Fast Marching Level Sets

theory & applications

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04351 Advanced Image Analysis

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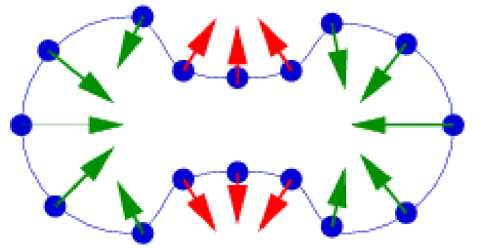


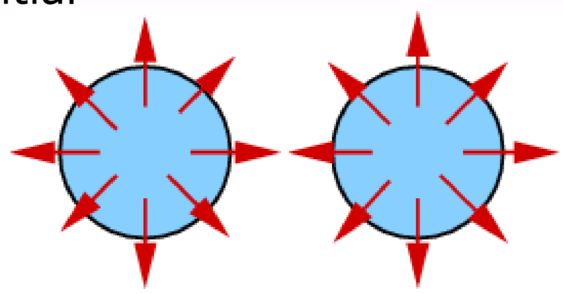
Introduction

- Level Sets: Smart handling of propagating contours
- Similarities to the classic snake:
  - Consists of a moving contour
  - Can be made dependent on user defined terms
- Advantages to the classic snake:
  - Marker-less
  - Stepsize-less
  - Handles sharp corners
  - Handles changes in topology (i.e. splits, merges)

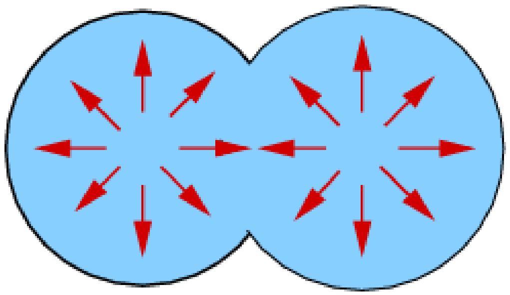
Consequences of marker-based evolution

- Example: Propagation of two flames

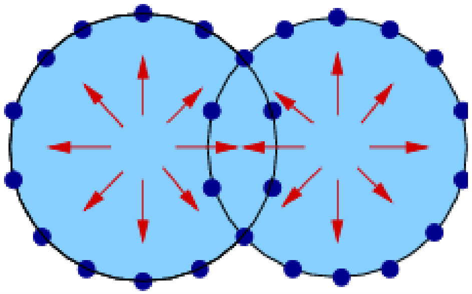
Initial



Later

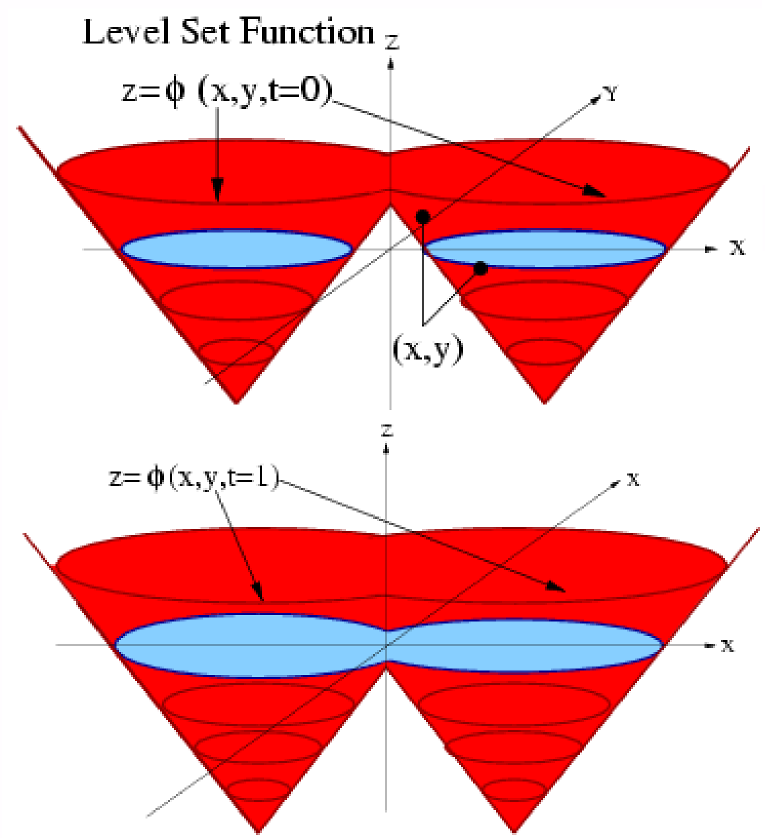


Later (marker-based solution)



A level set representation

- Adding an extra dimension to the problem



The level set function:

$$z = \Phi(x,y,t)$$

Contour at time *t:*

$$0 = \Phi(x,y,t)$$

The level set PDE:

$$\Phi_t + F|\nabla\Phi| = 0$$

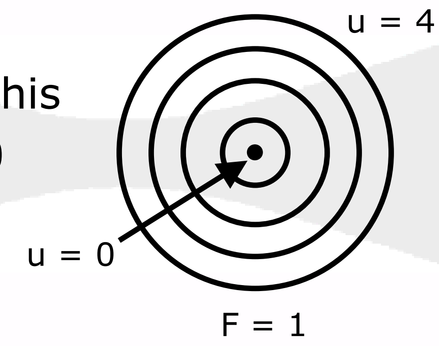
$$\text{given } \Phi(x,y,t=0)$$

Fast marching

- Observations
  - If F preserves sign, then the level set function $\Phi(x,y,t) = 0$ becomes single-valued in *t*, i.e. each pixel is visited once
  - This leads to the simpler Eikonal equation formulation

    $$|\nabla u(x,y)|F = 1$$

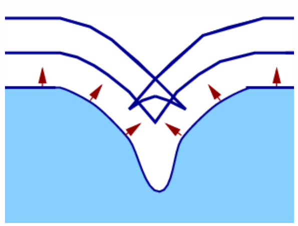
    where *u* is the arrival time of the front
  - Combined with an optimal sorting technique, this leads to a very fast solution to $\Phi(x,y,t=t_0) = 0$



