on a simplistic view on a model, cf. the end of this section.

In this section we briefly list some of the issues in the determination of the appropriate level of detail, in particular in relation to the Balmorel model.

### **2.1 Time structure**

For a model that shall reflect the longer-term development it will be quite natural to present results for annual values, say, over the period 2000 - 2030. However, this does not mean that the model in its internal mechanisms does not take into account that the individual year is constituted of months, weeks, days, etc. that are not identical. To the contrary, some models of relevance for the electricity sector operate on basic time scales that are hours, minutes, seconds, or even fractions of seconds.

Therefore we focus on the time steps used in the model. Some models use one year as the basic time step. However, in order to be able to analyse the effects of seasonal and daily variations in demand, wind power production etc., the Balmorel model can use smaller time steps than a year. This may be important for numerous reasons, for instance to get a better view on the actual transmission pattern during a year.

Consider Figure 1 where monthly values of transmission between Denmark and Sweden are shown. The 1998 net value of transmission is close to zero; however, as seen a transmission capacity sufficient to transmit at least 700 GWh per month is necessary in order to accommodate the actually observed transmission without bottleneck effects. Using a basic time unit shorter than one month might further increase the minimum transmission capacity required.

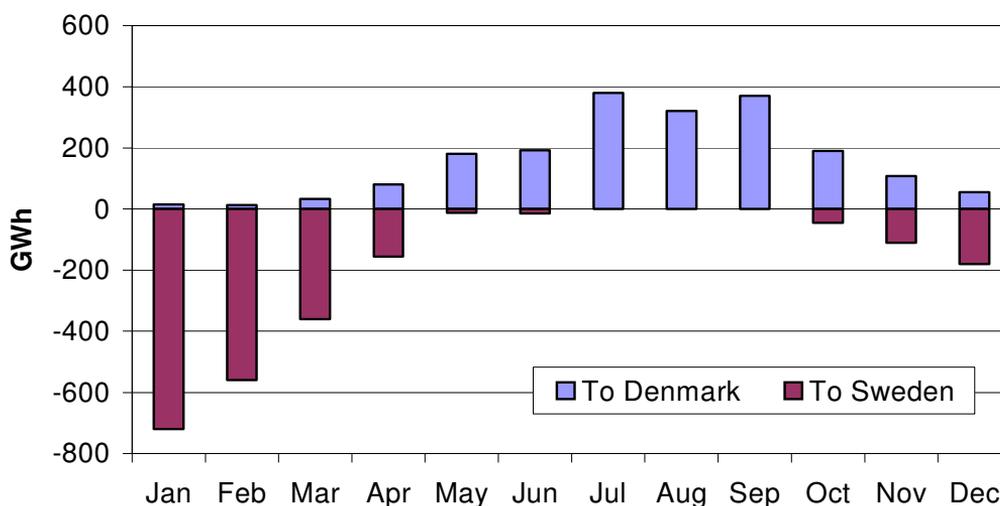


Figure 1 – Transmission between Denmark and Sweden in 1998, see Nordel (1999)

The dissolution of the time axis within the year must be adapted to the purpose of the modelling. Speaking of longer term models, as those in focus here, there are a number of reasons why the year should be divided into subperiod, e.g.:

- Differentiation between units: base load, peak load, etc.
- Differentiation between fuels (partly in consequence of the different units)
- Reflection of the interdependencies between heat and power in CHP modelling
- Reflect applications of storages on scales less than one year (hydro, heat)
- Reflect natural production patterns of unregulated technologies (wind, solar)

Apart from the question of how many subperiods to have within the year, one may consider the question of how the subperiods are linked. If there are no linkages, then the load duration curve technique seems adequate. Otherwise, e.g. in case of energy storages, more elaborate techniques are necessary, implying considerations on chronological models, feedback structures, delays, etc. In Galinis, Hindsberger, and Ravn (2000) such an analysis has been made.

In Section 4 we shall by example illustrate the importance of the subdivision of the year.

## 2.2 Geographical structure

The geographical area for the model is initially given as the Baltic Sea Region. Within this, the model has the countries in the region as a natural subdivision. The subdivision into countries is necessary since many questions of interest in relation to the present hot policy issues are related to the national level – national regulations, emissions policies, etc. Moreover, many of the input data are national in their character – relative to historically given supply systems, ways of organising the energy sector, cost levels, taxation, and other aspects.

However, a finer subdivision may be appropriate for some purposes. Thus, if looking at the electricity supply system it may be inappropriate to consider a country as a homogeneous area. In particular this holds true if larger parts of a country is geographically or electrically separated, as is the case with Russia/Kaliningrad region and Western/Eastern Denmark, respectively.

Considering CHP units, also the heat supply system may motivate a subdivision, viz., in the case where there are separate district heating areas, such that a dispatch of the heat supply between the production units located in separate areas is not possible.

The appropriate balance will have to be determined in accordance with, among other things, the objectives of the study, the availability of data, and the model solution capabilities such that no clear preference is possible.

## 2.3 Deterministic versus stochastic models

Most models for long-term analysis are deterministic in their construction. To the extent that the future is uncertain, various scenarios may be simulated, but for each scenario the model will typically be deterministic.

However, there are a number of reasons why the stochastics should be more systematically considered in modelling. Thus, in the Baltic Sea region, the hydropower is of significant importance for the power sector. However, the variations between the years of the hydropower potential are considerable. These variations explain part of the structural characteristics seen, e.g.:

- Variations of annual net exchange of power between countries
- Existence of certain technologies (e.g., peak and reserve units, electric boilers)
- Price differences between years

## 2.4 Data acquisition

Allowing more detail in the model structure implies the need for more data. This is not trivial. Some of the issues here are:

- Availability – do data exists?
- Reliability – can the data source be trusted?
- Confidentiality – may the data be used?
- Consistency – are the basic assumptions and methods for extraction similar?
- Maintainability – is data updated regularly?

The answers to these questions will to a large degree depend on the user's position in the energy sector.

## 2.5 The bottom up/top down perspectives

One of the present tendencies in the approaches towards modelling in relation to the energy sector is that the traditional distinctions, cf. Table 1, between bottom up and top down models become less clear. A number of models that integrate these two perspectives have now appeared and demonstrated that the approaches are not absolute alternatives.

The Balmorel model in its present stage is a bottom up model, emphasising production technologies, optimal distribution between production units to satisfy given demands,

etc. It is quite obvious that it will be possible to add a considerable amount of detail to such model, precisely because the modelling, according to the conventional philosophy of the bottom up approach indicated in Table 1, emphasises e.g. production technologies, where an abundance of detailed information of physical and technical nature is available.

*Table 1 – Traditional distinctions between bottom up and top down approaches in relation to the energy sector*

	(Early) Bottom Up	(Early) Top Down
Endogenisation of behaviour	Low	High
Details on non-energy sectors	Low	High
Details on energy end-uses	High	Low
Details on energy supply technologies	High	Low
Orientation towards prediction	Low	High

The question is how to match the levels of detail of the bottom up aspects to those of the top down aspects. One element of this is the attainment of a balanced model, i.e. an evaluation issue, another is whether it is possible at all to integrate the two perspectives, irrespective of the level of details on the bottom up side. A specific example in relation to the time structure will be indicated in Section 5.

## 2.6 Summarising

Though it seems obvious that more questions can be meaningfully answered by a highly detailed model, we do not believe that more details necessarily gives a more ‘correct’ model (whatever this might mean), and we do not believe that it necessarily gives a ‘better’ model either (again, whatever this might mean). And definitely there are a number of drawbacks associated with a detailed model. Thus, it is:

- More difficult to establish
- More difficult to verify
- More difficult to simulate/solve
- More difficult to maintain
- More difficult to modify
- More difficult to understand
- More difficult to interpret
- More difficult to communicate

The analysis presented in Section 4 was made to try to find where the balance for the level of details in the Balmorel model lies with respect to the time representation, viz., how to divide the year into subperiods.

### **3 Balmorel model overview**

The model, Balmorel version 1.0, used in the case analysis in chapter 4 must be considered as an early version of what to come as the project develops (see [www.balmorel.com](http://www.balmorel.com) for a description of the current model). According to the above classification, the model used is a bottom up model with emphasis on the modelling of the production technologies, on electricity and heat balance equations between demand and production (for electricity also for transmission) and on dynamics in relation to capacity investment and depreciation. A more detailed modelling of the demand side was not included in the model at this stage.

The model is a linear optimisation model. The model is formulated using the GAMS modelling language and solved by commercial solvers available for this system.

#### **3.1 Delimitation in time and space**

The main time step of the model is one year, covering the period 1995-2030. Each year can be subdivided further into two types of subperiods; seasons and hours. Seasons are included as a subdivision of the year in order to specify seasonal changes like monthly changes in consumption. Though any number of subperiods can be specified, 1 (no subdivision) though 12 (one per month) should be the rule.

Hour is a further subdivision of seasons. This is included in order to represent diurnal changes of parameters. As for seasons, any number could be used though the typical choices should be between 1 (no subdivision) and 24 (one per hour).

In relation to the geographical coverage, the model covers the countries bordering the Baltic Sea as well as Norway for a total of 10 countries. Of these, two is further subdivided into subnational entities. These countries are Denmark and Russia which have been split into eastern and western Denmark and main Russia and the Kaliningrad region respectively as these form major national parts (within the overall scope of the model) with no direct electricity transmission capacity in between.

## 3.2 Objective function and constraints

The objective function used for the optimisation is cost minimisation for the whole model area each year. That is, for each year the model determines the minimal cost of production and new investments given some constraints, which might concern the current production capacity, national or regional emission limits, etc. The production costs include fuel costs, operation and maintenance costs, as well as taxes on production and emission. The types of constraints in the model can be seen in Figure 2, which gives an overview of the model with emphasis on the constraints. The variables and dynamic parameters are introduced below.

## 3.3 Variables

The model is solved by optimisation for each year in the period specified and for each of those the optimal values for the decision variables listed below are found:

- Production of electricity in each sub-period of each technology type and country
- Production of heat in each sub-period of each technology type and country
- New production capacity built of each technology type in each country
- Transmission between all pairs of countries in each sub-period
- New transmission capacity built between each pair of countries

As a consequence of the values of the variables found, other relevant characteristics may be identified, e.g.:

- CO<sub>2</sub> emission per country
- Demand for investments per country
- Consumption of various fuels per country

Also several of the dual variables connected to the restrictions are of interest, e.g.:

- The expected price of electricity (from the electricity balance constraint)
- The expected price of heat (from the heat balance constraint)
- The shadowprice of CO<sub>2</sub> emission (from the emission level constraint)

## 3.4 Dynamics

There exists a link between each year due to the possible investments. Thus the total capacity at the start of each for each technology are defined as dynamic parameters (in

Figure 2 they are represented by the grey boxes). These values are exogenously given for the start year. For the following years they are found as the previous plus the capacity invested in during the previous year. This is for both production capacity and transmission capacity. For production capacity, a decommissioning of capacity is also given for each year and must be subtracted in order to find the capacity for the following year.

Decisions on investment in new capacities (production and transmission) are made on the basis of the information available during the ‘present’ year of simulation. In this sense, the model uses a myopic view. New capacity is built if it can produce cheaper electricity during one or more subperiods than the existing capacity. Here the cost of production by the ‘new’ capacity is the long-term marginal costs (i.e. the operations costs plus capital costs for the first year), while the existing units are willing to produce to their short-term marginal costs (i.e. just the operations costs).

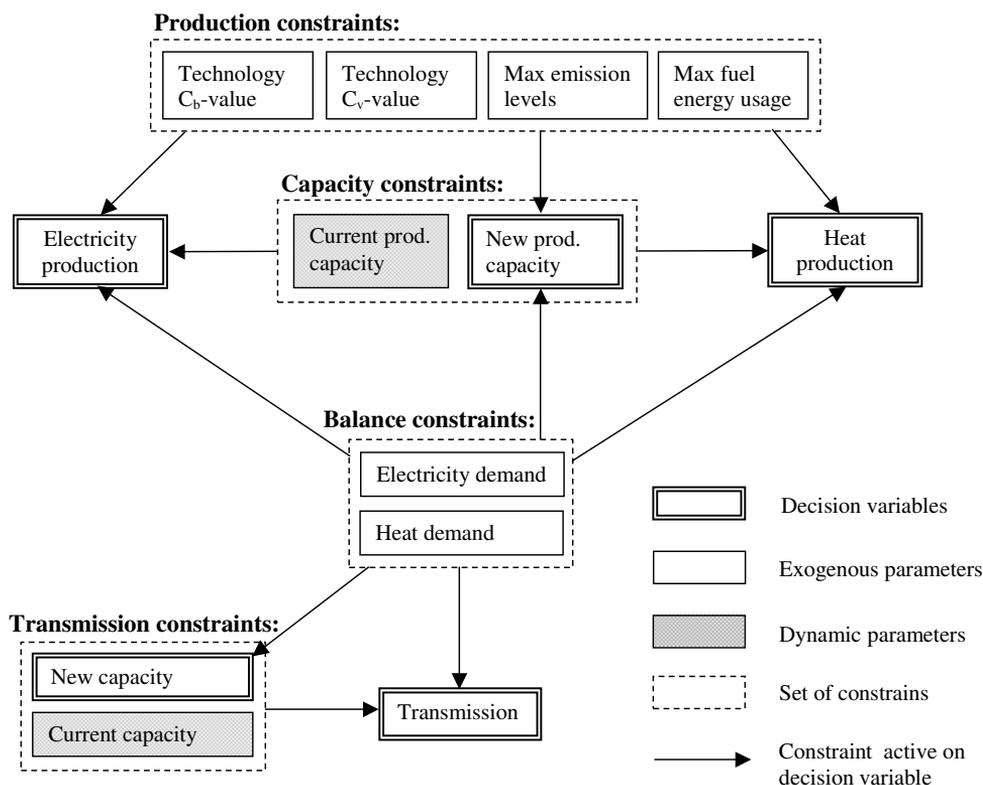


Figure 2 – An overview of the Balmorel model version 1.0

The quantities of new capacities are selected from a continuous range. Hence, the modelling disregards that production plants have some typical ‘minimal’, ‘maximal’ or ‘relevant’ magnitudes. This is done for reasons of efficiency in the model solution phase, and is consistent with the considerations below on representation of technologies.

### 3.5 Technologies

The model works with national capacities of each of the included technologies. I.e. the model has no information about single plants but looks at all similar plants as if it was one large unit (a consequence of the above described assumption of continuity of the decision variables for capacity sizes). Otherwise the model would no longer be a convex model – but a much harder to solve (and analyse) mixed integer problem.

In the model the technologies are divided into five different technology types as listed below:

- Pure electricity (includes thermal condensing, hydro, wind, photo-voltaic)
- CHP back pressure (fixed electricity/heat ratio)
- CHP extraction (variable electricity/heat ratio)
- Pure heat (boilers, solar heat, geothermal, all non-electricity based)
- Electric heat (production of heat by electricity, i.e. electric boilers or heat pumps)

Several production technologies of each type are defined; they differ with respect to efficiencies, the fuels they use, or otherwise. Each of the technologies is described by:

- Production areas (i.e.  $C_b$  and  $C_v$ -values for CHP plants)
- Fuel type used
- Efficiencies
- Emission of  $\text{NO}_x$
- Emission reduction (de-sulphuring and de- $\text{NO}_x$ )
- Investment costs
- Operations and maintenance costs

$\text{CO}_2$  emissions are derived from the amount and type of fuel used. To take de-sulphuring and de- $\text{NO}_x$  units into account,  $\text{SO}_2$  emissions are, unlike that of  $\text{CO}_2$  calculated from the amount of each fueltype used on each particular type of unit and  $\text{NO}_x$  emissions are calculated from the amount of fuel used at each particular type of unit.

### 3.6 Data

The data set used is based on that of the Baltic 21 – Energy (1998) report. A few extensions to this data set have been made—mainly concerning the transmission system and the addition of profiles defining the demand for subperiods of the year. This data has been assessed from data from Nordel (1999), UCPTE (1999), and national energy statistics.

## 4 Case analyses

In this paper we will present some analyses made using the Balmorel model focusing on the effects of changing the time resolution on overall cost, investments, fuel usage, and prices. In particular, going from yearly values to a further subdivision within a year is studied.

We emphasize that since the model is presently in an early verification phase, the results presented here may be taken only for their methodological implications, in particular concerning the directions of change of the indicators presented. Hence, nothing can be inferred from the absolute magnitude of those indicators – simply because the model is not presently fit for providing such magnitudes.

In the scenarios we have in general used the same assumptions as in the Baltic 21 – Energy (1998) report, e.g.:

- Simulation period is 1995 to 2030
- Initial production capacities in countries as in the report
- Growth in consumption and fuel prices as in the report
- Water inflow to hydro reservoirs as in the report
- Decommissioning of initial production capacities by using a linear function with the last parts being decommissioned in 2020
- All nuclear power is phased out in the period

Finally, we did not consider any kind of taxation in the calculations.

Five different scenarios have been used in this paper, one using yearly values, the others with a subdivision of the year as shown in the Table 2 below.

Table 2 – Scenarios included in the analysis

Scenario name:	# of seasons	# of hours/season
1-1	1	1
2-1	2	1
2-2	2	2
2-4	2	4
2-8	2	8

### 4.1 Total costs

The total annual system costs, which include all production costs and investment costs in the whole period, are shown in Figure 3.

We can see an increase in cost in 2003-2007 as the model here begins to invest in new capacity (i.e. in these years the model predicts that the current overcapacity of the system is gone due to decommissioning and increased demands). Another characteristic of the graph is the large drop in 2020, which is due to the fact that all initial capacity in this year have been decommissioned. All new requirements of investments are from this point only due to increased electricity and heat demands, which reduce the overall demand for new capacity compared with pre-2020.

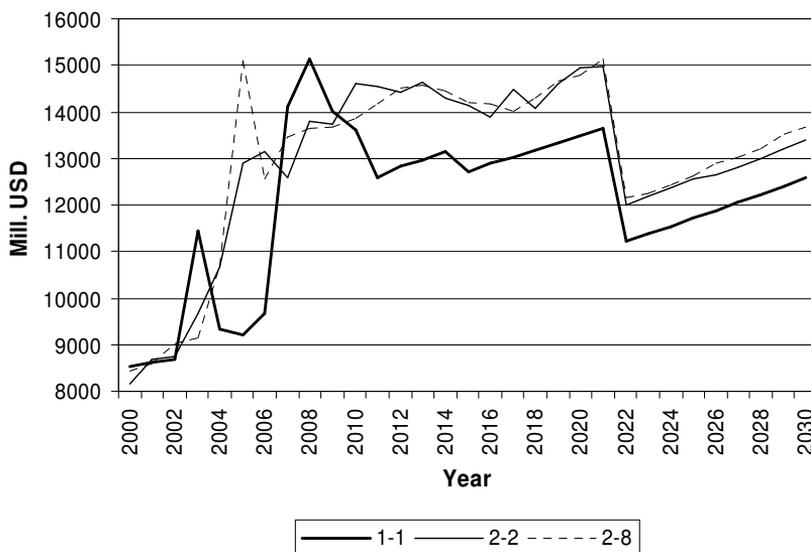


Figure 3 – System cost for each year in the period for different scenarios

In 2030 we end up with almost a 10% difference in the annual system costs between the 1-1 and 2-8 scenarios, while the difference the first years is much smaller. If we sum the costs of the whole period we get the numbers in Table 3.

We see that by using 1 period the total costs would be underestimated with about 8% compared to 2-8, which again should be an underestimation of the real value. The difference in costs is due to the extra capacity required to meet the higher demand in some of the subperiods. This is further investigated in the next section.

*Table 3 – Total system costs in the 1995-2030 period*

<b>Scenario</b>	<b>1-1</b>	<b>2-1</b>	<b>2-2</b>	<b>2-4</b>	<b>2-8</b>
System cost (in mill US\$)	373263	395661	399789	401162	402906
Index (1 period = 100)	100.00	106.00	107.11	107.47	107.94

*Table 4 – Investments (MW) in new production capacities by technology, 1995-2030*

<b>Production tech.</b>	<b>Type</b>	<b>Fuel</b>	<b>1-1</b>	<b>2-1</b>	<b>2-2</b>	<b>2-4</b>	<b>2-8</b>
CC-Co-B15	Elec	Gas	0	0	0	151	1302
ST-Cond1-C	Elec	Coal	8681	120	95	0	0
ST-Co-C-B15	Elec	Coal	0	901	1028	101	308
ST-CHP1-C	CHP	Coal	21322	27739	29641	30168	30375
ST-CHP1-B	CHP	Biomass	360	0	0	0	0
ST-CHP1-P	CHP	Peat	7960	7148	6933	6883	6447
CC-CHP-B15	CHP	Gas	0	217	12074	10109	9447
ST-CHP-C-B15	CHP	Coal	0	2324	141	1612	424
De-CHP-W-B95	CHP	Waste	666	516	308	716	616
GM-CHP-B15	CHP	Gas	27153	26820	22780	22551	23506
HO-C-New	Heat	Coal	9737	36566	36772	38110	37022
HO-B-New	Heat	Biomass	7445	19498	21792	20812	20709
HO-W-New	Heat	Waste	9281	8015	5477	4377	5685
HO-W-Old	Heat	Waste	2153	3568	6314	7007	5799
HO-P-Old	Heat	Peat	0	812	1027	1077	1513
HYDRO	Elec	Water	4832	4862	4862	4862	4862

<b>New production capacity, total</b>	99590	139108	149245	148535	148014
<b>Index (1period = 100)</b>	100.00	139.68	149.86	149.15	148.62

## 4.2 Investments

As seen in Table 4 the trend is that more technologies are introduced as the number of subperiods is increased. Also the need for installed capacity is considerably higher if the year is divided into two or more subperiods. The additional capacity introduced is for 4 or more periods almost 50% higher in MW, a huge number, which however, as it can be seen in Table 3, does not affect the total system costs with more than 7-8%.

We see that the more periods the more need for investments in CHP and heat-only boilers, the latter being used as “cheap” peak load units for the heat demand while CHP is chosen for its flexibility.

Note that the requirements for new capacity are higher for 4 periods (the 2-2 scenario) than for 16 (the 2-8 scenario). As this is due to larger investments in B15 technologies, i.e. a number of new and better technologies that are introduced in 2015, the explanation is that the 2-2 scenario invests more in technologies early, which after 2015 to a larger degree become inferior to the new technologies compared with the early investments in the 2-8 scenario.

*Table 5 – Investments (MW) in new transmission capacities, 1995-2030*

	<b>1-1</b>	<b>2-1</b>	<b>2-2</b>	<b>2-4</b>	<b>2-8</b>
<b>New transmission capacity</b>	9717	12601	14643	15161	14075
<b>Index (1 period = 100)</b>	100.00	129.68	150.69	156.03	144.84

If we look at the need for new transmission capacity, the picture is the same. In Table 5 we see the same underestimation of up to 56% of the needed capacity if the year is not subdivided into smaller periods. Compare also with the discussion around Figure 1. However, again we see a counterintuitive decrease in the capacity invested in from the 2-2 and 2-4 scenarios to the 2-8 scenario.

Figure 4 shows graphically a summary of the results from the Tables 3 through 5 though the scenario used as index is now the 2-8 scenario.

The observations are in general similar to those of the tables. However, the figure makes it easier to observe that for analyses concerning the system costs and the size of the investments the 2-2 scenario seems to perform just as well as scenarios with a larger number of subperiods. If we were to ask such question then, we might just as well stay with the simpler 2-2 representation rather than waste computation time and time gathering more detailed data for a more detailed subdivision of time.

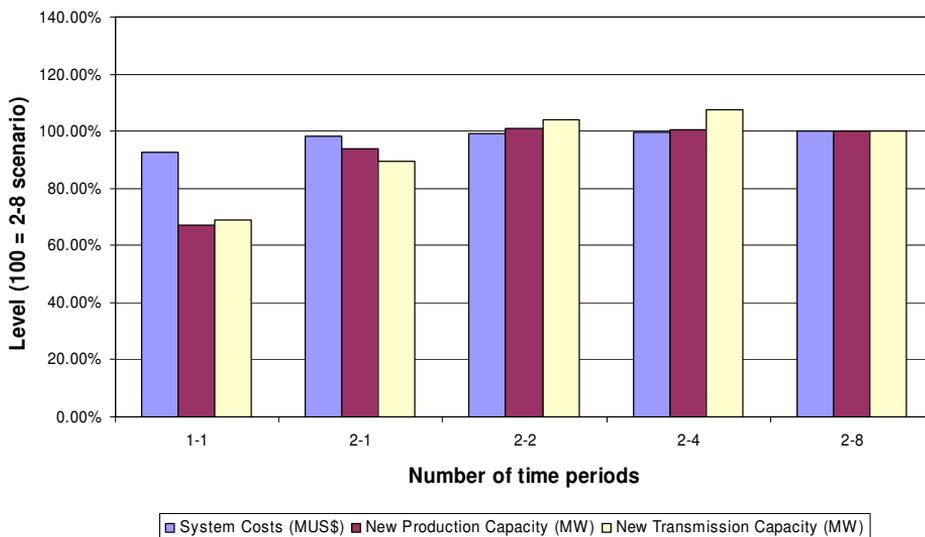


Figure 4 – Comparison between costs and investments

### 4.3 Fuels

We see from Figures 5 and 6 that the choice of fuel is not greatly affected by the time representation used. A few more fuels (shale and fuel oil) though are used in the 2-8 scenario (though to little to make it visible on the figures).

This shows that the diversity of fuels used in the solution tends to be bigger when more periods are modelled. This corresponds to what was observed in relation to the investments in the production technologies.

### 4.4 Electricity prices

The Figures 6 and 7 show annual weighted average electricity prices for the countries in the model area. These prices can be interpreted as expected prices on liberalised markets with perfect competition. Though the print does not make it possible to distinguish between the different countries it can be observed that electricity prices are more differentiated between the countries in the 2-8 scenario compared with the case with no subdivisions of the year. Also a more diversified development pattern over time is shown. The number of subperiods only to a limited degree affect the magnitude of the prices—less than 10% for most years—a conclusion that is in line with what has previously been observed in relation to costs, cf. Table 3.

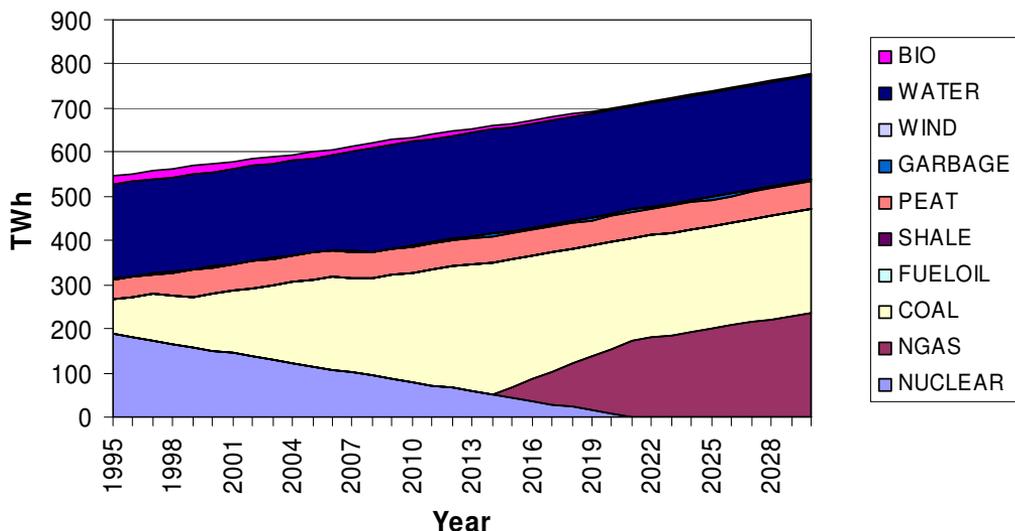


Figure 5 – Fuels used for electricity production in the region in the 1-1 scenario

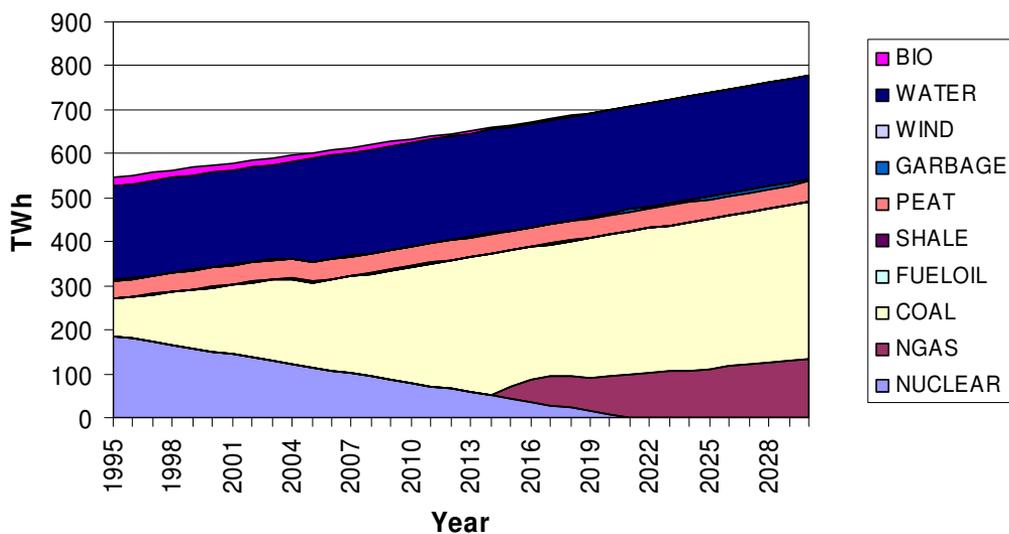


Figure 6 – Fuels used for electricity production in the region in the 2-8 scenario

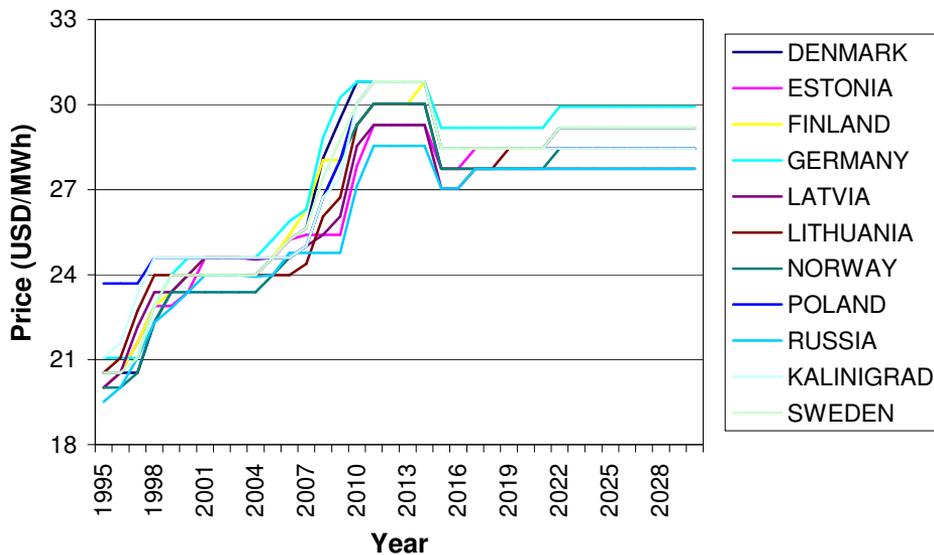


Figure 7 – Electricity price development (annual weighted average) in the 1-1 scenario

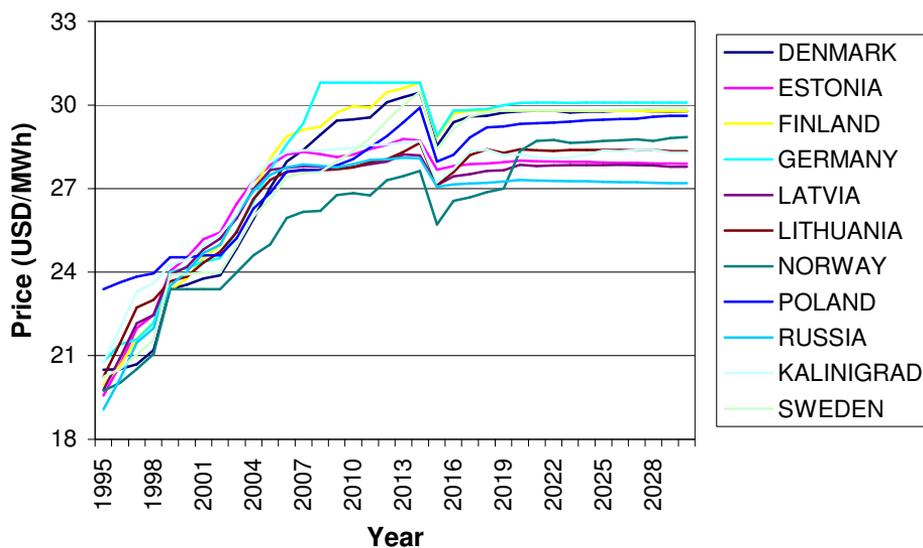


Figure 8 – Electricity price development (annual weighted average) in the 2-8 scenario

## 5 Conclusions

We have presented some reflections on the issue of the level of detail in models. By using the Balmorel as a practical case this has been illustrated by an analysis of which time scale to use for modelling the activities within the year.

Since the model is an early version without much validation and verification done, it is not possible to interpret meaningfully the absolute magnitudes in the tables and figures presented in Section 4. However, the direction of change, as the number of subperiods is changed, is believed to have validity.

The following observations have been made:

- The costs in the model increase with increasing number of subperiods.
- The diversification of production technologies applied increases with increasing number of subperiods.
- To the extent that there is an association between technologies and fuels, the diversification of fuels applied increases with increasing number of subperiods.
- To the extent that the technologies have different marginal costs of production (e.g. due to use of different fuels), the diversification of electricity prices will increase with increasing number of subperiods.

For further studies we advance the hypothesis that the above observations are not only specific for the present model but that they have more general validity. However, the issue is not trivial, as the included examples illustrate, see e.g. the discussion around Table 5.

Also, it is clear that the answer to the question of how to divide a year into subperiods is non-trivial, i.e. it cannot be answered beforehand.

Finally, the observations reveal a number of interesting questions of relevance for further modelling activities, and in particular also for the bottom up – top down distinctions.

1. What is, all other things being equal, the relevant number of subperiods? The present case study of Section 4 indicates that the effects of application of smaller subperiods are decreasing as the number of subperiods increases (which is consistent with intuition) – therefore a suitable balance should be attempted. Can there be given general advice on this, or how and to what extent does this depend on circumstances modelling (e.g. solution effort and bottom up data requirements)?

2. What is the relevant number of subperiods from the perspective of balancing this against other bottom up elements? An ideal could be that the details in the various sections of a model should be chosen to give a balanced totality. It seems not to be generally known what other refinements other model elements need, as the subperiods are made smaller. For example, hydropower, with the associated storages, has the tendency to level marginal production costs. How much detail is necessary on modelling of hydropower, if this tendency should be adequately reflected for a specific number of subperiods?
3. What is the relevant number of subperiods from the perspective of balancing this against top down elements in an integrated top down - bottom up model? Typically, a bottom up - top down linkage is the elasticity of electricity demand, and the number of elasticities should intuitively match the number of subperiods. If such subperiod elasticities are meaningful, then to which extent are they available (or how can they be made so)? Is it necessary or desirable to have the same subperiod division in the top down submodel as in the bottom up submodel?

As previously mentioned, the present version of the Balmorel model is a demand driven, bottom up version. The aim of the project is the development of a partial equilibrium model for the Baltic Sea region, and in the further work towards this, the above issues will be addressed, among others. The model and documentation of the project work is available at [www.balmorel.com](http://www.balmorel.com).

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**PAPER B**

**BOTTOM UP MODELLING OF AN  
INTEGRATED POWER MARKET  
WITH HYDRO RESERVOIRS**

Extended version of the paper: “Bottom up modelling of an integrated power market with hydro reservoirs” presented at the Second International Conference, “Simulation, Gaming, Training and Business Process Reengineering in Operations”. Riga Technical University, Riga, Latvia, September 8-9, 2000.

“A mathematician is a blind man in a dark room looking for a black cat that isn't there”

- *Charles R. Darwin*

# Bottom up modelling of an integrated power market with hydro reservoirs

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**Abstract:** The paper presents methodological reflections on bottom up modelling of an integrated power market. The geographical scope of the modelling is the Baltic Sea Region (BSR), i.e., of considerable extension. This paper will discuss the structure of sub periods and profiles of electricity and heat demands, which is appropriate for such model. Special emphasis will be devoted to methods for modification of the dispatch of electricity supply among the sub periods in order to reflect the operation of the pumped storage in Lithuania.

**Keywords:** Bottom up modelling, Power market, Hydro pumped storage, Optimisation, Baltic Sea Region.

## 1 Introduction

In line with the increased liberalisation of the energy markets and the similarly increased attention on environmental aspects a number of recent studies - in particular the Baltic 21 Energy and Baltic Ring studies – have demonstrated the need for a model for the analysis of the electricity and CHP (combined heat and power) sector, covering all the countries around the Baltic Sea. Such model should be capable of providing assistance for policy analyses in a long-term international perspective.

A modelling project with this scope is presently being carried out with support from the Danish Energy Research Programme in co-operation between research institutions around the Baltic Sea. The new model, named Balmorel, simulates the production of electricity and CHP heat and investment decisions concerning building of new capacities of different technologies. The main period of the model is one year, which can be divided into a number of sub periods that are composed of seasons and hours, e.g. 2 seasons and 4 hour types.

As in any modelling project, a key consideration concerns the level of detail in the model. In one perspective it is desirable to include as much detail as available in the belief that this will give maximum accuracy of the model. In another perspective it is necessary to keep modelling at a more aggregated level due to limitations in data acquisition, simulation or solution capabilities, and in order to keep the model and the results reasonably transparent. In any case there is the problem of having a degree of detail that is even over the different aspects of the model in order to get a balanced representation.

In this perspective the present paper discusses the structure of sub periods and profiles of electricity and heat demands, which are appropriate for a long-term model describing the development of national energy systems with cross-border trade and pumped storages. Special emphasis will be devoted to methods for modification of the distribution of electricity supply among the sub periods in order to reflect the operation of the pumped storage in Lithuania.

