

Analytical Performance Modeling of Hierarchical Interconnect Fabrics

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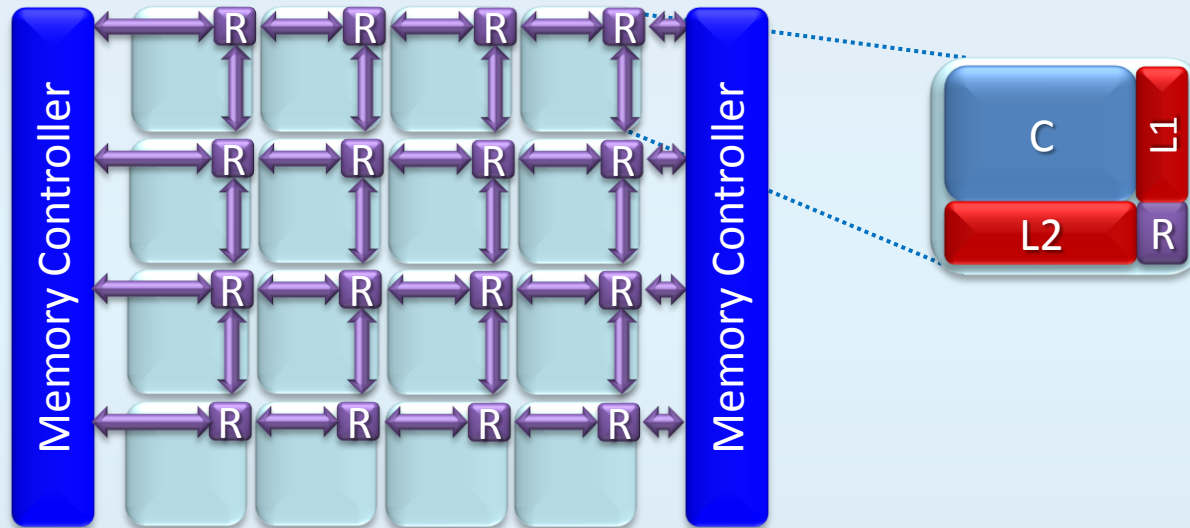
Supported by Intel Corporation

Outline

- **Introduction**
 - Hierarchical Chip Multiprocessors (CMPs)
 - Performance modeling for CMPs
 - The cyclic dependency between latency and traffic
- Analytical performance modeling
 - Modeling traffic
 - Modeling latency
 - Methods to resolve the dependency
- Results and conclusions

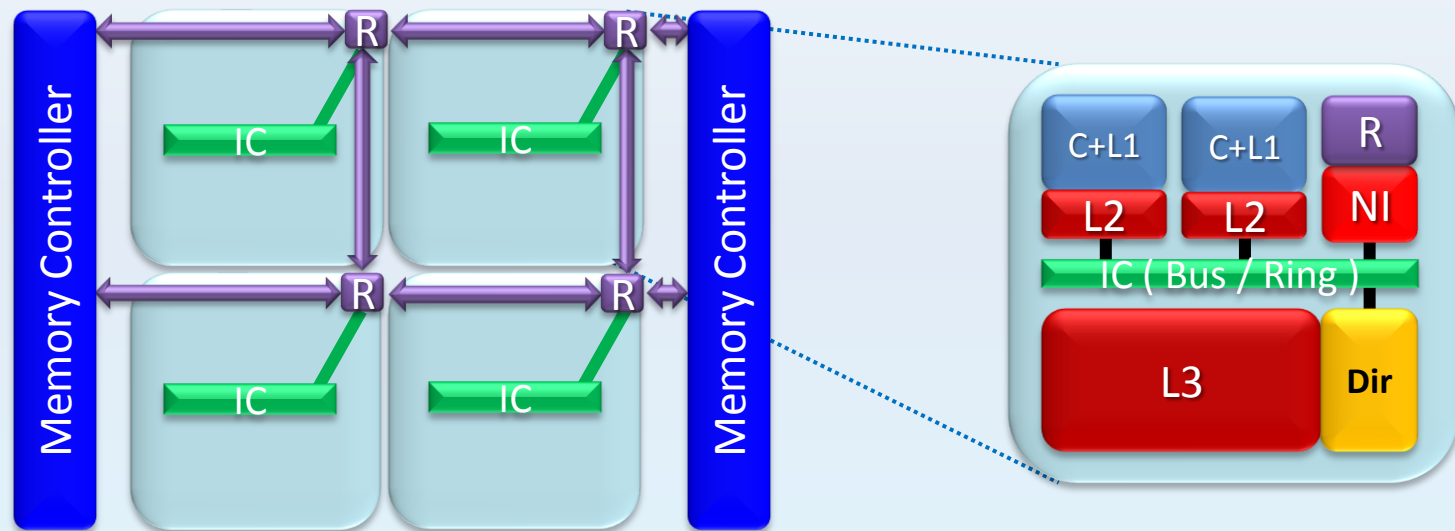
The trends in CMP design

- Hundreds of computing units per chip
 - Smaller, simpler, more power-efficient cores
- Advanced memory management
 - Larger on-chip cache
 - Increasing interconnect (IC) bandwidth
- Tiled architecture



Hierarchical interconnects

- Exploit locality of memory references*

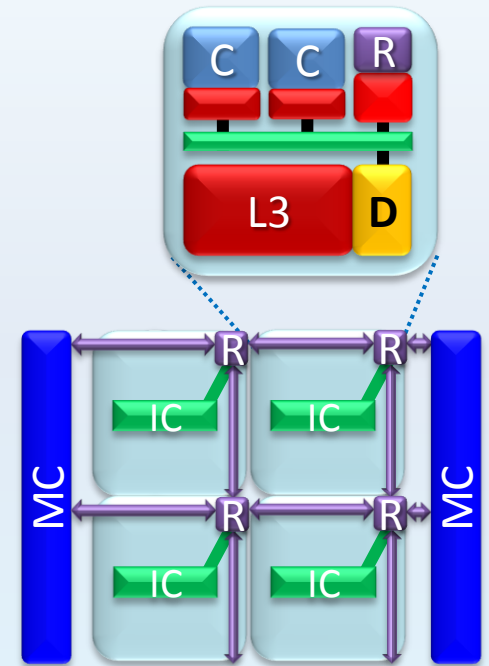


Tiled CMP with hierarchical interconnect

* *“Design and Evaluation of a Hierarchical On-Chip Interconnect for Next-Generation CMPs”, R.Das et al., HPCA, 2009*