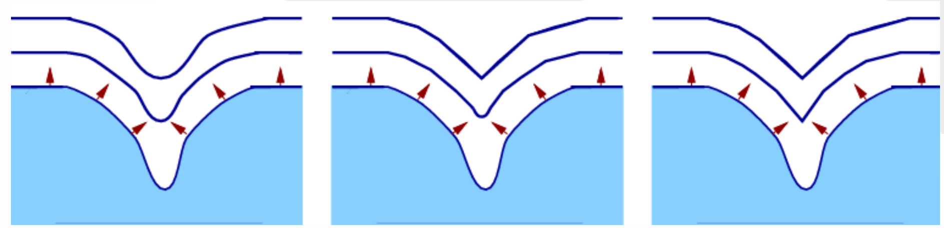
Huygen's Principle: "Every point on a primary wave front serves as the source of spherical secondary wavelets such that the primary wave front at some later time is the envelope of these wavelets..."

Propagation of concave contours

- The swallowtail problem
  - Imagine the propagation of a concave contour with F = 1
  - Adding viscosity in terms of curvature, leads to a simple curve, but the solution is somewhat smoothed
  - However, using Huygen's Principle the front should produce a sharp corner (which is no good in a PDE formulation)
- A solution
  - Let $\varepsilon \to 0$ or integrate the viscosity solution (or *weak solution*) into the numerical approximation of the gradient