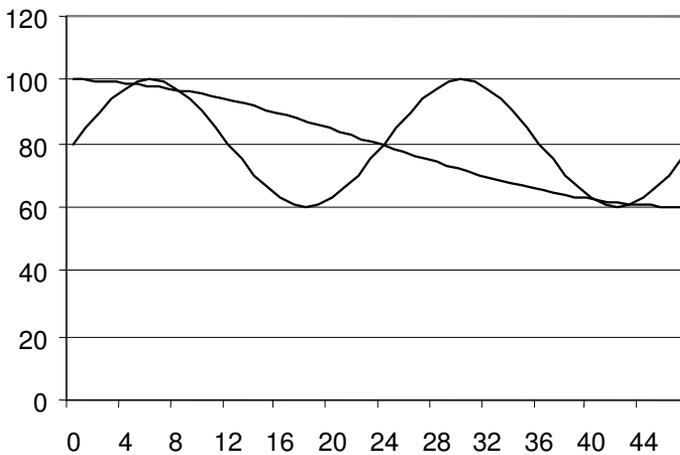


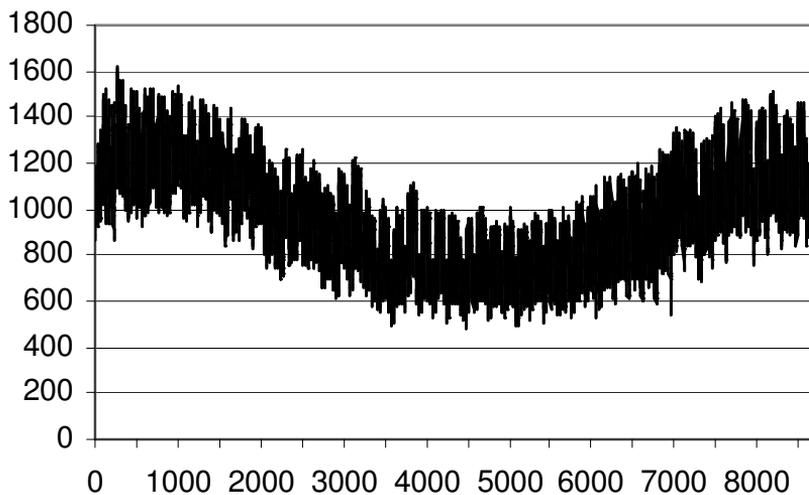
Figure 1: Sample demand curve and the corresponding duration curve (in MWh/h)

## 2 Economics of storage operation

Energy storage is important for several reasons. The demand of heat and electricity vary considerably between seasons, weekdays and hours of the day. Thus, in a system consisting of units with slow power output regulating capacities (e.g. nuclear power

plants) or very different marginal costs the balancing of production and demand may become difficult and/or expensive. Hence, storing the electrical energy may be an option. But energy storage is also important due to the increased application of renewable energy sources, of which several types have a very fluctuating power output (viz., wind turbines and solar power). Current technologies for storing electrical energy include batteries, kinetic energy storage, compressed air storage and hydro pumped storage plants, see e.g. Jensen and Sørensen (1984). This paper deals with the latter type and specifically with the Kruonis Hydro Pumped Storage Power Plant in Lithuania and how the operation of this plant could be modelled.

For the purpose of introducing the fundamentals of economics of power storage operation consider the following simple numerical example. For a 48-hour period the demand in each hour is known and given as in Figure 1 (a sine curve is used). The traditional load duration curve, where demands are sorted according to size, is shown as the decreasing curve in Figure 1. For comparison the actual Lithuanian electricity demand curve in 1999 and the corresponding duration curve are shown in Figures 2 and 3.



*Figure 2: Lithuanian electricity demand curve for 1999 in (MWh/h)*

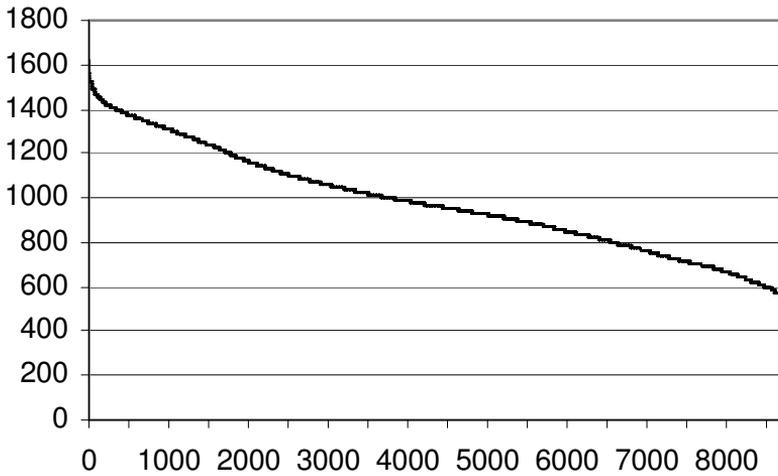


Figure 3: The duration curve corresponding to the curve in Figure 2 (in MWh/h)

The economics of the operation of the storage will depend on the electricity production system in combination with the demand characteristics. Assuming that the production system has a cost function such that the marginal cost depends linearly on the production level, the marginal cost of production will have a shape like the demand curve in Figure 1 where also the corresponding marginal price duration curve is shown.

It is easy to see that the optimal operation of a storage will imply that the storage is discharged during time periods when the marginal production costs (demand) are high, and charged during the periods when the marginal production costs (demand) are low.

Assume now that there is a storage with infinite capacity and no losses, neither due to the storage volume nor due to the charging or discharging of it. In this ideal case the use of the storage would imply a complete levelling over time of the production.

It is not difficult to realise that the following is the optimality condition for economic operation of the storage:

$$p_t^{in} = p_s^{out}, \forall t, \forall s \quad (1)$$

where  $p_t^{in}$  is the marginal production cost in a time period  $t$  when the storage is charged, and  $p_s^{out}$  is the marginal production cost in a time period  $s$  when the storage is discharged. (It is here assumed for simplicity that the marginal costs constitute a continuum such that equality may actually be obtained.)

If there is a loss or a cost associated with the use of the storage then the application of the storage will be less extreme. In the case of a cost  $a$ , such that one unit of energy taken out of the storage must pay a cost of  $a$ , then the storage will only be used to level out marginal cost differences that are greater than  $a$ , such that the optimality condition in (1) is now modified to:

$$p_t^{in} = p_s^{out} - a, \forall t, \forall s \quad (2)$$

Similarly, if there is a loss  $b \in [0,1)$  such that energy taken out of the storage is only  $(1-b)$  times the energy put into the storage then the storage will only partially level out marginal cost differences and the optimality condition is now:

$$p_t^{in} = p_s^{out} (1-b), \forall t, \forall s \quad (3)$$

Other storage losses or costs will not be considered within this context.

The consequences of a positive  $a$  are indicated on Figure 4 which shows the resulting cost duration curve relative to Figure 1 for  $a = 22$ . This is a typical illustration of the *peak shaving* of the marginal cost duration curve. The corresponding production pattern is shown in Figure 5 along with the storage contents assuming an initial and final storage content of 50 MWh equivalents. A similar illustration could be made relative to a positive  $b$ .

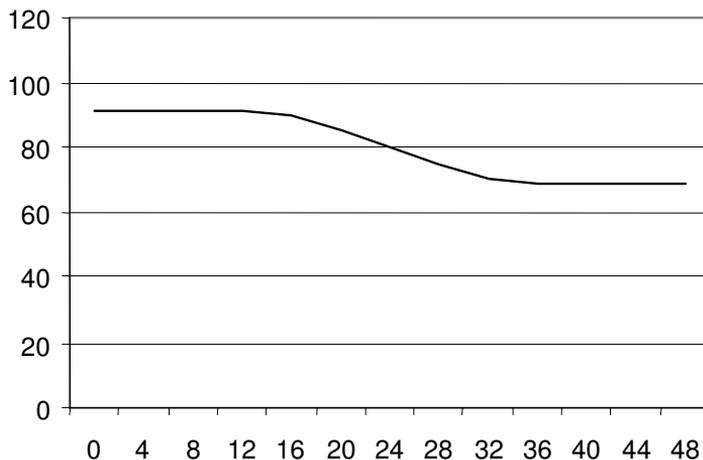


Figure 4: Marginal cost duration curve of a system with a pumped storage and a fixed cost (in USD/MWh)

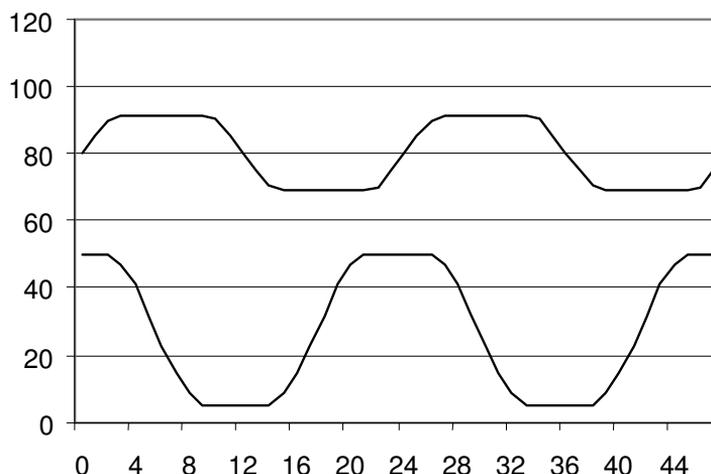


Figure 5: Production in MWh (upper graph) and pumped storage contents (lower graph) also in MWh, fixed cost case

Finally consider a storage with a finite storage capacity  $\bar{x}$ . Further analysis would show that in this case it is not possible to use the load *duration curve* technique illustrated above, and in particular also the peak shaving principle is invalid. In order to analyse the optimal economic operation of the storage it is necessary to perform a *chronological* analysis because the sequence of the different loads are of importance, a fact that is not reflected in the duration curve techniques. When using a chronological approach, restrictions on pumping and generation capacities can easily be handled. In the sequel this will be illustrated by a case study in relation to the Balmorel model.

### 3 The Balmorel model

The Balmorel model version 1.02 was used in the analyses. This version of the model has exogenously given demands for electricity and heat. It is a linear optimisation model written in the GAMS modelling language. The convex objective function used for the optimisation is cost minimisation for the whole Baltic Sea Region each year. It means that the model determines the minimal cost of production and new investments for each year given some constraints, which might concern the current production capacity, national or regional emission limits, etc. In this respect the model is a typical bottom up model. For a discussion of the top down and bottom up modelling approaches, see e.g. Wilson and Swisher (1993).

The optimisation period of the model is one year, which can be divided into a flexible number of sub periods (viz., seasons that are further subdivided into hours) making the model capable of handling seasonal and diurnal variations in the demand of electricity and heat. The model is described in detail in several documents, which are available from the homepage: [www.balmorel.com](http://www.balmorel.com). The problem to be addressed is then how to represent the time structure in order to get a satisfactory basis for the analysis, which in this case is a pumped storage.

## 4 Implementation of pumped storage

The applied version of the Balmorel model does not represent energy storages directly, but has been modified to include such a unit, which in this case was the Kruonis hydro pumped storage plant situated in Lithuania (see data in Table 1).

The geographical scope of the model was limited so that in most runs only Lithuania was included. The export from Lithuania was fixed to 250 MWh/h  $\approx$  2.2 TWh a year. Also a fixed production profile for the Ignalina nuclear power plant was used (approximating the actual production profile from 1999); thus, economic dispatch of this plant was excluded from the modelling. The model was solved for one year, thus excluding any investment possibility.

*Table 1: Data for the Kruonis Hydro Pumped Storage Plant used in the case study*

Pumping capacity	4 x 220 MW
Generating capacity	4 x 200 MW
Cycle efficiency	0.72
Storage capacity	4300 MWh (to be 8700)
Inflow	No natural inflow
Outflow, loss	No
Annual fixed cost	31 Lt/kW $\approx$ 7.75 USD/kW
Variable O&M cost	0

The cycle loss on the pumping operation was 28%, i.e.  $b = 0.28$  in (3), and with no significant variable costs,  $a = 0$  in (2). Restrictions on maximum energy level of the storage and the pumping and generating capacity were modelled.

To reflect the use of the storage for levelling out weekly and diurnal variations (as opposed to seasonal variations), a restriction was added so that the energy stored in the

plant at the start of each season should be equal to that at the end of that season. The maximum error introduced by disregarding continuity in storage from one season to the next one equals the capacity of the storage times the number of seasons, which turns out to be small compared with the total use of the storage during the year.

Two types of modelling of the hydro energy balance in the pumped storage plant were used:

$$V_{t+1} = V_t + i_t - o_t, \forall t \quad (4)$$

$$V_{lower} \leq V_t \leq V_{upper}, \forall t \quad (5)$$

$$\sum_t i_t - o_t = 0 \quad (6)$$

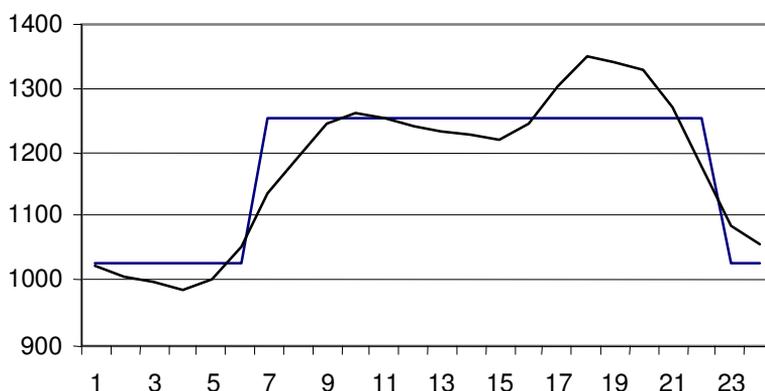
In (4)-(5),  $V_t$  is the energy contents of the storage at the beginning of period  $t$ , and  $V_{lower}$  and  $V_{upper}$  are the lower and upper limits of the storage, respectively. (In the model (4)-(5) initial and final conditions on the storage must be added, however, we will not discuss this here.) In (4) and (6),  $i_t$  is the energy put into the storage and  $o_t$  is the energy taken out of the storage during time period  $t$ . (6) is less accurate than (4)-(5) since it ignores storage limits. (6) is the basis for the duration curve techniques.

*Table 2: The time structures used in the analyses*

Name	Description
8760	All 8760 hours of the year chronological
1095	The year divided into 8 hours step
4-168	4 seasons each of 168 time periods (representing an average 168-hour week)
4-24	4 seasons each of 24 time periods (representing an average 24-hour day)
2-4	2 seasons each of 4 time periods
2-2	2 seasons each of 2 time periods
1-1	No subdivision of the year

## 5 Computational results

In the analyses several different time structures have been tried as listed in Table 2. The time structures were made from the hourly load profile of Lithuania in 1999. The 8760 time structure was this profile without any modifications while it in the 1095 time structure was divided into 8-hours steps (i.e. three each day) with the average load for that period. For the 4-168 and 4-24 scenarios, the year was split into four seasons each of three months' duration. For each season an average week and an average day was calculated. The average day profile for the summer and winter season was used for making the aggregated profiles 2-4 and 2-2 as indicated for the latter structure in Figure 6. Finally structure 1-1 with no subdivision of the year was tried.



*Figure 6: The demand profiles for a winter day in MWh/h for the 4-24 and the 2-2 time structures*

The model was then solved both with the pumped storage plant included and excluded and with Lithuania as the only country. The results can be seen in Table 3.

For solving the model, CPLEX 6.5.2 was used on a Pentium III 500 MHz computer with 128 MB RAM. Especially the 8760 hour scenario was large (the memory needed for generating the model was close to 200 MB), but nothing in the Balmorel model, GAMS nor CPLEX limited the level of detail. In Table 4 the computation time used for the different time structures can be seen. The time used by GAMS for generating the model is of the same order of magnitude as that used by CPLEX for solving the problem. It can be seen that the computation time is not very dependent on whether the

storage is modelled or not. But it is observed that the model is somewhat smaller when the storage is excluded.

*Table 3: Results for Lithuania*

Time structure	System costs, with storage (in MUSD)	System costs, without storage (in MUSD)	Generation from storage (in MWh)
8760	562.85	564.89	190878.11
1095	570.07	571.13	91572.48
4-168	551.83	553.23	168034.49
4-24	551.64	552.51	120595.02
2-4	546.75	546.88	63268.66
2-2	546.55	546.55	0.00
1-1	546.51	546.51	0.00

From Tables 3 and 4 it appears that the 1095 time structure is inferior to the others, since it both underestimates the use of the pumped storage and still takes longer time to solve than when using most other time structures.

From Table 3 it can be seen that the use of different time structures has little influence on the total system costs, viz., they are within the same 3 percent range. So when analysing the overall system the fineness of seasonal and diurnal variations does not need to be that high.

*Table 4: CPLEX computation time solving the model with/without the storage*

Time structure	Computation time, with storage (in secs.)	Computation time, without storage (in secs.)
8760	1004.75	841.95
1095	14.28	15.05
4-168	8.84	7.47
4-24	0.61	0.60
2-4	0.11	0.11
2-2	0.05	0.05
1-1	0.06	0.05

However, the situation is different when the interest is on a particular detail in the model. Thus, costs for the 8760 hours case without storage are 564.89 MUSD (MUSD=10<sup>6</sup> US\$), and thus the annual operational saving from the storage may be assessed to be approximately 2 MUSD based on the 8760 hours case. Since the storage is not used in the 2-2 and 1-1 cases these time structures implies vanishing savings. Note that the model may underestimate the use of the storage, since no start-up costs for the thermal power plants are included—neither are any restrictions on the regulating capabilities of these units.

Table 5 shows that when considering the energy storage, the level of fineness in the time structure need to be much higher in order to get reasonable results. This result is not surprising since the functioning of the storage precisely is linked to the variations over time.

*Table 5: Difference in system costs with/without storage for different time structures*

Time structure	Difference in system costs (in MUSD)	% of the 8760 time structure value
8760	2.04	100
4-168	1.40	69
4-24	0.87	23
2-4	0.13	6
2-2	0.00	0
1-1	0.00	0

In Figure 7 the results from Table 3 and Table 5 have been combined graphically, such that the left bars give the results from Table 5, and the similar percentage change of the system costs with storage (derived from Table 3) is given by the right bars.

The Balmorel model can, among many things, be used for assessing the marginal production cost of electricity. In Figure 8 the marginal production costs for the 4-24 case are shown for Lithuania, both with and without the storage. It is seen that the storage is able to ‘buy’ electricity at night at a price 28 percent lower than the price to which it can sell the generated power during peak hours. It is seen that the storage capacity is not used fully, since the marginal cost difference in that case would have been higher in some hours. Figure 8 also indicates that a storage can take over

production from typical peak units (usually condensing oil or gas fuelled power plants). This will influence fuel usage and thus also emissions (CO<sub>2</sub>, SO<sub>2</sub>, and others).

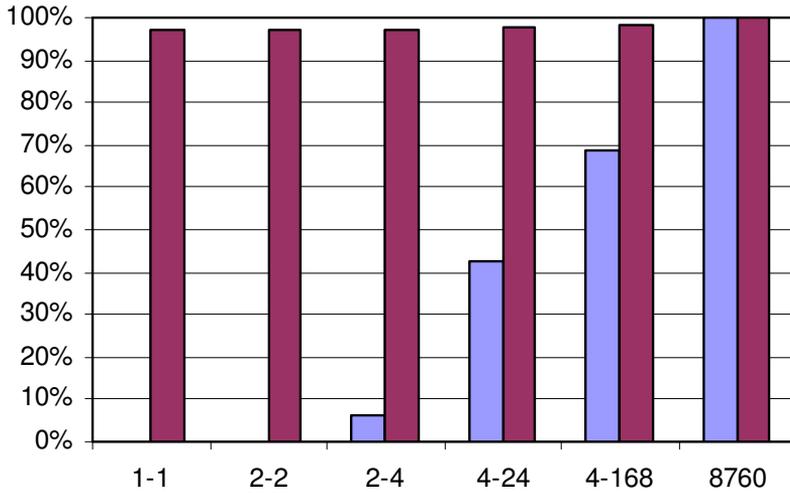


Figure 7: Comparison of a total system costs (right bar) and hydro storage benefits (left bar) for different time structures, relative to 8760 structure

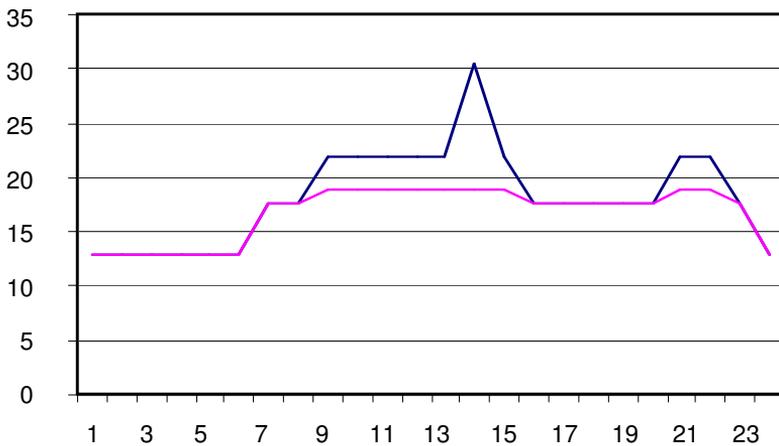


Figure 8: The Lithuanian marginal production cost in US\$/MWh an average winter day with (lower graph) respective without (upper) storage

It was tried to use the 4-168 time structure in the Balmorel model including Estonia, Latvia, and the Russian region Kaliningrad in addition to Lithuania. A result was an increase in computation time from 8.84 seconds to 140.01 seconds. In this scenario the storage was used for storing less than 1100 MWh electricity for the whole year. A reason for this is that Latvian hydropower, with a capacity of 1500 MW, can level out the market price to such a degree that the price difference only on few occasions was bigger than the pumped storage loss. So if no bottlenecks occur in the transmission network, a pumped storage is not expected to be used much in such a power market with extensive hydro reservoirs.

In other systems this need not hold true. Thus, considering the Nordic power exchange, Nord Pool, fluctuations in prices of more than 28% may be observed, even though large hydro power capacities exist in this area. In Figure 9 the Nord Pool spot system price of electricity in EUR/MWh in week 22 of 2000 is shown. It can be seen that the price varies more than 28% during most days, which should make the Kruonis hydro pumped storage advantageous if integrated into this system.

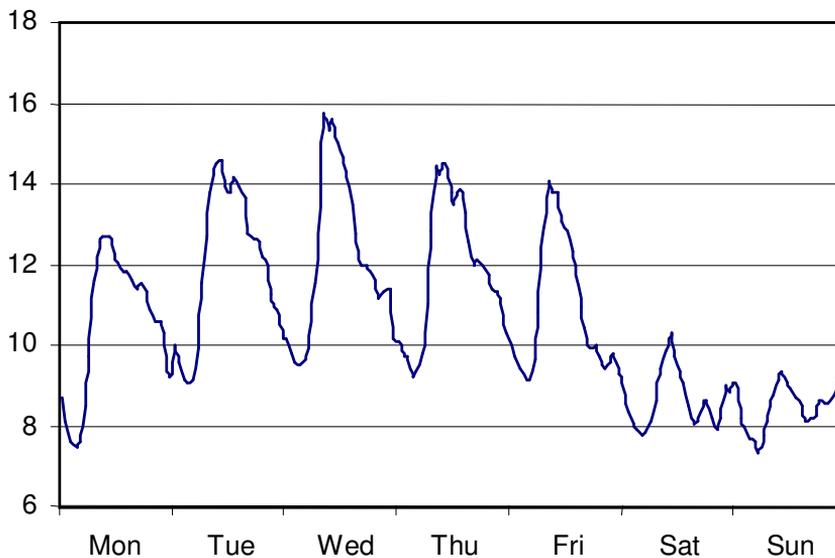


Figure 9: The Nord Pool spot system price in EUR/MWh in week 22, 2000

Of all the scenarios the storage capacity of 4300 MWh equivalents were only used fully in the 8760 and the 4-168 scenarios. So for the more aggregated time structures no large error will be made by assuming infinite capacity ( $V_{upper}$  in (5)) of the pumped storage. This allows the use of the duration curve peak shaving technique for these

time structures, i.e. modelling the hydro storage using (6) rather than (4)-(5). This hypothesis was confirmed in the sense that the total costs and the use of the storage were close between models (6) and (4)-(5). However, it was observed in all the cases compared, that the computation time was higher with (6) than with (4)-(5), and since the formulation (4)-(5) is the more theoretically satisfactory there seems to be no need to prefer (6) to (4)-(5).

## 6 Conclusions

The paper has analysed the modelling and functioning of a hydro pumped storage unit. As overture the functioning of a hydro pumped storage in the electrical system was considered with emphasis on the illustration of the peak shaving mechanism of the storage. In particular this was related to the duration curve technique, which is illustrative and intuitively appealing for simple analyses. However, the duration curve technique neglects the chronological nature of the storage problem and thus is an approximation.

It is clear from the presentation of the computation time that the fine time structure within the year has a significant influence on this. The temptation to use a coarse subdivision of the year (in particular when analysing a larger geographical area as illustrated), must in any particular case be balanced against the needed accuracy of the results, as just discussed. Two representations of the storage have been analysed, and it appears that the more theoretically satisfactory one is, luckily, the less computationally demanding.

As the goal of the work is the modelling of the CHP sector in a large geographical area, attention has been devoted to the trade off between accuracy of results and computation time. Here it may be concluded from Table 3 that the aggregation of time has little (viz., less than 3%) influence on the total costs of the Lithuanian system.

On the other hand, if the economy of the storage alone is considered, then the 8760 hours case implies a positive saving while the 2-2 and 1-1 cases imply no savings. Hence, apparently the time structure must be finer for the analysis of individual plants than for the analysis of costs for the whole system. Moreover, as Table 5 shows, it is necessary with a quite fine time structure in order to approach the 8760 hours accuracy. Also with respect to representing the functioning of the storage a fine time structure is

necessary, cf. the last column of Table 3, although this is not as outspoken as for the costs.

The contrast between the conclusions for the total systems cost and for the cost relative to the individual technology is clearly exposed in Figure 7. The appropriate time structure therefore depends on the specific purpose of the study in question.

## 7 References

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**PAPER C**

**DETERMINISTIC MODELLING OF  
HYDROPOWER IN HYDRO-  
THERMAL SYSTEMS**

“As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality”

- *Albert Einstein*

# Deterministic modelling of hydropower in hydro-thermal systems

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**Abstract:** This paper discusses the level of detail needed of the hydropower modelling for addressing different questions such as system costs, production patterns, and price developments. Both details in terms of subdivision of the year and of the number of restrictions in the mathematical model are considered. The influences of these are considered along with the influence of the inflow to the hydropower reservoirs. It is concluded that a relative high level of detail is needed for giving quality predictions of the expected power price over the year. For most other analyses a quite simple representation is sufficient.

**Keywords:** Modelling, deterministic vs. stochastic, hydro-thermal systems.

## 1 Introduction

In 1999 the hydropower production in the Nordic countries made up 55% of the total electricity production. The inflow to the hydropower reservoirs—and thus the production varies considerably from year to year. In 1998 the inflow to the Nordic reservoirs was more than 211 GWh while it in 1996 was less than 153 GWh. This variation affects the whole Nordic energy system and adequate modelling of hydropower is therefore needed when doing analyses on the energy system.

The purpose of this paper is to investigate the level of detail of the hydropower system modelling needed for different analyses. The investigation is based on results from the Balmorel model version 1.02, which uses a composite representation of the hydropower, i.e. all hydropower reservoirs of a region are treated as one reservoir. Inflows to the reservoirs add energy to the content while production by the hydropower plants will lower the content. Reasons for using the quite simple representation are to reduce the complexity of the model (i.e. to make computations faster) and that it may be hard to find data justifying more detailed modelling for a larger area.

In this paper three different composite hydropower models have been investigated. In the first, all inflows are available at the start of the year and no reservoir size restrictions are modelled. The others let the inflow follow a seasonal profile and include various reservoir restrictions. Also the implications of different numbers of subdivisions of the year have been investigated, dividing the year into monthly periods, bimonthly periods, the four seasons, and two half-years.

The results of the different models, inflows, and subdivisions will be compared both with each other and with actual data from Nordel, which is an organisation of transmission system operators (TSOs) in the Nordic countries.

The next section will present the models, the scope of the analysis, as well as the general assumptions made. In Section 3 the scenarios will be presented while in Section 4, the model results will be given. Finally, the overall conclusions can be found in Section 5.

## 2 Hydropower modelling

The models used in this paper are modified versions of the Balmorel version 1.02 [1]. The Balmorel model version 1.02 is a linear optimisation model. The objective function is minimization of the generation and transmission costs in the area modelled. Both electricity and district heating production are modelled due to the relative large amount of combined heat and power generation in Denmark, Finland, and parts of Sweden. The model is implemented in the GAMS modelling language.

The largest differences between the models in this paper and the Balmorel model are in the modelling of hydropower and the dataset, which is new. Also investments have been excluded, since focus has been on comparing equal production and transmission systems only varying with respect to the representation of hydropower.

One model uses the same hydropower modelling as the Balmorel model. In the others, the modelling of hydropower includes a representation of the hydro reservoirs. The water inflows to the reservoirs are now specified on seasonal basis. The usage of the water can be restricted by a minimum respectively a maximum level of water in the reservoirs at each season start. Also, in two of the models a minimum water flow requirement are added saying that the production of hydropower must not be lower than a certain percentage of the total hydropower capacity in each region.

Finally electricity storage is possible, i.e. by using electricity for pumping the reservoir level of a reservoir can be increased allowing a larger production later on. The pumped storages in the Nordic countries are usually used for handling seasonal variations, since the large reservoirs easily can handle diurnal variations in demand. Countries with small hydro reservoirs will on the other hand find diurnal storages more useful. An analysis of modelling diurnal storages in the Balmorel model can be found in [2].

Assuming an objective function minimizing the production costs for the whole year the restrictions can mathematically be formulated as:

$$\forall s,r: \quad RES\_LEVEL_{s+1,r} = RES\_LEVEL_{s,r} + INFLOW_{s,r} \quad (1)$$

$$+ \sum_t PUMPED_{s+1,t,r} - \sum_t PROD_{s,t,r}$$

$$\forall s,r: \quad MIN\_LEVEL_r \leq RES\_LEVEL_{s,r} \leq MAX\_LEVEL_r \quad (2)$$

$$\forall s,t,r: \quad MIN\_FLOW_r \times CAPACITY_r \leq PROD_{s,t,r} \leq CAPACITY_r \quad (3)$$

The indices used above are:

$r$	regions
$s$	season
$t$	time periods

The symbols used are described below.

$RES\_LEVEL_{s,r}$	Energy level of reservoir in region $r$ at the start of season $s$
$INFLOW_{s,r}$	Energy equivalent of the inflow in season $s$ to the reservoir in region $r$
$PUMPED_{s,t,r}$	Energy equivalent of water pumped up into the reservoir in time period $t$ in season $s$
$PROD_{s,t,r}$	Production of hydropower in region $r$ in time period $t$ in season $s$
$MIN\_LEVEL_r$	Minimum level of energy that can be stored at the start of each season in region $r$
$MAX\_LEVEL_r$	Maximum level of energy that can be stored at the start of each season in region $r$
$MIN\_FLOW_r$	Minimum percentage of production capacity that must be used at all times
$CAPACITY_r$	Production capacity of hydropower units in region $r$

The pumped storage modelling can increase the reservoir level of a season with the energy equivalent of the electricity used for pumping in the time periods of the season adjusted by the pumping loss (usually in the range 0.70-0.85). In this way the reservoirs can be used as energy storage for levelling out both diurnal and seasonal electricity prices. The minimum flow restriction still applies. Thus the model may be forced to have a certain hydropower production while it at the same time period pump water from sea level to the reservoir.

The `MIN_LEVEL`, `MAX_LEVEL`, and `MIN_FLOW` parameters describe the operation of the hydropower system. As seen in (2) and (3) they are in this analysis not depending on  $s$  and  $t$ . The minimum and maximum levels have been estimated from Nordel data showing the last 10 years' reservoir levels. In general the reservoir level of each region typically vary between 20 and 95 percent of the total reservoir capacity.

The `MIN_FLOW` requirements differ, as no data has been available to access the numerical values of these parameters. Instead these values are estimates based on the ratio between reservoir size and production capacity. Between 15 and 20 percent of the production capacity were forced to produce at any time. While these values are open for discussion, higher values in the order of 25 and 35 percent have been tried and did return inferior results for years with little inflow when compared with historical data (see discussion around Figure 7). On the other hand, values less than 15 and 20 percent respectively made the (3) restriction inactive almost all the time.

The `MIN_FLOW` parameter is included for in a simple way to represent the production from run-of-river plants (i.e. hydropower plants with no reservoirs) as well as minimum release requirements for the hydropower plants with reservoirs, which have been enforced for ecological and recreational reasons.

Other parameters can be included. For instance the difference between the energy level at the start of the year and that at the end of the year, which is otherwise assumed to be zero. This is necessary in order to recreate the historical production levels.

For this analysis the model covers the Nordic countries (except Iceland), with Norway, Sweden, and Denmark split up in 4, 4, and 2 regions respectively to represent bottlenecks in the transmission network. Finland is treated as one region. This gives 11 regions (index  $r$ ) in total. The model is solved for one year only with the year subdivided into several seasons (index  $s$ ) and time periods (index  $t$ ). Different numbers of seasons were used in the analyses (though 12 was most common), while the number of time periods was 12 in all cases.

The dataset includes data for one year. Both supply and demand data were for the year 1999. The main data source was the Nordel annual report 1999 supplemented with national reports on power and district heating supply.

### 3 Scenarios

As mentioned, three different models of hydropower have been used in the analysis denoted A, B, and BN. The model A uses the same hydropower modelling as the Balmorel model version 1. It has all the yearly inflow available at season one while the restrictions in (2) does not exist.

In model B all restrictions (1) – (3) are included. Finally, as a variation of this, model BN excludes the lower bound restriction (3). For the B and BN models the inflows are divided into seasons following historical accounts of the years mentioned above. For this analysis the MAX\_LEVEL, MIN\_LEVEL, and MIN\_FLOW parameters used are the same in all scenarios where they appear.

Three different inflows sequences have been used in the analysis. The wet year scenarios (denoted WET) have the inflow follow the actual inflow from 1999. The normal year scenarios (denoted NOR) simulate the inflow from 1993 while the dry year scenarios (denoted DRY) use the inflow profile from 1996.

Finally, the number of seasons used in the computations varies between 12 (most common), 6, 4, and 2.

### 4 Results

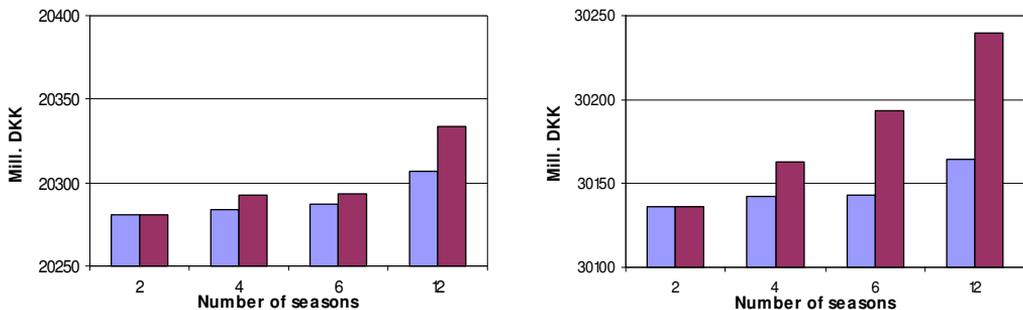
This section presents some results of the different scenarios. Firstly, the total system costs will be discussed. In Table 1 it can be seen that the total system costs on a dry year is higher than the costs on a wet year. This is intuitively clear, since the missing and almost free hydropower production must be substituted with more expensive fossil production. Comparing the A and B models, the system costs of the latter are higher as these problems includes more restrictions, i.e. (2) and (3) than found in A. Similarly, compared the model B results, the system costs decrease when the minimum flow requirement (3) is removed (viz. the BN model), as the model in these scenarios is less restrictive.

*Table 1 – Total system costs for the Nordic Region in million DKK*

	Hydro inflow	A	B	BN
DRY	153 TWh	30164.43	30239.52	30157.52
NOR	194 TWh	21752.63	22115.32	22048.99
WET	208 TWh	20306.85	20333.43	20332.73

Most important is the fact that the amount of inflow affects the system much more than the decision on how to model the hydropower (i.e. either A, B, or BN). Going from a wet year to a dry year increases the costs by almost 50% while the difference in costs between A, B, and BN is less than 1%. It can be seen that the normal inflow (in this case simulated by the 1993 inflow – average inflow for the last 10 years is 196 TWh) is closer to the wet year inflow than to the dry year inflow. Hence, the costs of the normal year scenarios must be expected to be closer to those of the wet year scenarios as seen.

Changing the number of seasons in the models will also affect the system costs as shown in Figure 1. It can be seen that increasing the number of subdivisions of the year from two 6-month periods to twelve monthly periods will increase the estimated system costs. With more subperiods the model will have more peak periods and to fulfil the demand it will have to produce on the more expensive units otherwise not needed. This explains the increase in the expected costs. Numerically, going from 2 to 12 seasonal subdivisions results in an increase of less than a half percent. It looks like a further increase would be obtained if an even finer seasonal subdivision was used (e.g. weekly periods). In percent, this increase is however not assumed to have any large effect on the overall system costs. Hence, it must be concluded that the number of seasons are of little importance when the system costs are to be found.



*Figure 1 – Total system costs for model A (left bars) and model B (right bars) for DRY inflow (left graph) and WET inflow (right graph) for different numbers of seasons*

Tables 2 and 3 show the total electricity production in TWh during the year. The main observation is like the one from Table 1, viz. that the differences between models A and B are insignificant while the influence of the annual inflow is of great importance. Another observation is that pumping power will not be used in the wet year scenarios. A pumping unit will not produce when the difference between the highest and lowest price in the regions with pumping capacities is less than the cost/loss of operating the pump. In these scenarios the only region to have pumping facilities is Norway South.

