Mathematical Modelling of Mandibular Metamorphosis (4M) Status Report

Abstract. This document gives a summary of the results obtained in the 4M project during the period January 2002 to March 2003 as required by the founding party (STVF) of the project. The majority of the milestones set out in the project description are completed with little deviation from the original time schedule and results are either published, accepted for, or in preparation for international journals and conferences. The remaining tasks are actively been processed and analysed with promising preliminary results. The project thus proceeds satisfactory in good agreement with its main objectives and innovative value.

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1) Status and summary on the results obtained

Five main tasks are specified in the project description. In the following the obtained results are summarized with references to published work.

Task 1: Evaluation of existing algorithms for generating landmarks correspondences based on clinically identified landmarks

The previous work of [Andresen et al. 2000[1]] on Geometry Constrained Diffusion (GCD) based registration of 3D mandibular surfaces is validated within acceptable error limits and reported in [Hilger et al.2003a]. The results are based on data fusion of the clinically identified landmarks and the segmented, smoothed mandibles from CT scans. The fusion of the data involves a recovery of a rigid-body transformation using the Iterative Closes Point (ICP) transformation and a free-deformation of the landmarks when mapping the landmarks for one coordinate frame of reference to another. [Hilger et al.2003a] moreover contains a robustness evaluation and density distribution analysis of the applied landmarks.

The clinically identified features are furthermore applied in [Krarup et al.2002a, Krarup et al.2002b] extending the paradigm of stable natural reference structures from 2D to 3D. In [Paulsen and Hilger 2003, Hilger et al.2003b] GCD is extended into a Bayesian framework of Markov Random Field relaxation. This allows for better integration and fusion of both prior knowledge and observational term.

Task 2: Robust alignment in 3D, identification of outlier areas of the mandible to be excluded from the growth analysis.

A method for resistant and robust alignment of sets of shapes wrt. position, rotation, and isotropic scaling based on minimization of absolute distances is presented in [Larsen and Eiriksson 2002]. This is done by casting the Generalized Procrustes Alignment (GPA) problem into a linear programming problem. In [Hilger et al.2002a] a connection between GPA and multiset analysis is reported. In the multiset formulation alignment may be performed under nonlinear metrics by applying an iterative weight allocation scheme. The scheme may be applied to detect outlier landmarks and regions of the mandibular surfaces. In [Wrobel et al.2002] the

[1] Andresen, P.R. et al., *Surface-bounded growth modelling applied to human mandibles*, IEEE Transactions on Medical Imaging, 19(11):1053-1063, 2000

symmetry of the mandibles is evaluated using an ICP based analysis. The results allow for an extraction of the governing metric that favours stable structures of the surface landmarks.

Task 3: Methods for 3D shape factorisation and decomposition

In [Larsen 2002, Larsen et al.2002, Larsen and Hilger 2003a] decompositioning in non-Euclidean metrics is extended to 3D. 3D Q-MAF is extended to 3D Q-MNF decomposition that allows for integration and usage of the independent metrics e.g. derived by robustness analysis of the surface landmarks. Analysis applying the Q-MAF/MNF decomposition in a growth modelling study is presented in [Hilger et al.2003c]. In [Larsen and Hilger 2003a] and in [Larsen and Hilger 2003b] decompositioning and generative modelling is analysed under the agenda of probabilistic model selection and regularization.

Task 4: Implementation and investigation of multiset and multiway methods for analysis on intra-patients variations and metamorphosis

A tool for non-linear multiset decompositioning is developed, see the theory in [Hilger 2001]. Results are presented in [Hilger et al.2002a]. New extensions are obtained and include the possibility of specifying a neighbourhood structure in the organization of the multisets previously not possible. The new extension is expected to aid in the temporal analysis of intra-patient and inter-patient variations.

The application of multiway analysis is yet to be applied but initial studies of the theory and methodology on multiway variance decompositioning is completed. Moreover, related multiway regression-based techniques have been coupled to the regression in Task 5. The above multiset and multiway methods are complex flexible systems. Due to the sparseness of the available data set, heavy regularization is necessary to handle the curse of dimensionality. Therefore, a compromise between the degrees of freedom in the applied tools must be made in favour on robustness in probing for generic results.

Task 5: Implementation and investigation of the use of non-linear regression techniques based on generalization of the alternating conditional expectations algorithm to 3D shape modelling.

Non-linear regression is also challenged by the sparseness of the data and preliminary results shows no significant decrease in residual variance when applied to the growth studies. This is not surprising in light of the general consensus on linear growth in Procrustes tangent space derived using even non-Euclidean metrics for decompositioning. The method of Partial Least Squares (PLS) regression, related to multiset analysis, is thus adapted and applied in direct targeting the decompositioning wrt. shape and form coupling to mandibular growth and metamorphosis. In [Hilger et al. 2003a] results are reported consistent with previous finding of a single dominating linear subspace of shape variation correlated to shape centroid size and subject age. Using the PLS based method a higher correlation coefficient is obtained over the traditional decompositioning that maximized variance in the training data. In [Hilger et al. 2003a] the regression was based on the clinically identified landmarks. In preparation is an analysis and application of PLS regression on the mandibular surfaces represented by the densely sampled homologous semi-landmarks derived via GCD.