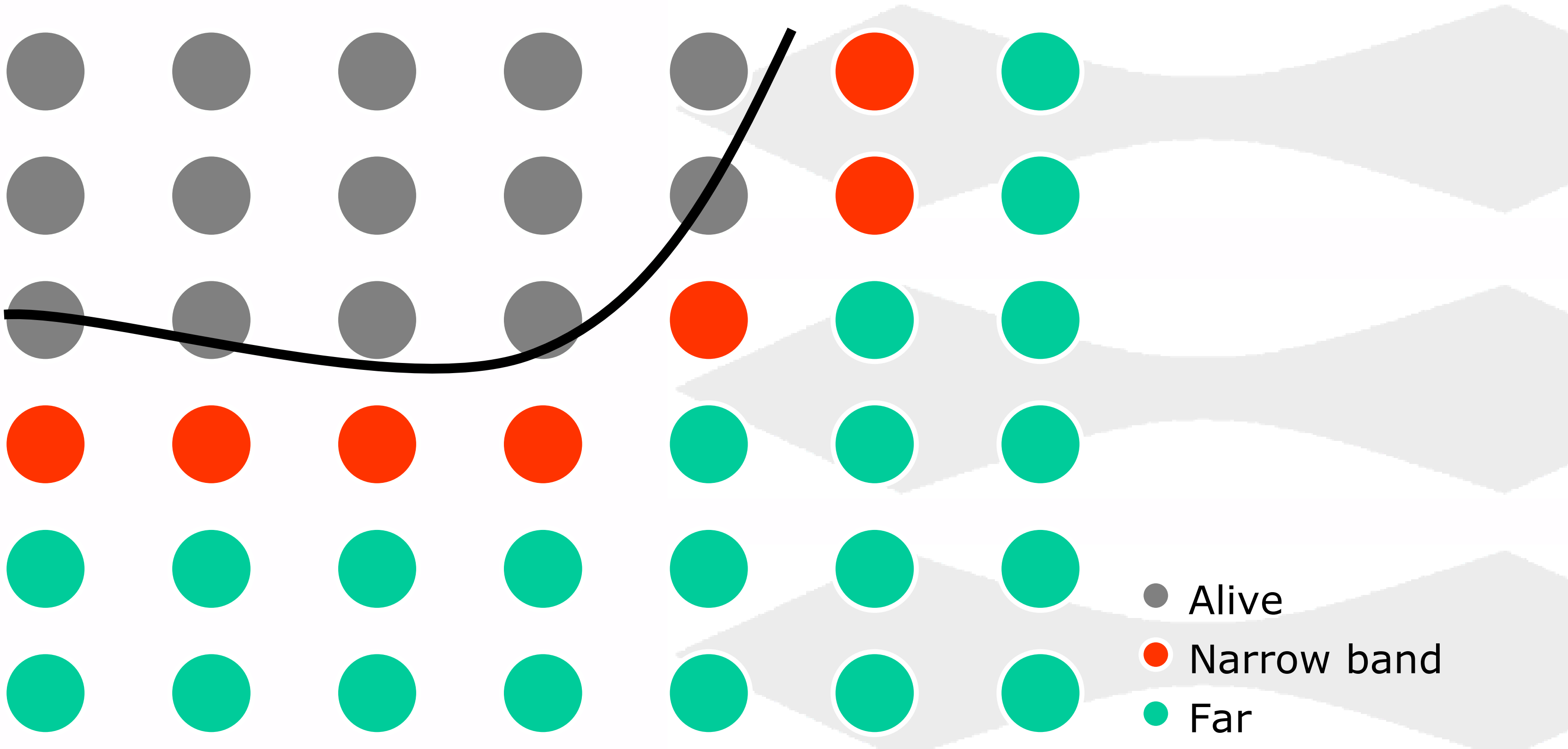


Swallowtail, F = 1



Front propagating with viscosity, $F = 1 + \varepsilon\kappa$

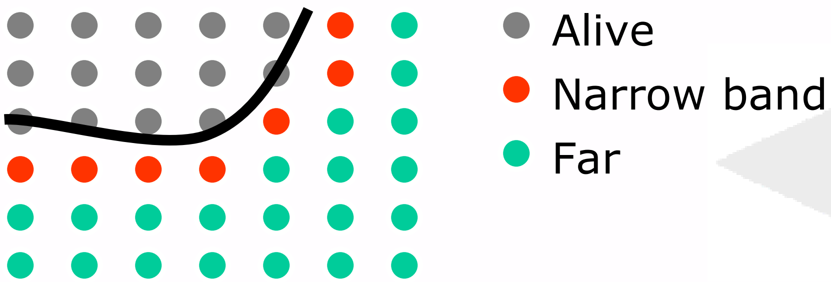
The Fast Marching algorithm



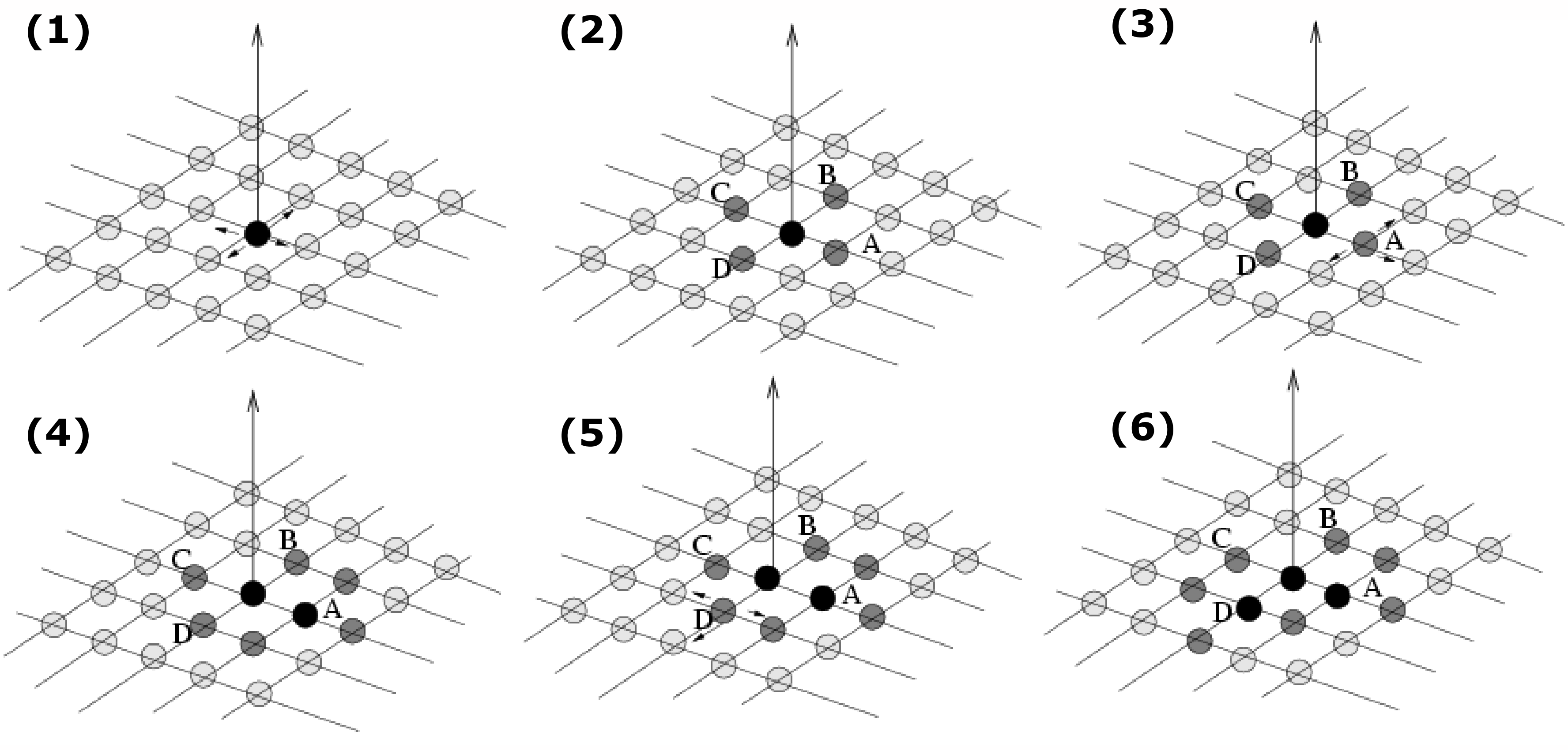
The Fast Marching algorithm

- Loop {
    1. Let *Trial* be the point in **Narrow band** with smallest u
    2. Move *Trial* from **Narrow band** to **Alive**
    3. Move all neighbors of *Trial* to **Narrow band**
    4. Recompute u for all neighbors of *Trial*