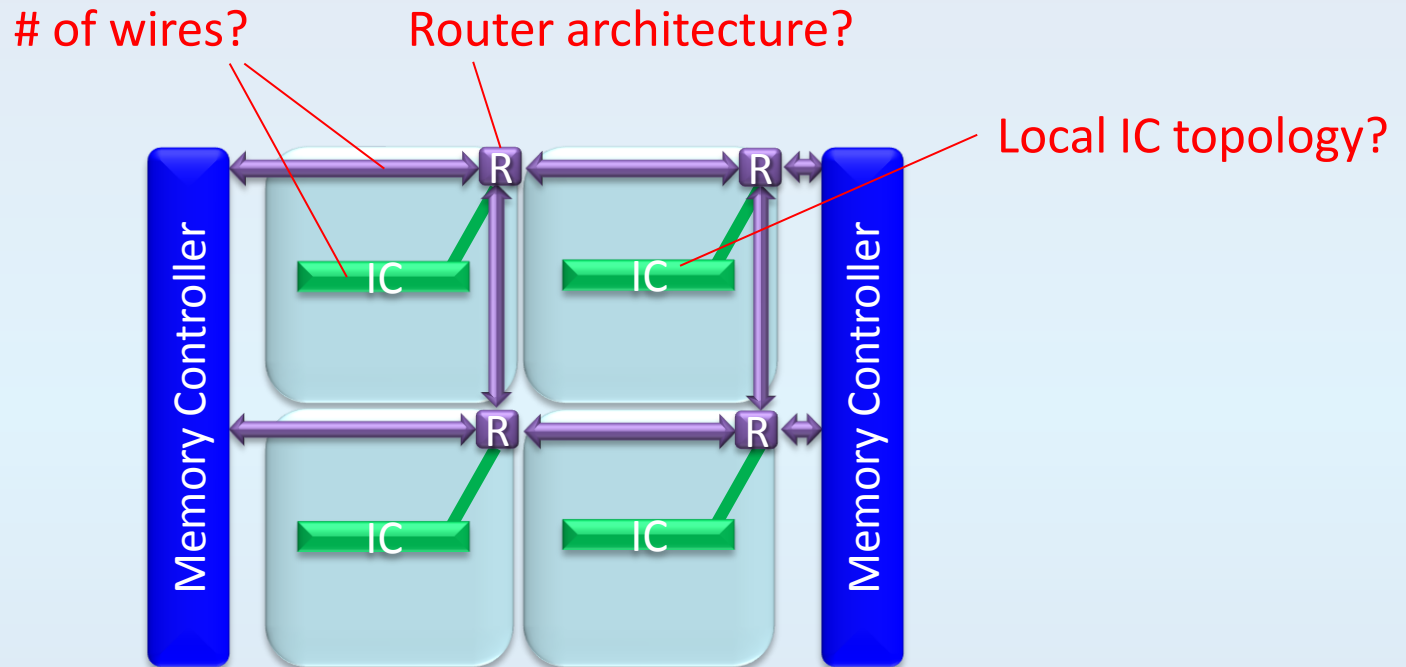
Design of CMP architecture

- Goal: efficient use of chip resources
 - Maximize performance
 - Fit area/power/thermal budget
- Multidimensional exploration space
(#cores / cache size /
memory hierarchy / IC topologies /...)
- Means: automated design space exploration
 - Analytical performance models are essential



Contention modeling

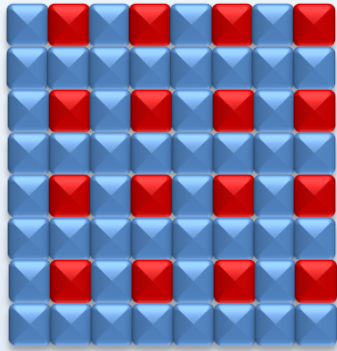
- Contention impacts CMP performance
- Crucial evaluating hierarchical interconnects
 - Is the required bandwidth sustainable?