2) Summary on possible deviations from the original time schedule

Only minor changes and deviations have been made to the original time schedule. Validation of GCD registration turned out to be more demanding than expected due to challenges in fusion and integration of data. These challenges are now overcome. Furthermore, additional attained milestones where achieved during the project period and are presented in the following section.

3) Time schedule for the remaining part of the project

The time schedule for the 4M with revisions is presented below. Focus is directed at Task 4 and 5 in the remaining part of the project. This is in agreement with the original activity plan.

	2002				2003			
Objective	1	2	3	4	1	2	3	4
Task 1	IMM/LAB3D			IMM	IMM/LAB3D			
Task 2	IMM	IMM	IMM					
Task 3		IMM	IMM	IMM				
Task 4				IMM	IMM	IMM	IMM	
Task 5					IMM	IMM	IMM	IMM
M-I	IMM	IMM			IMM			IMM/LAB3D
M-II	IMM	IMM						
M-III				IMM	IMM			IMM/LAB3D

M-I, Additional attained milestone: Development of robust tools, independent of coordinate frame of reference, for density and inter-correlations analyses of clinically identified landmarks in both 2D and 3D, see [Larsen et al.2002, Larsen and Hilger 2003a, Hilger et al.2003a].
M-II, Additional attained milestone: Fusion of non-Euclidean metric in a noise robust texture model for Active Appearance Models, see [Hilger et al.2002b].
M-III, Additional attained milestone: Development and implementation of tools for coorespondence registration that allows for integration of prior knowledge and observational models terms by solving the aperture and 3D interpolation problem simultaneously, see [Paulsen and Hilger 2003, Hilger et al.2003b].

4) Associated Educational Parts
There are no associated educational parts in the form of PhD study related to the 4M project.

5) Collaboration with national and international partners
Collaboration with national partners has led to several invited talks and seminars and involves exposure of 4M in biomedical related societies. Future invited talks involve both national and international exposure.

Invited talks

- Larsen R., Swedish Society for Automated Image Analysis, SSAB'02, Symposium on Image Analysis, Lund, March 7-8, 2002. Title of talk (keynote address): *Deformable models with biomedical and industrial applications*
- Hilger, K. B. et al, *Linear and Nonlinear Multiset Canonical Correlation Analysis*, Invited talk, the Eleventh International Workshop on Matrices and Statistics, 2002
- Larsen R., *Statistical Shape Analysis*. Tutorial, the Copenhagen Image and Signal Processing Graduate School 02 * Bornholm PhD-Workshop, April 8-12, 2002
- Hilger K.B., Invited talk, Danish Society for Biomedical Engineering Meeting on Biomedical Image Analysis, November 27, *2002, Mathematical Modelling of Mandibular Metamorphosis from 3D CT*
- Larsen R., Invited talk '*Den Store Kemometridag',* Kolding, October 22, 2002. Title of Talk: *Multivariate analysis of images with applications to biomedicine, biometrics, and chemometrics* .

Future invited talks

- Hilger K.B., Invited talk and tutorial for The Copenhagen Image and Signal Processing Graduate School, annual Ph.D. Workshop, Ebeltoft, May 5-8, 2003
- Larsen R., *3D statistical shape analysis*. Invited talk and tutorial for The 13th Scandinavian Conference on Image Analysis (SCIA), Gothenburg, Sweden, June 29 – July 2, 2003

Miscellaneous

- PhD course on "Statistical Learning" using T. Hastie, R. Tibshirani and J. Friedman, "The Elements of Statistical Learning", 2001, was held at IMM during the first half of 2002, and organized by K. B. Hilger
- Oxford University has invited K.B. Hilger to visit their statistics department during the summer term 2003. The contact person is the expert statistician Brian Ripley, Professor of Applied Statistics at the Department of Statistics, University of Oxford and a member of St. Peter's College
- Stanford University has invited R. Larsen to visit their statistics department in the last half of 2003. The contact person is the expert statistician Trevor Hastie, Professor of Applied Statistics at the Department of Statistics and the Division of Biostatistics of the Health, Research and Policy Department in the Stanford School of Medicine, Stanford University

6) List of 4M Publications

Published (and accepted) papers

Journal papers

- R. Larsen, Decomposition using Maximum Autocorrelation Factors, *Journal of Chemometrics*, vol. 16(8-10), pp. 427-435, John Wiley & Sons, 2002