*Table 2 – Yearly production by technology in TWh – Model A*

	Wet year				Dry year			
	Denmark	Finland	Norway	Sweden	Denmark	Finland	Norway	Sweden
<b>Electricity only units</b>	5.82	43.08	122.55	141.90	13.92	56.12	91.10	131.38
- Nuclear units	0.00	20.58	0.00	71.11	0.00	20.58	0.00	71.11
- Conventional thermal	2.73	9.80	0.00	0.00	10.84	23.08	0.82	8.49
- Hydro units	0.00	12.62	122.53	70.38	0.00	12.39	90.24	51.35
- Wind units	3.08	0.07	0.03	0.41	3.08	0.07	0.03	0.41
<b>CHP units</b>	35.20	16.83	0.72	11.53	40.26	27.93	0.72	16.95
- Extraction units	30.29	10.92	0.00	5.43	35.35	22.03	0.00	10.86
- Backpressure units	4.91	5.91	0.72	6.10	4.91	5.91	0.72	6.10
<b>Electricity usage</b>	0.00	0.00	-4.29	-1.12	0.00	0.00	-4.31	-1.12
- Electric heating units	0.00	0.00	-4.29	-1.12	0.00	0.00	-4.29	-1.12
- Pumping power	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00

*Table 3 – Yearly production by technology in TWh – Model B*

	Wet year				Dry year			
	Denmark	Finland	Norway	Sweden	Denmark	Finland	Norway	Sweden
<b>Electricity only units</b>	6.00	43.51	122.47	141.92	13.90	55.34	92.46	131.40
- Nuclear units	0.00	20.58	0.00	71.11	0.00	20.58	0.00	71.11
- Conventional thermal	2.92	10.23	0.00	0.00	10.82	22.29	2.22	8.52
- Hydro units	0.00	12.62	122.44	70.39	0.00	12.39	90.22	51.35
- Wind units	3.08	0.07	0.03	0.41	3.08	0.07	0.03	0.41
<b>CHP units</b>	34.97	16.82	0.72	11.29	40.26	27.27	0.72	16.89
- Extraction units	30.06	10.92	0.00	5.20	35.35	21.36	0.00	10.79
- Backpressure units	4.91	5.91	0.72	6.10	4.91	5.91	0.72	6.10
<b>Electricity usage</b>	0.00	0.00	-4.29	-1.12	0.00	0.00	-4.31	-1.12
- Electric heating units	0.00	0.00	-4.29	-1.12	0.00	0.00	-4.29	-1.12
- Pumping power	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.00

In Table 1 it was shown that the inflow to the hydropower reservoirs in the dry year (1996) was about 55 TWh less than in the wet year (1999). For comparison the annual Danish consumption in 1999 was less than 35 TWh. Therefore the amount of hydro inflow will greatly affect the energy flows in the international transmission system. In wet years hydropower from Norway and the northern parts of Sweden will be sent south to Denmark and the southern parts of Norway and Sweden. In dry years a mixture of thermal condensing and thermal extraction type CHP plants in Denmark,

Finland, and Sweden produces the “missing” hydropower production sending electricity from Denmark to Sweden and Norway. The transmission patterns described here can be seen in Table 4.

Table 4 shows that areas with high capacity of thermal power (the Danish regions, Finland, and Sweden Mid) export considerably more in the dry year. The total sum of transmission by the regions is also higher in the dry year with 85 TWh compared to about 65 TWh in the wet year.

*Table 4 – Yearly import/export of electricity in TWh between regions – Model B*

	Dry year		Wet year	
	Import	Export	Import	Export
Denmark East	0.00	9.05	0.00	0.65
Denmark West	0.70	6.07	0.91	1.48
Finland	0.00	10.01	12.59	0.00
Norway North	3.92	3.29	0.01	2.32
Norway Rana	4.69	6.54	1.52	6.48
Norway Mid	9.17	0.01	5.96	0.00
Norway South	19.84	0.00	1.40	3.10
Sweden U. Norrland	5.51	12.49	1.65	14.74
Sweden L. Norrland	9.93	17.70	3.73	20.37
Sweden Mid	17.97	19.05	21.56	16.25
Sweden South	12.75	0.29	16.07	0.00
Total sum	84.50	84.50	65.40	65.40

*Table 5 – Total sum of import/export in TWh for different numbers of seasons*

	2	4	6	12
<b>DRY_B</b>	87.32	86.74	86.71	84.50
<b>WET_B</b>	64.43	64.92	65.06	65.40

Table 5 shows no clear tendencies regarding the influence of the number of seasons for the total amount of transmission in the Nordic countries. For the dry year scenarios the amount transmitted is getting slightly lower as the number of periods are increased. The opposite is the case for the wet year scenarios.

In the following, price estimates of electricity are shown. It has been assumed that all electricity is sold on a power exchange e.g. as Nord Pool, and that perfect competition exists on the market. Hence, the electricity price has been assumed to equal the

marginal cost of production in the specific time period. In the model this value has been calculated from the marginal value of the equilibrium restriction in the model, which states that the electricity supplied should equal the demand.

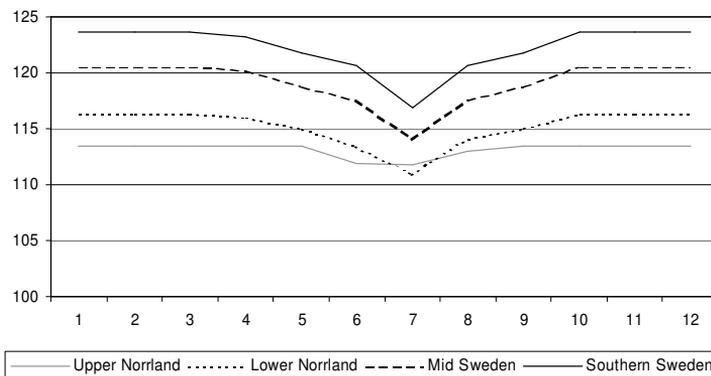


Figure 2 – Average price estimates for model WET\_A in DKK/MWh

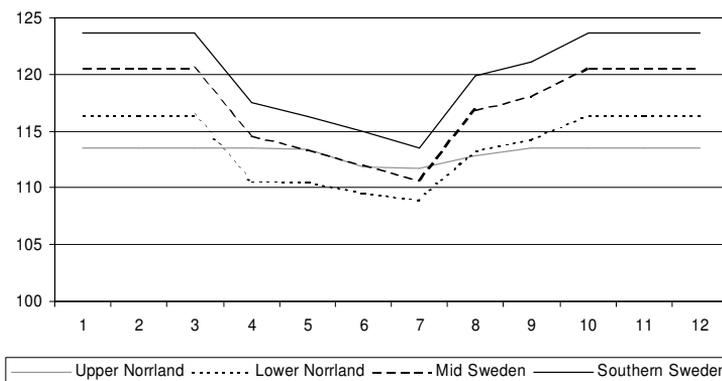


Figure 3 – Average price estimates for model WET\_B in DKK/MWh

Figures 2 to 4 show the results for the A, B, and BN models with a wet year inflow for the 4 regions in Sweden. The average Nord Pool price for Sweden in 1999 representing the wet year was about 104 DKK/MWh [4], i.e. in general about 10% lower than the model results.

If comparing the figures it can be seen that the all have the same shape—the typical curve with low prices during summer and high prices at winter. Also the order of

magnitude is similar with an average price around 115 DKK/MWh and variations over the year of between 5% and 10%.

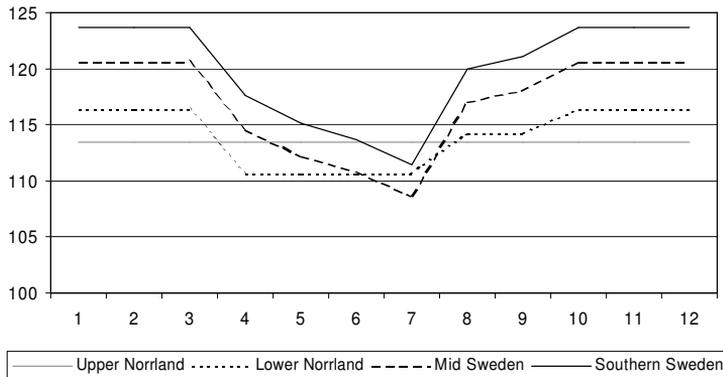


Figure 4 – Average price estimates for model WET\_BN in DKK/MWh

For all three models, it can be seen that the order of the price curves of the regions changes over the year. During winter the Upper Norrland region has the lower price indicating a net export from North to South. In the summer this change and a net export from Lower Norrland and sometimes also from Mid Sweden to Upper Norrland is seen. Also, it can be seen that in the Upper Norrland region, which is an area with large hydro resources, a stable or almost stable price over the year can be maintained.

In general, the price differences between the regions in Sweden are small with the variations being due to transmission losses and costs between regions. For the other regions Denmark and Finland look like southern Sweden, that is with the seasonal variations in the price, while the 4 regions in Norway look like the northern (Norrland) regions of Sweden and thus with an almost stable price over the year.

Figure 4 shows the WET\_BN model results, i.e. the model without the MIN\_FLOW restriction. The figure indicates a less differentiated price over the year when compared to the DRY\_B model. This complies well with the intuition, as this model has more freedom to dispatch the hydropower production between the seasons.

In Figure 5 the average price of the regions in Sweden as found by the model is compared with the historical price at Nord Pool, the Nordic power exchange [4]. While the monthly price level during winter fits very well with history, the model seriously underestimates the decrease in price during the summer.

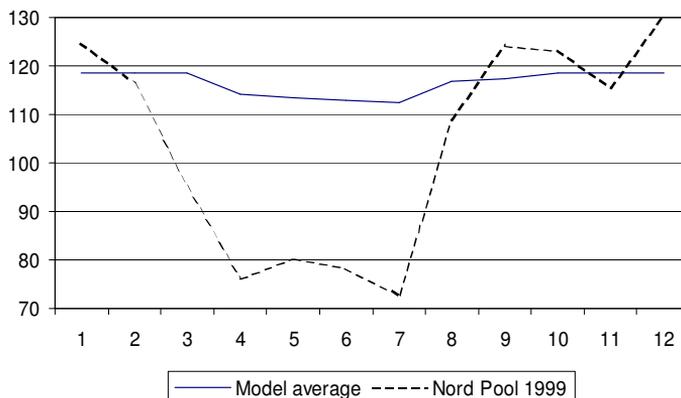


Figure 5 – The monthly prices in Sweden as found as the average of the four Swedish regions in the WET\_B model and as seen historically in 1999 at Nord Pool [4]

The Figures 6 through 8 show the average monthly electricity prices for the regions in Sweden for the dry inflow scenarios. For comparison, the historical Nord Pool price in Sweden averaged about 225 DKK/MWh during 1996, which has been used as the dry inflow profile [4]. The general shape of the price curves is as for the wet scenarios: low prices during summer and high prices at winter. The order of magnitude for all three dry models is similar with prices around 250 DKK/MWh during winter and in the order 190 – 210 DKK/MWh in the summer, with model B having the higher prices.

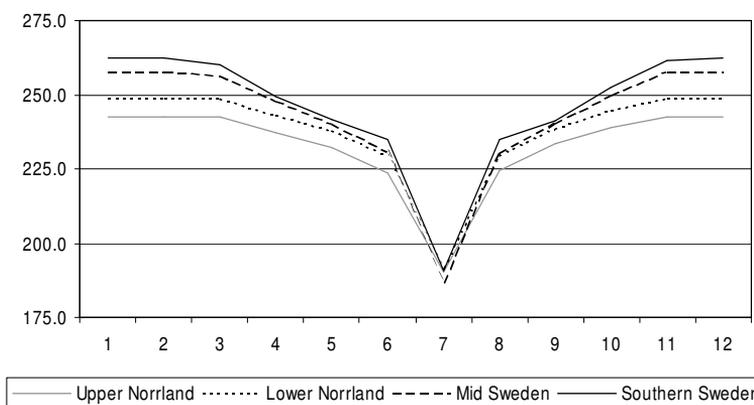


Figure 6 – Average price estimates for model DRY\_A in DKK/MWh

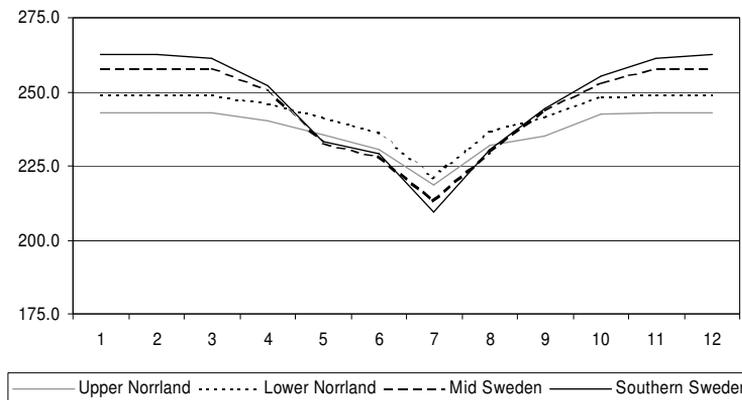


Figure 7 – Average price estimates for model DRY\_B in DKK/MWh

As a note to Figure 7, the same simulation was tried with higher MIN\_FLOW requirements as mentioned in Section 2, which resulted in prices varying between 360 and 200 DKK/MWh with an average around 300 DKK/MWh. As this differ far more from the historical average, it was chosen not to use such high percentages, though this was the only scenario, where the results varied that much depending on the percentages used in the restriction.

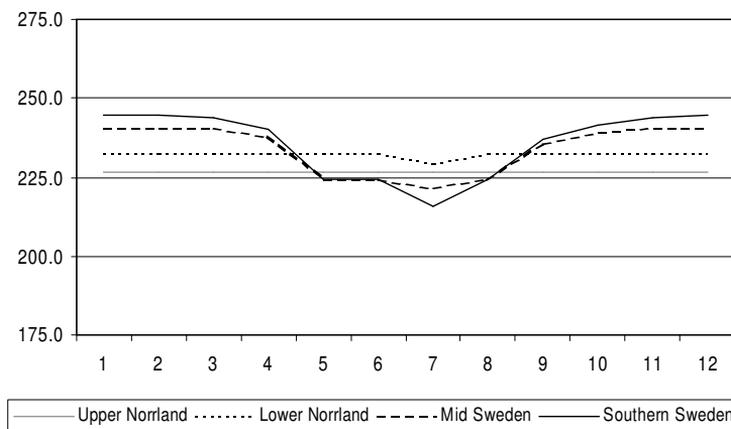
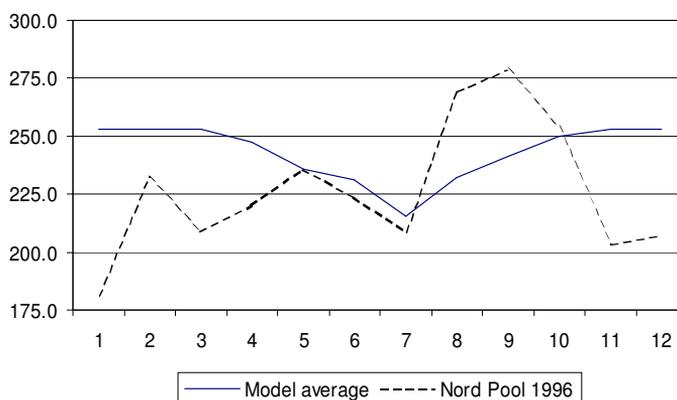


Figure 8 – Average price estimates for model DRY\_BN in DKK/MWh

Also for the dry scenarios, the BN model can be seen to have a less differentiated price over the year than the B model.

Overall, when looking at the results for Figures 6 through 8, the yearly averages comply well with the historical average for Sweden of 225 DKK/MWh though a direct comparison is impossible, as the production capacities and demands of electricity used in the scenarios have been on 1999 levels. Still, the yearly average prices match well with history. But when looking at monthly prices, which for Sweden in 1996 are shown in Figure 9, it can be seen that the historical trend showed increasing prices during summer. So also for the dry year scenarios, the models' predicted monthly prices cannot be trusted.



*Figure 9 – The monthly prices in Sweden as found as the average of the four Swedish regions in the DRY\_B model and as seen historically in 1996 at Nord Pool [4]*

Overall, it can be observed that compared with the small differences seen between the system costs results in Tables 1 through 3 the choice of model type (viz. A, B, or BN) influence more on the electricity prices.

The expected price of electricity calculated by the model was expected to grow with the number of subperiods chosen. On the right graph of Figure 10 it can be seen that for the B model, this happens when using the dry year scenario as the average prices will be about 10% higher when monthly periods are used instead of half-year periods. This is because the restrictions on the reservoir levels are getting more and more active as the number of subdivisions is increased.

However, as shown in the left graph of Figure 10, the average of the price estimates of model DRY\_A was generally unchanged with very small increases and decreases seen. That not much happens is because no monthly reservoir constraints have to be met.

Also, the reason that the price in some areas actually can drop as the number of subdivisions is increased is because the supply curve is not convex.

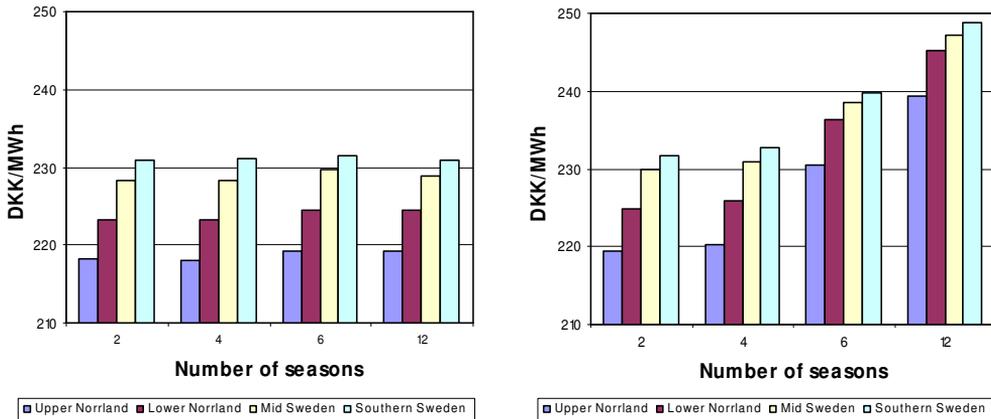


Figure 10 – Annual price estimates for the Swedish regions in models DRY\_A (left graph) and DRY\_B (right graph) for different numbers of seasons

An example of a non-convex supply curve  $s$  is shown in Figure 11. If the demand is  $d$  for all the year, i.e. the year is treated as one season, the average spot price is 100. If the year instead is split into two halves with the demands  $d_1$  and  $d_2$ , where  $\frac{1}{2} \times (d_1 + d_2) = d$ , the average weighed spot price is  $\frac{1}{2} \times (0.80 \times 10 + 1.20 \times 100) = 64$ . Also, this affects the income of power producers selling the power. When using one season this is  $100 \times 100 = 10000$ . With the two seasons the power producers income would only be 6400. This means that the model in some rare instances may predict a smaller revenue of the power produced as their predicted income is lower while their predicted costs are higher (as shown in Figure 1) when the model uses a finer division of time.

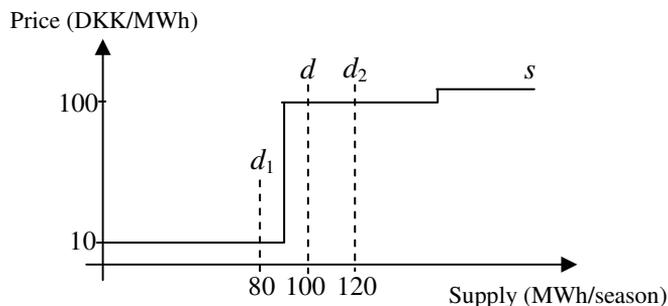


Figure 11 – A non-convex supply curve may lead to unexpected results

Table 6 shows the price span between the lowest and the highest prices predicted by the models for different scenarios. The numbers shown are from the northern Sweden (Upper Norrland) and the South Sweden for both WET\_B and DRY\_B scenarios and with either 2 or 12 seasonal subperiods each of 12 time periods. In general only a slightly larger price span is observed in 12 seasonal WET\_B scenarios compared with those with only 2 seasons. With so much water in the system, the reservoirs are capable of levelling out the price in general. In the dry years, the number of subdivisions is more important. Here the expected span between the highest and lowest prices over the year is 2-4 times larger when using 12 seasons than when using just 2 seasons.

Table 6 – The price span between highest and lowest price over the year for different scenarios

	Upper Norrland				South Sweden			
	DRY_B 2S	DRY_B 12S	WET_B 2S	WET_B 12S	DRY_B 2S	DRY_B 12S	WET_B 2S	WET_B 12S
Max	220	243	113	113	240	265	124	124
Average	219	239	113	113	232	249	123	120
Min	211	202	113	113	199	185	115	111

Finally, in addition to the observations in Figure 10, the table shows that, while the average price over the year is considerably higher for the 12 seasonal scenarios than the 2 seasonal scenarios for the dry scenarios, none or small differences are seen for the wet scenarios.

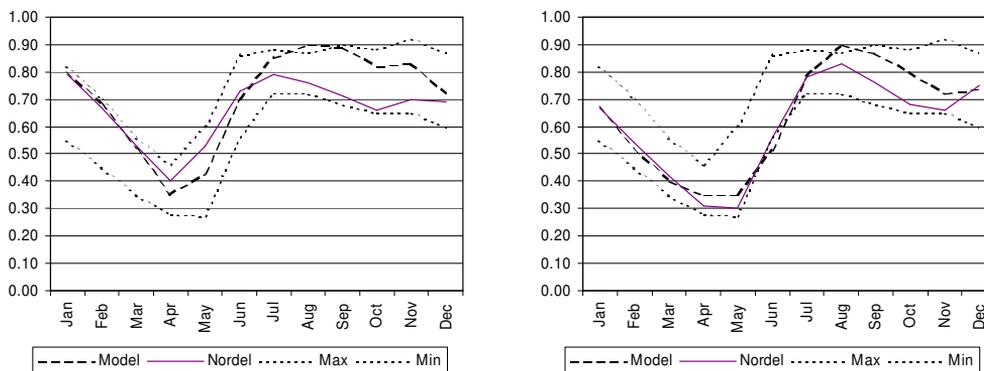


Figure 12 – Hydro reservoir contents for Finland, left graph shows the model B results for the WET\_B scenario and the actual reservoir level in 1999 [3] while the right graph shows the DRY\_B results and similar the historical level in 1996 [3]

Figure 12 shows the hydro reservoir level as predicted by model B for Finland for the WET\_B and DRY\_B scenarios. For comparison, the actual reservoir levels for Finland in 1999 (wet) and 1996 (dry) as given by Nordel [3] have been added together with the minimum and maximum levels observed in Finland in the last decade.

The results show that while the reservoir level predicted by the model falls well within the interval of historical observations, they are quite far from the actual historical levels for the year, which was used as inflow scenario.

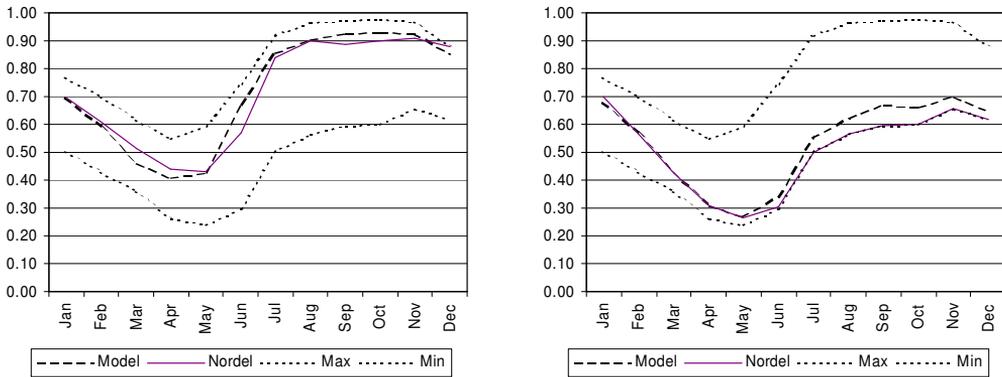


Figure 13 – Hydro reservoir contents for Norway, left graph shows the model B results for the WET\_B scenario and the actual reservoir level in 1999 [3] while the right graph shows the DRY\_B results and similar the historical level in 1996 [3]

For Norway, the results are much closer to the historical values, as shown in Figure 13. A main difference between Norway and Finland is the size of the reservoirs compared with the installed capacity, where Norway has the largest reservoirs per MW of capacity. This may make it easier to obtain results closer to those observed in real life.

Table 7 – Time usage in seconds by CPLEX for solving the models

	2 seasons	12 seasons
DRY_A	3.40	26.31
DRY_B	3.41	33.83
WET_B	3.46	38.28

The last type of results to be discussed is the computation time. In this paper the models has been solved using the CPLEX 6.5.2 solver on a 500 MHz Pentium III

computer. Table 7 shows the resource usage of CPLEX for solving the model for different number of seasons for some scenarios.

It can be seen that reducing the number of seasons (and thus approximately the number of constraints) by a factor 6 will reduce the time needed for solving the model by a factor 10. However, as expected the amount of inflow does not influence on the computation time.

In general, the computation times are very short, and few would reject the most complex of the models, the 12-season model B, due to the long computations. However, really fast computations are needed, e.g. if the model is a submodel of a larger model, which requires the submodel to be solved over and over again. An example of this could be a stochastic decomposition program.

## 5 Conclusions

In this paper various ways of modelling hydropower have been analysed using different inflow scenarios and a different temporal resolution. The results of these analyses have been compared with each other and to some extent, with historical data for the region.

It is clear that adding the restrictions (2) and (3), i.e. the B model, gives a better representation of hydropower. Also, the results indicates, that compared with the A model results, the extra restrictions results in better price estimates.

Especially when looking at the average yearly price the model estimates, not only for the B model, are reasonable when compared with historical data. When looking at monthly prices however, the estimated prices vary too little in the wet scenarios and have a clearly wrong profile for the dry scenarios. Still, it is expected that little can be done in improving the results in a deterministic model formulation.

Extending the model to a stochastic model could be one way of improving the price estimates as the stochastic nature of the inflow will be taking into account rather than now, where the models have perfect foresight of the yearly inflow.

As the type of modelling, i.e. whether the A or B models were run, had little effect on the computation time and as the additional demand for data for the B model is relative

small (the data needed is generally public available, e.g. from Nordel), one should use the B model if addressing questions related to prices, though with the shown limitations in mind.

Finally, the results of the 2 seasonal model A were often very close in percent to those of 12 seasonal model B, especially for the wet inflow scenarios and for questions related to the overall system costs, annual net transmissions, and annual average prices. So if quick analyses, e.g. as part of a larger model framework, should be made concerning those types of questions, using the small 2 seasonal A type modelling might be an option. Also, if a larger area is to be analysed, e.g. the whole Baltic Sea Region or even all of Europe, this type of hydropower modelling and resolution may be preferable.

## 6 References

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**PAPER D**

**MULTIRESOLUTION MODELING  
OF HYDRO-THERMAL SYSTEMS**

The paper was presented at the 22<sup>nd</sup> IEEE international conference "Power Industry – Computer Applications, PICA 2001", Sydney, Australia, May 20-24, 2001.

Mathematics is the art of giving the same name to different things.

- *Jules Henri Poincare*

# Multiresolution modeling of hydro-thermal systems

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**Abstract:** This paper discusses how modeling and solution techniques can be combined in relation to solving large hydro-thermal models. It is discussed how detailed production unit data, with focus on combined heat and power (CHP) units, from a specific geographical area can be converted for use in a less-detailed model covering a larger geographical region. A computational case is included where the technique is used on the Nordic countries (large area, low resolution) including the Elkraft System area (small area, high resolution). The large areas representation uses the publicly available Balmorel model and the local area representation uses the utility based SEVS model.

**Keywords:** Hydro-thermal systems, multiresolution modeling, combined heat and power, optimization, approximation.

## 1 Introduction

An important implication of liberalization of the power markets as seen in e.g. Northern Europe has been that international trade has increased. While it earlier often was sufficient for many purposes to consider the power system at a national level, now multi-regional, multi country analyses of trade and environmental policies are of greater interest. In pace with this, the need for methods for analyzing larger power systems has become urgent.

Still such systems are very large and cannot always be modeled in as much detail as wanted or as traditionally used. The reasons are among others the lack of detailed data and the lack of computational methods. However, a reason may also be that limited

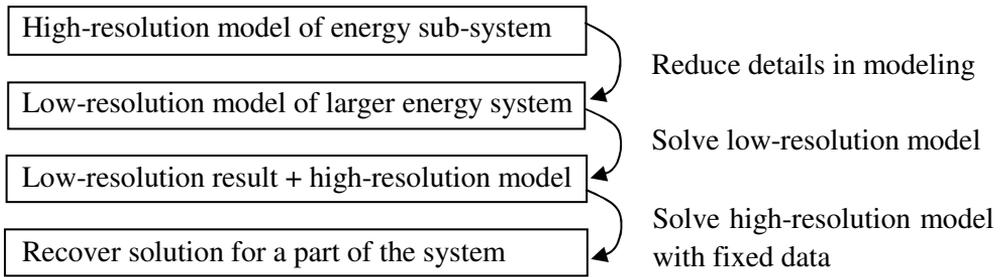
insight in and feeling with the power systems of neighboring countries makes it impossible to validate and interpret the results of simulations at a very detailed level of representation. In Denmark, for instance the power system is primarily based on thermal fuels supplemented with renewable energy (in particular wind energy) while in the neighboring Nordic countries hydropower is much more important. Hence, Danish analysts may not be interested in a very detailed model for the other countries, but rather prefer a model that is in sufficient detail to permit analysis of only those international phenomena that influence the performance of the Danish system.

Modeling should therefore permit a differentiated representation according to the needs. In relation to the above example, it would be required to have many details in the representation of the Danish system, while less detail should be included in the Nordic model.

In Eriksen et al. (1996) this goal was pursued by using a Lagrangean relaxation type method in relation to a detailed unit commitment problem of a limited area in order to derive appropriate price signals to be used in a hydro-thermal model covering the Nordic countries. Solution of the latter then provided transmission quantities, which were taken as input to the unit commitment problem, which then provided a detailed solution for the limited area.

Within other application areas approaches toward such problems have been developed. A common technique and methodology in optimization is aggregation and disaggregation, see e.g. Rogers et al. (1991). However, as the approach by Eriksen et al. indicates this is not the only way to handle the underlying problem described at the beginning of this introduction. Therefore the terminology multiresolution modeling, see Davis and Bigelow (1998), has been adopted to emphasize that the issue is primarily one of modeling, and also to avoid any indication that the most detailed modeling is necessarily the best one.

The present paper discusses a method suited for analyzing a power system in a geographically small area in much detail related to time and technology. The area is part of a larger area with which it interacts through transmission. The idea is to split the problem into two – a detailed thermal part and a more aggregated hydro-thermal system. The multiresolution method presented in this paper can be outlined as below.



In the next section a mathematical model of the problem will be formulated followed in Section 3 by a discussion on how to convert elements in the high-resolution representation of the energy system into the lower resolution needed for the multi-regional model. Section 4 is a short introduction to the Balmorel and SEVS models, which are used in the case in Section 5. This case covers the system in the Nordic countries at a low-resolution level with a detailed solution for the Elkraft System area. Finally in Section 6 some conclusive remarks are made

## 2 Some considerations on model details

A hydro-thermal energy system with multiple regions can be modeled as illustrated below:

$$\min \sum_r \sum_s \sum_h \sum_u c(r, u, p_E(r, s, h, u) + p_H(r, s, h, u)) \quad (1)$$

$$\forall r, s, h \quad \sum_u p_E(r, s, h, u) = d_E(r, s, h) + \sum_{r' \neq r} x(r, r', s, h) \quad (2)$$

$$\forall r, s, h \quad \sum_u p_H(r, s, h, u) = d_H(r, s, h) \quad (3)$$

$$\forall r, r', s, h \quad \underline{x}_{r',r} \leq x(r, r', s, h) \leq \bar{x}_{r',r} \quad r \neq r' \quad (4)$$

$$\text{other thermal constraints} \quad (5)$$

$$\forall r, s, u' \quad R(u', s+1) = R(u', s) + I_{u',s} - \sum_{u'} p_E(r, s, h, u') \quad (6)$$

$$\text{other hydro constraints} \quad (7)$$

The symbols used have the following meaning:

$r$	Region index
$s$	Season index
$h$	Hour index
$u$	Production unit index – all units
$u'$	Production unit index – hydropower only (i.e. $u' \subseteq u$ )
$c(r,u,p(\dots))$	Production cost, region $r$ , unit $u$ , production level $p$
$p_E(r,s,h,u)$	Electricity production level, region $r$ , season $s$ , hour $h$ , unit $u$
$p_H(r,s,h,u)$	Heat production level, region $r$ , season $s$ , hour $h$ , unit $u$
$x(r,r',s,h)$	Transmission between regions $r'$ and $r$
$\underline{x}_{r,r'}$	Minimum transmission level
$\bar{x}_{r,r'}$	Maximum transmission level
$d_E(r,s,h)$	Electricity demand, region $r$ , season $s$ , hour $h$
$d_H(r,s,h)$	Heat demand, region $r$ , season $s$ , hour $h$
$R(u',s)$	Reservoir level of hydropower unit $u'$ at start of season $s$
$I(u',s)$	Inflow to reservoir of hydropower unit $u'$ at season $s$

The model above has a representation of time consisting of hours, to indicate the shorter time intervals e.g. hours of the day, and seasons to represent the variations over the year. The former is in particular important in the thermal subsystem, and the latter in relation to hydro systems with capacities big enough to save water throughout a year. In a simple version, a year could be modeled as two seasons (summer and winter) each with two representative hour types (day and night). That level of detail might be sufficient for some analyses, but a more detailed level, e.g. 52 seasons (all weeks) each of 168 hours (all hours of the week) might be appropriate for other analyses. See Galinis et al. (2000) for one such analysis.

The production costs in (1) are individually specified for the units, even if they are technically identical, due to regional variations in fuel prices (e.g. of biomass, natural gas, etc.). In addition to fuel costs, operation and maintenance costs are included. In particular this includes startup costs, although this is not explicitly specified in the model above. Production of electricity and heat in each region should equal the demand, cf. (2) and (3). Transmission of electricity between regions is possible within the limits given by (4). The thermal constraints mentioned in (5) can include capacity limits, ratio between electricity and heat on CHP units, ramping rates, spinning reserve constraints, etc. see e.g. Sen and Kothari (1998) and Sheble and Fahd (1994).

Restriction (6) is the hydro energy constraint that models the variation of the water inflow over the year. Other hydro constraints, which could be included in (7) could be maximum and minimum reservoir levels depending on the seasons, minimum water flow requirements, etc.

As is easily imagined, the model (1) - (7) may be large if a large geographical area is covered (many regions) and if the time is represented in a detailed way (many seasons and many hours). An additional problem related to a large-scale version of the model (1) - (7) is that it may be difficult to get reasonably accurate data. Therefore it may be relevant to consider an aggregate or otherwise simplified version of model (1) - (7), this will be considered next.

### 3 Approximation of data

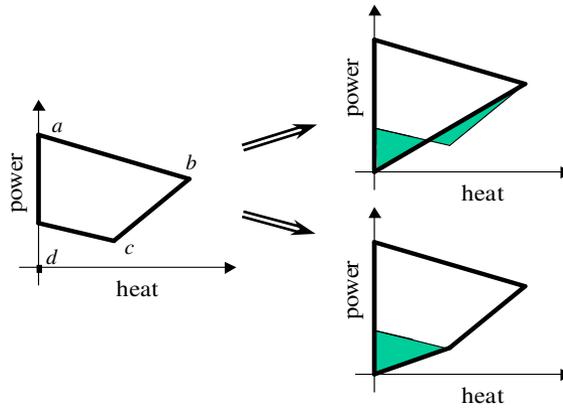
A basic element in the modeling of thermal systems is the unit commitment, see e.g. Sen and Kothari (1998) and Sheble and Fahd (1994). In a unit commitment startup costs are included as well as integer constraints in relation to the modeling of minimum (positive) production levels, minimum up and downtime, and other aspects. The practical implication of this is that the time required to solve such optimization problems to true optimality grows exponentially with the problem size. Therefore often heuristics are used to speed up solution time towards a near optimal solution.

Relaxing the integer constraints (i.e., disregarding the units commitment aspect of the problems) speeds up the solution time considerably.

In this paper one step further is taken. The low-resolution model as described in the next section is a linear programming (LP) model. Such a model is faster to solve than non-convex and/or non-linear models and that makes it possible to model very large systems within the limits of LP. Since it often will be impossible to obtain data for larger regions with a precision that justifies use of non-linear programming, this approximation is often acceptable. The rest of the section presents how the restrictions and the objective function can be modeled as a convex LP problem and more specifically how it is done in the low-resolution model.

The principles in the transformation that will be presented here is that a linear model should be obtained. The emphasis in the presentation will be on the transformation of

the constraints on the thermal production units, represented in (5), where in particular combined heat and power (CHP) units will be considered.



*Figure 1 – Different way of modeling the possible combinations of production of heat and power of a thermal HP plant. The error is indicated by the shaded areas*

On Figure 1 the left graph shows the production area (viz. the feasible combinations of production of heat and electricity) of an extraction type CHP plant, while the two graphs to the right show two ways of modeling this in an LP model. The production area is the point  $d$  and the area within the thick lines. The shaded areas show the modeling error done in each case. In the upper right graph the production area has been transformed into a triangle. The lower right graph includes one restriction more, but makes sure that all of the original production area is included in the resulting model i.e. the so-called convex hull.

Whether one approximation is better the other depends on the role of the unit in the energy system. Base loads units will normally produce at near full electricity load as permitted by the heat production. For such units the modeling error in the region between the points  $a$  and  $b$  should be minimal. Indeed, as illustrated on Figure 1, the error could be zero here. For load following or peak units the situation is different. Here there is often a heat load, which implies a certain production of heat. Therefore, at low electricity loads the electricity production will be as low as permitted by the heat production. For such units therefore the modeling error should be small (or zero if possible) around point  $c$  in Figure 1. By using two constraints rather than one around  $c$ , the error to the right of point  $c$  may indeed be eliminated and in that case the lower approximation should be preferred. However, the error to the left of point  $c$  (i.e., where both heat and electricity productions are low) cannot be eliminated.