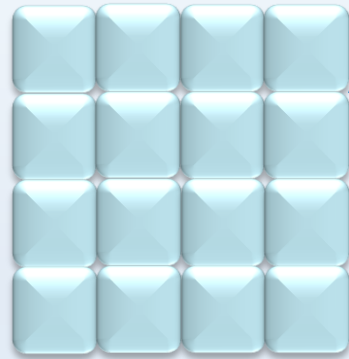
Motivational example

48 cores, 16 cache modules

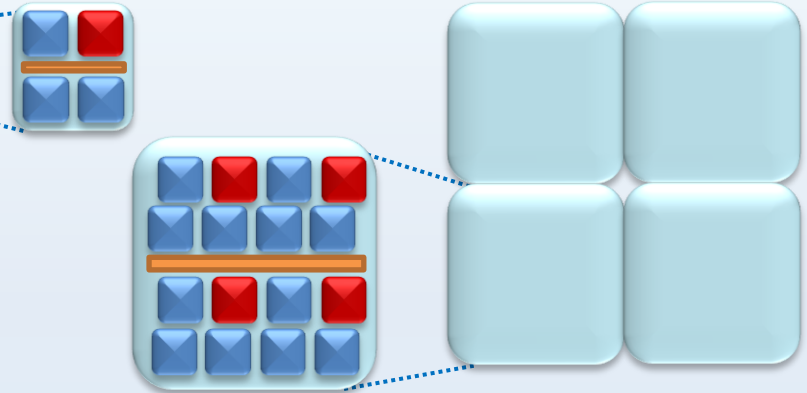
Legend:  core  cache  IC



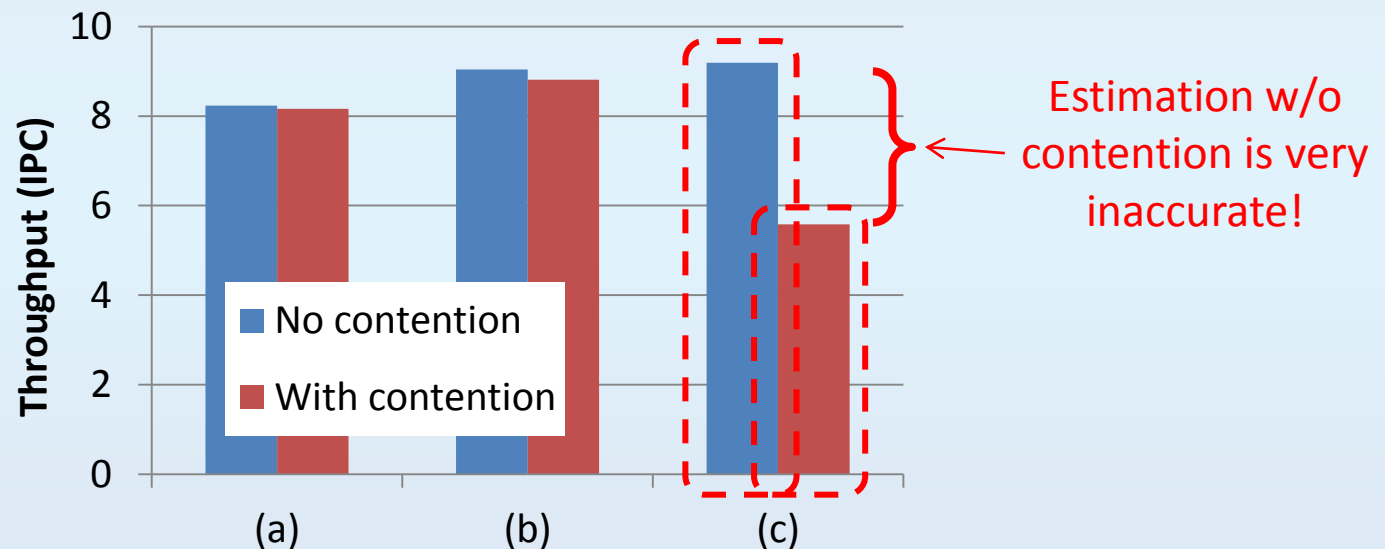
(a) 8x8 mesh



(b) 4x4 mesh with bus clusters



(c) 2x2 mesh with bus clusters

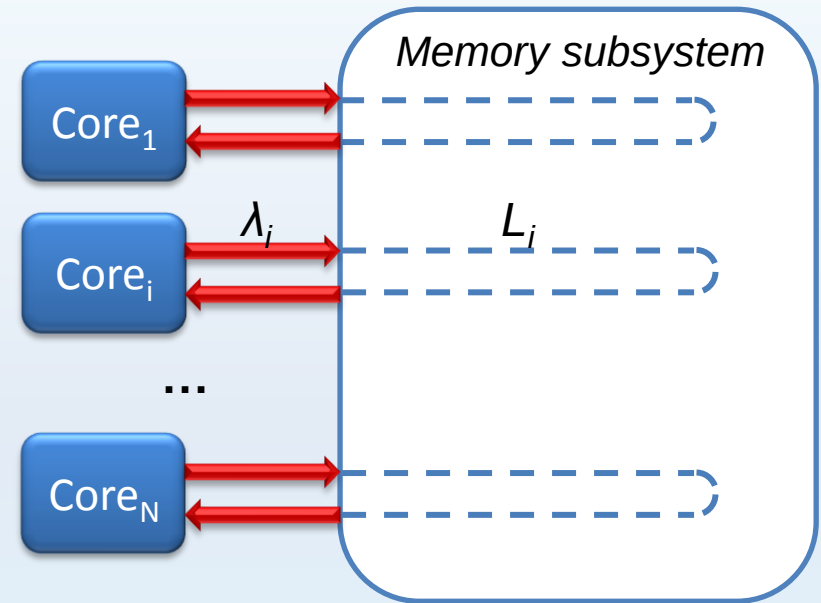


Analytical modeling of CMP performance

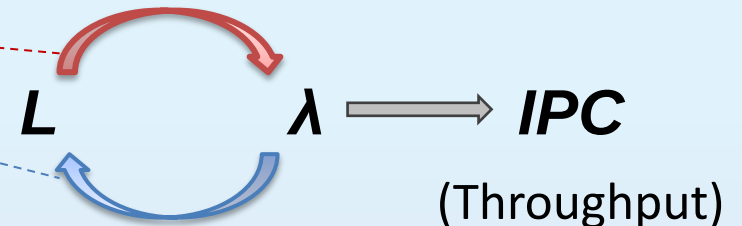
- Analytical models for ICs:
 - Latency L as a function of traffic λ
 - λ defined by the workload

Emphasis: λ depends on L !

$\lambda \uparrow \rightarrow L \uparrow \rightarrow \lambda \downarrow \rightarrow L \downarrow \rightarrow \lambda \uparrow \rightarrow \dots$



- This work: resolve the cyclic dependency of traffic and latency
 - Formulate λ as a function of L
 - Add existing model for $L(\lambda)$
 - Resolve the system efficiently



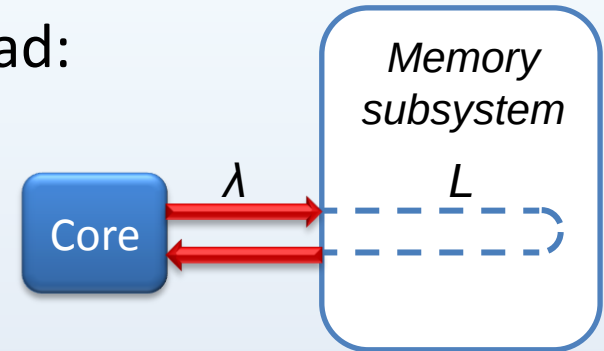
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 - **Modeling latency**
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Modeling memory traffic

Parameters of core executing some workload:

1. CPI_0 - *ideal* Cycles Per Instruction
2. MPI - # Memory references Per Instruction



Real performance of in-order core:

$$CPI = CPI_0 + \underbrace{AccessRate \cdot AccessCost}_{\text{Memory access penalty}} = CPI_0 + MPI \cdot \underbrace{L}_{\text{Average latency of memory access}}$$

Traffic to memory (probability of a memory reference per cycle):

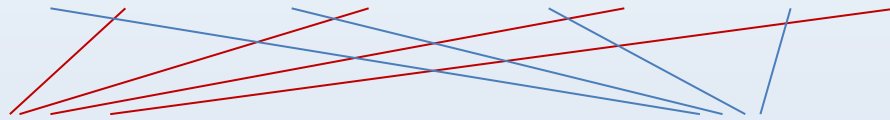
$$\lambda = MPI \cdot IPC = MPI \cdot \frac{1}{CPI} = \frac{MPI}{CPI_0 + MPI \cdot L}$$

Modeling average memory latency

- Average latency of memory requests for a core:

$$L = L_{hop-count} + L_{contention}$$

$$L_{hop-count} = p_{L1} \cdot L_{L1} + p_{L2} \cdot L_{L2} + p_{L3} \cdot L_{L3} + p_{MC} \cdot L_{MC}$$