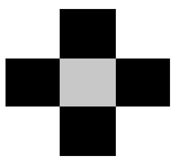
  }



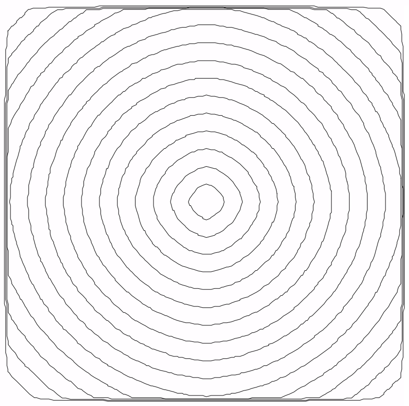
Fast marching – cont'd

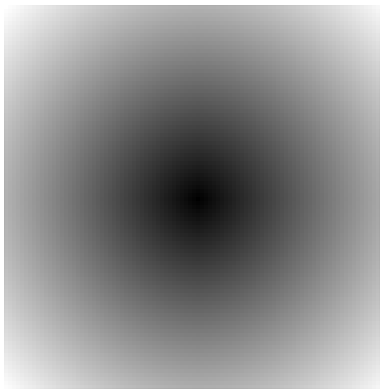


Constant speed marching

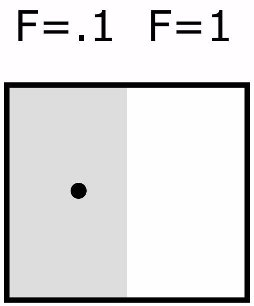


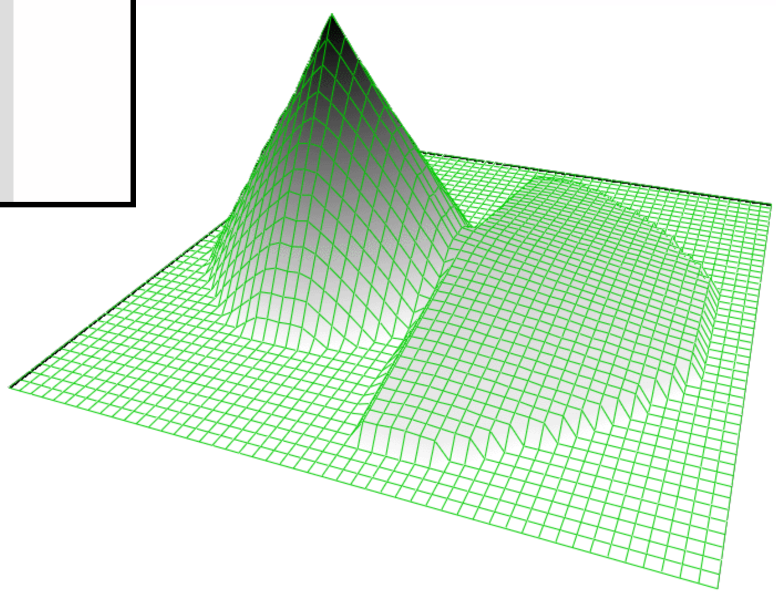
F=1



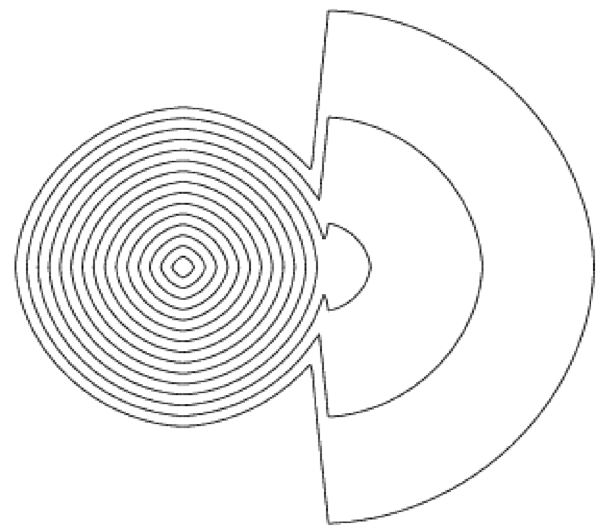


The parking lot example

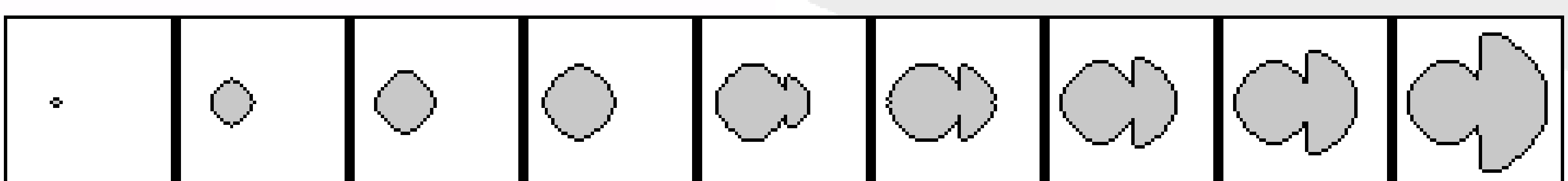




Level set function



Iso contours



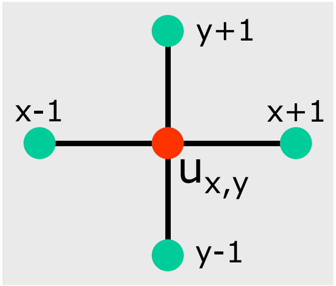
Fast marching in details

- How to: ”4) Recompute *u* for all neighbors of *Trial*”
  - That is: solve the Eikonal eq. $|\nabla u|F = 1$ at all neighbors
  - The correct viscosity solution is obtained by:

$$\max(\max(u_{x,y} - u_{x-1,y},0), -\min(u_{x+1,y} - u_{x,y},0))^2 + \max(\max(u_{x,y} - u_{x,y-1},0), -\min(u_{x,y+1} - u_{x,y},0))^2 = 1/F^2$$