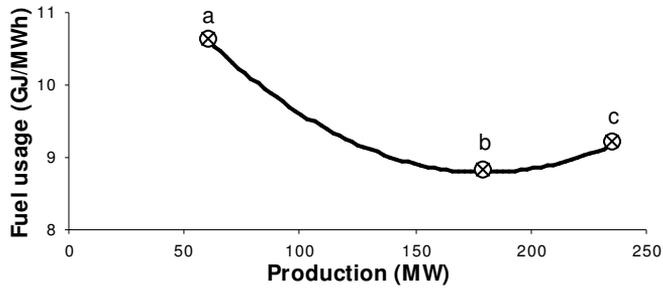


Figure 2 – The fuel usage as a function of the power output of a thermal power plant

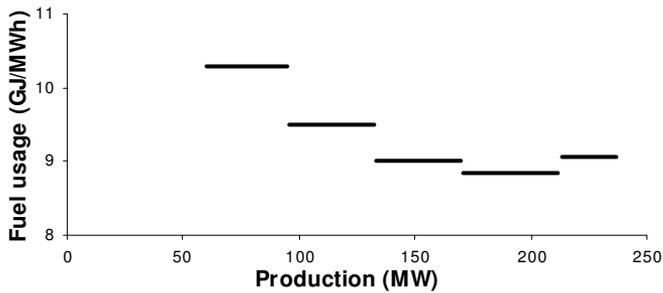


Figure 3 – Piecewise constant representation of the unit from Figure 2

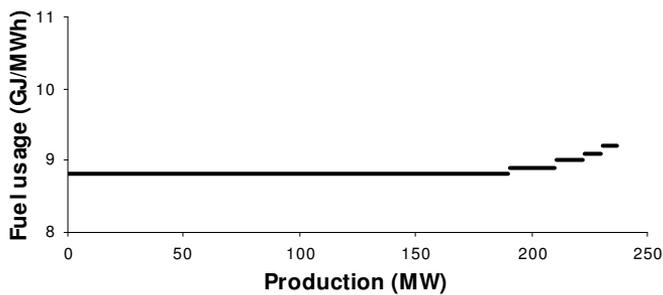


Figure 4 – Piecewise constant representation of the unit from Figure 2 but with increasing fuel usage

Figure 2 shows how the fuel usage varies for different loads of a typical condensing thermal power plant. The representation of this curve in a linear model poses two difficulties. One is that the curve is nonlinear. This may be overcome by using a piecewise representation as in Figure 3. However, this does not solve the other

difficulty, which is that the rate of fuel usage (and the proportional marginal production cost) should be non-decreasing. Therefore, only one value could be used, corresponding to e.g. point  $c$  in Figure 2. However, using point  $c$  for peak load units, which seldom produces at full load, gives an optimistic view of the use of such units. A better representation is indicated in Figure 4. Here, the approximation is quite good for points between  $b$  and  $c$ , but optimistic for points between  $a$  and  $b$ .

As concerns the start costs for thermal units, these must be neglected, as they are impossible to represent in a satisfactory way in a linear model. However, some other elements found in unit commitment and economic dispatch modeling, e.g. ramping constraints, are linear in their nature or may be given a linear representation in a satisfactory way and can therefore be included in the aggregate model.

## 4 The models

The multiresolution modeling technique presented in this paper is based on two models used by Elkraft System. A thermal unit commitment model SEVS and a slightly modified version of the Balmorel model, see Balmorel (2001). They will be briefly described below.

SEVS has been developed by the utility for analyzing the heat and power production system in the scale of one month or one year, using a time resolution of one hour. SEVS is an optimization model minimizing the overall production cost given technical constraints. Optimization is based on Lagrangean relaxation that determines the unit commitment. The economic dispatch is then made, using the committed combined heat and power plants (CHP). The model includes transmission constraints to neighboring countries, wind power, stochastic forced outages, emission constraints and other features relevant for daily operation and it has been in continuous operation for several years.

The low-resolution model covering a geographical larger area and—if wanted—a longer time span was developed from the Balmorel model. This model, developed by a research team financed in part by the Danish Energy Research Program, is publicly available. The modified model has a more detailed modeling of hydropower but excludes the investment decisions otherwise found by the Balmorel model. The model is multi-regional with transmission constraints between each region. Within each region the district heating production is modeled due to the large proportion of

combined heat and power production in Denmark and the neighboring countries. The model is solved for one year with the year subdivided into seasons and each season divided into different hour types composing a diurnal profile. The number of seasons and hour types can be chosen freely but influences of course the computation time.

## 5 Results

This section will present a case where the multi-resolution technique has been applied to system consisting of four Nordic countries, viz. Denmark, Finland, Norway, and Sweden. The region is interesting as test case because of its mixture of production technologies (hydropower, nuclear power, thermal condensing, CHP, and wind), cf. Table 1. Also as per October 2000 the power pool Nord Pool covers the whole region with the national electricity markets liberalized or under liberalization.

*Table 1 – Production capacities in MW by different technologies in the Nordic region and the Elkraft System area, which covers Eastern Denmark, see Nordel (2000).*

	Elkraft System Area	Nordic Region
Nuclear power	0	12092
Thermal cond.	1153	9471
Thermal CHP	3120	15979
Hydropower	0	46756
Windpower	386	2033

This case presented may be briefly described as follows. First SEVS unit data for 1999 was converted for use in the Balmorel model as described in Section 3. For the three other countries in the model, other data sources, giving the appropriate level of detail, were used. Then the Balmorel model was solved for the Nordic countries with some countries being divided into several regions to reflect bottlenecks in the national transmission network. Thus a total of 11 regions were used in the model. The year was divided into 6 bimonthly seasons for which the inflow to the hydro reservoirs was specified for each country. Each season was further subdivided into 12 hour types giving diurnal load profiles as shown in Figures 5 and 6.

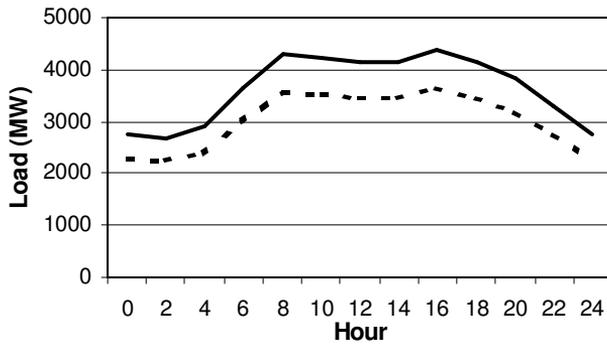


Figure 5 – Danish power load profile for the January/February season (upper) and the July/August season (lower).

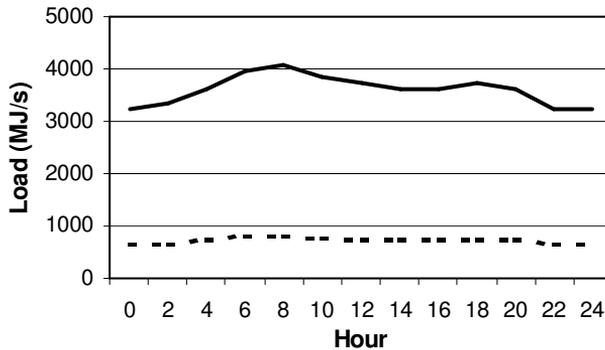


Figure 6 – Danish heat load profile for the January/February season (upper) and the July/August season (lower).

Figure 7 shows some of the results of the Balmorel simulation. It can be seen that the model predicted a greater Finnish import while Sweden exports more, mainly due to a greater production on fossil fueled units. With respect to the fuels used for power production the figure indicates the model gives a reasonable result. A further subdivision of fuels and units, and as seen below, of the net import may show a greater difference between the model results and historical records.

While the net import/export of the counties in Figure 7 looks close to what has been historically observed, the figures for transmission between countries, as seen in Tables 2 and 3, are less consistent. A reason for this can be an inadequate modeling of the transmission system (costs and constrains) and the use of market power by dominant producers on the market, which is not reflected in the model.

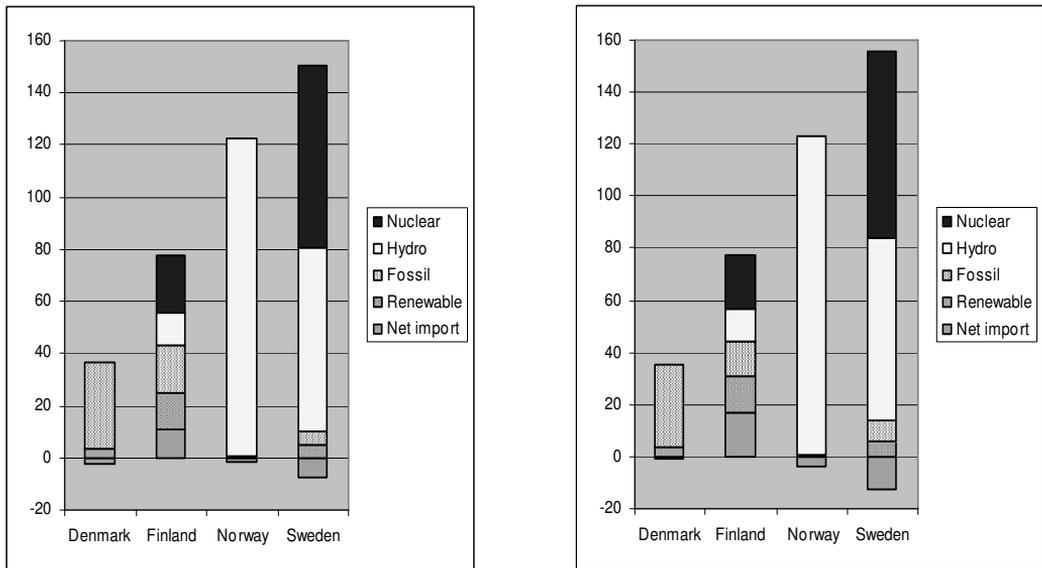


Figure 7 – Left, historical production by fuels cf. Nordel (2000) while the right graph shows the model result

Table 2 – Export in GWh from row country to column country as predicted by model using 1999 data

	Denmark	Finland	Norway	Sweden
Denmark	0	0	0	3225
Finland	0	0	0	0
Norway	1195	613	0	2410
Sweden	5728	10879	559	0

Table 3 – Export in GWh from row country to column country in 1999 as given by Nordel (2000)

	Denmark	Finland	Norway	Sweden
Denmark	0	0	622	1614
Finland	0	0	104	825
Norway	2759	107	0	5904
Sweden	2046	6737	5929	0

The model was solved using the CPLEX 6.5.2 solver and required less than 20 seconds to be solved on a Pentium III (500 MHz) computer. As it can be seen LP models can be solved very fast. Increasing the level of detail by adding new regions and more seasons and hours compared with the model run above can be done without getting an unacceptable high computation time.

The results from the Balmorel model is now used as exogenous parameters for SEVS. Two simulations with SEVS were performed. One where SEVS found the transmission given the prices in the neighboring regions as calculated by Balmorel and one where the transmission with other regions was fixed to the values given by the Balmorel model.

Table 4 shows how much electricity that was produced on different technologies. Balmorel is the results from the low-resolution model run above, while SEVS 1 is a SEVS run with the export price to southern Sweden set to 14.9 EUR/MWh, a value found by the Balmorel model. SEVS 2 on the other hand had a fixed export to Sweden of the same amount as found in the Balmorel results. (Observe that Elkraft System area in the model is connected only to Sweden.)

*Table 4 – Electricity production in the Elkraft area by different technologies*

	Balmorel	SEVS 1	SEVS 2
Condensing	4929	6660	5366
CHP-central	9711	10268	9436
CHP-decentral	2895	2732	2732
Wind	736	737	737
Net. export	3871	5997	3871

It can be observed that the general division of the production seems to follow the same pattern for the three model runs above. The main observation when comparing the Balmorel results with those of SEVS runs is that with the export price calculated in Balmorel is used as input parameter to the SEVS simulation (SEVS 1) then SEVS overestimates the transmission to Sweden. The major explanation for the discrepancy is that it is a general observation that price signals are less accurate for such coordination than quantity signals.

In relation to the fuels used for electricity production, cf. Table 5, it can be seen that oil—generally used in peak load units—are only used in the SEVS 2 model run.

*Table 5 – Electricity production by different fuels in the Elkraft area*

	Balmorel	SEVS 1	SEVS 2
Coal	9866	10896	9073
Natural gas	2223	2720	2733
Orimulsion	4568	5148	4197
Oil	0	2	637
Waste	391	411	411
Straw	487	483	483
Wind	736	737	737

The profile used for transmission to Sweden made export take place at peak hours. Though this sometimes is the case, more often it is not. The oil fueled peak load units were therefore needed much more in the SEVS 2 than in the SEVS 1 scenario. This emphasizes the need to be quite careful when modeling the variation over the day, viz., the load pattern and in particular the export quantities or prices used as a signal for exporting.

Making a simulation in SEVS for one year takes approximately 30 seconds, which is fast given the level of detail in this model. The use of certain heuristics to obtain close to optimal solutions makes this possible.

Comparing the solution times for the two models it is seen that they are approximately equal. This indicates a proper balance between the level of detail and complication of the two models.

## 6 Conclusions

A multiresolution modeling technique has been introduced for analyses on hydro-thermal systems. Methods for converting detailed information from the high-resolution model for use in the low-resolution model has been presented. The method has been applied to the Nordic system, for which a linear programming model was used. The high-resolution subsystem was modeled as a mixed integer unit commitment problem.

Comparisons of the results show that in many respects the results are similar. Obviously details differ, however, major trends are reasonably close. In particular it has been noted that care must be taken in handling the time profiles in relation to price and/or quantities.

As also shown, it is important which results from the low-resolution model that are to be used as input to the high-resolution model. In particular, the input reflecting the transmission conditions can consist of price signals or quantity signal. As demonstrated, the quantity signal provides better correspondence, as also expected.

It should be emphasized that the case is not only one of reduction of computational time. For many real cases there is an independent point in aiming at a model that is not too complex. This may be because data is not available, or it may be because there is insufficient experience in interpreting the results of a detailed model.

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**PAPER E**

**CO-EXISTENCE OF ELECTRICITY,  
TEP, AND TGC MARKETS IN THE  
BALTIC SEA REGION**

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*"Obvious is the most dangerous word in mathematics"*

*- Eric Temple Bell*

# Co-existence of Electricity, TEP, and TGC Markets in the Baltic Sea Region

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**Abstract:** This paper analyses the application of two policy instruments, tradable emission permits (TEPs) and tradable green certificates (TGCs) to the electricity sector in an international context. The paper contains an explicit modelling at two levels of abstraction, one suitable for defining and analysing basic functionalities and one suitable for numerical analysis in relation to countries in the Baltic Sea Region. Emphasis is on estimating implications in quantitative terms for countries in the Baltic Sea Region in 2010 when the TEP market in the analyse extends to four Nordic countries (Denmark, Finland, Norway, Sweden), and the TGC market extends to North European EU countries (Denmark, Finland, Sweden, Germany). The study concludes that within the range of goals stipulated in the EU draft directive (23.6% renewable energy) and the Kyoto targets for emissions, the following prices are affected significantly: from -2 to +10 Euro/MWh for electricity spot prices, TGC prices up to 50 Euro/MWh, TEP prices up to 18 Euro/t CO<sub>2</sub> and up to +15 Euro/MWh on the consumer cost. It is shown that such price changes have important consequences for the production and investment patterns in the electricity sector, and the resulting patterns will be clearly different according to the specific numerical targets for the two goals. An immediate consequence is increased pressure on transmission lines. Further, the introduction of TEP and TGC markets will imply a restructuring of the electricity sector, e.g. (depending on the specific combination of targets) by a significant increase in wind power capacities. However, this will have to be counterbalanced by access to production technologies that have fast regulation properties and/or that may maintain voltage stability. However, the price signals of TGCs (and to some extent also TEPs) that will enhance wind power investments will simultaneously hamper investments in technologies that are a precondition for extensive use of wind power technologies.

# 1 Introduction

The reduction in greenhouse gas emissions is an increasingly important issue in the energy and environment policies of the European Union, accession countries and other industrialised countries. The Kyoto Protocol and the related emission targets set the agenda for the future energy policy in the region (European Commission (2001)). The Kyoto Protocol introduced the concept of flexible mechanisms, and agreements about the rules for these mechanisms have recently been discussed at the seventh session of the Conference of the Parties in Marrakesh in 2001. With the decisions in Marrakesh, the road has been paved for emission trading, joint implementation and clean development mechanisms.

Also promotion of renewable energy is high on the agenda in Europe (European Commission (2000 a)). Renewable energy can contribute to reducing greenhouse gas emissions, but it also has other assets. In its green book on security of supply the EU points at the positive effect of renewables on diversification of energy supply (European Commission (2000 b)). In the long run, renewable energy may be a better answer to the climate problem and to a sustainable energy system than traditional energy transformation based on fossil fuels. In the short run, renewables cannot compete with other greenhouse gas reduction options, and this has led to the development of separate promotion schemes for renewables. One of the schemes that are being discussed internationally is a market for renewable energy certificates.

The questions of emission reduction and renewable energy promotion are on the agenda in other contexts as well. Thus, the Nordic Ministers of Energy have recently emphasised the possibilities of using the Baltic Sea Region as a testing ground for joint implementation. They have further pointed at the possibilities of emission trading between the Nordic countries and a Nordic market for renewable energy certificates and agreed to support the development of a sustainable electricity market around the Baltic Sea. This may be seen as a commitment to developing a testing ground for the Kyoto mechanisms in the Baltic Sea Region (Nordic Council of Ministers (2001)). These initiatives are sustained by national investigations and proposals, e.g. Energistyrelsen (1999) in Denmark and Miljödepartementet (2001) and Näringsdepartementet (2001) in Sweden.

The introduction of systems for tradable emission permits (TEPs) or tradable green certificates (TGCs) are immediate suggestions for instruments of international cooperation on these issues. The introduction of TEPs and TGCs will create two new

interdependent products, and they will in turn interact with the electricity market. In the short run, this will affect emissions through changes in electricity production, consumption and trade. In the longer run, the production capacities for the various types of electricity production capacities will be affected, and in particular favour renewables to the extent warranted by the price signals.

As the initiatives in the Baltic Sea Region illustrate, international initiatives and cooperation on emission reduction, introduction of renewables, and the existence and enhancement of electricity markets need not have the same geographical coverage. In particular, cooperation may involve EU and well as non-EU countries. Also with respect to the constitution of the energy systems and the economic conditions, the cooperating countries may differ vastly.

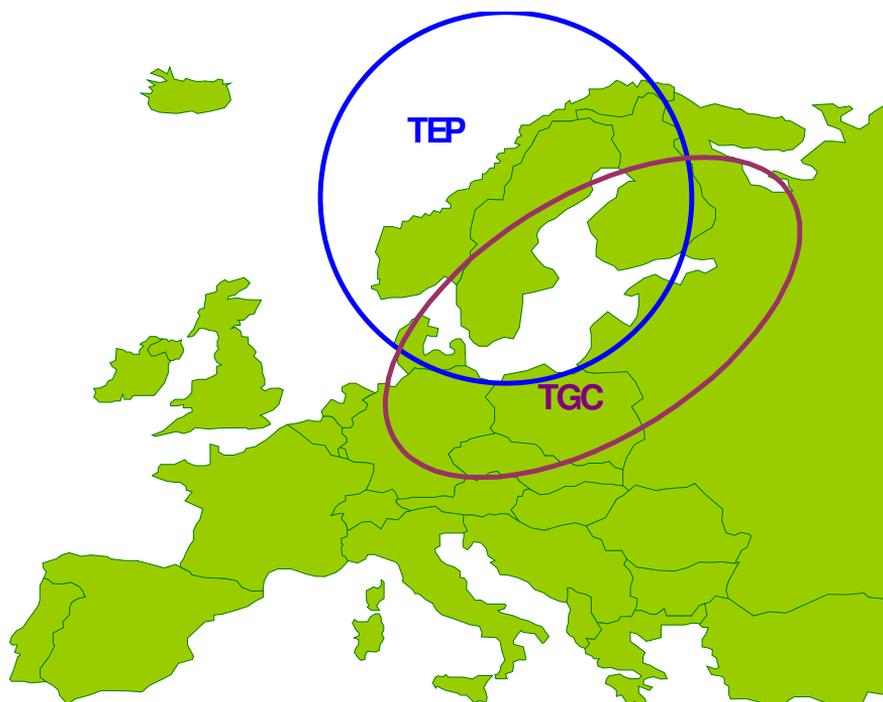
Therefore a number of unanswered questions remain in relation to the specific architecture and to economic efficiency issues. In part this is because a number of practical implementation issues must be settled. However, it is also because there is an intrinsic interplay between the introduction of TEP and TGC systems and key evaluation parameters. Relevant evaluation parameters are economic efficiency, consumer prices, producer profits and distribution of costs and benefits between the countries participating in such international cooperation.

This is demonstrated in a number of papers dealing with the questions concerning two instruments (TEP and TGC) and/or two or more countries. Thus, in the treatment of the Danish green certificate system, Amundsen and Mortensen (2000) demonstrate among other things that the effects of an increase in the percentage requirement of green electricity's share of the total electricity consumption are most inconclusive. Morthorst (2001, 2002) deals with TEPs and TGCs in an international context and concludes among other things that the application of the two instruments must be coordinated in order for the benefits to be distributed internationally in proportion to the level of ambition of the national targets. Jensen and Skytte (2002 a, 2002 b) also demonstrate that there is no simple relationship between targets for emission and renewables and consumers' cost of electricity.

A common lesson from these theoretical investigations is therefore that some of the consequences of introducing TEP and TGC systems in an international context cannot be assessed on theoretical grounds alone.

On this background the purpose of the present study is to apply an empirical model to investigate some of the consequences of introducing such systems. In particular,

trading of TEP, TGC and electricity will be analysed in a situation where the corresponding markets do not necessarily coincide, i.e. in the countries near the Baltic Sea.



*Figure 1, International markets in addition to the electricity market*

The paper presents the theoretical framework and modelling in Section 2. A numerical model covering the Baltic Sea Region is established in Section 3. The spot market for electricity extends to all countries considered (subject to transmission constraints). The TEP market consists in the present analysis of four Nordic countries. The TGC market here consists of the North European EU countries shown in Figure 1. Section 4 presents the results of the calculations. The results include the various prices (TEP, TGC, electricity spot price, consumer price and consumers' cost) relative to introducing emission trading and trade of renewable energy certificates in this region. The consequences for profits for owners of the various types of production technology are analysed. Further, implications for the physical constitution of the electricity sector, including investments in new technologies and transmission between countries, are investigated. Section 5 points to the perspectives of the analysis.

## 2 Theoretical Framework and Modelling

The international dimension in focus is represented by the definition of a number of countries and the exchange of TEPs, TGCs and electricity between them. Therefore an initial description will be given as motivation for the formulation of an adequate mathematical model. The description is given at a fairly abstract level, and a number of details included in the model used for the numerical analysis are not commented on here, see Section 3.

In the Kyoto Protocol, the emission limitations are specified at the level of countries. However, national emission limits are not strict and may be violated provided the surplus amount is counterbalanced by a similar reduction in another country, i.e. the flexible mechanism. An international system of TEP administration aims at ensuring that the sum of national emissions does not violate the sum of national emission limits.

This analysis assumes that a share of the national limit of each country has been allocated to the electricity sector. At present, this assumption is unrealistic, although some countries (e.g. Denmark) have initially implemented such sector emission limits. International exchange of TEPs is furthermore assumed to be permitted between the electricity sectors of the participating countries. The electricity producers in the countries may only cause emissions if they are associated with the acquisition of a corresponding amount of TEPs; the total amount of TEPs corresponds to the total emission allowed the electricity sector in the countries in the emission bubble. In each country, the TEPs corresponding to the national quota allocated to the electricity sector may be given to the producers (grandfathering) or bought from the state.

The system of TGCs originates from the requirements in some of the countries that a certain share of electricity must come from renewable energy. This binds the consumers to match the purchase of electricity with the purchase of a proportional amount of TGCs. This involves a system that certifies green electricity, it involves the issuing of a number of TGCs to the production unit owners proportional to the amount of electricity produced, and it involves a market for the exchange of TGCs. The producers may sell the TGCs to the consumers. International exchange of TGCs may also take place if agreed between countries so that the TGCs bought by a consumer in one country may originate from the consumer's country or any other country in the TGC bubble. Observe that in this analysis it is assumed that CO<sub>2</sub>-reduction due to increased renewable energy production is handled exclusively through the TEP system and not as a part of TGCs.

Finally, the international exchange also involves electricity. The physical basis is that the electricity production systems in the various countries are connected by an electricity transmission system with given characteristics (capacities) for the individual transmission lines. The supply systems in the various countries have technologies based on renewable energy sources (producing green electricity and associated TGCs) as well as other technologies (e.g. based on fossil or nuclear fuels - some of them emitting greenhouse gases). In the following, they are referred to as renewable and traditional technologies, respectively. This classification is a matter of definition. Thus, examples of traditional technologies without emission are nuclear and (depending on adopted definition) large-scale hydro. Examples of renewable technologies with physical emission are those based on incineration. The market side of electricity consists of an international spot market where demand and supply are cleared by the spot price in each country. The two types of technologies sell their electricity on the same spot market. International transmission is possible on market conditions. Electricity prices will thus be the same within two countries connected by a transmission line if the transmission capacity is sufficiently large. If the transmitted quantity is equal to the transmission capacity, prices may differ, with the import country having the higher price.

Thus, the theoretical set-up contains three distinct international markets: the spot market for electricity, the TEP market in relation to emissions, and the TGC market in relation to renewable energy. As described, the markets need not have the same geographical extension.

The above elements are integrated into a model in the context of maximising the sum of consumers' and producers' surplus, i.e. it involves an integrated demand and supply system. The specifics of the system are the incorporation of environmental and energy constraints in an international context with trade possibilities. Thus, in addition to the basic constraints of physical nature, describing the supply and demand equations and the production units' technical characteristics, three sets of additional elements must be introduced. One element must express the possibility of international electricity trade within the technical possibilities of transmission. The second element must express that for the countries in a defined bubble, a minimum share of the electrical energy consumed must come from renewable sources, and that trade in TGCs may take place between those countries. The third element must express that each country in a defined bubble has imposed limits on the emission from the electricity sector, but that trade in TEPs may take place between those countries.

At the present level of specification, the ideas of the above model are believed to be close to the ideas behind the reasoning in Morthorst (2001, 2002), Jensen and Skytte (2002 a, 2002 b) and Amundsen and Mortensen (2000). Thus, the verbal description of the various instruments and mechanisms in relation to TEPs and TGCs seem almost identical. The exception is Amundsen and Mortensen (2000), where minimum and maximum prices of TGCs, and a maximum price of TEPs are introduced. The quoted papers differ with respect to emphasis on international aspects; thus, the papers by Morthorst (2001, 2002) treat several countries in equal detail, the Amundsen and Mortensen (2000) paper treats one country but analyses import and export, while the papers by Jensen and Skytte (2002 a, 2002 b) focus on one country. All the quoted papers assume a convex model in the sense of the model shown in the Appendix. The Amundsen and Mortensen (2000) and the Jensen and Skytte (2002 a, 2002 b) papers contain explicit mathematical models, while the other papers rely on verbal and graphic reasoning.

In the Appendix, a mathematical model is formulated for the above-mentioned framework. The model is formulated explicitly as a one period static model with possibilities of investment in new production technologies. The Appendix also stipulates basic properties of the model. Basic assumptions for the derivation are that the market is assumed perfectly competitive. This permits the formulation as an optimisation problem and the derivation of prices. Prices are found as optimal Lagrange multiplier values so that the electricity prices observed on the spot market are identical to marginal production costs (short-term marginal costs if there is sufficient capacity, long-term marginal costs if there is a shortage). The prices of TEPs and TGCs reflect marginal costs associated with the constraints on emission and renewable energy application, respectively. Further, convexity assumptions imply certain monotone properties of the prices. Trade in electricity is represented directly in the model. Trade in TEPs and TGCs is not represented directly. However, the quantities traded may be derived from the optimal solution as discussed in the Appendix.

As outlined in the introduction, the motivation for the introduction of international markets for TEPs and TGCs is that this is a way of obtaining economic efficiency in the attainment of the goals of emission and renewable energy share. Thus, the more countries participating in a bubble, the higher the efficiency gain may be. The same observation applies to an international electricity market. This feature is well understood and may be formally derived from the model in the Appendix, but it will not be analysed here.

As concerns the emission goal, the TEP price will increase if the permitted amount of emissions is decreased. The cost of buying TEPs on the international market will be internalised in the electricity price on the spot market. Thus, the higher the TEP price is, the higher the electricity spot price is.

However, with respect to the renewable energy goal the situation is reversed. Thus, if the share of renewable energy is increased, the electricity supply offered on the spot market increases. The assumption of low marginal production costs of renewable electricity implies that traditional electricity will be replaced by renewable electricity, and the spot price will decrease. The TGC price in turn will increase if the share of renewable energy is increased.

The model also permits the calculation of consumers' and producers' surplus, and thus the distributional effects of the introduction of goals for emission limitation and renewable electricity production. Finally, the model is dynamic in the sense that it identifies new investments, and hence a description of the changes of the physical side of the supply system is possible.

### 3 Numerical Model and Data

The above quite general understanding has been applied to an empirical analysis of countries close to the Baltic Sea. The numerical model to be used incorporates the empirical details that are not necessary for presentation of the general ideas and properties of the model in Section 2, but that are essential in order to reach conclusions of a quantitative nature.

The obvious shortcoming of the model in the Appendix is the abstract formulation with functions that have not been explicitly specified (e.g. the demand function in country  $c$  is given as  $D^c$ , and the electricity production cost function for unit  $i$  is given as  $f_i$ ) and therefore allow for a wide variety of instances. In a numerical model, the abstract functions must obviously be specified.

However, apart from this, the model in the Appendix has a number of structural shortcomings. Some of them will tend to bias the quantitative results in the direction of underestimating prices. The most important ones relate to the representation of the electricity system. First, the time dimension (not explicitly indicated in the Appendix) involves only one time period, which may then be interpreted to be one year. For

electricity systems, many important features involve shorter time intervals reflecting in particular the variation on electricity demand over the day and year, and forced electricity production from wind turbines, etc. Secondly, relevant constraints on electricity production units link individual units together, e.g. requirements of reserve capacity; also the linkage to the heat demand side (through combined heat and power units) is omitted in the formulation. Thirdly, losses in distribution and transmission are not represented. Fourthly, countries are represented in the geographical dimension. However, in many applications the electricity system must be represented in more geographical detail. Other important aspects relate to the cost structure; in particular taxes and tariffs may influence the results significantly.

The numerical analysis was made using the Balmorel model and adapting it to the specific purpose. This model represents the principles of the Appendix and permits specification of additional elements to overcome the structural shortcomings mentioned of the model in the Appendix.

The Balmorel model was developed recently in cooperation between various organisations in the countries around the Baltic Sea (Ravn et al. (2001), Ravn (2001)). The model contains a specification of geographically distinct entities and covers (at least parts of) the countries around the Baltic Sea, and it also includes Norway. On the supply side, it describes possibilities and restrictions in relation to generation technologies and resources, transmission and distribution constraints and costs, and different national characteristics (costs, taxes, environmental policies, etc.). In the time dimension, the model covers the large perspective (up to 2030) with a subdivision of the year into a number of sub-periods. This number may be chosen according to the character of the analysis and the data available; for the present study, a division into four sub-periods was used.

The model has a specification of the electricity and combined heat and power (CHP) production system based on ten different classes of technology (including thermal, condensing and backpressure types and renewable technologies based on wind, hydro and solar sources). The model permits specification of production and investment costs dependent on the year and the country. As regards production, both short-term marginal costs (fuel, operations and maintenance costs) and investment costs (long-term marginal costs) are represented. The physical constraints represented include generation possibilities of the different technologies according to for example installed capacities and fuel availability. Also transmission and distribution constraints are satisfied along with balance between supply and demand, appropriately taking into account losses and limitations.

The model determines the following entities: Generation of electricity and heat distinguished by technology and fuel; Consumption of electricity and heat; Electricity transmission; Emissions; Investments in generation and transmission capacities; Prices of electricity and heat. All these entities are specified with respect to time period within the year and geographical entity. The variables are determined to either maximise the sum of producers' and consumers' surplus or to minimise the costs in the supply system. Properties of the solution are: Equilibrium in each sub-period between the marginal cost of electricity of distinct regions taking into account transmission losses, storage possibilities, costs and constraints; Equilibrium between short-term and long-term marginal generation costs in each geographical entity so that long-term marginal costs prevail in periods in which capacity is extended, and short-term marginal costs prevail in periods with surplus capacity.

For the purpose of the present study, the base version of the model structure was supplemented in order to represent the quota and trade mechanisms related to TEPs and TGCs. This was done in accordance with the ideas outlined in the Appendix. Thus, the model captures the spirit of the driving mechanisms of the model. However, as described, a substantial additional amount of empirical information is represented in the empirical model. As an indication of the size of the model it can be mentioned that for the present analysis it contains approximately 90,000 equations and 100,000 variables.

The data for the model are based on a variety of sources. In relation to technologies, international databases were used, including the Baltic 21-Energy study (Baltic 21 (1998)). Concerning other information such as electricity and heat demand, transmission networks, and taxes, international sources were used, e.g. IEA energy balances. In addition to these open sources, the Balmorel project involved data collection by local participants in the countries around the Baltic Sea.

While the Balmorel model originally contains a quite detailed description of the electricity and CHP sector, it was found necessary to supplement it with respect to renewable energy, which is in particular focus here. This was done by adapting data from the Rebus project (Voogt et al. (2001)) which provides insight into the effects of implementing targets for renewable electricity generation at EU member state level and the impact of introducing renewable burden-sharing systems within the EU, e.g. TGCs. As part of that project, a database was developed describing the costs and potentials for renewable electricity. Wind turbines, small hydro, solar, biomass/wood, solid agricultural wastes, solid industrial wastes, wave and tidal, geothermal, large hydro,

biogas, and municipal solid waste were included in the definition of renewable sources of electricity.

The model represents Denmark, Estonia, Finland, Germany, Latvia, Lithuania, Norway, Poland and Sweden. These countries are linked in a common electricity market in the model. The renewable energy bubble consists of the EU countries (Denmark, Finland, Sweden, Germany), while the emission bubble consists of the Nordic countries (Denmark, Finland, Norway, Sweden). Figure 1 shows this.

The specification of goals for renewable electrical energy is based on the goal (23.6%) stated in an EU draft directive from 2000 (European Commission (2000 a)) (see Voogt et al. 2001, p. 22). This is taken as the most ambitious renewable energy goal. The specification of goals for emission reduction is modified from the Kyoto targets (UNFCCC 2001). Since only the electricity and CHP sector is modelled here, it has been assumed that the national quotas have been further distributed on sectors in proportions corresponding to the historical (1990) CO<sub>2</sub> emissions. For the emission bubble a 55.88 Mt CO<sub>2</sub> limit is used as the most ambitious goal. This corresponds to the allowed emission from the four Nordic countries included in the study.

Based on the most ambitious goals, 30 different cases consisting of combinations of emission and renewable energy goals were defined. These are made up of six different levels of the renewable energy target of the EU countries included combined with five different levels of the amount of permitted emission of the Nordic countries included. The corresponding levels are shown in Table 1. Note the identification of the four extreme cases TEP0-TGC0, TEP1-TGC0, TEP0-TGC1 and TEP1-TGC1.

*Table 1, The different levels of required renewable energy share in EU and Nordic emission limits*

Nordic CO <sub>2</sub> emission limit (in Mt)	EU renewable energy share of demand (in %)
100.6 (TEP0)	0.0 (TGC0)
89.4	4.7
78.2	9.4
67.1	14.2
55.9 (TEP1)	18.9
	23.6 (TGC1)

The calculations are performed for the year 2010. Investment may be made in new production technology in relation to the capacities existing in 2000. As currency Euro 2000 has been used (denoted EUR00 in the following).

## 4 Simulation Results

The presentation of the results of the model calculations includes prices, different producer type incentives for investments, new production technologies, electricity transmission between countries, and trade in TEPs and TGCs.

Graphic illustrations of prices are given in figures 2 through 5. Figure 2 shows the average annual electricity spot price for Eastern Denmark. From the least ambitious case (high emission level and low renewable energy requirements, front left corner in the figure; cf. also TEP0-TGC0 in Table 1) a price of 21 EUR00/MWh is seen. Adding strong renewable energy requirements makes the price drop about 10% to 19 EUR00/MWh, while the price would grow by 50% to more than 31 EUR00/MWh if a strong limit were enforced on emission instead. If ambitious goals are adopted on both renewable energy and emission, the price will be approximately the same as without any goals. The relatively flat shape in the direction of increased renewable share is due to the already existing renewable energy production. This is also clearly visible on Figure 3, which shows the calculated common EU prices of TGCs. A TGC price of 50 EUR00/MWh can be seen corresponding to the EU draft directive on renewable energy. It is virtually independent of the emission constraints.

Similarly, Figure 4 shows the calculated common prices of TEPs in the Nordic countries. It can be seen that the price of the emission permits may rise to 18 EUR00/t CO<sub>2</sub> for the cases with strong emission quotas but only weak requirements of the renewable energy share of the electricity supply. As expected, the price drops as the renewable energy share is increased, and very sharply. Finally Figure 5 shows the aggregate consumers' cost of electricity in Eastern Denmark (i.e. the sum of the electricity spot price and the price of the TGCs, where the latter is weighted by the appropriate share). This corresponds to the consumers' marginal cost of using electricity assuming no taxes or distribution costs are added. As seen, the aggregate cost tends to increase with increasing TGCs and decreasing TEPs, up to almost 15 EUR00/MWh.

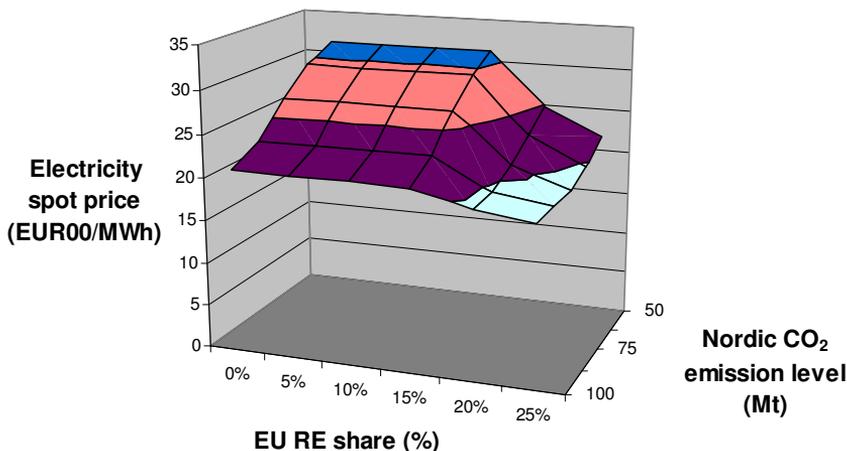


Figure 2, The annual weighted average spot price in Eastern Denmark

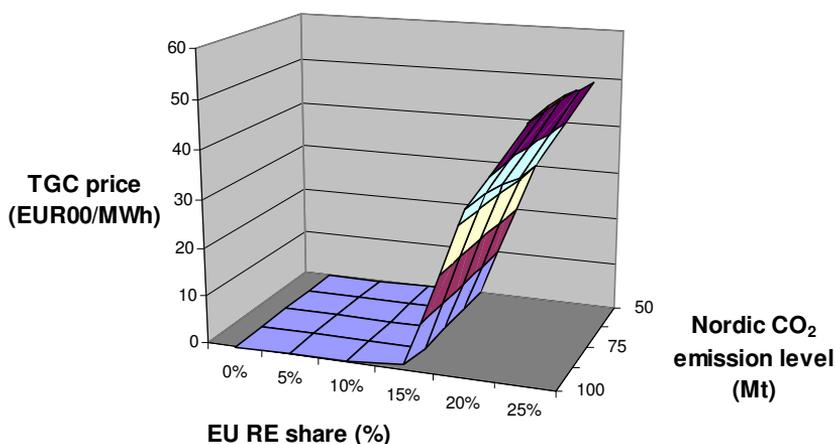


Figure 3, The price of TGC in the EU countries

Observations of the spot price and the aggregate consumers’ cost for the other Nordic countries will look very similar to the ones for Eastern Denmark (Figures 2 and 5), and the graphs are not given, spot prices for all countries are given in Table 2. Taking into account that the annual amount of electricity consumption in the Nordic countries is approximately 400 TWh, the indicated marginal price increase of 15 EURO/MWh will imply an increase in consumers’ cost of electricity of up to 6 billion EURO.