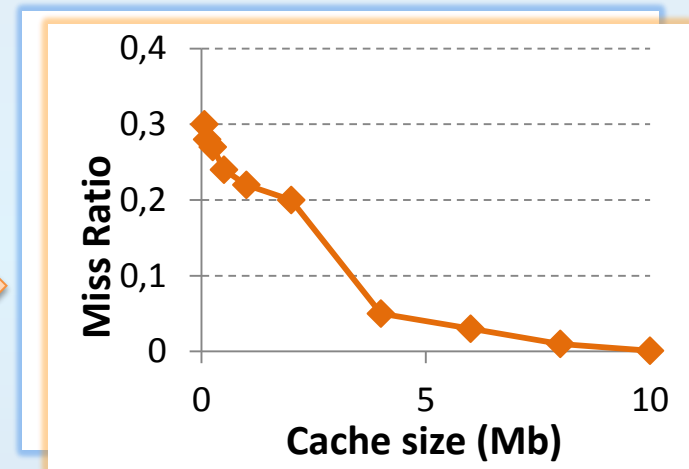
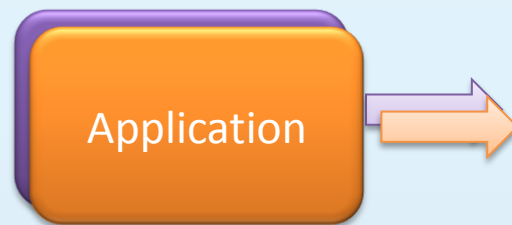


Latencies are calculated using

- Cache latencies
- Interconnect topology
- Routing algorithm (XY)

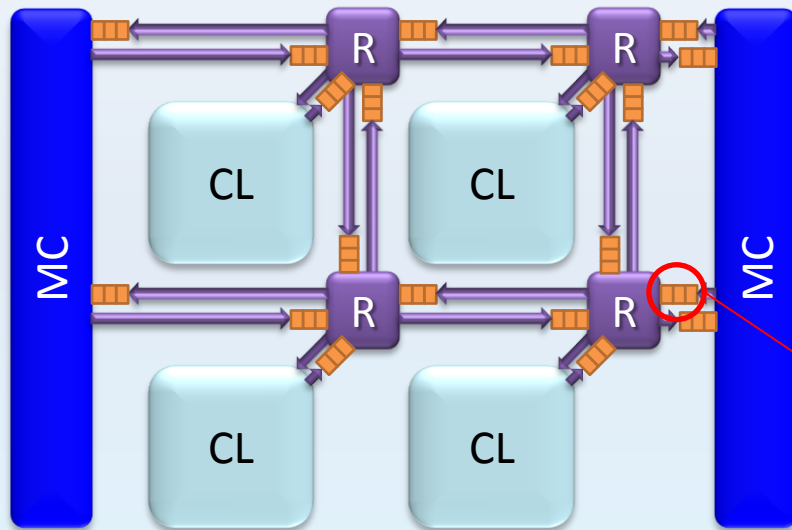
Probabilities are calculated using

- Miss ratio dependency on cache size

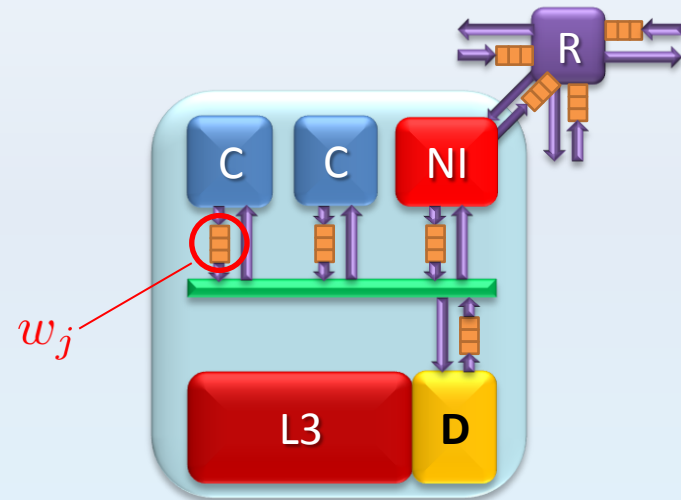


Modeling contention latency

*“An Analytical Approach for Network-on-Chip Performance Analysis”,
Ogras et al., TCAD, 2010 (Best Paper Award)*



Mesh NoC



Bus-based cluster

Delays in queues are defined by extending M/G/1 queuing model:

$$\bar{w}_q = W(\lambda_1, \dots, \lambda_N) \quad \Rightarrow \quad L_c = L(\lambda_1, \dots, \lambda_N)$$

The cyclic dependency of L and λ

System of non-linear equations

$$\forall c = 1, \dots, N : \begin{cases} L_c = L(\lambda_1, \dots, \lambda_N) \\ \lambda_c = \lambda(L_c) \end{cases}$$

Analytical model for latency

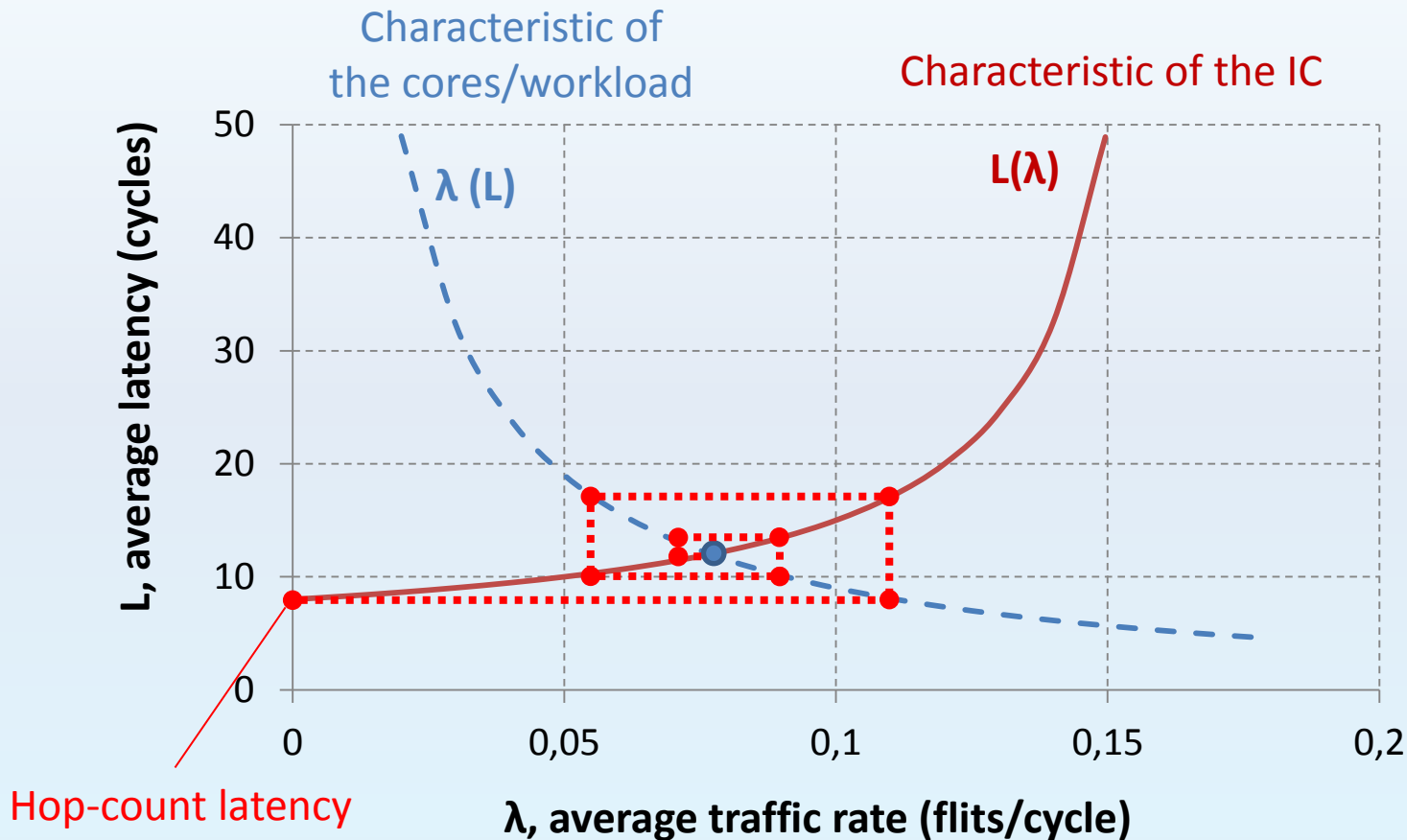
$$\begin{aligned} \tau_j &= TN_j + T \sum_{k=1, k \neq j}^P c_{jk} N_k + R \\ f_{ij} &= \frac{\lambda_{ij}}{\sum_{k=1}^P \lambda_{ik}} \quad c_{ij} = \sum_{k=1}^P f_{ik} f_{jk} \\ &\dots \end{aligned}$$