  - Example:

$$(u_{x,y} - u_{x+1,y})^2+(u_{x,y} - u_{x,y-1})^2 = 1/F^2$$



Fast marching in details

- The correct viscosity solution (upwind solution) in 1D:

$$\max(\ \max(u_x - u_{x-1},0),\ -\min(u_{x+1} - u_x,0)\ )^2 = 1/F^2$$

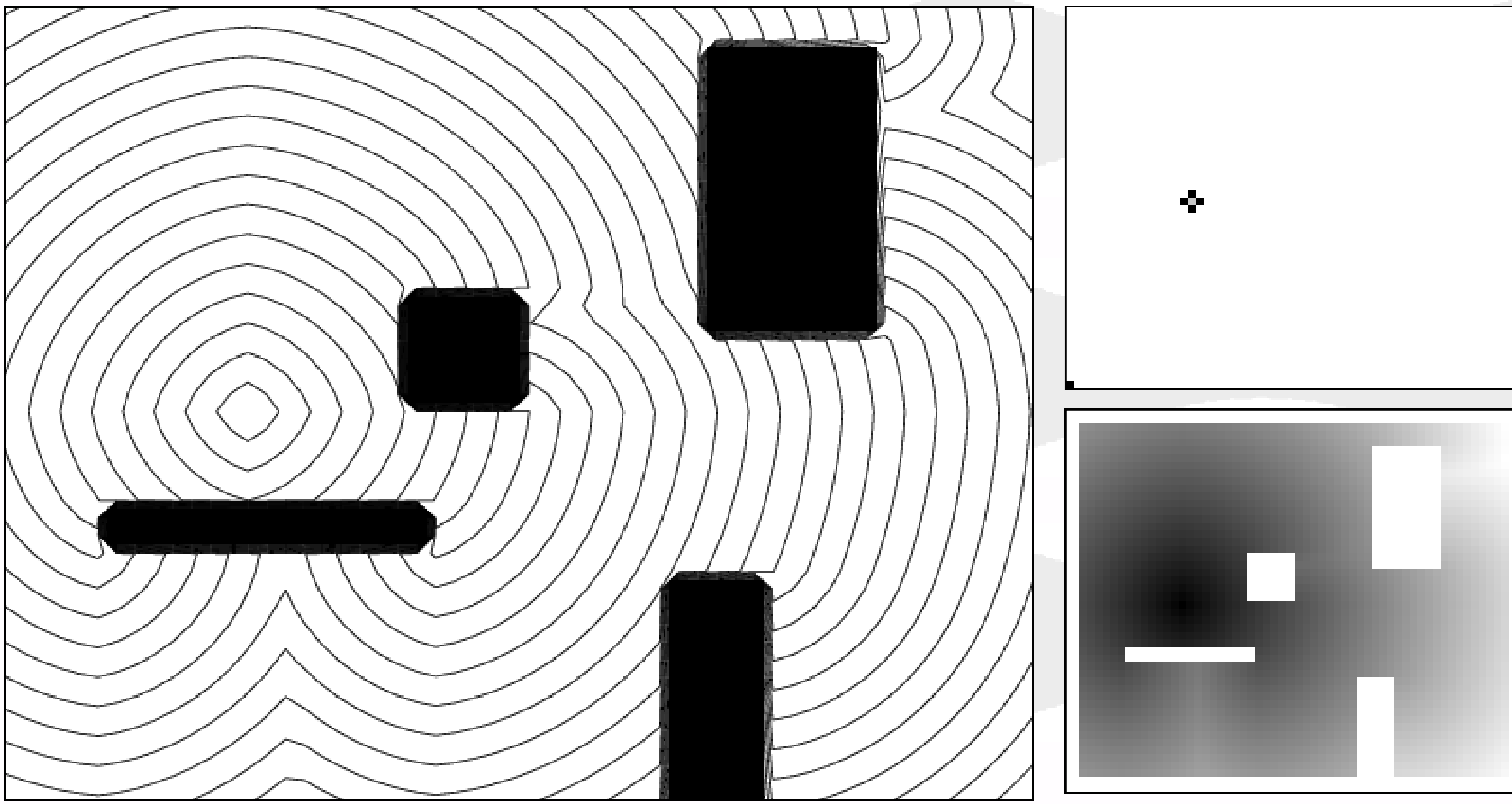
i.e. $u_x - u_{x-1} > 0 \Rightarrow u_x > u_{x-1}$ or $u_{x+1} - u_x < 0 \Rightarrow u_x > u_{x+1}$

- Observations
  - The inner max/min pair selects the upwind solution
  - The outer max pair selects the largest upwind solution
  - Only the largest solution to the quadratic satisfy the upwind condition.