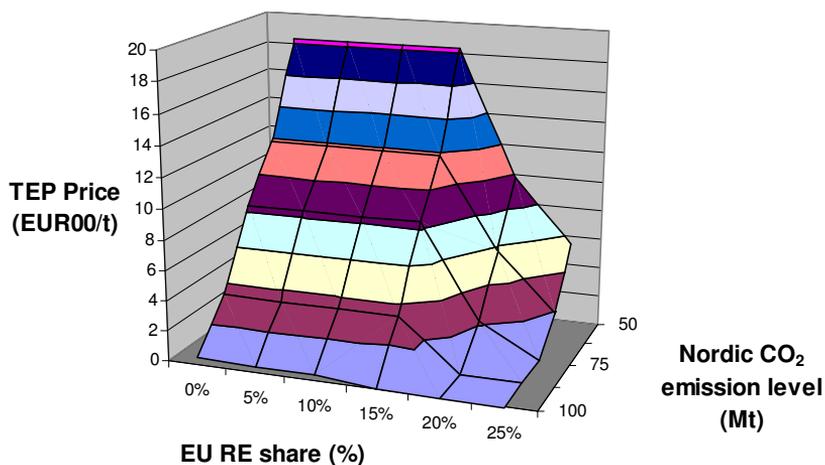


Figure 4, The TEP price in the Nordic countries

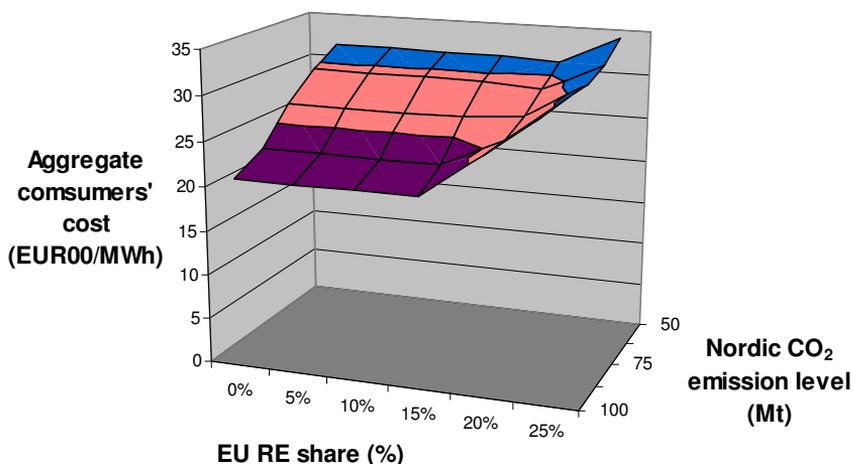


Figure 5, The aggregate consumers' cost (weighted spot price + TGC) in Eastern Denmark

However, it should be noted that the spot price in Germany does not change significantly when the CO<sub>2</sub> allowance in the CO<sub>2</sub> bubble is changed. This is because Germany is outside the CO<sub>2</sub> bubble. If the transmission capacities were unlimited, the price would be the same everywhere due to the assumption of perfect market conditions. However, the German market is large in comparison with the transmission

capacity linking Germany to other countries within the model (capacities of approximately 120,000 MW production and 4,000 MW transmission). Transmission is discussed in further detail below.

As seen, the theoretically derived results of monotone connections between quotas and prices (cf. the Appendix) are confirmed. As a special point, note that for the aggregate consumers' cost, Figure 5, there is not complete monotonicity. Thus, around 55 Mt CO<sub>2</sub> emission allowance and 18.9% renewable energy requirements the price drops slightly (hardly visible). The possibility of such non-monotone relationship is predicted and analysed in Jensen and Skytte (2002 a, 2002 b). Though non-monotonic price relations are seen in the simulation results these are observed to be of minor relevance in the overall picture.

*Table 2, Annual weighted average spot prices 2010 (in EUR00/MWh)*

Scenario	DK-E	DK-W	Estonia	Finland	Germany	Latvia	Lithuania	Norway	Poland	Sweden
TEP0-TGC0	21	21	27	19	31	29	32	21	20	21
TEP0-TGC1	19	18	27	15	23	29	32	17	20	17
TEP1-TGC0	31	30	27	29	32	29	32	26	20	27
TEP1-TGC1	22	21	27	19	23	29	32	21	20	20

As concerns the producers' surplus, the picture is complicated. The surplus gained by the owner of a particular production unit depends on the cost of production (and for new investments also on investment costs) and on one or two prices in the TEP and TGC market. For production units based on renewable energy, the relevant prices are the spot price and the TGC price. For production units with emissions, the relevant prices are the spot price and the TEP price. For production units that have no emissions and do not qualify for a TGC (e.g. nuclear or large hydro), the relevant price is the spot price. Hence, the various types of production technology will be quite differently affected by the policy measures adopted. An illustration is given in Figure 6, which refers to the situation in Sweden. The graph shows the variable production costs for different types of technology in two cases: no emission limitation (TEP0) and maximum emission limitation (TEP1). Since the cost of acquiring the necessary TEPs associated with production is internalised into the electricity production cost, the cost will increase for the production types that have emissions. The graph also shows the electricity spot price in Sweden in four extreme cases.

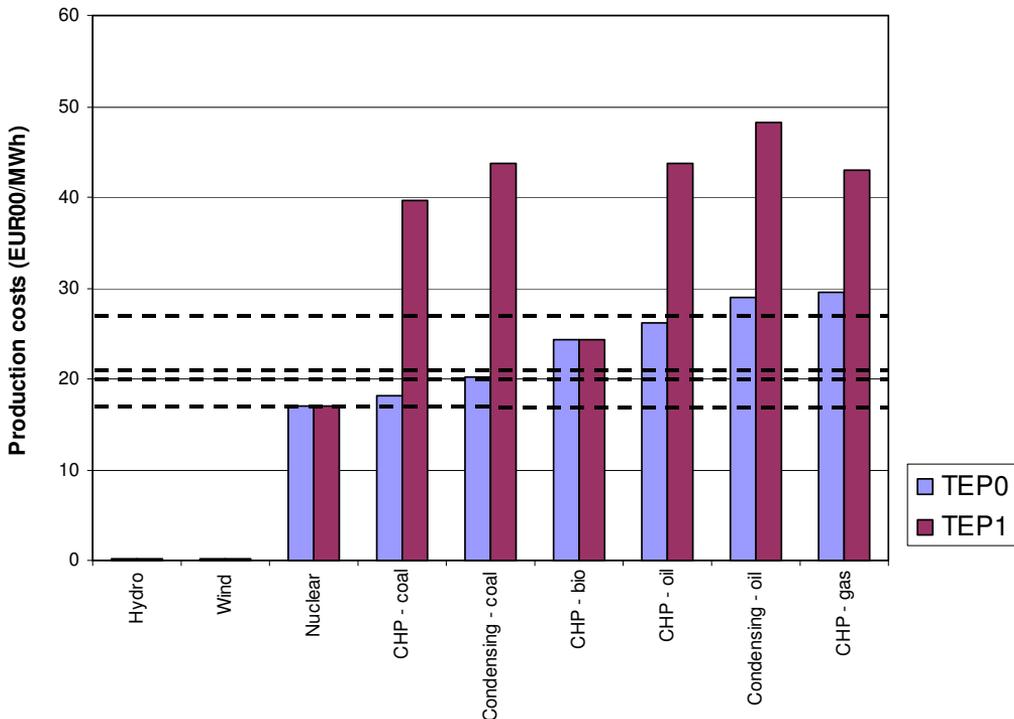


Figure 6, Variable production costs without and with  $CO_2$  emission limits (bars), and the spot prices (horizontal lines (TEP0-TGC0: 21 EUR00/MWh, TEP0-TGC1: 17 EUR00/MWh, TEP1-TGC0: 27 EUR00/MWh and TEP1-TGC1: 20 EUR00/MWh). Sweden 2010.

As seen, the variations on the spot price imply that some of the technology types may change the situation from earning money to losing money. Heavy restrictions on emissions penalise the fossil-fuelled technologies significantly, and the associated increase in the spot price does not compensate for this. For non-emitting technologies (renewable and nuclear) the cost is not affected by emission limitations, but the income is. Thus, apart from the effect of the policy measures on the redistribution between producers and consumers, there is a substantial redistribution between the owners of the different types of technology.

The consequence of high profitability for an individual type of technology in a given country is that more capacity of this type will eventually be established. This effect is illustrated in Table 3 where the production in 2010 at units constructed between 2000

and 2010 is shown for three cases. It can be observed that only for the TEP1-TGC0 case is the dominant investment in hydropower in Norway. The TEP0-TGC1 and the TEP1-TGC1 in contrast motivate investments in wind power, biomass and waste-fuelled technologies in Denmark, Finland, Germany and Sweden. For countries outside both bubbles no extensive investments take place (not shown in the table).

The redistribution of income between different technologies and the resulting establishment of more capacity of the above mentioned technologies might have negative effect on the possibility of maintaining stable electricity conditions. Thus, enhancement of wind power in a situation with high renewable demand share combined with lower spot market prices will make it unattractive to invest in production capacity with fast regulation properties. Since capacity with fast regulation properties is necessary in an energy system with large amounts of wind power, the result may be reduced stability in the electricity system.

*Table 3, 2010 production at selected plants in some countries constructed between 2000 and 2010 in four cases (in TWh)*

<b>TEP1-TGC0</b>	Denmark	Finland	Germany	Norway	Sweden
Wind	0.7	0	0	0	0
Hydro	0	0	0	15.5	0
Bio + waste	0.2	0	0.9	0	0
Total	0.9	0	0.9	15.5	0

<b>TEP0-TGC1</b>	Denmark	Finland	Germany	Norway	Sweden
Wind	9.6	6.0	10.1	0	2.7
Hydro	0	0	0	0	0
Bio + waste	8.5	3.9	33.1	0	4.8
Total	18.1	9.9	43.2	0	7.6

<b>TEP1-TGC1</b>	Denmark	Finland	Germany	Norway	Sweden
Wind	9.6	6.0	9.4	0	2.7
Hydro	0	0	0	0	0
Bio + waste	8.6	8.7	28.6	0	5.3
Total	18.2	14.8	38.0	0	8.1

Table 4 shows - in relation to the TEP1-TGC1 case, where production of electricity from renewable energy sources will take place within the TGC bubble and compares this with the number of certificates (in TWh equivalents, that are to be bought in each of the countries. The table indicates that both Denmark and Finland produce an electricity surplus of around 10 TWh annually, while Germany and Sweden import similar amounts. With a TGC price of 50 EUR00/MWh, a yearly money flow of 500 million EUR00 from each of the deficit countries will go to each of the surplus countries. In addition, the payment for electricity is according to the spot price.

Transmission between countries is motivated by spot price differences across borders with a transmission line. Table 2 shows the annual weighted average spot prices in the different countries in four of the cases (see identification in Table 1). The results primarily indicate that a stronger interconnection between Germany and Poland and Poland and the Baltic countries could be suggested. Also stronger connections between the Nordic countries and Central Europe (Germany and Poland) may be interesting, depending on the case.

*Table 4, National renewable energy production and demand in 2010 in the TEP1-TGC1 case*

	Denmark	Finland	Germany	Sweden
2010 Demand (TWh)	41.4	83.9	641.0	167.6
RE target (%)	29.0	31.3	12.5	60.0
RE target (TWh)	12.0	26.3	80.1	100.6
RE production (TWh)	22.2	36.5	69.1	91.1
RE production surplus (TWh)	10.2	10.2	-11.0	-9.5

*Table 5, Import and export in Sweden and Poland (TWh)*

	Poland		Sweden	
	Import	Export	Import	Export
TEP0-TGC0	1.6	3.3	10.0	2.5
TEP1-TGC0	0.0	8.6	20.6	12.8
TEP0-TGC1	6.4	2.7	6.4	11.4
TEP1-TGC1	1.6	2.7	4.9	8.5

From the relations between the environmental goals and the spot prices, it is seen that environmental goals affect transmission between countries within and outside a bubble. As an example consider Sweden (within) and Poland (outside), and assume for the sake of convenience that Poland does not have environmental goals. A transmission line links the two countries. Without environmental goals in Sweden and the other Nordic countries, the spot price will be lower in Poland, implying import of electricity to Sweden from Poland (possibly with transit to other countries). If the emission goal is strengthened in Sweden (and other countries in the bubble), the spot price in Sweden will increase further, and there is an economic motivation to increase the import even more if possible. This is indicated in Table 5. Growth in export to neighbouring countries from Poland can be seen, while Sweden representing a transmission entry to the emission bubble increases its import. The net balance between import and export, though, is the same since much of the import to Sweden is exported to other Nordic countries also at high prices. In other words, the strengthening of the emission goal in the Nordic countries implies (within the transmission possibilities) substitution of Nordic production by Polish production. The net impact on the environment in terms of emission depends on the electricity production system in the two countries. If, on the other hand, the renewable energy goal is strengthened in Sweden (and other countries in the renewable energy bubble), then the spot price in Sweden will decrease. This will imply an economic motivation to decrease the import to Sweden from Poland and, if the renewable energy goal is high, even to change to net export from Sweden to Poland. Thus, the strengthening of the renewable energy goal in the renewable energy bubble will imply substitution of Polish production by Swedish production. The net change in renewable energy production again depends on the electricity production system in the two countries. This can also be seen in Table 5.

## 5 Conclusions and Perspectives

The paper has addressed a situation where goals for limitations on CO<sub>2</sub> emissions and/or introduction of renewable energy have been implemented through the establishment of international systems of exchange of TEPs and/or TGCs. Thus, one or two international markets are assumed to have been established in addition to the electricity market. The situation has been explicitly modelled at two levels of abstraction, one suitable for defining and analysing basic functionalities and one suitable for numerical analysis in relation to countries in the Baltic Sea Region.

The numerical simulations contribute an estimate of the prices of TEPs, TGCs, and electricity spot price in the region depending on the assumptions regarding target setting for renewable energy and emission limitation. The results have been commented on in detail in Section 4; a general conclusion is that within the range of goals stipulated in the EU draft directive (23.6% renewable energy) and the Kyoto targets for emissions, prices are affected significantly: from -2 to +10 EUR00/MWh for electricity spot prices, TGC prices up to 50 EUR00/MWh, TEP prices up to 18 EUR00/t CO<sub>2</sub> and up to +15 EUR00/MWh on the consumer costs. This estimated increase will result in increased consumers' cost of electricity in the Nordic countries of up to 6 billion EURO annually.

It has been shown that such price changes have important consequences for the production and investment patterns in the electricity sector. The quantitative effects in these directions have been estimated, and as shown, the resulting patterns will be clearly different according to the specific numerical targets for the two goals. This in turn will determine the international exchange of electricity and the international trade in TEPs and TGCs. Thus, unlike before, when the location of production capacity was determined to a large extent by national energy self-sufficiency, the motivation for establishing new production technology is now also determined by international arrangements in relation to renewable energy and emission limitations. As shown, an immediate consequence is increased pressure on transmission lines. The transmission quantities indicated in the analysis will clearly motivate or force investments in increased international transmission capacity. If this does not take place and result in a segmentation of the electricity spot market, some of the efficiency gains, which are the motivation for the introduction of TEP and TGC markets, will be lost.

There are other perspectives of the restructuring of the electricity system that may result from the introduction of TEP and TGC markets. Thus, a significant increase in wind power will have to be counterbalanced by measures such as access to production technologies with fast regulating properties and/or that may maintain voltage stability. However, one consequence of the pursuit of a renewable energy goal is to reduce the spot price of electricity – therefore the motivation for investments in traditional technologies with such desirable qualities will be lower. In other words, the price signals of TGCs (and to some extent also TEPs) that will enhance wind power investments will simultaneously hamper investments in technologies that are a precondition for extensive use of wind power technologies.

## 6 Acknowledgements

The authors are grateful to Poul Erik Morthorst for inspiring discussions during the early stages of the work and to two anonymous referees for constructive comments at later stages.

## Appendix A

The purpose of this appendix is to define a mathematical model of the problem of introduction of tradable emission permits (TEP) and tradable green certificates (TGC) markets in addition to the electricity market in an international context. Further a compact derivation is given of the prices in the markets, their relationship with the quotas and the international trade. Also expressions for producers' and consumers' surplus are derived along with relaxation and monotonicity properties.

Consider the following model:

$$\max \left[ \sum_{c \in C} D^c(d^c) - \sum_{i \in I} f_i(g_i) \right] \quad (1)$$

$$\sum_{i \in I} g_i^c + \sum_{a \in C} (x^{(a,c)} - x^{(c,a)}) = d^c + o_1^c, c \in C \quad (2)$$

$$\sum_{c \in C_R} \sum_{i \in I_R} g_i^c \geq \sum_{c \in C_R} \alpha^c d^c + o_2 \quad (3)$$

$$\sum_{c \in C_M} \sum_{i \in I_M} \phi_i(g_i^c) \leq \sum_{c \in C_M} \bar{m}^c \quad (4)$$

$$0 \leq x^{(a,b)} \leq \bar{x}^{(a,b)}, a \in C, b \in C \quad (5)$$

$$\Phi_i(g_i) \leq 0, i \in I \quad (6)$$

Here the individual production units are identified by the index  $i$ , and the index set  $I$  holds all units. Each unit in  $I$  is classified as either renewable or emitting, indicated by belonging to one of the index sets  $I_R$  and  $I_M$ , respectively, where  $I_R$  and  $I_M$  are mutually

exclusive subsets of  $I$ , and together constitute  $I$ . The set  $C$  is the set of countries  $c$ . Two subsets are defined on  $C$  viz.,  $C_R$  holding the countries in the renewable bubble,  $C_M$  holding the countries in the emission bubble.  $C_R$  and  $C_M$  need not be mutually exclusive, nor together constitute  $C$ . The electricity production of unit  $i$  is denoted  $g_i$ ; the notation  $g_i^c$  indicates that unit  $i$  is located in country  $c$ . A notation like  $\sum_{i \in I} g_i^c$  is used to indicate the summation over those  $i$  that are located in  $c$ .

The function  $D^c$  describes for country  $c$  the consumers' benefit as a function of electricity consumption  $d^c$ . The cost of the production  $g_i$  on unit  $i$  is given by  $f_i(g_i)$  and the associated emission by  $\phi_i(g_i)$ ;  $\phi_i(g_i) \geq 0$  for all units, and by definition  $\phi_i(g_i) = 0$  for  $i \in I_R$ . Electricity export from country  $a$  to country  $b$  is indicated by  $x^{(a,b)}$ .

The variables in the model (1) - (6) are production  $g_i^c$ , transmission  $x^{(a,b)}$ , and consumption  $d^c$ . The objective function in (1) describes the sum of producers' and consumers' surplus, which is to be maximised. Eq. (2) describes the balance between supply and demand of electricity in country  $c$ . The parameters  $o_1^c$  and  $o_2$  will be discussed below; they take the value zero. As seen, international transmission is permitted within the limits given in Eq. (5); transmission from a country to itself is not possible, i.e.,  $\bar{x}^{(c,c)} = 0$ . Eq. (3) describes the requirement that a certain part of total consumption in the countries in  $C_R$  (derived from the quantities  $\alpha^c d^c$  in the individual countries) must be covered by renewable electricity. Eq. (4) describes the limitation of total emission in the countries in  $C_M$  where  $\bar{m}^c$  is the quantity in country  $c$ . Eq. (6) represents all other constraints on the individual production units.

Associate the Lagrange multipliers  $\lambda^c$ ,  $\rho$ , and  $\mu$  to (2), (3) and (4), respectively, and define the Lagrangian as

$$\begin{aligned}
 L = & \sum_{c \in C} D^c(d^c) - \sum_{i \in I} f_i(g_i) \\
 & + \left( \sum_{c \in C} \lambda^c \left( \sum_{i \in I} g_i^c + \sum_{a \in C} (x^{(a,c)} - x^{(c,a)}) - d^c - o_1^c \right) \right. \\
 & + \left( \rho \sum_{c \in C} \left( \sum_{i \in I_R} g_i^c - \alpha^c d^c \right) - o_2 \right. \\
 & \left. \left. - \left( \mu \sum_{c \in C_M} \left( \sum_{i \in I_M} \phi_i(g_i) - \bar{m}^c \right) \right) \right) \right) \quad (7)
 \end{aligned}$$

For simplicity, Eqs. (5) and (6) have not been included in the definition of the Lagrangian. Eq. (5) will be discussed later.

Now assume that all the functions are once continuously differentiable, that a regularity condition holds and that the solution and the Lagrange multipliers are unique. Then the following interpretations may be given in relation to the optimal solution and Lagrange multipliers.

The value  $\partial L / \partial o_1^c = \lambda^c$  is the marginal cost of electricity production in country  $c$ , i.e. the additional cost of producing one more unit of electricity. This value may further be taken as the spot price of electricity in that country. Observe that this marginal cost disregards the additional cost associated with the requirement given in (3) and that it can therefore not be interpreted as the marginal cost of satisfying increased consumption, see below.

The value  $\partial L / \partial o_2 = \rho$  may be interpreted as the marginal cost of increasing the production of renewable electricity. This value may further be taken as the price of the TGC. Observe that this value is not the total marginal cost of the renewable energy production, but only that part which is in addition to the marginal cost given by  $\lambda^c$  for the country  $c$  considered.

The marginal cost associated with increasing production of renewable electricity by a small amount and at the same time increasing consumption in country  $c$  by the same amount is given as  $\partial L / \partial o_1^c + \partial L / \partial o_2 = \lambda^c + \rho$ .

The marginal cost of satisfying increased consumption in country  $c$  is given as  $\partial L / \partial d^c = \lambda^c + \alpha^c \rho$ . This may be taken as the consumers' combined electricity and TGC price, i.e. the consumers' marginal cost of acquiring electricity.

The marginal cost of increasing  $\alpha^c$  is given as  $\partial L / \partial \alpha^c = \rho d^c$ .

The marginal cost of increasing emission is given as  $\partial L / \partial \bar{m}^c = -\mu$ . The marginal cost of reducing emission is then  $\mu$ . This value may further be taken as the price of the TEP. Observe that this value is the same for all countries in  $C_M$ , in contrast to the results relative to renewable energy.

Now consider countries  $a$  and  $b$  that have a transmission line between them. Let  $\lambda^a$  and  $\lambda^b$  be the associated multipliers relative to (2). If transmission between the two countries is not actively constrained by (5), the optimality condition specifies that the values  $\partial L/\partial x^{(a,b)} = \lambda^a - \lambda^b$  and  $\partial L/\partial x^{(b,a)} = \lambda^b - \lambda^a$  are zero, i.e. the spot prices are identical in the two countries. If on the other hand  $\lambda^a < \lambda^b$ , then country  $a$  has maximum export  $\bar{x}^{(a,b)}$  to country  $b$  and if  $\lambda^a > \lambda^b$  the transmission is  $\bar{x}^{(b,a)}$ , i.e. maximum in the other direction.

The international trade of electricity is given by the optimal values of  $x^{(a,b)}$ . Assuming that all emission requires a corresponding TEP, the need for TEP in country  $c$  is given as  $\sum_{i \in I_M} \phi_i(g_i^c)$ . It is assumed that the quantity of TEP issued in country  $c$  corresponds to  $\bar{m}^c$ . The net import of TEP to country  $c$  is therefore  $(\sum_{i \in I_M} \phi_i(g_i^c) - \bar{m}^c)$ . The need for TGC in country  $c$  is given as  $\alpha^c d^c$ ; the net import of TGC to country  $c$  is therefore  $(\sum_{i \in I_R} g_i^c - \alpha^c d^c)$ .

For countries  $c \in C_R$  the consumers' total cost of acquiring electricity is  $(d^c(\lambda^c + \alpha^c \rho))$ , and their surplus is  $(D^c(d^c) - d^c(\lambda^c + \alpha^c \rho))$ . For countries not in  $C_R$  the same expressions apply with  $\alpha^c = 0$ . Total consumers' surplus is found by summation over all indexes  $c$ .

Producer's surplus with production quantity  $g_i^c$  on unit  $i$  is  $(\lambda^c g_i^c - f_i(g_i^c) + \rho g_i^c - \mu \phi_i(g_i^c))$ . The penultimate term represents the income from sale of TGC (zero if  $i \in I_M$ ). The last term represents the cost of acquiring TEP corresponding to the emission (zero if  $i \in I_R$ ). If grand fathering is assumed such that the owner of unit  $i \in I_M$  has a permit of  $\bar{m}_i^c$  then this producer's surplus is  $(\lambda^c g_i^c - f_i(g_i^c) + (m_i^c - \phi_i(g_i^c))\mu)$ . Total producers' surplus is found by summation over all indexes  $(c,i)$ .

The following are basic properties of the model (1) - (6).

Eq. (4) may be seen as a combination (relaxation) of a number of equations  $\sum_{i \in I_M} \phi_i(g_i^c) \leq \bar{m}^c$ , one for each country in  $C_M$ . Hence, the model (1) - (6) may be seen as one of cooperation between the countries in  $C_M$  in contrast to the model where

each country has individual limits  $\bar{m}^c$ . From properties of relaxation it follows that the total production cost (i.e., the optimal value of Eq. (1)) is not larger with cooperation as in (1) - (6) than with individual limits. Similar considerations apply to Eq. (3).

Now assume in addition to the above that all functions involved are convex, except  $D^c$ , which is assumed concave. Then the following holds true: the value of  $\lambda^c$  increases weakly with increasing  $d^c$ ; the value of  $\rho$  increases weakly with increasing  $\alpha^c$ ; the value of  $\mu$  increases weakly with decreasing  $\bar{m}^c$ .

In relation to investments in new electricity production technology, the following clarification may be made. Let the capacity already existing at the beginning of the period be given by  $\bar{g}_i$  for unit  $i$ , then this is included in (6) as  $g_i \leq \bar{g}_i$ . New capacity may be constructed at specified costs. Therefore one possible specification of the combined costs of production and investment is the following. Assume that production on a new unit  $i$  takes place at a constant marginal cost of  $\beta_i$ , and that new capacity may be constructed at a cost of  $\gamma_i$ . Then the cost function  $f_i$  for this unit is given as  $(\beta_i g_i + \gamma_i g_i)$ . With such or any other convex continuously differentiable form of  $f_i$  the above conclusions hold true.

Based on these observations the extension to a multi-year dynamic model, where new capacity may be invested at the beginning of each year, is straightforward.

Finally also observe that the extension to a situation where each year is subdivided into time segments to reflect diurnal and seasonal variations is straightforward, although tedious.

Also with such extensions the above conclusions hold true. A description of the more detailed representation of the dynamic production, transmission and demand systems used in the numerical model calculations may be found in 'The Balmorel Model: Theoretical Background', see [www.Balmorel.com](http://www.Balmorel.com).

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**PAPER F**

**STOCHASTIC MEDIUM-TERM  
MODELLING OF THE NORDIC  
POWER SYSTEM**

Presented at the seminar “Investments and Risk management in a liberalised electricity market”, Copenhagen, Denmark, 20-21 September 2001.

“People working with stochastic programming don't usually stutter. That would be awkward, as they have to say things like nonanticipativity”

- *Magnus Hindsberger*

# Stochastic medium-term modelling of the Nordic power system

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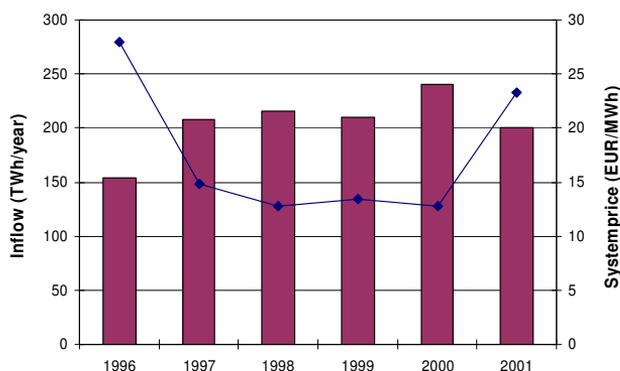
**Abstract:** This paper presents a stochastic model for analysing the effects of varying precipitation on the Nordic power system. The model is formulated as a multistage stochastic linear program and solved by the ReSa sampling based Benders decomposition method. The stochastic parameter representing the inflow to the hydro reservoirs was split into two, to describe inflow from rain and snowmelt respectively. This ensured serial independence, which is needed to allow ReSa (or any similar sampling based algorithm) to be used. Also, it reduced the uncertainty in the model to a level similar to the one faced by decision-makers.

**Keywords:** Stochastic modelling, hydro-thermal power systems, decomposition.

## 1 Introduction

On a liberalized electricity markets, such as the Nordic, it is essential for the actors to have tools for analysing the future price development on the medium-term scale (i.e. up to 1-3 years time horizon). This is needed to do optimal physical trade on the spot market, on longer-term contracts, and for financial products used to reduce risk (e.g. futures). Also, the regulating authorities are interested in being able to predict the price development and the need for new production capacity.

A common power pool, Nord Pool, exists for the four Nordic countries: Denmark, Finland, Norway, and Sweden. Overall, the Nordic power system is a hydro-thermal system with about half the electricity production coming from hydroelectric power plants, which yearly production capacity is highly affected by the amount of precipitation, and thus the amount of inflow to the reservoirs, during the year. This clearly affects the price on the power pool as shown in Figure 1. A tool for price predictions in this area will thus have to handle the stochastic behaviour of the inflow.



*Figure 1 – Average Nordic spot price and yearly inflow are negatively correlated (data from Nordel and Nord Pool)*

A common used tool in the Nordic countries is the EMPS model; see Botnen et al. (1992). Its main focus is hydropower and for a thermal dominated country like Denmark, the representation of thermal power plants and in particular, combined heat and power plants (CHP) is too weak for many analyses. Using different models to represent each type of system (hydro- vs. thermal-dominated) is one approach that has been used; see e.g. Eriksen et al. (1996) and Hindsberger and Ravn (2001).

In this paper a single model for medium-term analyses of the Nordic power system will be formulated. It has been sought to include an evenly representation of hydroelectric and thermal based (CHP included) power production.

The model is formulated as a multistage stochastic linear program. Within the time scope (one year) and the geographical delimitation (the Nordic countries) of the model the main uncertain parameter is the inflow. As a random phenomenon of nature, the expected inflow can be described by a distribution function and thus a stochastic model can be used to deal with the uncertainty. Since the stochastic outcomes are revealed over time, a multistage stochastic model formulation is normally used.

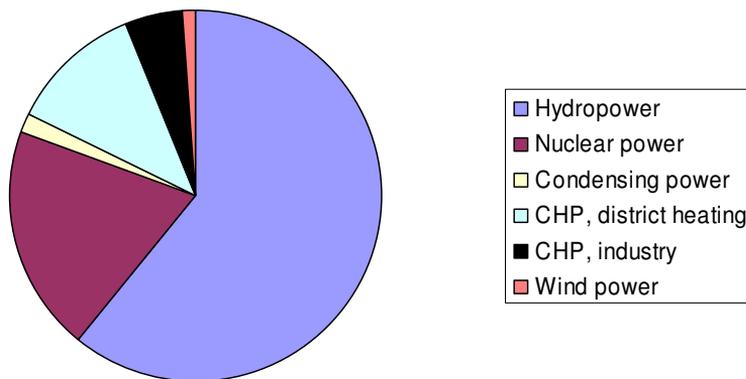
Two implementations of the model have been made. Both will cover a year either with 12 stages each corresponding to a month or with only 6 stages corresponding to bimonthly periods. The inflow to the hydro reservoirs each stage is the only stochastic parameter. As the size of the 12-stage model is considerable, it is planned to use sampling based Benders decomposition methods for solving it. It requires though that

the stochastic parameters are serially independent. In the discussion of the modelling of the inflow in Section 5, this property of the parameters is therefore emphasised.

Below the Nordic energy system is introduced. In section 3 the composite modelling of hydropower in medium-term models is described. Section 4 introduces multistage stochastic modelling and after this the modelling of the inflow will be discussed with focus on how serial independence can be obtained. Scenario generation is addressed in section 6 while in section 7 a full mathematical model of the system will be formulated. Finally, some computational results and conclusions are given in sections 8 and 9.

## 2 The Nordic electricity system

The Nordic energy system modelled here consists of the countries Denmark, Finland, Norway, and Sweden. Iceland, also part of the Nordic countries, has been omitted, since it is not electrically connected to any other country. An international power pool exists, see Nord Pool (2001), covering all four of the modelled countries.



*Figure 2 – Production by technology in the Nordic countries in 2000*

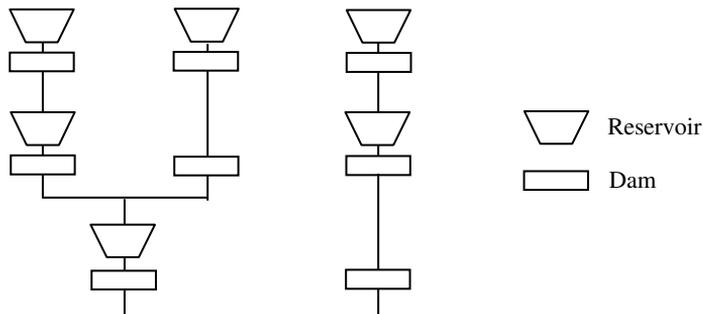
Figure 2 shows the production in year 2000 by the different technologies in the region as given in Nordel (2001). This year was a very wet year. In a normal year, hydropower would contribute with about 50% of the total production. The other half of the production will in general be evenly split between nuclear power and other thermal power, mainly combined heat and power (CHP) production, except for the small contribution of wind power (a little more than 1% in 2000).

These numbers are for the combined Nordic system. Looking at individual countries large differences can be seen between the production systems. E.g. Norway has about 99% hydropower while Denmark use thermal power, mainly CHP, apart from some 15% windpower. Finland and Sweden both have all types of technologies.

### 3 Hydropower modelling

In countries with hydropower this production capacity most often consists of several reservoirs and dams, of which some may be in serial connection along a river chain. An example of a hydropower system is shown in Figure 3.

Some dams may have a reservoir and thus be able to store water for later production while others, the so-called run-of-river plants, have no or little reservoir capacity, and must therefore produce as they receive the inflow. The river chains adds to the complexity as up-river units upon production will release an amount of water, which will be available with some delay to the next unit down-stream. Thus, there are both temporal and spatial connections between the units.



*Figure 3 – System of hydropower plants*

Modelling all this in detail for larger hydropower systems, like those in countries like Brazil and Norway, will result in huge models. Finding the optimal production levels, e.g. hourly, in such systems may be possible, but not for many timesteps.

For longer-term modelling though, like the medium-term perspective to be discussed here, less detail is required in order to be able to solve the model. One way of reducing detail is to look at longer intervals than hourly timesteps.

Another way of reducing details is by using the one-dam or composite representation. In this all dams with reservoirs are treated as one big dam with a reservoir. The measurement of the stored water is often in energy equivalent. The reservoir receives potential energy from the inflow and releases this energy as the dam produces electricity. An upper and a lower bound limit the amount of energy that can be stored. Run-of-river plants can be treated as fixed production like production from wind turbines and thus excluded from the one-dam modelling.

When using the composite representation a generation function for the dam must be found. This is the relationship between the power generation and the water head, i.e. the difference in elevation from the water levels on both sides of the dam. The easy solution is to use a fixed value taken as the average of different water heads. This is obviously not true and better solutions can be found e.g. as in da Cruz and Soares (1995), though this requires a good knowledge of the individual power plants, which may not be available in a deregulated system as such information may be considered confidential. Here it has been chosen to use the fixed value, partly due to missing data, and partly to keep the model simple.

In Arvanitidis and Rosing (1970) it is argued that a composite modelling is reasonable for longer-term models, when the inflow is uncertain and the market to be satisfied is flexible, e.g. when a deregulated market with complete competition exists. This has till now been the normal picture in the Nordic power system; see e.g. Hjalmarsson (2000). As the operations of individual units are not intended results of the model, rather monthly values concerning energy productions by groups of units, it has been chosen to use the composite representation for modelling this system.

## 4 The stochastic framework

For the model to be developed it has been chosen to use the composite reservoir type as described above. This section will introduce the stochastic framework to be used to describe the inflow to the reservoir.

The inflow is treated as a stochastic parameter,  $\omega$ . A linear representation of the power system will be used similar to the one used in the Balmorel model, see Ravn et al. (2001), though the level of detail is greatly reduced. This allow the whole system to be modelled as a multistage stochastic linear programming model with the stages

corresponding to monthly or bimonthly timesteps,  $t$ , for which the realisation of the stochastic parameter  $\omega_t$  is revealed.

The multistage problem formulation creates a nested structure of the model. A master problem at stage  $t-1$  thus has a number of subproblems at stage  $t$ , which each are master problems for stage  $t+1$  subproblems. The number of subproblems is given by the number of possible realisations of the stochastic parameters. Thus, in case the probability distribution describing the possible outcomes is continuous, this needs to be discretised in order to have a finite number of subproblems.