$$\lambda = \frac{MPI}{CPI_0 + MPI \cdot L}$$

- Solve using numerical methods
- General methods are very slow
 - 10x10 mesh (**10K vars./eqns.**) – MATLAB timeout after few hours
- Proposed methods:
 - Fixed-point iteration
 - Bisection search for λ

Any “black-box”
model for $L(\lambda)$!

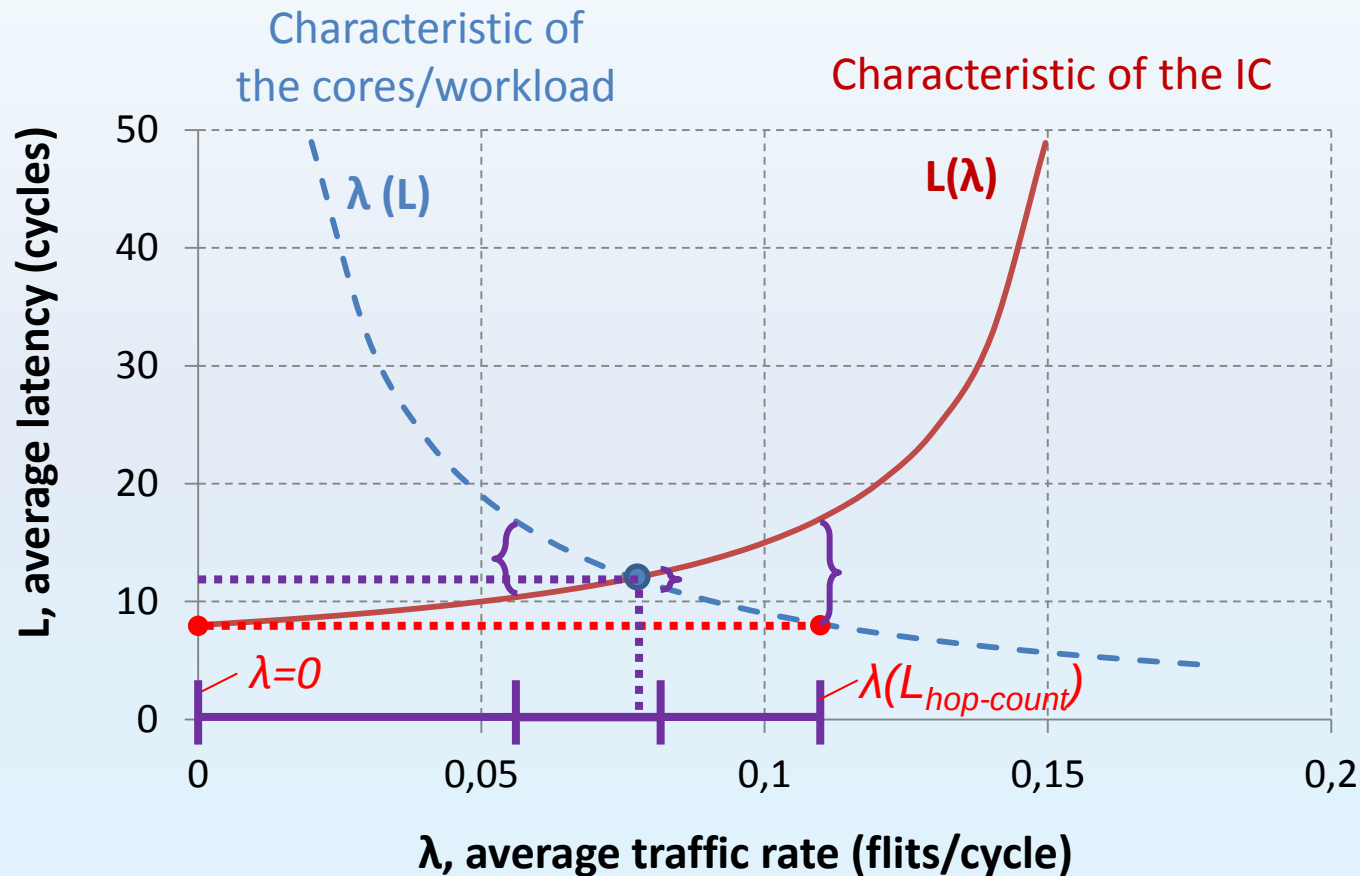
Fixed-point iteration



- + Fast (10x10 mesh in several ms)
- + Converges to the exact solution

– May not converge for high λ

Bisection search for λ



- Fast, as fixed-point
- Always converges to an approximate solution (good for homogeneous clusters)