    Ex.: $(u_x - u_{x-1})^2 = 1/F^2 \Rightarrow u_x = u_{x-1} \pm 1/F^2$, $F>0$

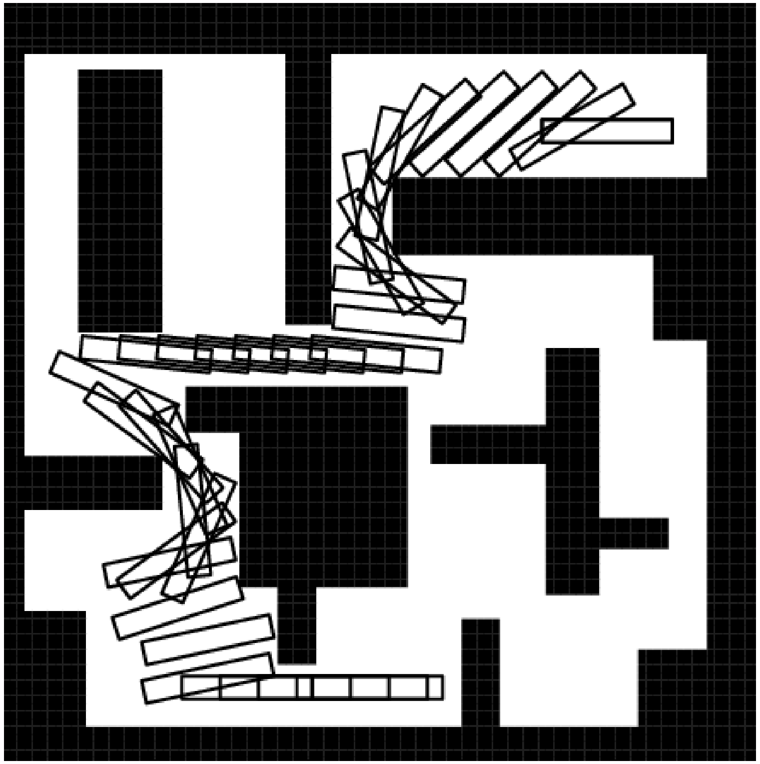
Applications

Shortest path calculation with obstacles on a 60x50 grid



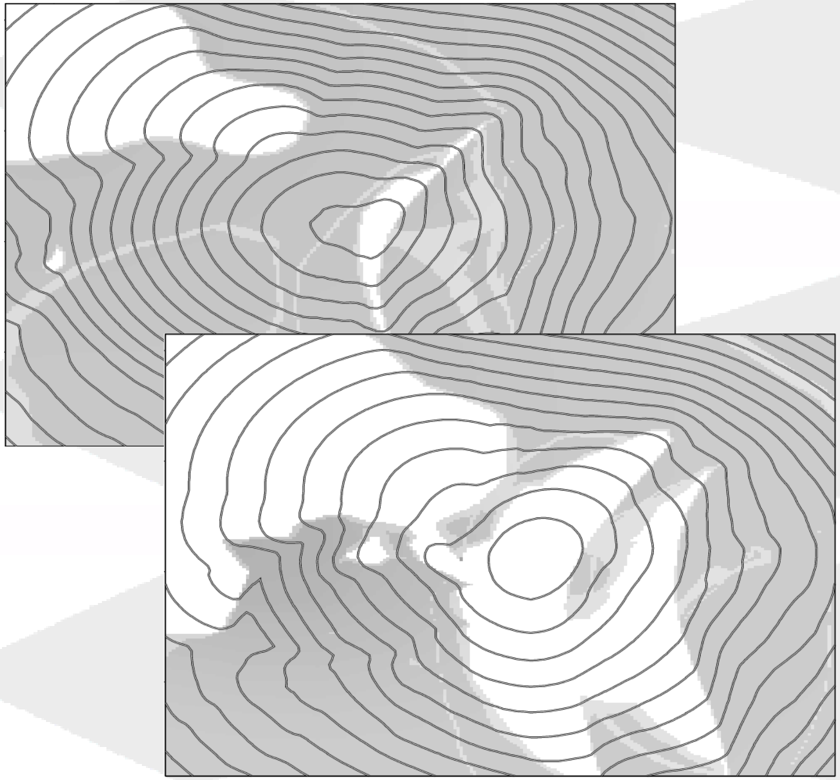
Applications – cont'd

Robotics / Navigation



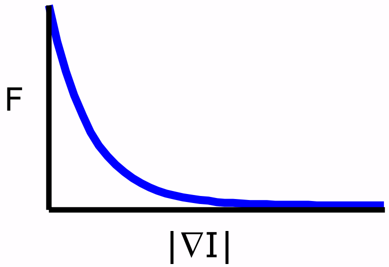
[Sethian et. al.]

Seismic travel times



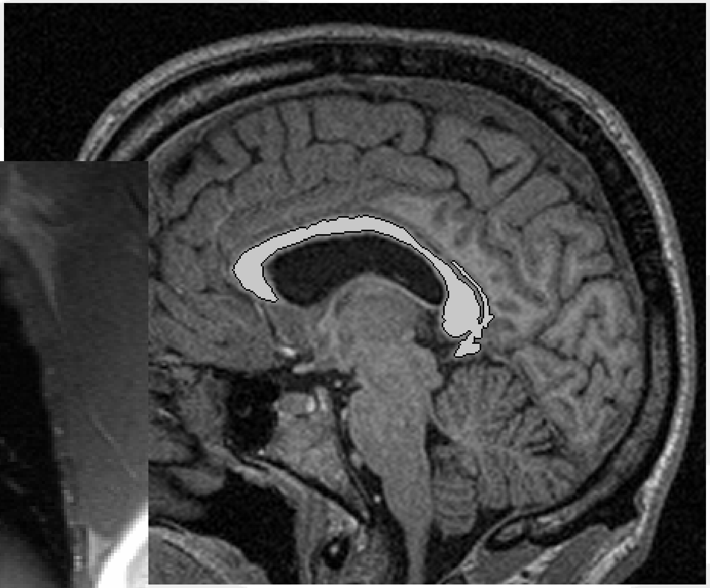
[Sethian et. al.]