In general, a multistage stochastic linear programming problem can be formulated as:

$P_1$ :

$$\text{minimise } c_1^\top x_1 + Q_1(x_1) \quad (4.1)$$

$$\text{subject to } A_1 x_1 = b_1 \quad (4.2)$$

$$x_1 \geq 0 \quad (4.3)$$

with

$$Q_1(x_{t-1}) = E_{\omega_t} [Q_{\eta, \omega_t}(x_{t-1}, \omega_t)] = \sum_{\omega_t \in \Omega_t} p(\omega_t) Q_{t, \omega_t}(x_{t-1}, \omega_t) \quad (4.4)$$

where  $Q_{t, \omega_t}(x_{t-1}, \omega_t)$  is the optimal solution value for the problem  $P_t$  given by

$P_t$ :

$$Q_{t, \omega_t}(x_{t-1}, \omega_t) = \text{minimise } c_t(\omega_t)^\top x_t + Q_t(x_t) \quad (4.5)$$

$$\text{subject to } A_t x_t = b_t(\omega_t) - T_{t-1}(\omega_t) x_{t-1} \quad (4.6)$$

$$x_t \geq 0 \quad (4.7)$$

Here  $\omega_t$  describes one realisation of the stochastic parameters from the set of all possible outcomes  $\Omega_t$  at stage  $t$ . The function  $Q_t(x_{t-1})$  that is part of the objective function is the so-called future cost function, or cost-to-go function. This is the average costs of all possible realisations of  $\Omega_t$ , i.e. the stage  $t$  subproblems, each which includes their future cost functions of later stages.

The nested structure of the formulation can be utilised by decomposition algorithms like nested Benders decomposition; see Birge (1995). The sizes of the problems that can be solved are still limited. If sampling based versions like SDDP, see e.g. Pereira and Pinto (1991) and Velásquez, Restopo and Campo (1999) are used instead, much larger problems can be solved, though this requires serial independence.

Serial independence exists if the probability of the realisation of outcome  $\omega_t$  is not affected by the realisations of  $\omega_{t-1}$ ,  $\omega_{t-2}$ , etc.

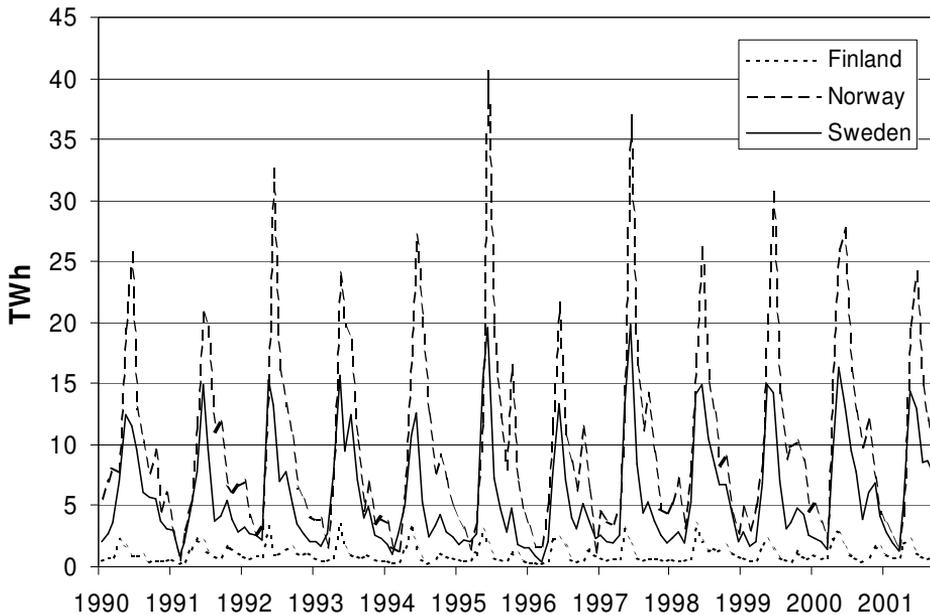


Figure 4 – Examples of inflow data for the Nordic countries 1990-2001.

## 5 Inflow modelling

Countries having hydropower plants for electricity generation usually have records of inflow to the hydropower reservoirs for many years back. From Nordel (2002) examples of such data showing the monthly inflows to the Nordic countries in recent years can be found in Figure 4.

By analysing the monthly data from Figure 4 the correlations between the three hydropower countries (there is no hydropower in Denmark) are found:

Norway – Sweden:	0.89
Norway – Finland:	0.50
Sweden – Finland:	0.74

It can be seen that the correlation between Norway and Sweden is quite strong. Therefore the inflow may be treated as one instead of two different stochastic variables, which will reduce the number of subproblems at each stage of the model. The correlation with Finland is less strong, but since Finland only produce 6% of the hydropower in the region, it is assumed that the Finland inflows follow the trend of Norway and Sweden. Thus all inflows may be described by one stochastic parameter reducing the complexity of the problem considerably. On the downside, it will however affect some of the results, e.g. the transmission between the countries, though this should not have any large implication on prices assuming no major transmission constraints.

Figure 5 sketches to the left a one-dam representation. The inflow to the reservoir  $i_t$ , is a stochastic parameter. In some countries the hydro inflow mostly depends of the amount of rain that falls each month, while it other places may be the snow that falls during winter that makes up most of the yearly inflow. While inflow from rain becomes available to the system with little delay, snow is stored in the mountains for months and is released in often large amounts as spring arrives.

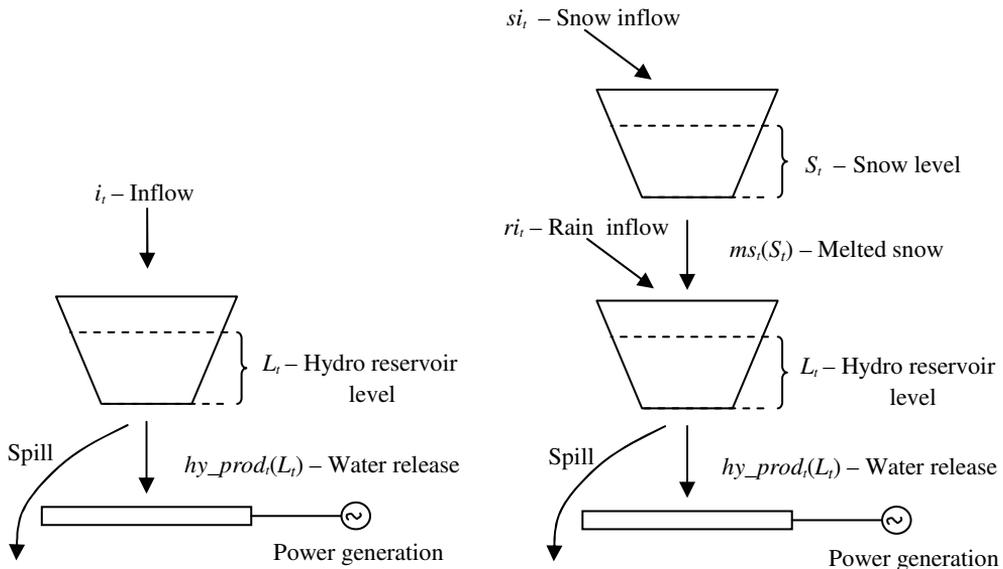


Figure 5 – One-reservoir representation of larger hydropower system (left figure) and a possible two-reservoir representation (right figure)

The right diagram of Figure 5 shows how the inflow may be split into a snow part and a rain part by adding a second reservoir. Why should this be considered?

According to Andreassen and Udnæs (2001) the snowmelt contributes to about half the inflow in Norway. The melting snow often causes flooding during spring while the rain inflow mainly falls in autumn occasionally causing flooding at that time of the year. It is assumed that it is the same for Finland and Sweden. Thus using the snow reservoir model may be reasonable, though it adds another stochastic parameter compared with the one-storage model.

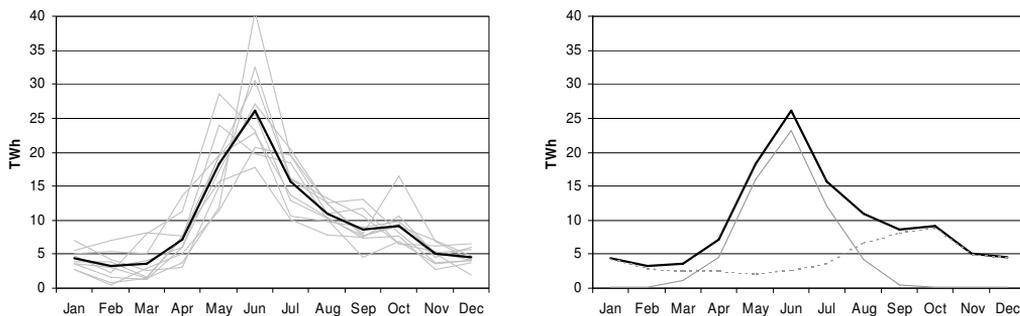


Figure 6 – Inflow sequences for Norway (left) and total average inflow as a rain and a snow part (right)

The left graph of Figure 6 shows the inflow sequences of Norway for 1990-1999 with the bold black line representing the average. The shape corresponds to similar figures from Sweden and Finland and fits well with the description of the Norwegian inflow above. The inflow can be split into a rain part and a snow part as in the right graph of Figure 6. Here the grey full line is the snow inflow while the dotted grey line represents the rain. The bold black line is the average from the left figure. The division of the inflow shown above has been made by hand to get two curves with the same amount of inflow over the year.

As described in the previous section, serial independence for the stochastic parameters is needed, since sampling techniques is to be used for the larger of the models. Using a single stochastic parameter would be troublesome with respect to this. The time when melting starts will be delayed if much snow has fallen. Thus the amount of melted snow each month,  $ms_t$ , depends on the level of snow,  $S_t$ . As a result, serial independence cannot be assumed for systems where snow have a large contribution to the total inflow, if a one-reservoir representation of the inflow is used.

If the modelling period starts as the melting may start, the amount of snow that eventually will become inflow is known to a certain degree. On Figure 6 this corresponds to the area beneath the dotted grey curve on the right graph. When it will start to melt is unknown, as is the actual amounts that will melt at each stage. If the model is started with a high initial level of snow, the melt percentages should be lower in order to make the snow melt later than otherwise. Further snow inflow, i.e.  $si_t$ , is assumed to be zero. By adjusting the parameters depending on the observed snow level at the start of the simulation, serial independence can now be assumed for snow. It is thus chosen to use the two-reservoir rain/snow representation for the model.

The rain inflow, on the other hand, should also be serial independent. The amount of inflow from rain may vary freely with the average given by the full grey line of Figure 6. It is known that a rainy day is more likely to be succeeded by another rainy day than if the day was dry. Similar the autocorrelation of weekly inflows are quite strong. In Førland and Nordli (1993) though, it is found that for Norway only few larger regions had an autocorrelation larger than 0.2-0.3 for precipitation of successive months. And this was only for a few of the winter months. Data for Sweden, see Vedin et al. (1991) shows a similar low autocorrelation in most cases.

Based on these references it is assumed that the autocorrelation of monthly or bimonthly inflows from rain are so small that serial independence in practice also exists for rain. The two-reservoir representation thus ensures serial independence and allows sampling methods to be used for solving.

Apart from making the inflow modelling serially independent the two-storage modelling also to reflect that the snow level is known by decision-makers. The decision-makers know that they will at least get this amount of inflow in addition to the wholly unpredictable rain inflow. This reduces the uncertainty for the decision-maker and should be reflected in the model rather than modelling the inflow as one random and unpredictable parameter.

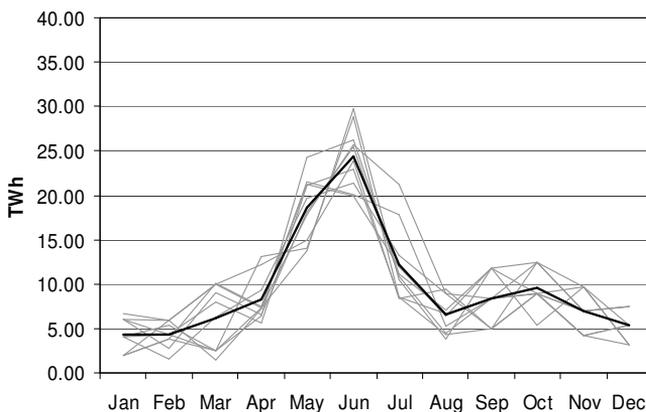
## 6 Scenario generation

This section addresses the discretisations of the stochastic parameters  $ri_t$  and  $si_t$  of the inflow model. It has been done more or less by hand in order to quickly get a reasonable set of data to use for trials and is thus not based on the typical theory behind scenario generation, see e.g. Høyland, Kaut, and Wallace (2000).

Three possible outcomes the stochastic parameters  $ri_t$  has been defined for each stage corresponding to the average rain inflow or a certain percentage above or below this level. For the spring and summer months three possible outcomes of  $si_t$  are defined. These are different percentages of the actual snow level that become available as inflow in that month. This value may both be zero (no melting) or 100 (all the remaining snow melts in this month).

For the 12-stage model the scenario tree representing all possible realizations of inflow will branch into 9 branches (i.e. all combinations of the rain and snow outcomes) at six of the stages. The remaining stages will have 3 branches, as the snow inflow does not differ in these. This results in about 130 million possible inflow scenarios. Using this inflow model 10 random inflow sequences have been generated. These are shown in Figure 7. Comparison with Figure 6 indicates both that this way of representation is suitable for modelling the inflow and also that the magnitude of the stochastic parameters is reasonable.

The 6-stage model has 6561 possible scenarios as snowmelt occur in three out of six stages only.



*Figure 7 – Random inflows generated for Norway*

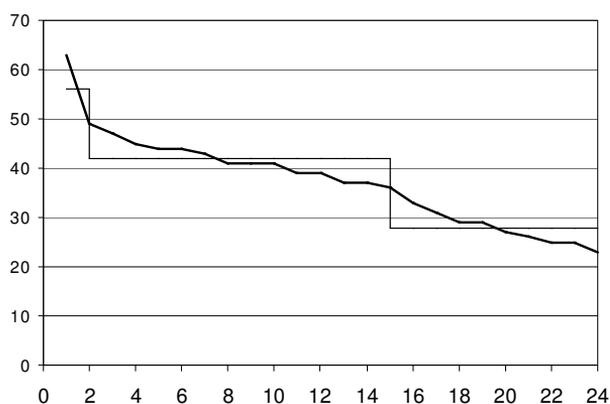
## 7 Model formulation

In this section the stochastic model of the Nordic hydro-thermal system will be formulated.

The overall geographical delimitation is the 4 Nordic countries that are part of Nord Pool. They have been further subdivided into 8 regions in total to better represent bottlenecks in the transmission network.

Due to the high proportion of CHP production in the regions, it has been decided to represent district heating endogenously in the model. Thus, the subproblems to be solved at each stage in this multistage model are dispatch models with 2 commodities; power and district heating. There are linear costs associated with production as well as linear constraints only on the system. The inflow during a given month is assumed to be available for use at the beginning of the next month at earliest. The two stochastic parameters contributing to the inflow has been described earlier.

Each stage, monthly or bimonthly, is split into 3 load blocks representing peak hours, daylight hours and night hours. The demand for both power and heat varies for each load block and is approximated from the load duration curve of that country. An example of a load duration curve of an average day is shown in Figure 8. The thin line of the figure shows the three load block approximation of the load duration curve.



*Figure 8 – Sketch of duration curve of the demand (bold line) that is split into three load blocks (thin line)*

The productions on nuclear power plants in the model are restricted as such plants in general have poor regulation capabilities. Thus the production on those in the 3 load blocks of a stage may not differ more than 10% from each other.

CHP plants can be divided into two groups; backpressure and extraction. The main difference is the relationship between the heat and power production. This has been

sketched in Figure 9. Here it can be seen that backpressure plants, typical small-scale district heating plants and industrial plants, produce heat and power in a fixed proportion. Extraction units, which typically the large (>100 MW) power stations, on the other hand have a greater degree of freedom.

To simplify the model, fixed production values have been used for small-scale CHP production and production from wind turbines. As backpressure units produce power and heat at a fixed ratio, the power output will follow the heat demand, which is given exogenously in the model. Thus the production for these units will be fixed.

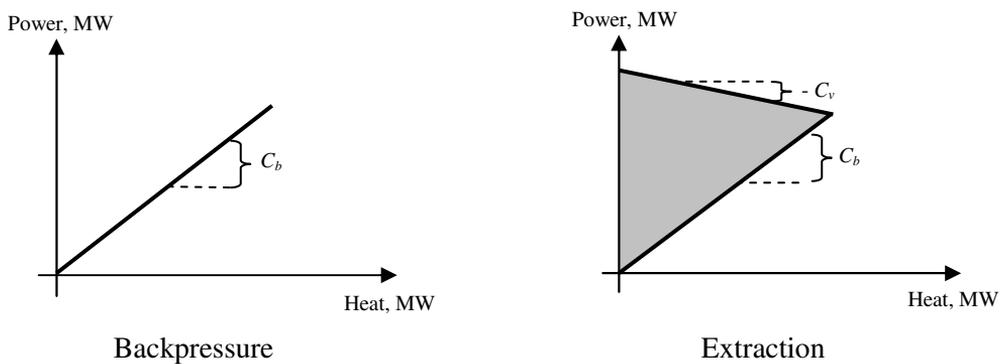


Figure 9 – Backpressure and extraction CHP production units

For wind turbines the capacity is still limited and making the production a stochastic parameter would add little in precision but make the model much harder to solve. But for future scenarios, e.g. 2020 where the installed capacity of wind turbines in the region may exceed 10000 MW, this may be needed for some analyses.

The model formulation for the month  $m$  subproblem will now be given below:

#### Sets and indices:

- $m$  month index, all months =  $M$
- $t$  hour type index, all hour types =  $T$
- $r$  regional index, all regions =  $R$ . Also used regional indices are  $r'$ ,  $r_1$ , and  $r_2$
- $u$  production unit index, all units =  $U$
- $U^{HY}$  subset of  $U$  that includes the hydropower units only
- $U^{EX}$  subset of  $U$  that includes the extraction type CHP units only
- $U^{NU}$  subset of  $U$  that includes the nuclear units only

**Decision variables:**

$P^E(r,u,m,t)$	production of electricity by unit $u$ , in MW
$P^H(r,u,m,t)$	production of heat by unit $u$ , in MW
$RL(r,m)$	reservoir level of region $r$ at end of month $m$ , in GWh equivalent
$SL(r,m)$	snow level of region $r$ at end of month $m$ , in GWh equivalent
$AP(r,u,m)$	average production of unit $u$ of region $r$ during $m$
$X(r_1,r_2,m,t)$	transmission from region $r_1$ to region $r_2$ ( $r_1 \neq r_2$ ) at time $t$ , in MWh/h

**Functions:**

$C(P^E, P^H)$	cost of producing $P^E$ and $P^H$ . Assumed linear function, in \$/GWh
$Q(RL, SL)$	approximation of future costs, i.e. the expected costs for all later stages assuming reservoir levels as given. The function is generated by the solution method from iteration to iteration, in \$/GWh

**Parameters:**

$D^E(r,m,t)$	electricity demand at time $t$ in region $r$ , in MWh/h
$D^H(r,m,t)$	heat demand, central heating network, at time $t$ in region $r$ , in MWh/h
$FP(r,m,t)$	fixed electricity production, at time $t$ in region $r$ , in MWh/h
$wght(t)$	the number of hours per month this hour type $t$ represents, in h
$InitRL(r,m)$	initial reservoir level in $r$ at beginning of month, in GWh equivalent
$InitSL(r,m)$	initial level of snow in $r$ at beginning of month, in GWh equivalent
$MaxI(r,m)$	maximum possible inflow to reservoirs in $r$ at end of month $m$ , in GWh equivalent
$MinI(r,m)$	minimum possible inflow to reservoirs in $r$ at end of month $m$ , in GWh equivalent
$C_b(u)$	slope of $C_b$ -curve of extraction unit $u \in U^{EX}$ (see Figure 9)
$C_v(u)$	positive slope of $C_v$ -curve of extraction unit $u \in U^{EX}$ (see Figure 9)
$MaxRL(r,m)$	max reservoir level in region $r$ at end of month $m$ , in GWh equivalent
$MinRL(r,m)$	min reservoir level in region $r$ at end of month $m$ , in GWh equivalent
$MinFlow(r)$	min generation by hydropower units in region $r$ in percent
$MaxP^E(r,u)$	max production of electricity in region $r$ by units $u$ , in MW
$MaxP^H(r,u)$	max production of heat in region $r$ by units $u$ , in MW
$MaxX(r_1,r_2)$	max transmission from region $r_1$ to region $r_2$ (zero for $r_1 = r_2$ ), in MW

**Stochastic parameters:**

$RI(r,m)$	amount of rain inflow in region $r$ that becomes available for production this month, in GWh equivalent
$MeltS(r,m)$	amount of $InitSL$ that melts in region $r$ and becomes available for production this month, in percent

**Equations:**

- (7.1) Objective function  
 (7.2) Electricity equilibrium condition  
 (7.3) Heat equilibrium condition  
 (7.4) Reservoir level – dynamic  
 (7.5) Min reservoir level  
 (7.6) Max reservoir level  
 (7.7) Snow level – dynamic  
 (7.8) Extraction unit restriction –  $C_b$   
 (7.9) Extraction unit restriction –  $C_v$   
 (7.10) Average nuclear production  
 (7.11) Max nuclear production  
 (7.12) Min nuclear production  
 (7.13) Max production, electricity  
 (7.14) Max production, heat  
 (7.15) Transmission constraint  
 (7.16) Non-negative variables

**Model:**

For a fixed  $m \in M$  the singlestage (subproblem) model can now be formulated as:

$$\text{Minimise} \quad \sum_{r \in R} \sum_{u \in U} \sum_{m \in M} \sum_{t \in T} \left( \text{wght}(t) \times C(P^E(r, u, m, t), P^H(r, u, m, t)) \right) + Q(RL(r, m), SL(r, m)) \quad (7.1)$$

Subject to:

$$\forall r, m, t: \quad \sum_{u \in U} P^E(r, u, t) + \sum_{r' \in R} X(r', r, t) - \sum_{r' \in R} X(r, r', t) + FP(r, m, t) = D^E(r, m, t) \quad (7.2)$$

$$\forall r, m, t: \quad \sum_{u \in U} P^H(r, u, m, t) = D^H(r, m, t) \quad (7.3)$$

$$\forall r, m: \quad RL(r, m) = \text{Init}RL(r, m) + RI(r, m) + \text{Init}SL(r, m) \times \text{Melt}S(r, m) - \sum_{u \in U^{HY}} \sum_{t \in T} \left( \text{wght}(t) \times P^E(r, u, m, t) \right) \quad (7.4)$$

$$\forall r, m: \quad RL(r, m) \geq \text{Min}RL(r, m) - \text{Min}I(r, m) \quad (7.5)$$

$$\forall r, m: \quad RL(r, m) \leq \text{Max}RL(r, m) - \text{Max}I(r, m) \quad (7.6)$$

$$\forall r, m: \quad SL(r, m) = \text{InitSL}(r, m) - \text{InitSL}(r, m) \times \text{MeltS}(r, m) \quad (7.7)$$

$$\forall r, u \in U^{HY}, m, t: \quad P^E(r, u, m, t) \geq \text{MinFlow}(r) \times \text{MaxP}^E(r, u) \quad (7.8)$$

$$\forall r, u \in U^{EX}, m, t: \quad P^E(r, u, m, t) \geq C_b(u) \times P^H(r, u, m, t) \quad (7.9)$$

$$\forall r, u \in U^{EX}, m, t: \quad P^E(r, u, m, t) \leq \text{MaxP}^E(r, u) - (C_v(u) \times P^H(r, u, m, t)) \quad (7.10)$$

$$\forall r, u \in U^{NU}, m: \quad \sum_{t \in T} (P^E(r, u, m, t) \times \text{wght}(t)) = AP(r, u, m) \quad (7.11)$$

$$\forall r, u \in U^{NU}, m, t: \quad P^E(r, u, m, t) \geq 0.95 \times AP(r, u, m) \quad (7.12)$$

$$\forall r, u \in U^{NU}, m, t: \quad P^E(r, u, m, t) \leq 1.05 \times AP(r, u, m) \quad (7.13)$$

$$\forall r, u, m, t: \quad P^E(r, u, m, t) \leq \text{MaxP}^E(r, u) \quad (7.14)$$

$$\forall r, u, m, t: \quad P^H(r, u, m, t) \leq \text{MaxP}^H(r, u) \quad (7.15)$$

$$\forall r, r', m, t: \quad X(r, r', m, t) \leq \text{MaxX}(r, r') \quad (7.16)$$

$$\forall r, r', u, t, m: \quad P^E(r, u, m, t) \geq 0, \quad P^H(r, u, m, t) \geq 0, \quad RL(r, m) \geq 0 \quad (7.17)$$

$$SL(r, m) \geq 0, \quad X(r, r', m, t) \geq 0$$

## 8 Computational results

In this section some computational results will be shown. The models will be solved using the ReSa algorithm presented in Hindsberger and Philpott (2001). First a comparison between the results of the 6-stage and the 12-stage models will be made.

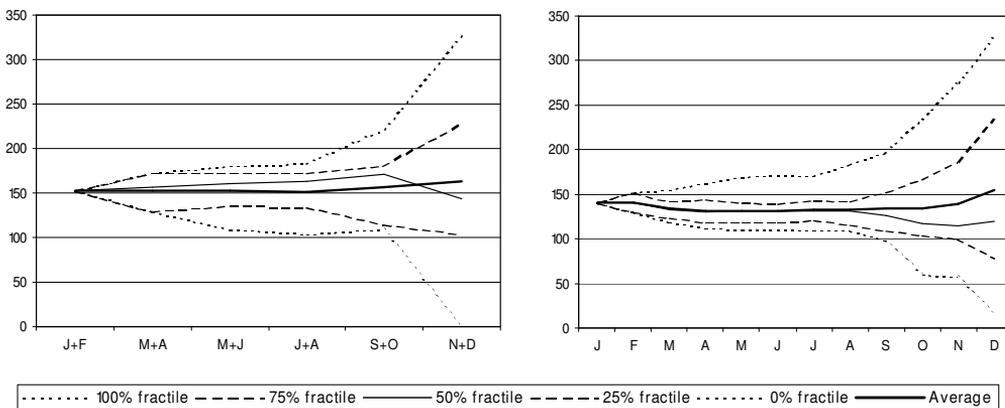
*Table 1 – Comparison between 6-stage and 12-stage model results*

	Costs (mill. DKK)	Time (minutes)	Spot price NO-S (DKK/MWh)			Spot price DK-E (DKK/MWh)		
			Max	Average	Min	Max	Average	Min
6-stage	17267	43.28	324.74	154.95	0.60	324.64	157.13	104.77
12-stage	16784	143.59	324.74	136.44	16.48	239.61	140.98	101.09

As seen in Table 1 the total expected system costs (corresponding to the objective function value) is roughly the same for the two models. There is however a considerable gain in computation time when using a 6-stage model rather than a 12-stage model as indicated.

The sampling method used for solving the models was forced to run for 25 iterations. The changes in the objective function value for the last 10 iterations though were only 1%. Had the model been stopped after 15 iterations instead, the running time of the solution method would have been 22 and 65 minutes instead.

Prices have been estimated for each stage for 100 random realisations of the inflow given a fixed initial reservoir level. Looking at the prices, the results show that the 6-stage model, compared with the 12-stage model, results in a larger span of possible prices in the future for the hydro-dominated Norway-South region (NO-S). For the thermal dominated area Denmark-East (DK-E) the opposite holds. For both regions though the 6-stage model estimates a higher average price than the 12-stage model.



*Figure 10 – Spot price estimates (100 samples) for NO-S in DKK/MWh for the 6-stage (left) and 12-stage (right) models*

In Figure 10 fractiles showing the predicted prices of the models for NO-S region are shown given a fixed initial reservoir and snow level. The 12-stage model clearly shows expected lower prices during summer than winter as seen historically (upper, right graph of Figure 11). This is less clear for the 6-stage model. Still, for many analyses, e.g. those more related to estimating the capacity requirements of the system than actual price estimation, the 6-stage modelling should be sufficient.

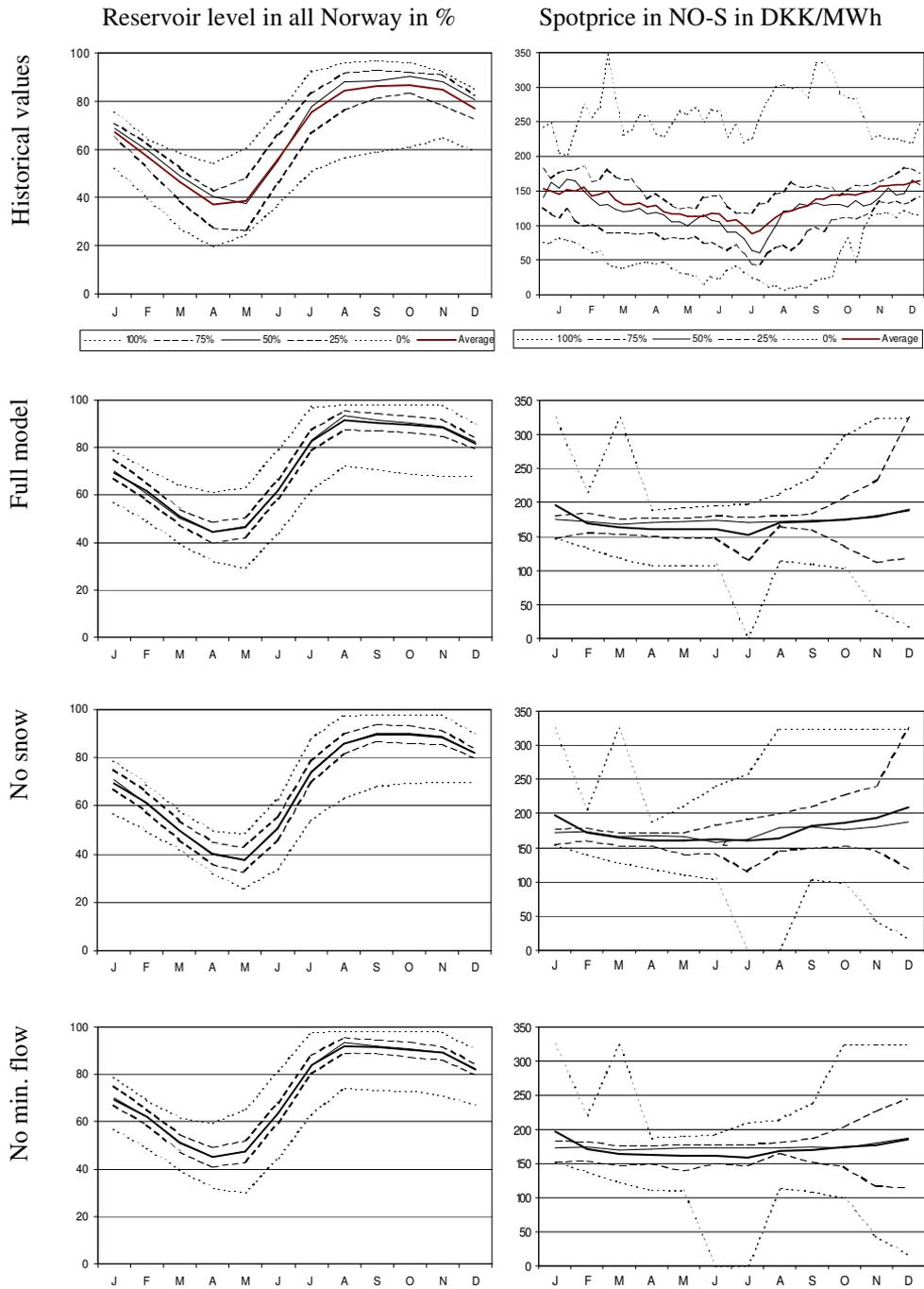


Figure 11 – Comparison between 12-stage model variants

Now the results of the full 12-stage model (denoted FM in the following) will be compared with those of some reduced models. In one, the modelling of snow has been excluded. Instead, only one stochastic parameter is used, and the inflow is received directly in the hydro reservoir. This variant is denoted NS. Also a variant excluding the minimum flow requirements of equation (7.8) has been tried. This is denoted NM.

*Table 2 – Comparison of the results for different variants of the 12-stage model*

	Costs (mill. DKK)	Time (minutes)	Spot price NO-S (DKK/MWh)			Spot price DK-E (DKK/MWh)		
			Max	Average	Min	Max	Average	Min
FM	16784	143.59	324.74	136.44	16.48	239.61	140.98	101.09
NS	16952	103.35	324.74	141.79	0.60	324.48	147.47	101.09
NM	16785	141.13	324.74	137.08	16.48	324.48	142.82	101.09

Table 2 shows only a negligible difference between the FM and the NM variants while the NS variant predicts a higher total cost. The differences in costs between the FM and the NS variants can be interpreted as the value of additional information (i.e. the value saved by using knowledge of the snow levels). The difference though is small—approximate 1%. As expected the NS variant was solved faster than the others as it has only one stochastic variable (with 3 possible outcomes) compared with two (and a total of 9 possible outcomes) for the others. Again the models were run for 25 iterations.

Figure 11 shows graphs for the three different versions of the 12-stage model together with historical data for comparison. The historical values (fractiles) shown for Norway are from 1991-2001 using data from Nordel (2002) and Nord pool (2001). Different initial levels of both the hydro and the snow reservoirs have been used.

Looking at the reservoir content no big differences can be observed for the variants, all three fit reasonably well with historical values.

For the spot price estimates a price drop of about 45 DKK/MWh during summer can be seen for all three variants while the historical average shows a 55 DKK/MWh drop. This indicates a reasonable estimation the seasonal variations in price. However, the average price levels are in general higher. This may be because the production system as it was in 2000 has been used. It differ from the average situation in the 1990's as the production capacity in the Nordic countries in general has been kept at the same level while consumption has increase some 10%. Hence the price level has shown a growing trend in the last couple of years, as the marginal units now are more expensive than in the mid 1990's.

For August, the last month with possible snowmelt, a large difference in the results between the FM/NM scenarios and the NS scenario. From Figure 6 (right) it can be seen that the average inflow should consist of 4 TWh of snowmelt and 7 TWh of rain. The FM and NM scenarios know beforehand whether they will receive the 4 TWh or not and thus only have 7 TWh on average with a possible deviation on this. The NS scenarios have an 11 TWh inflow on average and with the same deviation, this gives a larger span of possible values, where some may cause the model to reach the upper reservoir level and force production at a very low price. In general NS has a larger span in its price estimates in all months.

Also, note that compared with history, the models all start with the same reservoir and snow levels. Thus more extreme values can be expected if these starting points were varied also, though the occurrences would be rare.

Comparing the results on the short term (i.e. the first 2 months or so) little variations are observed between the model variants. This time horizon is important e.g. for here-and-now hydro release decisions and a rough model formulation (like NS) may be sufficient for such analyses. But the longer-term perspective is interesting in other cases, e.g. for evaluation of medium-term power contracts. In those cases, the full 12-stage model should be used.

## 9 Conclusions

A stochastic model of the Nordic hydro-thermal system has been established so that the stochastic parameters are serially independent. This allows the model to be solved using efficient sampling based algorithms.

Numerical results show that the model behaves well when compared with historical observations. When only 6 stages are used the model is solved 3 times faster with little loss in the quality of many types of results. Only when it comes to predicting the drop in the expected spot price during summer, the 6-stage model deviates considerably from the 12-stage model results. The 12-stage model does catch this property, though the drop is less than the one historically observed.

The minimum flow constraint is shown to have little effect on the results while the modelling of snow reservoirs reduce the span between the highest and lowest of the expected future spot prices considerably. For modelling future years, this may not be

relevant, but for price prediction models of the current year, where the snow level is known, the addition of snow reservoirs should be considered in the future.

There are several ways to improve the model further e.g. by using better scenario generation. Also adding reservoir minimum and maximum levels and minimum flow constraints based on monthly values rather than using the same value for all the year should be considered.

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**PAPER G**

**RESA: A METHOD FOR SOLVING  
MULTISTAGE STOCHASTIC  
LINEAR PROGRAMS**

This paper was presented at the conference “Stochastic Programming ‘01”, Berlin, Germany, August 2001.

“It is hard to foretell—especially about the future”

- *Robert S. Petersen*

# ReSa: A method for solving multistage stochastic linear programs

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**Abstract:** This paper presents a new sampling scheme for solving large multistage stochastic LP-models using Benders decomposition. The approach is compared with two alternative sampling approaches by applying the methods to a large hydro-thermal scheduling model with stochastic inflows. For this class of problems, the new scheme performs better.

**Keywords:** Decomposition, multistage stochastic linear programming, sampling.

## 1 Introduction

Multistage stochastic linear programming is becoming a popular technique for solving convex optimisation problems involving planning under uncertainty, where stochastic outcomes are revealed over time. When the number of state variables is large a stochastic programming framework has some advantages over dynamic programming, which suffers from the so-called “curse of dimensionality” when the state variables are discretised.

Discretisation can be avoided to some extent by using stochastic Benders decomposition, a technique originally developed for two-stage stochastic linear programming by Van-Slyke and Wets (1969), and extended to multi-stage problems by Birge (1985). Stochastic Benders decomposition approximates the future cost function at each stage by a piecewise affine function of the decision variables that is computed using the optimal dual variables computed from the linear programs that are solved in the immediately subsequent stage. Even with this approximation, many linear programs must be solved to solve such a problem, and recent efforts in this area have been focussed on using sampling to reduce this computational effort.

The first method to use sampling in a multistage stochastic programming application was the Stochastic Dual Dynamic Programming (SDDP) algorithm of Pereira and Pinto (1991). The Abridged Nested Decomposition (AND) method by Donohue and Birge (2001) improved the performance of SDDP by adapting the sampling scheme to fit the structure of the scenario tree. In particular AND performs well on bushy scenario trees, while SDDP is most suited for trees with many stages and few random outcomes per stage. In this paper we investigate a variation of SDDP that we call Reduced Sampling (ReSa). This method incorporates some of the ideas from AND into a sampling scheme for trees with many stages and few random outcomes per stage.

The paper is laid out as follows. In the next three sections the SDDP, the AND, and the ReSa methods will be presented. Section 5 will present some computational results of the performance of the algorithms on a hydro reservoir optimisation problem while finally in section 6 some concluding remarks will be given.