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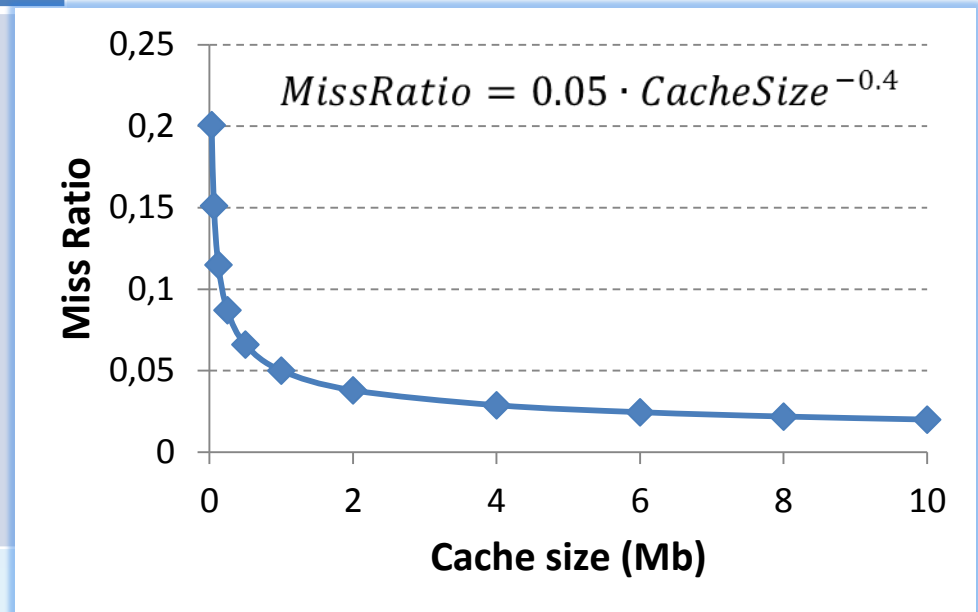
Performance of analytical methods

Test	Mesh	Cont. lat.	Num. of var./eqn.	Runtime (sec)		
				MATLAB	Fixed-Point	Bisection
T1	2 x 2	5%	236	0.023	0.001	0.001
T2	4 x 4	13%	1224	1.412	0.001	0.002
T3	6 x 6	8%	3108	30.831	0.002	0.003
T4	8 x 8	12%	6128	408.539	0.006	0.010
T5	10 x 10	23%	10260	Timeout (1hr)	0.010	0.012
T6	10 x 10	46%	10260	Timeout (1hr)	0.022	0.015
T7	10 x 10	55%	10260	Timeout (1hr)	NA	0.016

Case study: performance exploration

Parameter	Value
Chip area	350 mm ²
Core area	1.25 mm ²
Core IPC ₀	2.0
MPI	0.5
L1 size	64, 128 Kb
L2 size	64 Kb to 3 Mb
Memory density	1 mm ² / Mb
Mesh dimensions	2x2 to 16x16
MC latency	100 cycles