Conference papers

- R. Paulsen and K.B. Hilger, *Shape Modelling Using Markov Random Field Restoration of Point Correspondences*, accepted for Information Processing in Medical Imaging (IPMI) 2003
- K. B. Hilger, A. A. Nielsen, R. Larsen, K. Conradsen, *Linear and Nonlinear Multiset Canonical Correlation Analysis*, Eleventh International Workshop on Matrices and Statistics, Informatics and Mathematical Modelling, Technical University of Denmark, DTU, 2002
- R. Larsen, H. Eiriksson, *L1 Generalized Procrustes 2D Shape Alignment*, Eleventh International Workshop on Matrices and Statistics, Informatics and Mathematical Modelling, Technical University of Denmark, DTU, 2002
- S. Krarup, T.A. Darvann, P. Larsen, J.L. Marsh, S. Kreiborg, *Three-Dimensional Analysis of Mandibular Growth and Tooth Eruption*, Scandefa Dental Fair, Copenhagen, Denmark, 2002
- K. B. Hilger, M. B. Stegmann, R. Larsen, *A Noise Robust Statistical Texture Model*, Medical Image Computing and Computer-Assisted Intervention - MICCAI 2002, 5th Int. Conference, Tokyo, Japan, vol. 2, pp. 444-451, 2002
- R. Larsen, K. B. Hilger, M. C. Wrobel, *Statistical 2D and 3D shape analysis using Non-Euclidean Metrics*, Medical Image Computing and Computer-Assisted Intervention - MICCAI 2002, 5th Int. Conference, Tokyo, Japan, Springer, 2002
- R. R. Paulsen, R. Larsen, S. Laugesen, C. Nielsen, B. K. Ersbøll, *Building and Testing a Statistical Shape Model of the Human Ear Canal*, Medical Image Computing and Computer-Assisted Intervention - MICCAI 2002, 5th Int. Conference, Tokyo, Japan,, Springer, 2002
- Wrobel M et al. *Mathematical Modelling of Mandibular Metamorphosis*, The Medical and Computer Graphics Visionday at DTU, IMM, 2002
- S. Krarup, T.A. Darvann, P. Larsen, J.L. Marsh, S. Kreiborg, *Three-Dimensional Analysis of Mandibular Growth and Tooth Eruption*, American Dental Association, New Orleans, USA, 2002

Technical reports

- *Mathematical Modelling of Mandibular Metamorphosis (4M) Project Description*, 4M Technical Report 1, Informatics and Mathematical Modelling, Technical University of Denmark, 2002
- *Mathematical Modelling of Mandibular Metamorphosis (4M) Status Report*, 4M Technical Report 2, Informatics and Mathematical Modelling, Technical University of Denmark, 2003

Submitted papers

Journal papers

- K.B. Hilger, R. Larsen and M. Wrobel, Growth Modelling of Human Mandibles using Non-Euclidean Metrics*,* invited submission to: *the Journal of Medical Image Analysis* (MEDIA), 2003
- R. Larsen and K.B. Hilger, Statistical 2D and 3D shape analysis using Non-Euclidean Metrics*,* invited submission to *the Journal of Medical Image Analysis* (MEDIA), 2003

Conference papers

- K.B. Hilger, R.R. Paulsen and R. Larsen, *Markov Random Field Restoration of Point Correspondences for Active Shape Modelling,* Medical Image Computing and Computer-Assisted Intervention - MICCAI 2003
- K.B. Hilger and R.Larsen, S. Krarup, S. Kreiborg, and T. Darvann., *Active Shape Analysis of Mandibular Growth*, Medical Image Computing and Computer-Assisted Intervention - MICCAI 2003
- R. Larsen and K.B. Hilger, *Probabilistic Generative Modelling,* Scandinavian Conference on Image Analysis (SCIA) 2003

Unpublished papers

- T.A. Darvann., *LM, Software for registration and analysis of 3D mandibles*, 3D-Laboratorium Department of Pediatric Dentistry and Clinical Genetics, School of Dentistry, University of Copenhagen, 2002

Work in preparation

Work is in preparation for journal and conference articles in order to further report and elaborate on the results obtained in the 4M project. Emphasis is on the results attained from the analyses related to Task 4 and Task 5.

7) Spin-off and innovative value

The project has been a driving force behind the establishment of a research group at IMM focusing on 3D/4D medical image analysis with applications to cranio-facial, cardiac, facial, and retinal modelling. The group presently encompass R.Larsen, K. B. Hilger and 4 Ph.D. students. The group is internationally renowned an presently have 1 visiting Ph.D. student (February-June 2003) and 2 international students (from Ohio State University and Lund University) have submitted applications for Ph.D. stipends.

In collaboration with professors Mads Nielsen, the IT University of Copenhagen and Olaf Paulsen, the Neurological Research Unit, Copenhagen University Hospital a bid has been made to host the world premier conference in medical imaging MICCAI in Copenhagen in 2006. The bid is under evaluation.

The tools developed, implemented and evaluated in 4M holds a wide range of usage in e.g. analysis of new datasets. In particular, the methods for CCA, MRF, and growth decomposition are expected to be very valuable in future projects. Already, the methods are being applied in other shape and biomedical related PhD and Master projects at IMM, DTU. Moreover the tools are not just applicable for the specific problems in 4M, but are generic and allows for better integration and fusion of data originating from multiple sources in most problems involving exploratory data mining analysis.

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