## 2 Stochastic Dual Dynamic Programming

In its multistage form, Benders decomposition is a nested decomposition algorithm, which we shall describe briefly. For details the reader can refer to the text by Birge and Louveaux (1997). At any given stage  $t-1$  there is a master problem having a number of subproblems at stage  $t$ , each of which are master problems for stage  $t+1$  subproblems. In the following, the value of  $Q_t(x_t)$  for the final stage  $t = T$  is defined to be zero. The stage one problem is:

$$P_1: \quad \text{minimize} \quad c_1^\top x_1 + Q_2(x_1) \quad (2.1)$$

$$\text{subject to} \quad A_1 x_1 = b_1 \quad (2.2)$$

$$x_1 \geq 0 \quad (2.3)$$

with

$$Q_t(x_{t-1}) = E_{\omega_t} [Q_{t,\omega_t}(x_{t-1}, \omega_t)] = \sum_{\omega_t \in \Omega_t} p(\omega_t) Q_{t,\omega_t}(x_{t-1}, \omega_t) \quad (2.4)$$

where  $p(\omega_t)$  is the probability of realisation of random outcome  $\omega_t$  and  $Q_{t,\omega_t}(x_{t-1}, \omega_t)$  is the optimal solution value for the problem  $P_t$  given by:

$$P_t: \quad Q_{t,\omega_t}(x_{t-1}, \omega_t) = \text{minimize} \quad c_t(\omega_t)^\top x_t + Q_{t+1}(x_t) \quad (2.5)$$

$$\text{subject to} \quad A_t x_t = b_t(\omega_t) - T_{t-1}(\omega_t) x_{t-1} \quad (2.6)$$

$$x_t \geq 0 \quad (2.7)$$

At stage  $t$  in a multistage stochastic linear program the future cost function,  $Q_{t+1}(x_t)$ , is a piecewise linear, convex function of  $x_t$ . Benders decomposition will for each iteration add cuts to each stage  $t$  approximating the  $Q_{t+1}(x_t)$  function as seen in Figure 1.

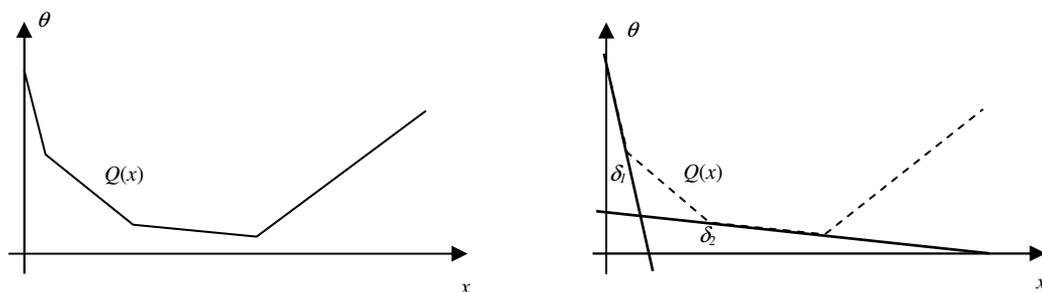


Figure 1 – An example of a future cost function (left) and an approximation by two cuts  $\delta_1$  and  $\delta_2$  (right)

```

1.  Initializations
Forward pass:
2.  Sample  $S$  scenarios forming the set  $SS$ 
    FOR  $t=1$  TO  $T$ 
      FOR  $s=1$  TO  $S$ 
        Solve problem  $s$  of  $SS$ 
      END
    END
3.  Calculate  $\bar{z}$  and  $\underline{z}$ 
4.  IF not converged
Backward pass:
5.  FOR  $t=T-1$  TO  $1$ 
      FOR  $s=1$  TO  $S$ 
        Solve all stage  $t+1$  subproblems of problem  $s$  of  $SS$ 
        Calculate and add cut to stage  $t$ 
      END
    END
6.  GOTO 2
    ENDIF
    
```

Figure 2 – The Stochastic Dual Dynamic Programming algorithm

Throughout this paper we assume relatively complete recourse, meaning that a stage  $t$  subproblem will have feasible solutions irrespective of the results of the previous stage 1 to  $t-1$  subproblems. Solving the stage 1 subproblem with the  $Q_2(x_1)$  approximation will create a lower bound  $\underline{z}$  of the problem since this is a relaxation of the problem. Also the expected value of any feasible solution to the full problem will be an upper bound  $\bar{z}$  since it is a minimization problem. The Benders decomposition algorithm terminates when  $\bar{z}$  equals  $\underline{z}$  (or is sufficiently close), in which case the problem is solved to optimality.

The SDDP method is based on Benders decomposition, but instead of solving all subproblems in the scenario tree at each iteration in order to calculate the cuts, only a subset are sampled and solved. The algorithm is outlined in Figure 2, and illustrated by the pictures in Figure 3, where the left figure is a full 5-stage scenario tree, with 3 possible outcomes each stage. The figure in the middle shows 3 randomly chosen scenarios. A scenario is here defined as a sequence of random outcomes leading from the stage 1 subproblem to a stage  $t$  subproblem. The black dots represent subproblems that will be solved during the forward pass while both black and grey subproblems will be solved during the backward pass of the algorithm.

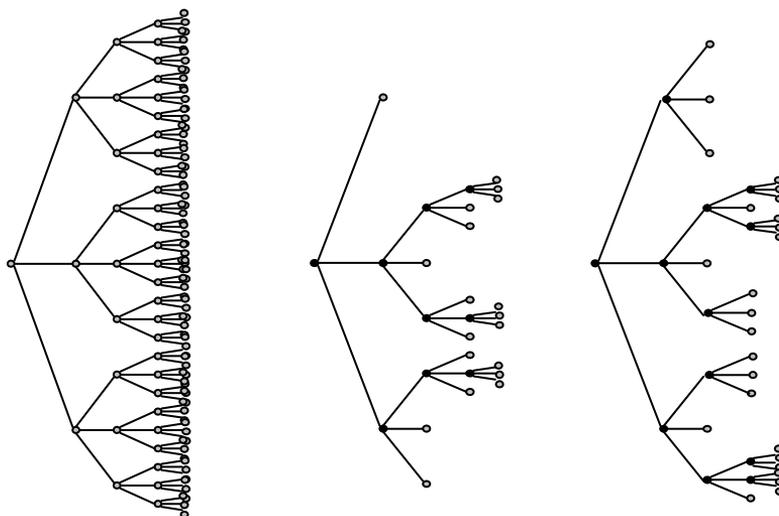


Figure 3 - Full scenario tree (left), SDDP (middle) and AND (right) samplings

An important assumption in SDDP and all other sampling techniques is that serial independence exists. If serial independence does not exist then cuts cannot be shared amongst all subproblems at the same stage. Serial independence can be assumed when the probabilities  $p(\omega_t)$  are independent of history, i.e. of the previous state  $\omega_{t-1}$ .

In SDDP, calculation of the lower bound  $\underline{z}$  is the same as in Benders decomposition. But since only a subset of all stage  $t$  problems are solved at each iteration, the upper bound is only an estimate, and the value of  $\bar{z}$  may in some occasions be lower than  $\underline{z}$ . However, given a candidate solution  $x$ , the objective values

$$z_k = c^1 x_k^1 + \sum_{t=2}^N c^t (\xi_k^t) x_k^t \quad (2.8)$$

of scenarios  $k=1,2,\dots,K$ , provide independent identically distributed samples of the (random) objective value  $z(x)$ , with mean, say  $\mu$  and variance, say  $\sigma^2$ , so the estimate

$$\bar{z} = \frac{1}{K} \sum_{k=1}^K z_k \quad (2.9)$$

is asymptotically normally distributed with mean  $\mu$  and variance  $\sigma_z^2 = \sigma^2/K$ . Typically,  $\sigma^2$  is not known and so  $\sigma_z^2$  is estimated using

$$\sigma_z = \sqrt{\frac{1}{K^2} \sum_{k=1}^K (\bar{z} - z_k)^2} \quad (2.10)$$

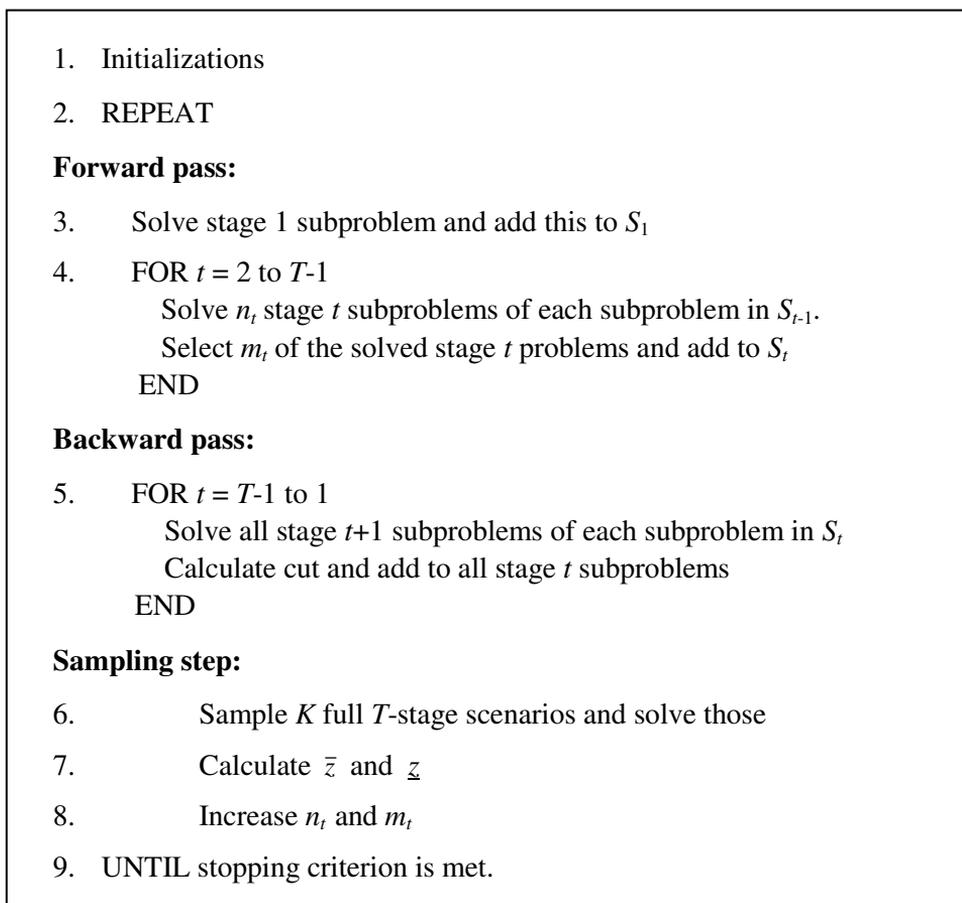
This can be used to construct a confidence interval  $[\bar{z} - 2\sigma_z, \bar{z} + 2\sigma_z]$  for the actual value of  $\bar{z}$ . If the lower bound  $\underline{z}$  lies within this confidence interval at the end of a forward pass then the algorithm is stopped.

### 3 Abridged Nested Decomposition

Abridged Nested Decomposition (AND) is a recent sampling method using the same basic idea as SDDP. Like SDDP, AND requires the assumption of serial independence, and uses a sample of all subproblems at each stage for calculating an approximation of the future cost function,  $Q$ .

The main difference between the two sampling techniques can be seen by examining the sampled scenario trees. Looking at Figure 3 it can be seen that the SDDP technique creates scenarios spanning  $T-1$  stages. In the AND technique some of the scenarios end before stage  $T-1$  is reached, like the upper branch on the right figure. This allows information in the form of cuts to be obtained from more parts of the scenario tree than in SDDP while having similar computation time. Therefore AND is more suited for solving problems with bushier scenario trees, while SDDP may perform better on long

narrow scenario trees. Another difference, which speeds up the algorithm, is that AND will initially start with a low sampling size, increasing the number of subproblems to be sampled for each iteration if the algorithm has not converged.



*Figure 4 – The Abridged Nested Decomposition algorithm*

Given integer values of  $n_t$  and  $m_t$  for  $t = 2 \dots T-1$ , the Abridged Nested Decomposition algorithm is outlined in Figure 4. In the algorithm  $m_t$  denotes the branching solutions, namely the subset of solved stage  $t$  subproblems for each of which  $n_{t+1}$  branches to stage  $t+1$  subproblems will be sampled and solved. The stage one subproblem is always considered to be a branching solution. Returning to the AND example in Figure 3 it can be seen that  $n_2 = 3$ ,  $m_2 = 2$ ,  $n_3 = 2$ ,  $m_3 = 2$ , and  $n_4 = 2$ .

In the SDDP algorithm  $S$  scenarios are randomly selected and solved. These can be used for calculating a statistical valid  $\bar{z}$  estimate. The forward pass in the AND algorithm does not allow such an estimate to be made as the generated scenario tree does not have scenarios that cover all stages. Instead a full forward pass similar to the one in SDDP is performed after the backward pass to allow  $\bar{z}$  to be computed during this sampling step. The sampling step is purely a device to enable the calculation of a statistical upper bound. Observe that this might be expensive, and so it need not be computed at every pass.

## 4 Reduced Sampling method

In this section we outline a new approach called Reduced Sampling (ReSa). The idea is to improve the efficiency of the SDDP method using some of the innovations of AND, in the hope of getting better performance than AND when solving long scenario trees, and better performance than SDDP when solving bushier scenario trees. The main feature of AND that we use is to limit the number of subproblems to be solved for the first iterations.

The algorithm is presented in Figure 5. Comparing it with the SDDP algorithm, it can be seen that the algorithms are very similar. The main difference is the addition of  $B_t$ , the number of randomly selected stage  $t$  problems, which were solved during the forward pass, for which a cut is calculated during the backward pass. The value of  $B_t$  is increased if improvements from iteration to iteration are insignificant. Note that for any  $t$ ,  $B_t$  cannot be higher than  $S$ . Compared with AND, the ReSa method calculates the  $\bar{z}$  estimate based on the forward pass, so no sampling pass is needed. There are no similarities in the way the scenarios to be solved are picked, though the idea of keeping the sample size small during the early iterations is the same.

Normally the values of  $B_t$  should be low in the beginning—in the order 1 to 5. As seen in step 6 of the algorithm, they can be increased if some criteria are met. For example, such a rule could be to increase  $B_t$  if the variance estimate of the  $\bar{z}$  calculation is increased compared with the value in the previous iteration or if the  $\bar{z}$  estimate itself is higher than the previous one. The idea is to add more sampled problems to solve as convergence slows down. On the other hand large initial gains can be obtained with very few cuts, so there is no reason to use too much computation power on the backward pass. The scenario trees illustrating this are shown in Figure 6.

```

1.   Initializations

Forward pass:

2.   Sample  $S$  scenarios forming the set  $SS$ 
     FOR  $t=1$  TO  $T$ 
       FOR  $s=1$  TO  $S$ 
         Solve problem  $s$  of  $SS$ 
       END
     END

3.   Calculate  $\bar{z}$  and  $\underline{z}$ 

Backward pass:

4.   IF not converged
5.     FOR  $t=T-1$  TO 1
       Sample  $B_t$  stage  $t$  problems from  $SS$  forming the set  $BS$ 
       FOR  $b=1$  TO  $B_t$ 
         Solve all stage  $t+1$  subproblems of problem  $b$  of  $BS$ 
         Calculate and add cut to stage  $t$ 
       END
     END

6.   IF (no improvements) AND ( $B_t < S$ )
        $B_t = B_t + 1$ 
     ENDIF

7.   GOTO 2

ENDIF

```

*Figure 5 – The Reduced Sampling algorithm*

In Figure 6 the middle graph shows the forward pass of 3 sampled scenarios (here of length  $T$  rather than  $T-1$  as in SDDP) used for estimating  $\bar{z}$ . The right graph shows the backward pass. First two stage  $T-1$  scenarios of those solved during the forward pass are randomly selected. The subscenarios of those are solved (here shown as black dots) and cuts for stage  $T-1$  are calculated and added. Next two stage  $T-2$  scenarios are randomly picked and the subscenarios of these are solved adding cuts to stage  $T-2$ , etc. Compared with SDDP fewer cuts are added, but also fewer scenarios are solved during the backward pass—in this case 33% less.

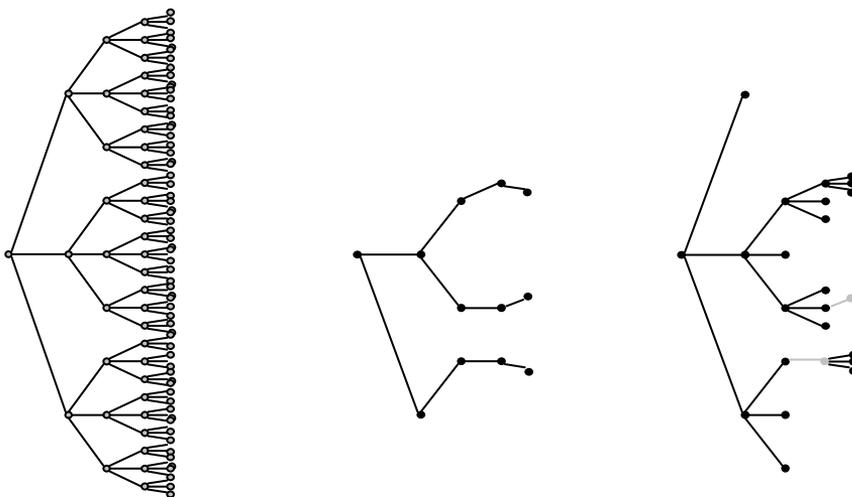


Figure 6 – Full scenario tree (left), ReSa forward pass (middle), and ReSa backward pass (right)

The ReSa algorithm is very well suited for long narrow scenario trees. For problems where the scenario tree is bushier, the  $B_t$  values will probably end up being equal to the  $S$  before the stopping criterion is met. Then the algorithm will perform just as SDDP. It may improve the speed of convergence to increase the number of  $S$  (and thus also the  $B_t$  values) at this point though the original value should be used for  $\bar{z}$  estimation.

## 5 Computational results

As a computational test case the three methods above were applied to a hydro reservoir management problem. This was modelled as a stochastic linear program with 12 stages, each corresponding to one month in time. The objective is to minimize the expected cost of production of a hydro-thermal power system. Though hydropower is the dominant technology, nuclear power plants, peak gas turbines and thermal cogeneration plants delivering district heating as well are included. The power system is split into 8 regions with different mixture of production capacity and with transmission constraints in between. In 5 of the 8 regions hydro reservoirs exist, which are operated independently, but the inflows to them all are controlled by a single stochastic parameter, which is the only stochastic parameter.

Three possible inflows are defined for each of the first 3 and the last 3 stages while the 6 remaining stages had 9 possible outcomes each. This gives a total of nearly 130 million possible scenarios. The size of the LP-subproblems to be solved at each stage of the scenarios are  $1089 \times 786$  not including the cuts that will be generated by the algorithms.

As stopping criterion the one presented in Section 2 was used. The model was formulated in GAMS. The solution methods were also written in GAMS with the subproblems being solved using CPLEX 6.5.2 on a 500 MHz Pentium computer.

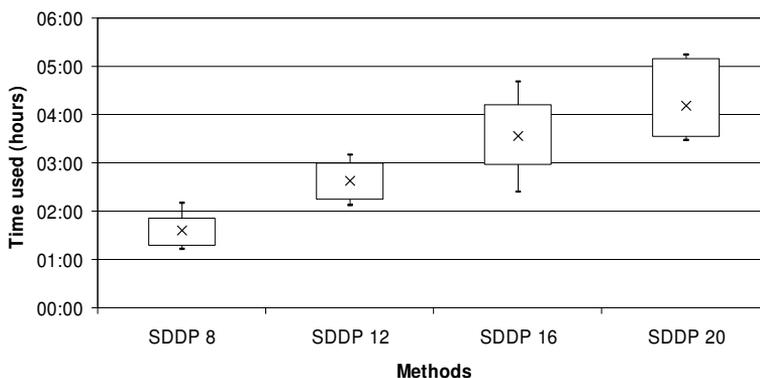


Figure 7 – Boxplot of computation times of SDDP for a different numbers of sampled scenarios

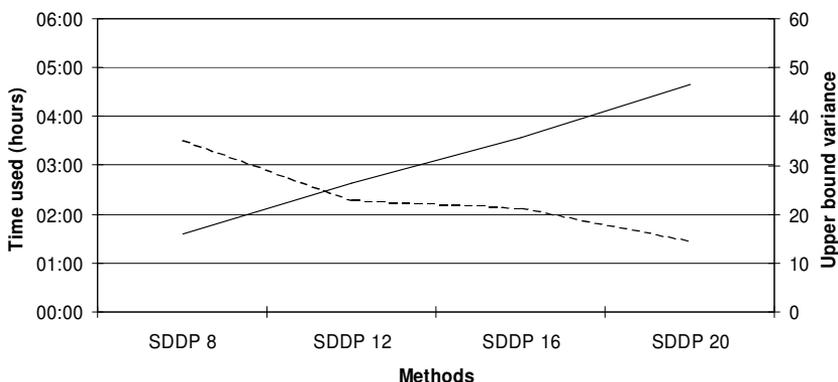


Figure 8 – Average SDDP computation time and upper bound variance estimate for different numbers of sampled scenarios

## 5.1 SDDP method

We first applied SDDP to the hydro-thermal model using different sampling sizes. Since the method is stochastic, the number of iterations needed to achieve convergence can vary, giving varying computation times. Therefore 10 computations have been made for each case. The boxplot in Figure 7 shows that SDDP gives very different computation times depending on the number of sampled scenarios. Values of 8, 12, 16, and 20 scenarios to be sampled have been tested. In a boxplot, the maximum and minimum values obtained are marked as the end points of the vertical lines. The upper and lower edges of a box show the 75% and 25% percentiles respectively, while the cross indicates the average value.

It can be seen that with fewer scenarios to be sampled, less computation time is needed to meet the convergence criterion. In particular the backward pass takes considerably longer with more scenarios sampled, and the initial gains can be achieved with relatively fewer sampled scenarios.

Figure 8 shows the average variance estimates of UB for the 4 test runs. The objective function value is around 2652, so it can be seen that the SDDP 8 runs especially have a large variance estimate (due to fewer samples). This variance estimate is part of the stopping criterion, where a larger variance estimate allows a larger gap between the lower bound and the upper bound estimate. Thus, in general, fewer iterations are needed before the stopping criterion is met. So SDDP 8 is faster than SDDP 20 not only because fewer subproblems are to be solved during the iterations, but also because it terminates more easily with a solution that allows for less confidence than that of SDDP 20. Hence, by using this stopping criterion, it is necessary to trade off the increase in computation time against the higher quality of the solution. It is expected that the variance estimate for larger sampling sizes will slowly converge on zero as the sample size tends to the number of possible scenarios.

## 5.2 AND method

A similar experiment has been carried out with the AND implementation. In Figure 9 results for this implementation with four different settings are shown. For all runs, the number of sampled scenarios in the sampling phase is 12. The number of branching solutions is either 1 (denoted LB) or 2 (denoted HB) with 2 subproblems being solved for each branching solution. These values (i.e. the number of branching solutions and the number of subproblems to be solved per branching solution) are either increased after each iteration (FI) or every second iteration (SI).

It can be seen that AND LB-SI gives the fastest convergence on average. Starting with few branching solutions and increasing the number slowly over the course of the algorithm, performs much better than a more rapid increase.

### 5.3 ReSa method

Here four different start values for  $B_t$  have been tested for a fixed forward sample size,  $S$ , of 12. In the first three cases  $B_t$  was set to 2, 4, and 6 for all  $t$ . The last trial ReSa V had values from 3 to 5 for different values of  $t$  so that  $B_t$  was 5 in the months with 9 outcomes, and 3 for the other months. The results of the four test runs, denoted ReSa 2, ReSa 4, ReSa 6, and ReSa V respectively, are shown in Figure 10.

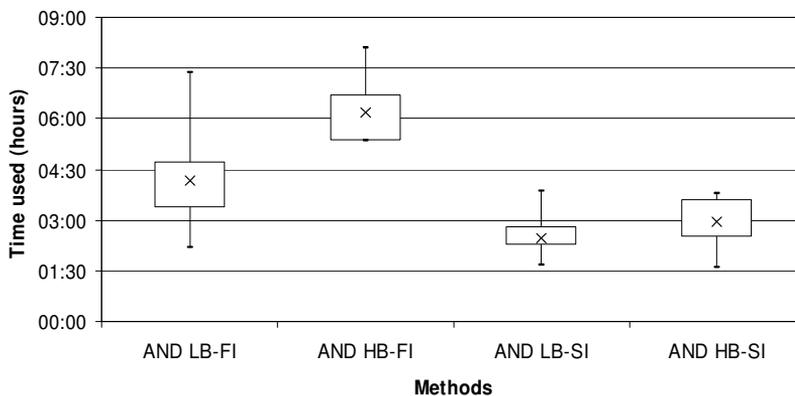


Figure 9 – Boxplot of computation times of AND with different parameter settings

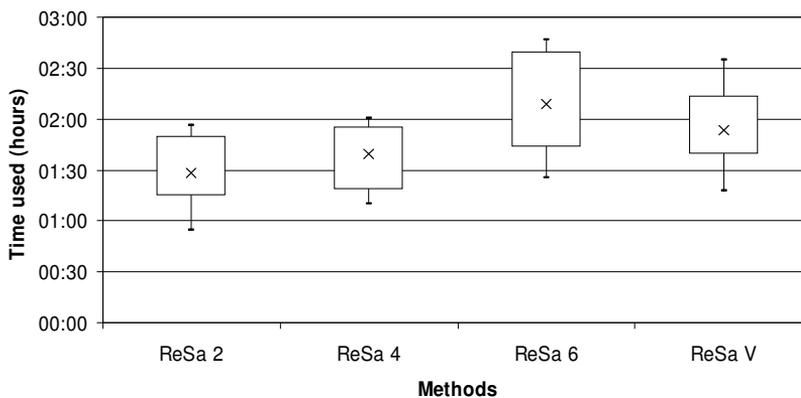


Figure 10 – Boxplot of computation times of ReSa with different initial values of  $B_t$

It can be seen that starting with the lowest initial  $B_t$  values gives the best performance. Here,  $B_t$  values are increased by one either if the variance estimate of  $\bar{z}$  increases compared with the previous iteration, or if the  $\bar{z}$  estimate itself is higher than the previous one. If both conditions apply then  $B_t$  is increased by two for all  $t$ . Other strategies that increase  $B_t$  faster or slower may be better but this has not been tested.

Figures 11 and 12 show the results for runs with different sample sizes,  $S = 12, 25, 37$  and 50. It can be seen that the computation time increases with increasing number of samples. As for SDDP the variance estimates drops, so this forces the algorithm to run longer. Hence, the growth in computation time is not linear.

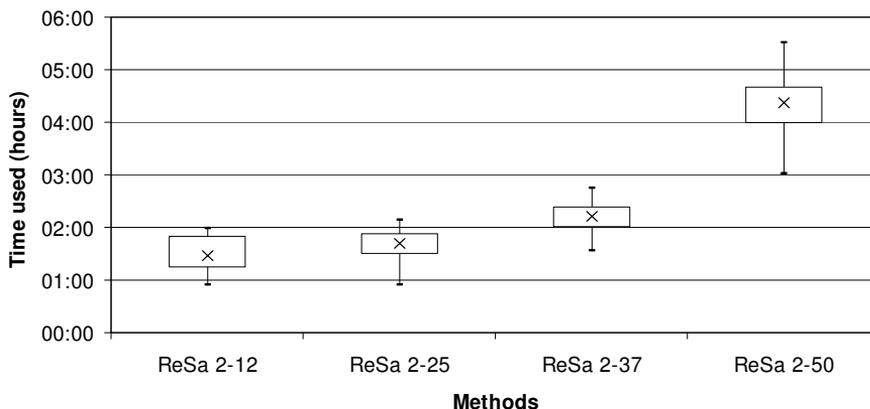


Figure 11 – Boxplot of computation times of ReSa with different sample sizes,  $S$

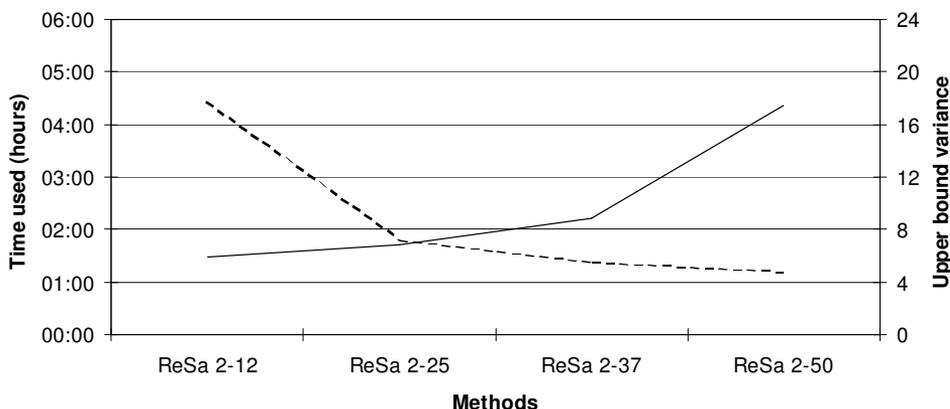


Figure 12 – Average computation times (left axis) and upper bound variance estimates (right axis) for ReSa with different sample sizes,  $S$

## 5.4 Comparison

Figure 13 shows the results of comparing each algorithm when it has the best parameter settings, namely SDDP 12, AND LB-SI and ReSa 2. Boxplots are used to indicate the computation times achieved in the 10 test runs of each algorithm. The better algorithm for this model looks like ReSa, though the average variance of the upper bound estimate (based on 12 samples in each case and shown as the triangles and measured by the right y-axis) is slightly higher than the one of AND.

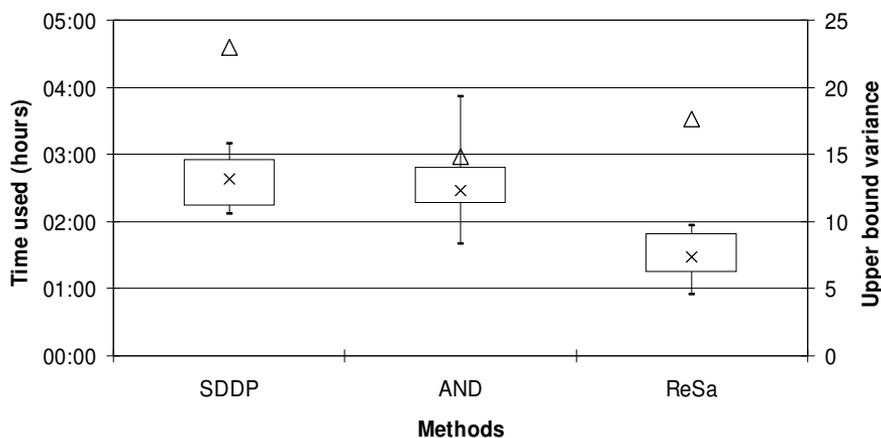


Figure 13 – Boxplot of computation times (left axis) of SDDP, AND, and ReSa each with 12 samples for upper bound estimation. The average upper bound variance is indicated with triangles (right axis).

Compared with SDDP 12, the AND and ReSa algorithms should initially have faster iterations since fewer cuts are calculated to approximate the future cost function. For the SDDP 12, AND LB-SI, and ReSa 2 runs this has been sketched in Figure 14 (right y-axis, in minutes). The computation time for each iteration grows quickly for AND. For ReSa the growth is less as the  $B_i$  values are increased slowly. Also, it can be observed that the computation time for SDDP increases too, though very little. This is due to the extra cuts being added to each subproblem, making it a little harder to solve each time.

In Figure 14 the average gaps between the upper bound estimate and the lower bound at each iteration have been plotted (left y-axis). While SDDP and ReSa have approximately the same gaps, the gap for AND is considerably lower. By using the ReSa method to solve the model, a large drop in computation time is gained for a slightly inferior representation of the future cost function. The AND method has a

smaller gap, but the fast growth in computation time makes this slightly inferior to ReSa as seen in Figure 13. This may indicate that an even slower increase of the number of branching solutions for AND may make it perform better overall.

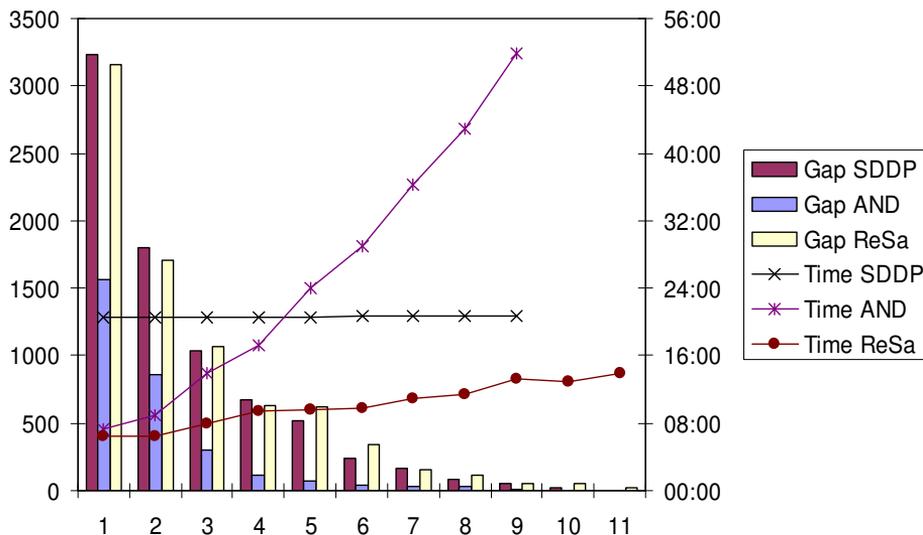


Figure 14 – Gap between UB estimate and LB (bars, left scale) for SDDP, AND, and ReSa compared with computation time in minutes for each iteration (lines, right scale)

## 6 Conclusions

This paper has presented a new algorithm for solving multistage stochastic linear programs. The performance of the algorithm has been compared with those of similar existing algorithms: the SDDP and the AND algorithms. The test case has been a hydro-thermal model of the Nordic power system.

From the results, it can be seen that ReSa outperforms SDDP and AND for solving this specific model. Compared with SDDP the ReSa method gained much in speed with little or no loss in the accuracy of the future cost function approximation. Compared with AND, ReSa also performed better, but other parameter settings for AND, basically slowing down the expansion of the scenario tree to be analysed, may improve the performance of AND to a level equal to or better than ReSa.

Also, using only 12 samples for estimating the upper bound is too low a value to confidently invoke the Central Limit Theorem, and thus the computation times vary

considerably. Future trials with more samples should be made to see if the results are consistent with the ones presented here. As shown 50 samples reduces the variance with 75%.

Another main conclusion is that choosing the right parameters, e.g. the number of scenarios for SDDP, branching solutions for AND, etc., is very important as the choice of these parameters highly affects the performance of the algorithms. The ReSa 2 was on average 31% faster than ReSa 6 while AND LB-SI on average runs 61 percent faster than AND HB-SI. Having implemented an algorithm, time should be used for finding the values giving the best performance of this. This is especially important when comparing methods. Also due to the stochastic nature of the solution algorithms, multiple runs should be made to see the variance of the computation time.

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**PAPER H**

**STOPPING CRITERIA IN  
SAMPLING STRATEGIES FOR  
MULTISTAGE SLP-PROBLEMS**

This paper was presented at the conference “Applied mathematical programming and modelling”, Varenna, Italy, June 2002.

“Famous remarks are very seldom quoted correctly”

- *Simeon Strunsky*

# Stopping criteria in sampling strategies for multistage SLP-problems

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**Abstract:** The most efficient optimisation methods for solving large multistage stochastic linear programming problems are those such as SDDP, Abridged Nested Decomposition, and ReSa, that combine sampling and Benders decomposition. The stopping criterion originally proposed for these methods can lead to premature termination when the sample size is small. In this paper, we compare the standard criterion with a more restrictive stopping rule that ensures that a candidate solution is close to optimal with a specified probability. We report the results of a series of computational experiments on a multi-stage stochastic linear programming model of a hydro-thermal electricity planning problem. The results confirm that the new stopping criterion terminates with better solutions, at some computational expense.

**Keywords:** Multistage stochastic linear programming; sampling; stopping criteria

## 1 Introduction

The two-stage stochastic Benders decomposition algorithm was developed by Van-Slyke and Wets (1969), who called it the L-shaped method. An equivalent decomposition was discovered earlier by Benders (1962) within the context of mixed integer programming. The two-stage stochastic Benders decomposition algorithm was first extended to multistage problems by Birge (1985), who called it nested Benders decomposition. Background knowledge of this is assumed in the following.

The termination criterion for stochastic Benders decomposition algorithms is provided by bounds on the optimal expected value. The optimal value calculated from the stage 1 master problem is a true lower bound on the optimal objective value, and since for any candidate feasible solution, all subproblems of the scenario tree are solved, one obtains an exact calculation of the expected value of this solution, giving an upper bound on the optimal solution value. If, after an iteration, the calculated upper and lower bounds are equal the algorithm terminates as this indicates that the optimal solution has been reached.

When the probability distributions of the random variables are finite, leading to a finite set of scenarios, multistage Benders decomposition algorithms can be shown to terminate in a finite number of steps. However for many practical problems, the dimension of the sample space makes an approach using all realizations of the scenarios intractable. In these circumstances it becomes necessary to resort to a sampling approach.

Sampling based multistage stochastic Benders decomposition algorithms, such as the SDDP method of Pereira and Pinto (1991), Abridged Nested Decomposition by Donohue and Birge (2001), and ReSa by Hindsberger & Philpott (2001), have shown to be very efficient in solving larger multistage stochastic linear programming problems.

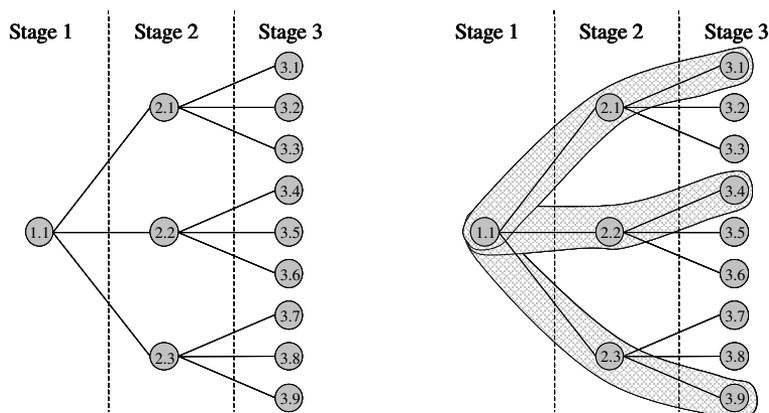


Figure 1 – Example of scenario sampling for a 3-stage problem

The sampling strategies work under the assumptions of relatively complete recourse, i.e. the subproblems at any stage  $t$  have feasible solutions regardless of any decisions

taken at earlier stages, and serial independence between stages, i.e. the probabilities of the stochastic outcomes at stage  $t$  are unaffected by the realisation of the stochastic outcomes at stage  $t-1$ .