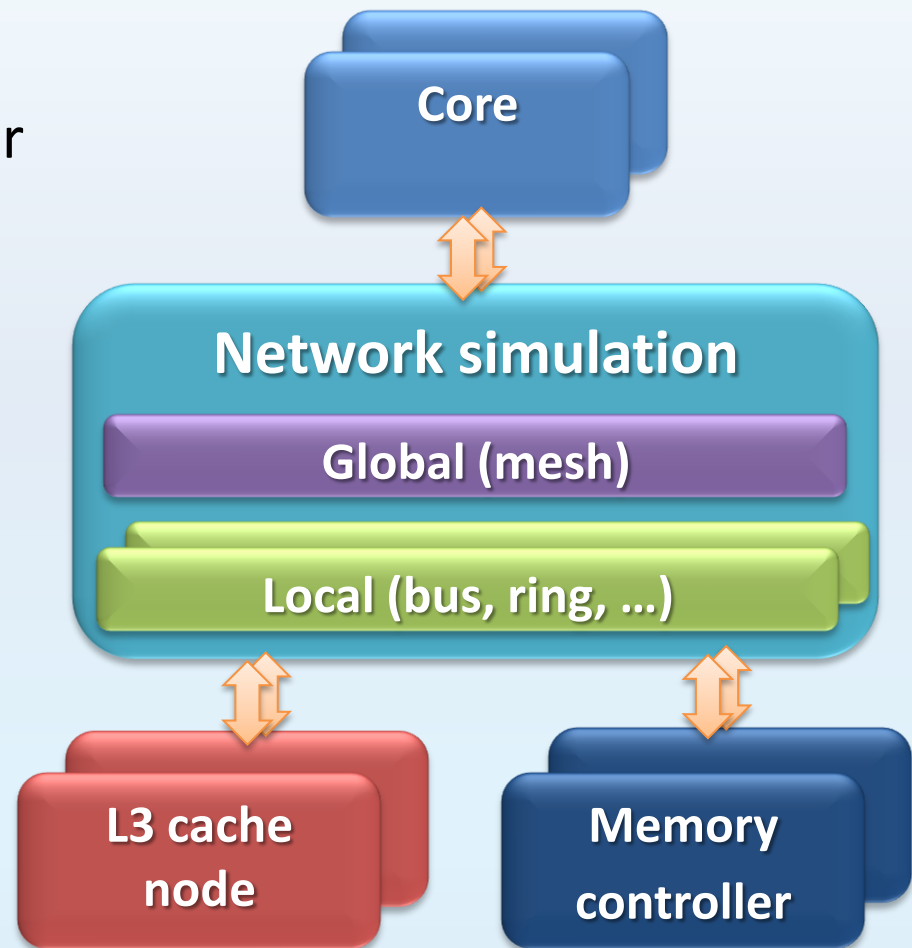
1062 configurations explored



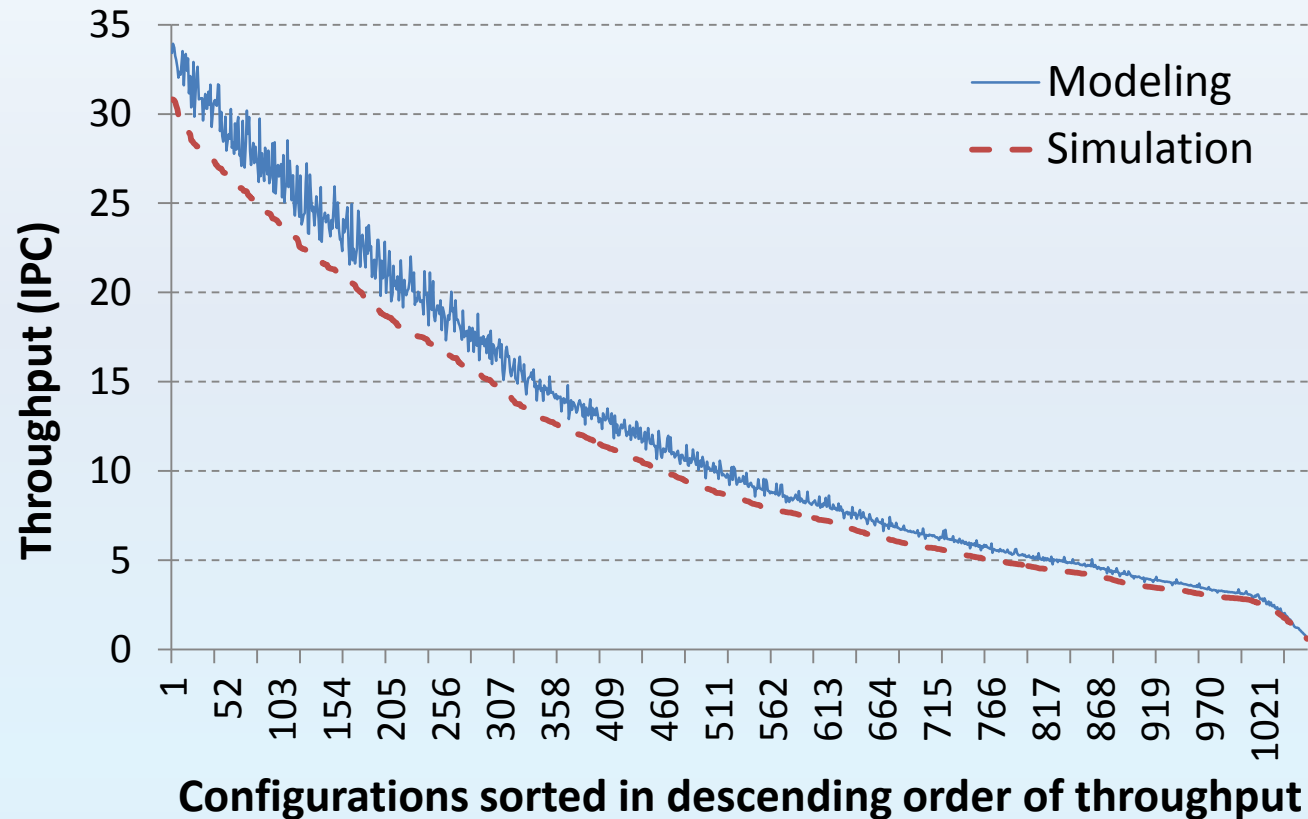
Cache Size	64K	128K	256K	512K	1M	2M	4M	8M
Area* (mm ²)	0.063	0.125	0.25	0.5	1.0	2.0	4.0	8.0
Latency (cycles)	2	3	4	5	6	7	8	9

Simulation environment

- Verify model by simulation
- Cycle-accurate NoC simulator
 - On top of BookSim 2.0
- Extensions
 - Hierarchical networks
 - Bus topologies
 - Probabilistic state-machines for cores and memories

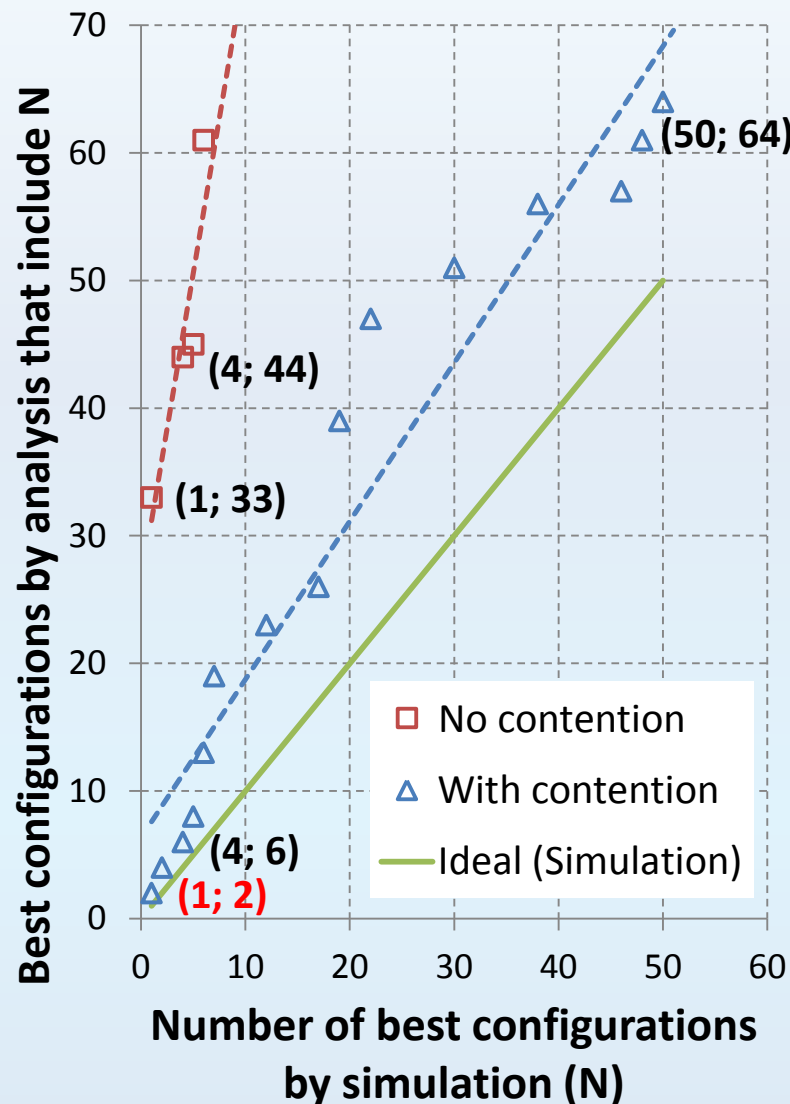


Faithfulness of the model

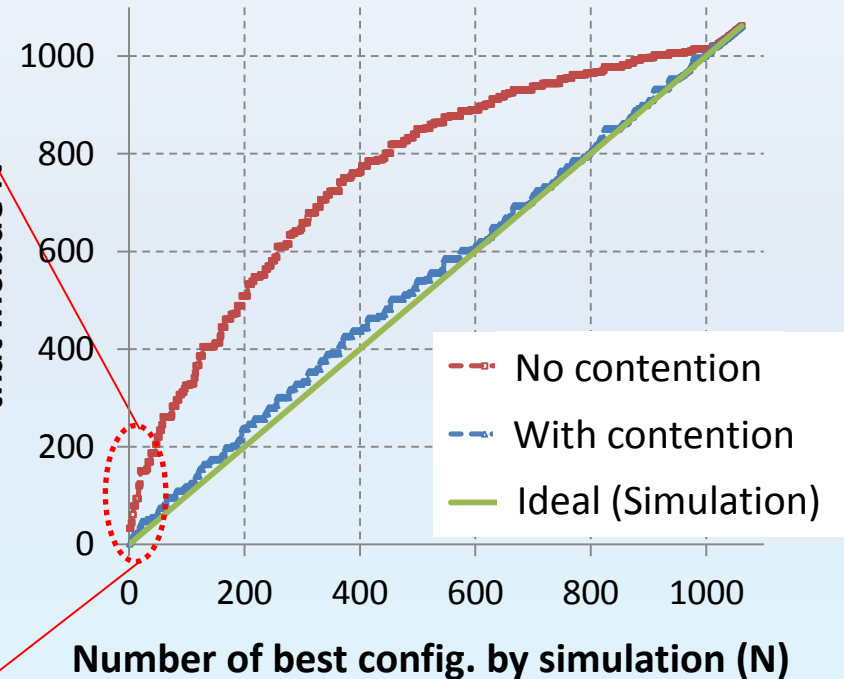


- Average difference in throughput is about 10%
- Corresponds to the error of the latency model