Applications – cont’d

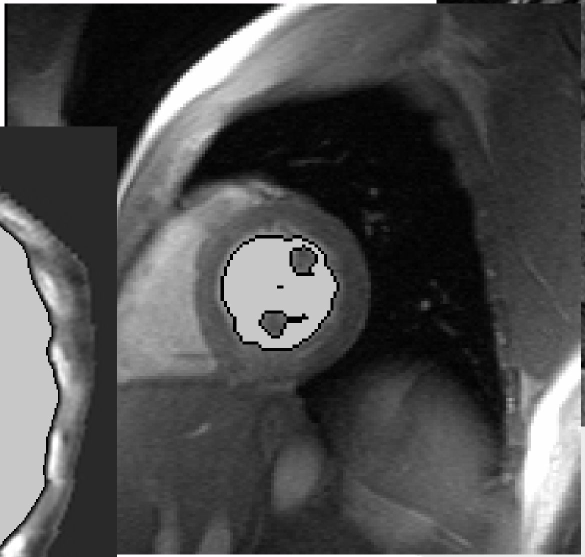


- Image segmentation: $F = e^{-k|\nabla I|}$

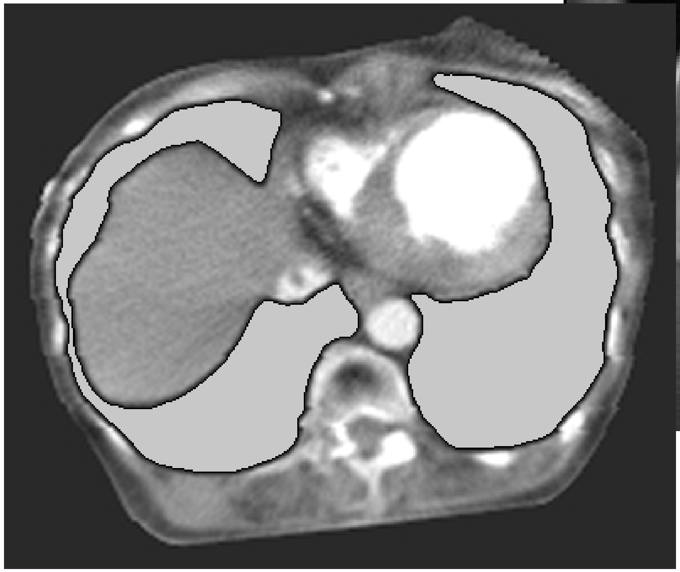
Brain, Corpus Callosum - MRI



Heart, Left ventricle- MRI

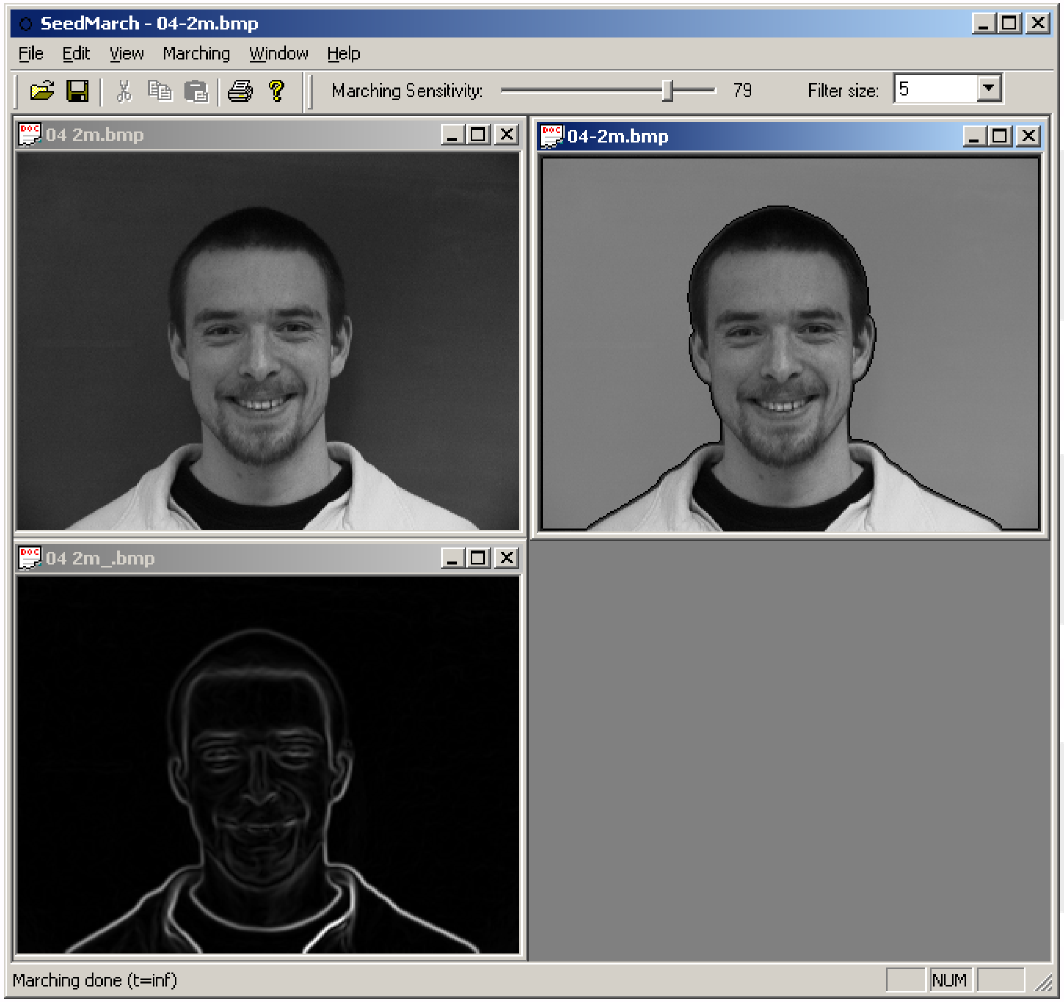


Lungs - CT



The SeedMarch application

- Face segmentation (3 mouse clicks)