These assumptions makes it possible to solve only a subset of the subproblems in the scenario tree in each iteration in order to improve the approximation of the future cost function. It turns out that large improvements to this approximation can be made with relative few samples, especially during the first iterations. Figure 1 shows a 3-stage example where to the right three samples are chosen out of the nine possible scenarios.

Sampling approaches will typically stop as a near-optimal solution has been found. How near to optimality is determined by the stopping criterion. In this paper we experiment with several stopping criteria for the ReSa algorithm applied to a multi-stage energy-planning model of the Nordic hydro-thermal system as a computational case.

The next section will discuss stopping criteria for sampling strategies followed in Section 3 by an introduction to the ReSa sampling based method. Section 4 presents some computational results, while in Section 5, some final remarks will be given.

## 2 Stopping criteria

As mentioned, sampling-based algorithms for multistage stochastic Benders decomposition only solve a subset of the subproblems in the scenario tree. Hence, an exact calculation of the expected value of any candidate solution (an upper bound) is no longer available. However, a simulation of any candidate solution will give independent identically distributed objective function values  $z_i$ ,  $i = 1, 2, \dots, N$ , with mean, say  $\mu$  and variance, say  $\sigma^2$ , so the estimate

$$\bar{z} = \frac{1}{N} \sum_{i \in N} z_i \quad (2.1)$$

is asymptotically normally distributed with mean  $\mu$  and variance  $\sigma_z^2 = \sigma^2/N$ . Since  $\mu$  is the expected value of a candidate solution it is an upper bound on the expected value of the optimal solution. A 95% confidence interval for  $\mu$  is given by:

$$\bar{z} - 2\sigma_z \leq \mu \leq \bar{z} + 2\sigma_z \quad (2.2)$$

In multistage stochastic Benders decomposition algorithms one can obtain a lower bound on the optimal value as long as each cut is guaranteed to be a lower bound on the expected future cost at each stage. This can be ensured by requiring that for each cut added at a node in the scenario tree, the algorithm must solve the subproblem at the next stage for every realisation of the random variables at that node. (These subproblems themselves may have an expected future cost bounded below by cuts, so their solution yields a lower bound of the expected future cost.) The value  $\underline{z}$  of the stage 1 master problem then yields a lower bound on the optimal value of the multistage stochastic program.

A multistage stochastic Benders decomposition algorithm without sampling terminates when  $\mu$  and  $\underline{z}$  are equal, or sufficiently close, say within  $\delta$  of each other. In the sampled case, the stopping criterion suggested by Pereira and Pinto (1991) and Donohue and Birge (2001) terminates the algorithm when

$$\bar{z} - 2\sigma_z \leq \underline{z} \leq \bar{z} + 2\sigma_z \quad (2.3)$$

Usually the sample size  $N$  is chosen before the algorithm is run, and  $\sigma^2$  is estimated using a sample variance. Observe that if  $\sigma_z^2$  is large (perhaps because  $N$  is chosen to be too small) then the algorithm might terminate very quickly with a large error. This is indicated on Figure 2.

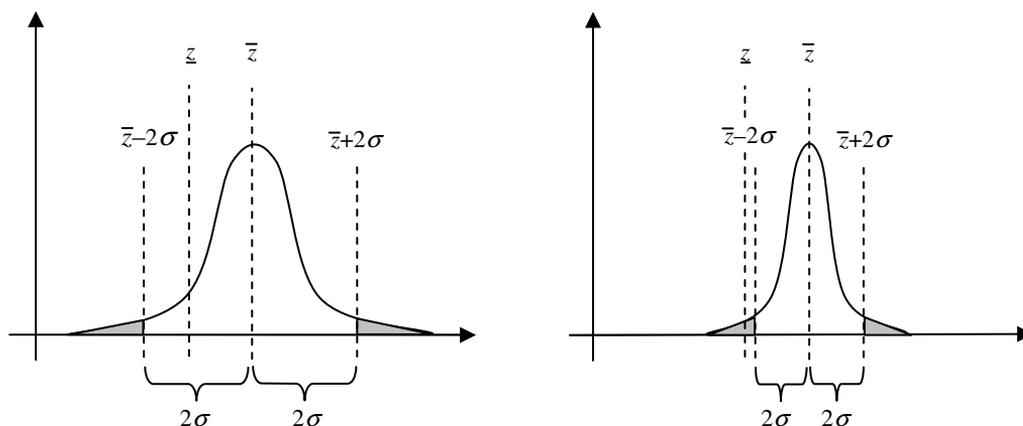


Figure 2 – Larger variance lead to earlier termination

The theory of stopping criteria in sampling-based algorithms has been explored in some depth by Morton (1998). He shows that a fixed choice of  $N$  for sampling based

algorithms such those referred to in section 1 can result in termination of the algorithm at a candidate solution outside the confidence interval defined by (2.2) with probability approaching 1 as the interval size decreases. He describes some procedures for increasing  $N$  as the algorithm proceeds to ensure termination within a given confidence interval with high probability.

To put this work into our context, suppose at the end of an iteration  $k$  that we have a feasible solution  $x_k$ , a true lower bound  $\underline{z}_k$  on the optimal value  $z(x_k^*)$ , and an estimate  $\bar{z}_k$  of  $z(x_k)$ . The stopping rule investigated in Morton's paper is to terminate the algorithm at the first iteration where (in our notation)  $\bar{z}_k - \underline{z}_k \leq 0$ .

First observe that this is a harder condition to satisfy than (2.3). Indeed Morton shows for a fixed choice of  $N$  how to construct sequences of  $\underline{z}_k$  and  $z(x_k)$  with  $z(x_k) - \underline{z}_k$  converging to zero for  $k \rightarrow \infty$ , but with a non-zero probability of failing to terminate. However, under the assumption that  $z(x_k) - \underline{z}_k$  converges to zero, Morton also shows that stopping the algorithm when  $\bar{z}_k - \underline{z}_k \leq \epsilon$  where  $\epsilon > 0$  can be shown to give finite termination with probability 1 if  $N$  is chosen appropriately. Morton's paper does not consider the much stronger stopping criterion that  $\bar{z}_k - \underline{z}_k \leq -\epsilon$  where  $\epsilon > 0$ . Using his arguments it is easy to show, under the assumption that  $z(x_k) - \underline{z}_k$  converges to zero, that asymptotic convergence is achieved for smaller corresponding choices of (increasing)  $N$ , but there is no proof of finite termination in this case.

In this paper we carry out a computational study of the case (leaving out the index  $k$ ):

$$\bar{z} - \underline{z} \leq \delta - 2\sigma_z \text{ for } \delta > 0 \tag{2.4}$$

Using this, we can get a guarantee at termination that the candidate solution has an expected value  $\mu$  that is within  $\delta$  of the optimal value. For example, since

$$\Pr(\mu \leq \bar{z} + 2\sigma_z) = 0.975, \tag{2.5}$$

if we terminate the iteration  $k$  where

$$\bar{z} + 2\sigma_z < \underline{z} + \delta \iff \bar{z} < \underline{z} + \delta - 2\sigma_z \tag{2.6}$$

then we are guaranteed that

$$\Pr(\mu \leq \underline{z} + \delta) = 0.975 \tag{2.7}$$

This criterion is sketched in Figure 3. For small values of  $\delta$  this might not be a very good stopping criterion, since as shown on the left graph of the figure  $z$  must be very close to  $\bar{z} + 2\sigma_z$  to terminate the algorithm. For the case  $\delta=0$  the criterion will reject 97.5 % of the iterations where the desired quality is actually obtained, which will lead to substantial computation times. Furthermore, since it is improbable that the lower bound will ever be exact in realistic applications of these methods, termination of the algorithm is very unlikely if  $\delta$  is chosen to be 0.

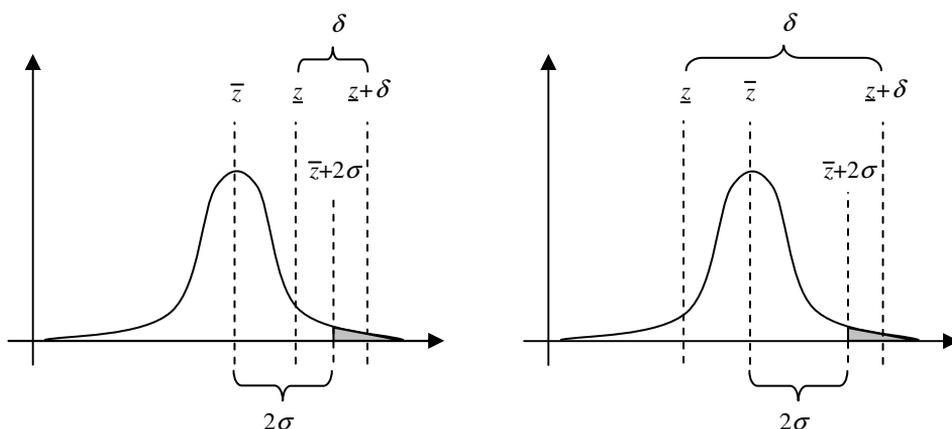


Figure 3 – Stopping criterion based on a normal distribution of  $\bar{z}$

The right graph shows a case with a larger  $\delta$ . Although  $z$  is now to the left of  $\bar{z}$ , the new stopping criterion still gives a guarantee of having  $\bar{z}$  within  $\delta$  of the optimal value with probability 0.975. It is important to note that if the sample size is large and held constant throughout the algorithm, then the original stopping criterion will be sufficient. However if the sample size is altered adaptively during the course of the algorithm, the original criterion can lead to early termination at a solution far from optimum, whereas the new criterion responds appropriately to alterations in sample size.

## 2.1 Non-statistical stopping criteria

Several other simple, but often efficient, stopping criteria can be defined using the information about the bounds obtained at each iteration. Some suggestions are:

- Stop if  $\underline{z}$  does not change for  $k$  iterations
- Stop if  $\underline{z}$  has improved less than  $q$  percent of current  $\underline{z}$  in the last  $k$  iterations
- Stop if the average of  $\bar{z}$  for the last  $k$  iterations is within  $q$  percent of the current  $\underline{z}$

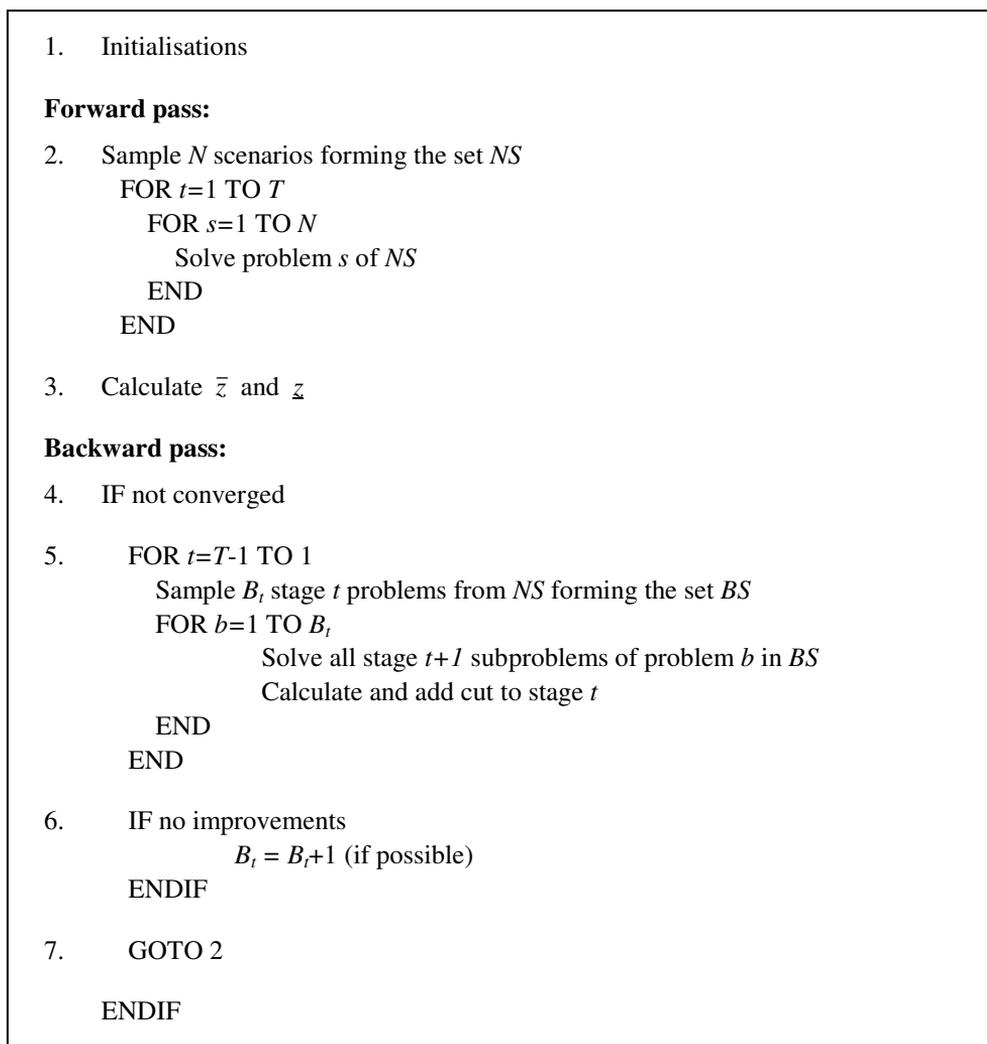
For larger problems it is unlikely that ReSa will stop early using the first of these criteria even for the case  $k = 2$  as small improvements of the  $\underline{z}$  estimate most likely will be made during each most iterations. The second criterion describes the rate of improvement of  $\underline{z}$ . If little improvement has been obtained for a number of iterations, it indicates that you may be close to the optimal solution. Finally the third of the criteria looks at the gap between  $\underline{z}$  and  $\bar{z}$ . If the average gap over some time is little, this may also mean that the current solution is close to optimal. These criteria will be used for comparison during the analysis later in this paper.

### 3 The Reduced Sampling algorithm

Reduced Sampling (ReSa) is a sampling-based Benders decomposition algorithm like the SDDP algorithm presented by Pereira and Pinto (1991). A pseudo code of the ReSa algorithm is shown in Figure 4, while a more detailed description can be found in Hindsberger and Philpott (2001).

In ReSa, a set  $SS$  of  $S$  sampled scenarios is created during the forward pass, where calculation of the upper and lower bounds is done precisely as in SDDP. But unlike SDDP, only a randomly selected subset  $BS \subseteq NS$  of those stage  $t$  problems, which were solved during the forward pass, are used during the backward pass to create cuts for the stage  $t-1$  problems. The number of elements in  $BS$  (which upper bound is  $N$ ) is increased if improvements from iteration to iteration are insignificant. This is done until all subproblems solved during the forward pass are also solved during the backward pass.

The idea is to use more subproblems to generate cuts as convergence slows down. On the other hand large initial gains can be obtained with very few cuts so there is no reason to use too much computation power on the backward pass initially. For very large problems, one may choose to increase the number of samples in the forward pass  $N$  cf. the discussion in the previous section. Assuming a finite set of scenarios, the algorithm will then clearly terminate in finite time, as it will evolve into the non-sampling nested Benders decomposition algorithm, for which this has been shown.



*Figure 4 – The Reduced Sampling algorithm*

## 4 Experimental Results

The model used in this analysis is formulated as a multistage stochastic linear programming problem. It covers the power and combined heat and power (CHP) production in the countries Denmark, Finland, Norway, and Sweden. To represent bottlenecks in the transmission system the countries are divided into regions with

transmission constraints between those. A total of 8 regions are represented in the model of which 5 have hydro reservoirs.

Two variants of the model have been tested; one of 12 stages with each stage corresponding to months and the other with 6 stages for each two month period. Within each stage the time has been divided into three segments to represent the variations of the power demand over the day, viz. peak, day, and nightly demand.

The total demand in the countries included is almost 400 TWh. About 200 TWh of the production will come from hydropower in years with normal inflow. This may vary with up to 40 TWh in wet or dry years.

To reduce the size of the model all reservoirs in each region are aggregated into a single reservoir for that region. The only stochastic parameters in the model are the inflow to the hydro reservoirs in each region. The 6-stage model has 3 stochastic outcomes (wet, normal, and dry) defined for each stage for a total of  $3^5=243$  possible scenarios. Depending on the stage either 3 or 9 stochastic outcomes are defined for the 12-stage model resulting in about 130 million possible scenarios.

Apart from hydropower, 10 thermal production technologies are included using different fuels and being either pure power producing or CHP plants, where the latter type also must produce district heat to meet the requirements of this in each of the included regions.

The model seeks to minimise the overall costs by finding for each stage and load level the optimal production levels of power and heat of each production technology as well as the transmission of power between regions. Also the optimal hydro reservoir levels in each region at the end of each stage are found.

The following analysis tests several stopping criteria on the 6-stage and 12-stage model. Both the 6-stage model and the 12-stage model were run 10 times stopping after 25 iterations in order to keep the computation time within reasonable limits. It was then analysed at which iteration during the 25 iterations each of the stopping criteria would have terminated the algorithm. The stopping criteria used in the analyses are shown in Table 1.

To solve the models the ReSa algorithm was used with  $N=50$  and  $B_i=2$  (see pseudo code in Figure 4 for explanation).

Table 1 – Stopping criteria used in the analysis

Criterion	Short name
Original proposed by Pereira & Pinto	P&P
Upper bound within 0.0% of current lower bound with probability 0.975	H&P 0.0
Upper bound within 0.1% of current lower bound with probability 0.975	H&P 0.1
Upper bound within 0.5% of current lower bound with probability 0.975	H&P 0.5
Upper bound within 1.0% of current lower bound with probability 0.975	H&P 1.0
Lower bound unchanged for 2 iterations	LB UC2
Lower bound unchanged for 3 iterations	LB UC3
Lower bound unchanged for 4 iterations	LB UC4
Lower bound unchanged for 5 iterations	LB UC5
Lower bound improved less than 1.0% in 3 iterations	LB3<1.0
Lower bound improved less than 1.0% in 5 iterations	LB5<1.0
Lower bound improved less than 0.1% in 3 iterations	LB3<0.1
Lower bound improved less than 0.1% in 5 iterations	LB5<0.1
Average upper bound for 3 iterations is within 1.0% of current lower bound	UB3<1.0
Average upper bound for 5 iterations is within 1.0% of current lower bound	UB5<1.0
Average upper bound for 3 iterations is within 0.1% of current lower bound	UB3<0.1
Average upper bound for 5 iterations is within 0.1% of current lower bound	UB5<0.1

For the 6-stage model Figure 5 shows at which iteration each of the stopping criteria would terminate the algorithm for each of 10 runs. The triangles on the graph indicate the average number of iterations before termination for the criteria.

It can be seen that the P&P criterion, see equation (2.3), stops on average after 3.2 iterations. The other statistical criterion, H&P, see equation (2.6), on average terminates after 24 iterations for  $\delta = 0\%$ . In two of the runs the algorithm terminated before the optimal solution was found with this criterion. This corresponds to the type 2 error of accepting a suboptimal solution for which  $\bar{z} < \underline{z} - 2\sigma_z$ , an outcome that should occur with probability 0.025. Increasing  $\delta$  to 1% makes the algorithm terminate after about 7 iterations.

The LB UC2 criterion stopped after 3 iterations in each of the 10 runs. Requiring the lower bound to be unchanged for 3 or more iterations increases the average number of iterations before termination to more than 15 (with a similar effect on the computation time). The remaining criteria stopped on average after about 6 iterations.

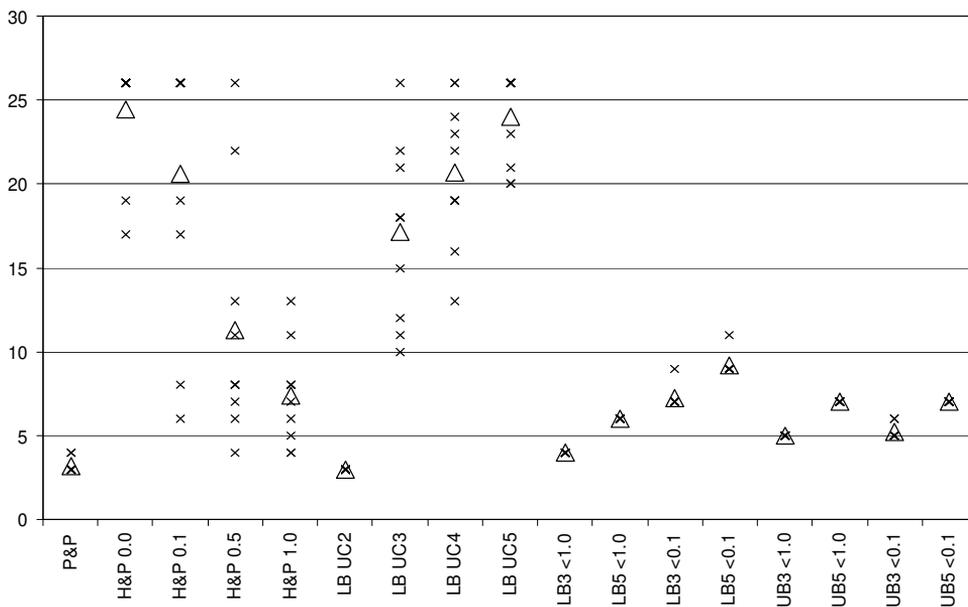


Figure 5 – Iteration at which the algorithm terminated for the 6-stage model. Each cross may represent several cases. The triangles represent the average of the 10 runs

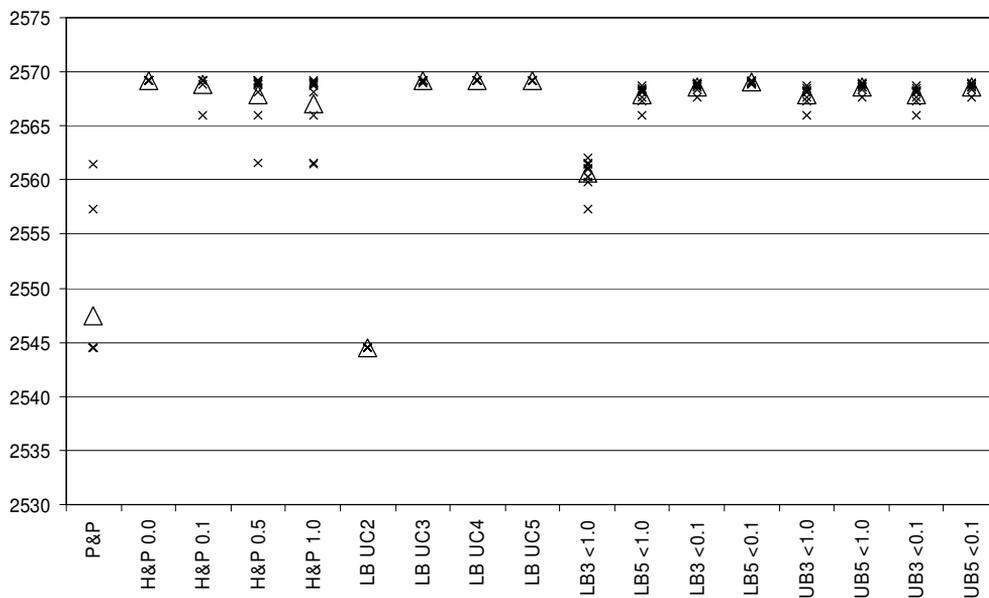


Figure 6 – Lower bound at the time of termination for the 6-stage model. The triangles represent the average of the 10 runs

Figure 6 is a similar figure showing the lower bound value  $z$  at the time of termination. The best value found during the 10 runs was 2569.1986. The true solution (obtained by applying nested Benders decomposition without sampling) gives the optimum solution of 2569.1987. Using the P&P criterion the algorithm terminated after about 3 minutes on average while the nested Benders decomposition algorithm required one hour and 15 minutes to solve the problem to optimality.

If the average lower bounds of Figure 6 are compared with the optimal solution, this value indicates how close in percent the different criteria are to the optimum. On Figure 7 this value is plotted comparing it with the average computation time used before termination. Thus, the trade-offs between computation time and quality can be seen.

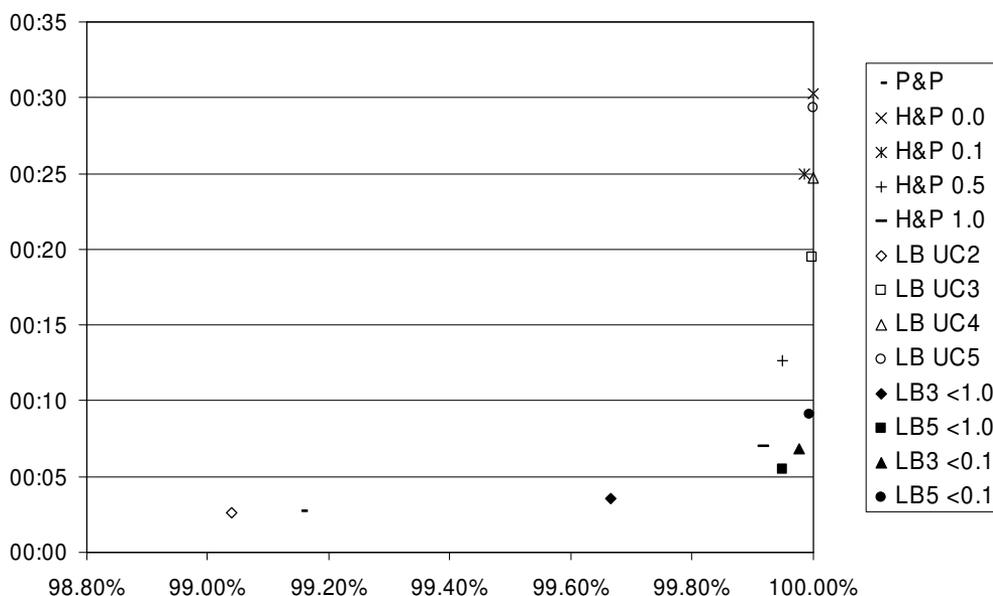


Figure 7 – Quality of solution vs. computation time (average of 10 runs – 6 stages)

The figure shows that in the model the P&P criterion terminates at least 0.8% from the optimum value on average. The H&P criteria ensures a much higher quality solution but takes up to 10 times longer (which would be even more if the algorithm was allowed to exceed 25 iterations). Apart from LB UC2, the rest of the LB UCx criteria perform very well having high quality solutions found faster than the H&P criteria, though the former do not provide guarantees on solution quality.

The other LB based criteria give an even better trade-off between time used and quality. The UB based criteria are not shown on the graph as their performance were very similar to the last four LB based criteria.

Figure 8 displays the average lower bound after each iteration of the 10 runs. It explains the shape of the graph in Figure 7. After 3 iterations the lower bound is still some way below the optimum, but after 6 iterations little further improvement takes place.

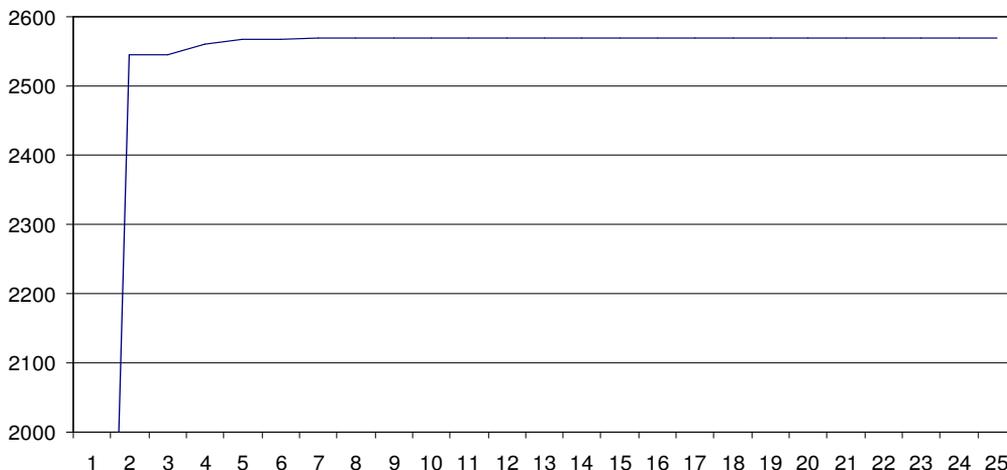


Figure 8 – Average lower bound after each iteration

A similar 10 run analysis has been made using the 12-stage model. Figure 9 shows at which iteration the stopping criteria would terminate the algorithm when solving this model. It can be seen that none of the LB UC2 through UC5 criteria ever terminated the algorithm. This is a consequence of having a much larger model where small improvements can be made iteration after iteration.

On Figure 10 a trade-off graph similar to the one in Figure 7 can be seen for the 12-stage model. The solution quality is here compared with best value found after a 100 iterations run as the true optimal cannot be found. The highest value of the lower bound found during the 10 runs of 25 iterations was 14690.2564 while the lower bound after the 100 iterations run was 14691.3224. So it seems reasonable to assume that little additional improvement is possible above this and thus this value has been used for estimating the quality of the solutions.

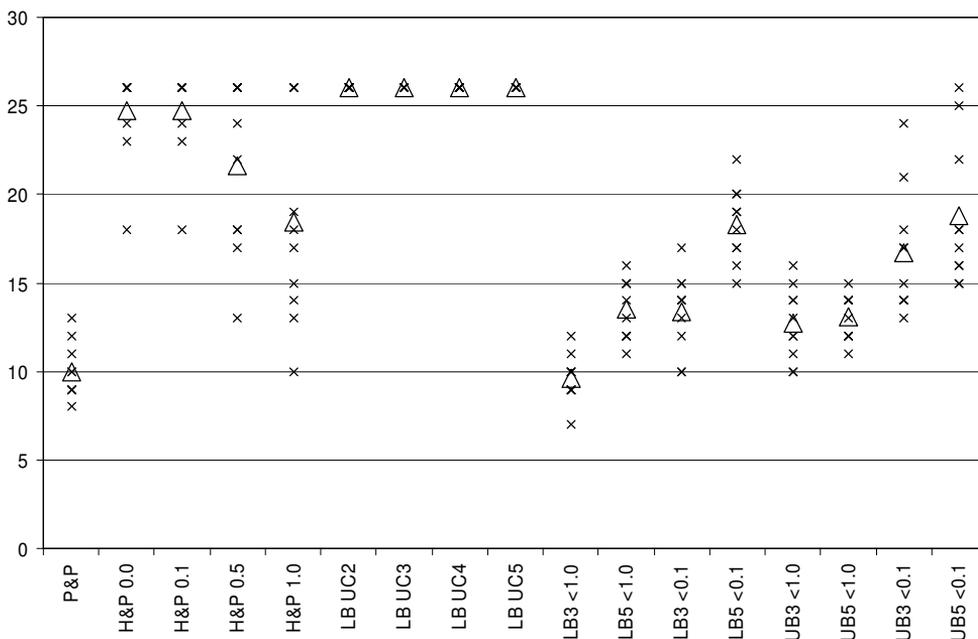


Figure 9 – Iteration at which the algorithm terminated for the 12-stage model. Each cross may represent several cases. The triangles represent the average of the 10 runs

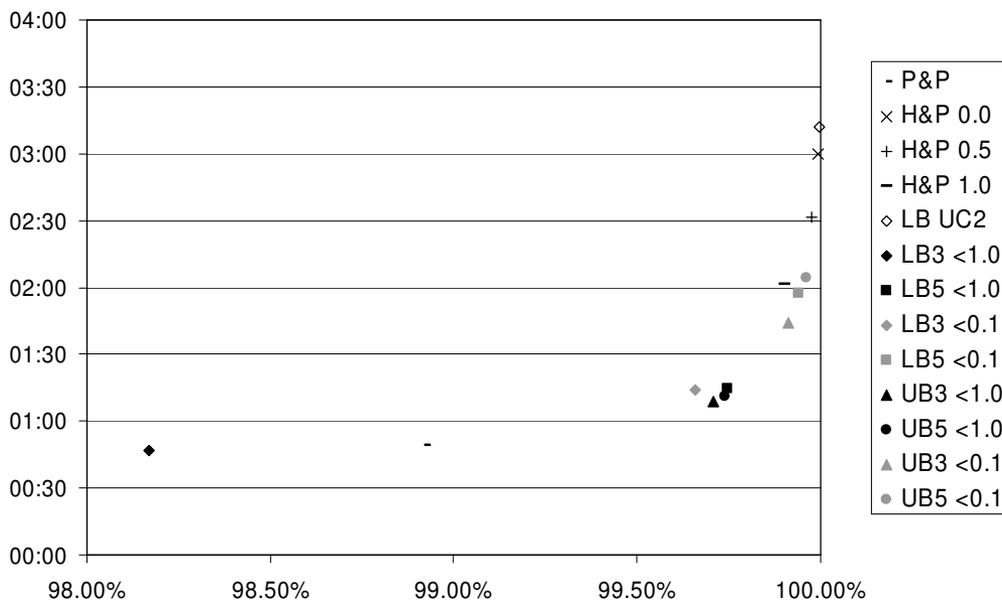


Figure 10 – Quality of solution vs. computation time (average of 10 runs – 12 stage)

From Figure 10 it can be seen that the P&P criterion on average terminates when the objective function value is about 1% from the optimum. The H&P criteria return higher quality solutions though the model runs take considerably longer. The non-statistical criteria perform well for this model terminating most of the runs just as the “exponential” growth in computation time starts to be felt. So some of the non-statistical criteria look superior in terms of the trade-off between quality and time to compute, but they lack the quality assessment as the H&P criteria do provide.

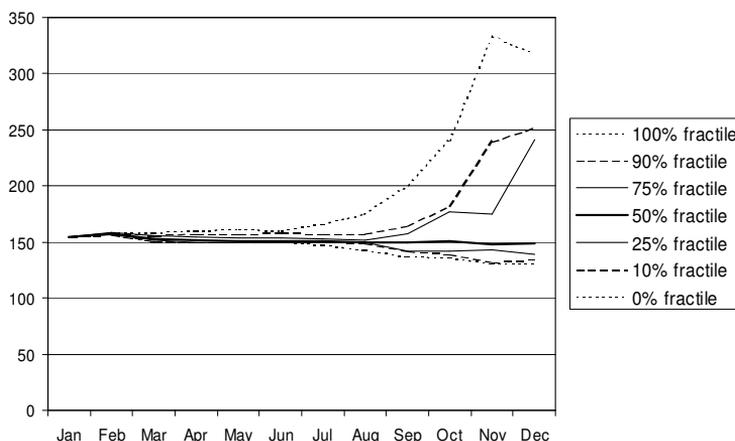


Figure 11 – Price estimates for eastern Denmark (DKK/MWh) of 100 simulations when using the expected future cost approximation obtained after 100 iterations

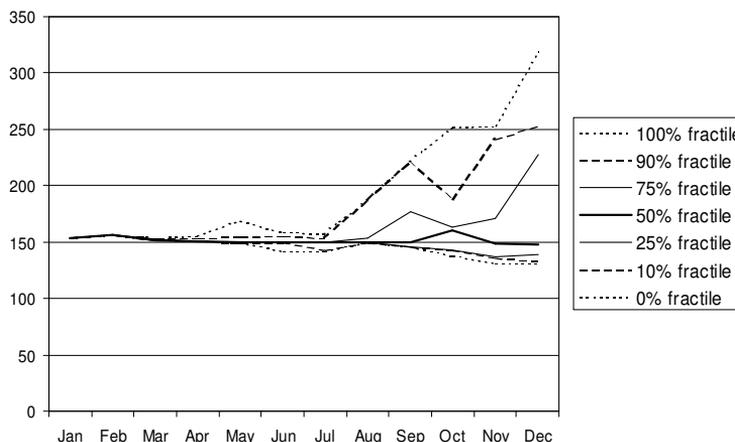


Figure 12 – Price estimates for eastern Denmark (DKK/MWh) of 100 simulations when using the expected future cost approximation obtained using the P&P criterion

Looking at the lower bound estimates (in this model corresponding to the total costs of power production in the system), we have seen that all criteria in reality could achieve results within 1% of the optimum. In many cases this quality would be sufficient, and hence, any of the criteria could be used and those, which terminates fastest might be preferred.

However, as seen in Figures 11 and 12, other types of results may depend more on the stopping criterion chosen. In Figure 11 the price fractiles obtained for using the 12-stage model for price estimation in eastern Denmark has been shown. Here 100 simulations of inflow have been made using the model with the future cost approximation as it was after the model had been forced to run for 100 iterations. Similarly, Figure 12 shows the similar case, but here the approximation of the future cost function is of less quality, as the model has terminated using the P&P criterion. While small differences in the results appear for the first months, later on, and especially in October, larger variations (about 10%) exist. If the model was to be used for managing power contracts months ahead, using the P&P criterion would likely return results of unacceptable quality.

## 5 Conclusions

This paper has compared some different stopping criteria to be used in sampling-based multistage Benders decomposition algorithms such as ReSa. Since the performance of such algorithms is dependent on varying the sample size, it is important to ensure that the termination criterion chosen can adapt to this. Such a criterion, H&P, is presented in the paper. With this satisfied stronger bounds on the degree of suboptimality of a candidate solution are obtained, but might give rise to longer (and possibly infinite) computation times. This observation is borne out in the reported numerical results.

Generally, the curves in Figures 7 and 10 show that the quality of the candidate solution obtained depends on the computation time and thus the choice of stopping criterion. From the figures, it can also be seen that results within about 99.9% of optimality for the 6-stage model and 99.5% of optimality for the 12-stage model can be obtained very fast, while the computation time needed for further improvements beyond this level becomes prohibitive.

The percentages given above for the two cases are not exact by any means and may depend on the users perception. However, if no quality assessment is wanted, one

might prefer to use a stopping criterion that would terminate the algorithm on average around those points as this indicates a good trade-off between quality and time. Otherwise, if quality assessment is sought, a criterion like H&P should be used.

Another observation in relation to the choice of stopping criterion is that the right choice depends on the actual problem. Hence, a criterion superior for one problem may be inferior to others for other problems using the users perception of the “right” trade-off. An example is the performance of the LB UC2 criterion. It terminated the 6-stage model very early and never terminated the 12-stage model within the 25 iterations. Trials with different criteria and parameter settings are necessary should be made to find the best-suited criterion for the problem in focus.

Further research in this field could, when looking at the practical side, be directed into looking at variance reduction techniques of different kinds, as the use of those may speed up the algorithms. However, it is not clear how the use of such techniques will affect the theoretical foundation of the different criteria, for instance whether the upper bound estimate still is asymptotically normally distributed with mean  $\mu$  and variance  $\sigma_z^2 = \sigma^2/N$ .

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