Upper left: original image

Upper right: segmentation

Lower left: gradient image

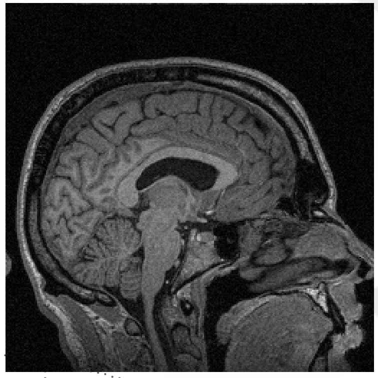
Summary

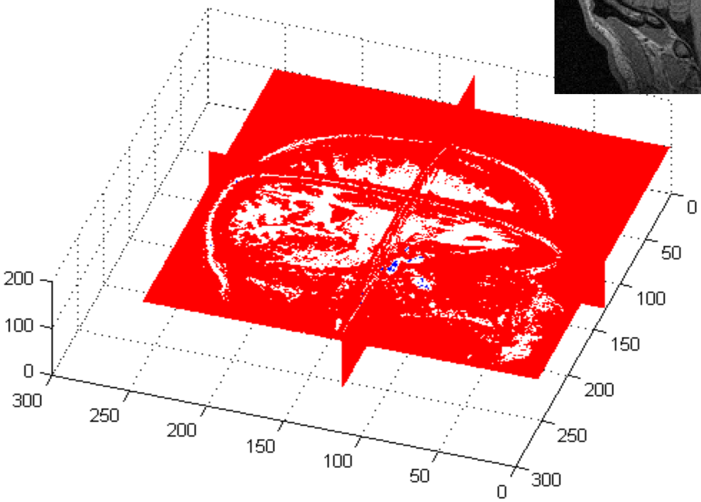
- Fast Marching methods are:
  - Very efficient – also in 3D
  - Of an ’open architecture’ in terms of speed functions
  - Dealing with sharp corners and changes in topology
  - Widely applicable: image segmentation, robotics, seismic travel times, shortest path finding etc.
  - In essence a smart way to solve the Eikonal equation

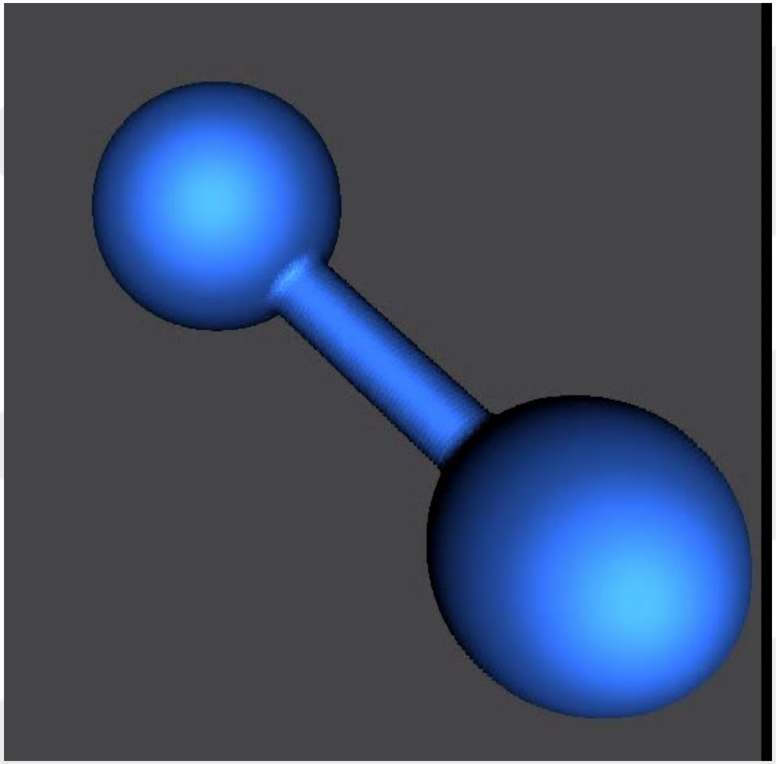
- Discussion
  - Can inherently only propagate monotonically
  - The more general – but slower – level set formulation handles non-monotonically advancing fronts
  - Stop criteria selection is crucial w.r.t. image segmentation

Project proposal I

- A 3D Fast Marching Implementation



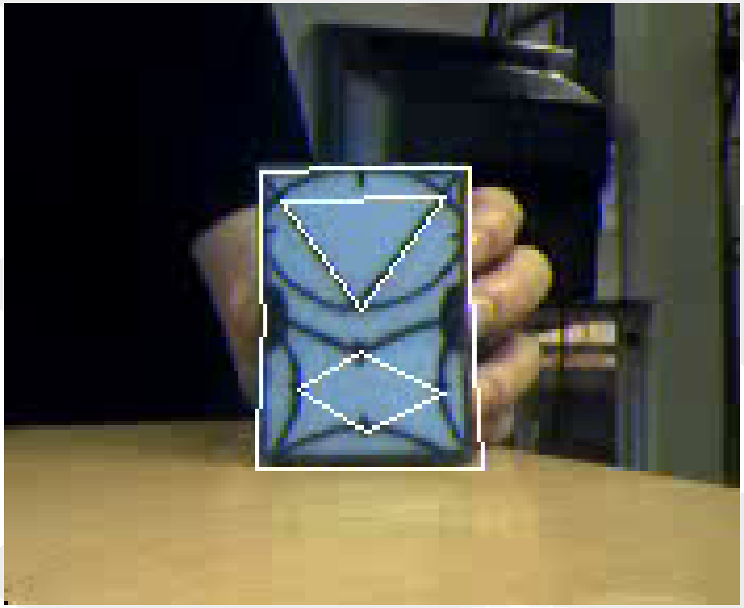




[Bærentzen]

Project proposal II

- Tracking using Active Appearance Models



[Nielsen, Schiøler & Wrobel]

The End

- Acknowledgements

  - M.Sc., Ph.D. Stud. Andreas Bærentzen, IMM
  - Selected illustrations from publications of J.A. Sethian

- References

  - J.A. Sethian, *"Level Set Methods and Fast Marching Methods"*, Cambridge University Press, 1999
  - http://www.math.berkeley.edu/~sethian/