Best-throughput ordering



Best configurations by analysis that include N



Simulation time: 5.5 hours
Modeling time: 16.8 sec (>1000x faster)

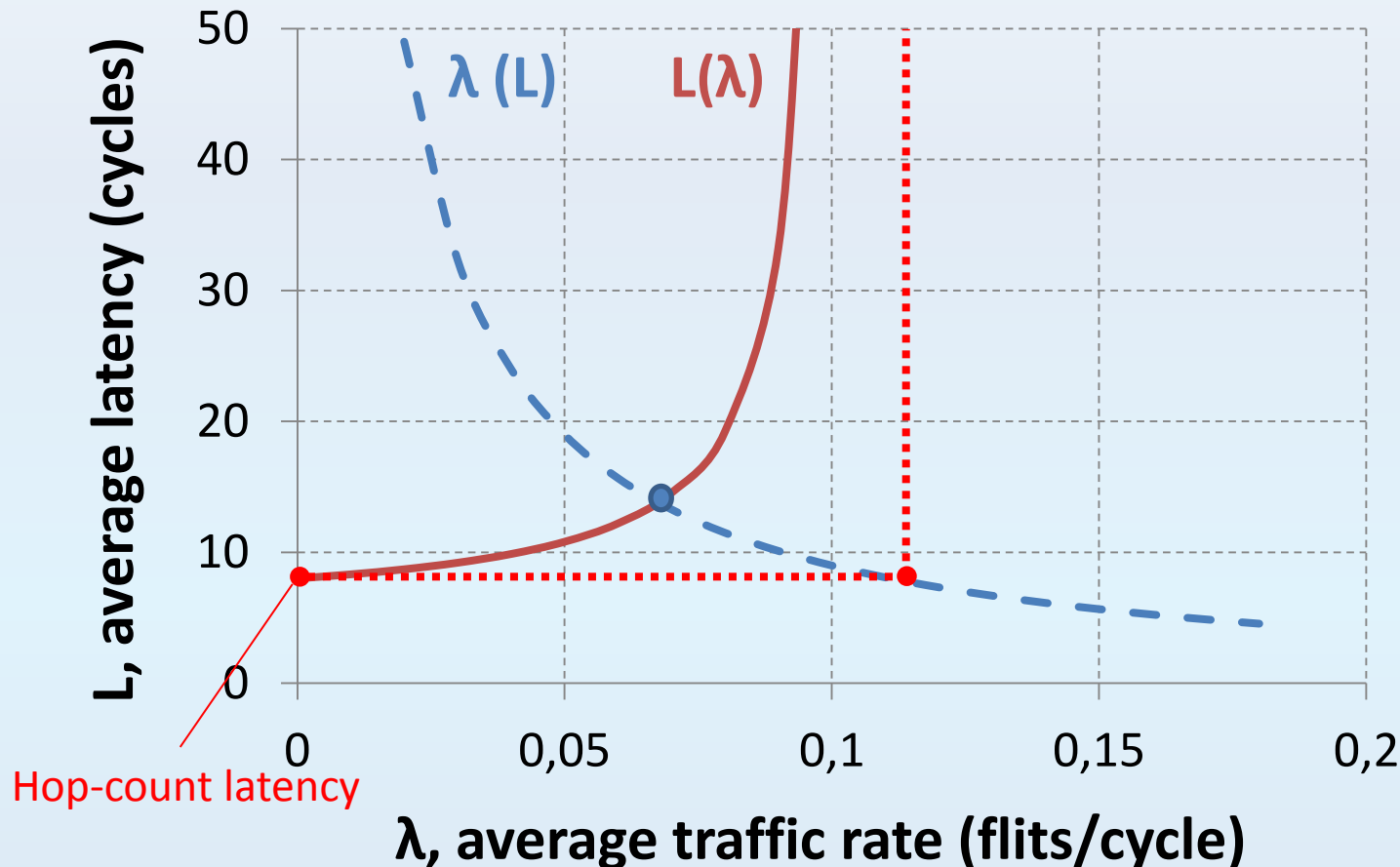
Conclusions

- Analytical modeling of contention in CMPs is essential
- *There exists cyclic dependency between latency and traffic of memory requests*
- This dependency can be efficiently resolved using numerical methods (fixed-point, bisection)
- Precision of the model is significantly improved
- Current work: out-of-order cores, heterogeneity

Backup

Fixed-point convergence issues

Sufficient for convergence of $\bar{x}_{n+1} = F(\bar{x}_n)$: $\sum_i \left| \frac{\partial F}{\partial x_i} \right| < 1$.



Bisection search

Latency model

$$L_c = L(\lambda_1, \dots, \lambda_N)$$



$$L(\bar{\lambda}) = \frac{1}{N} \sum_{c=1}^N L_c(\bar{\lambda}), \quad L^*(\bar{\lambda}) = \frac{1}{N} \sum_{c=1}^N L_c^*(\lambda_c)$$



$$F(\bar{\lambda}) = L(\bar{\lambda}) - L^*(\bar{\lambda})$$

Traffic model

$$\lambda_c = \frac{MPI}{CPI_0 + MPI \cdot L_c}$$



$$L_c^*(\lambda_c) = \frac{1}{\lambda_c} - \frac{1}{MPI \cdot IPC_0}$$



$$L(\bar{\lambda}) = \frac{1}{N} \sum_{c=1}^N L_c(\bar{\lambda}), \quad L^*(\bar{\lambda}) = \frac{1}{N} \sum_{c=1}^N L_c^*(\lambda_c)$$

Average latency calculation

- Average Memory Access Time (AMAT):

$$L_{hop-count} = p_{L1} \cdot L_{L1} + p_{L2} \cdot L_{L2} + p_{L3} \cdot L_{L3} + p_{MC} \cdot L_{MC}$$

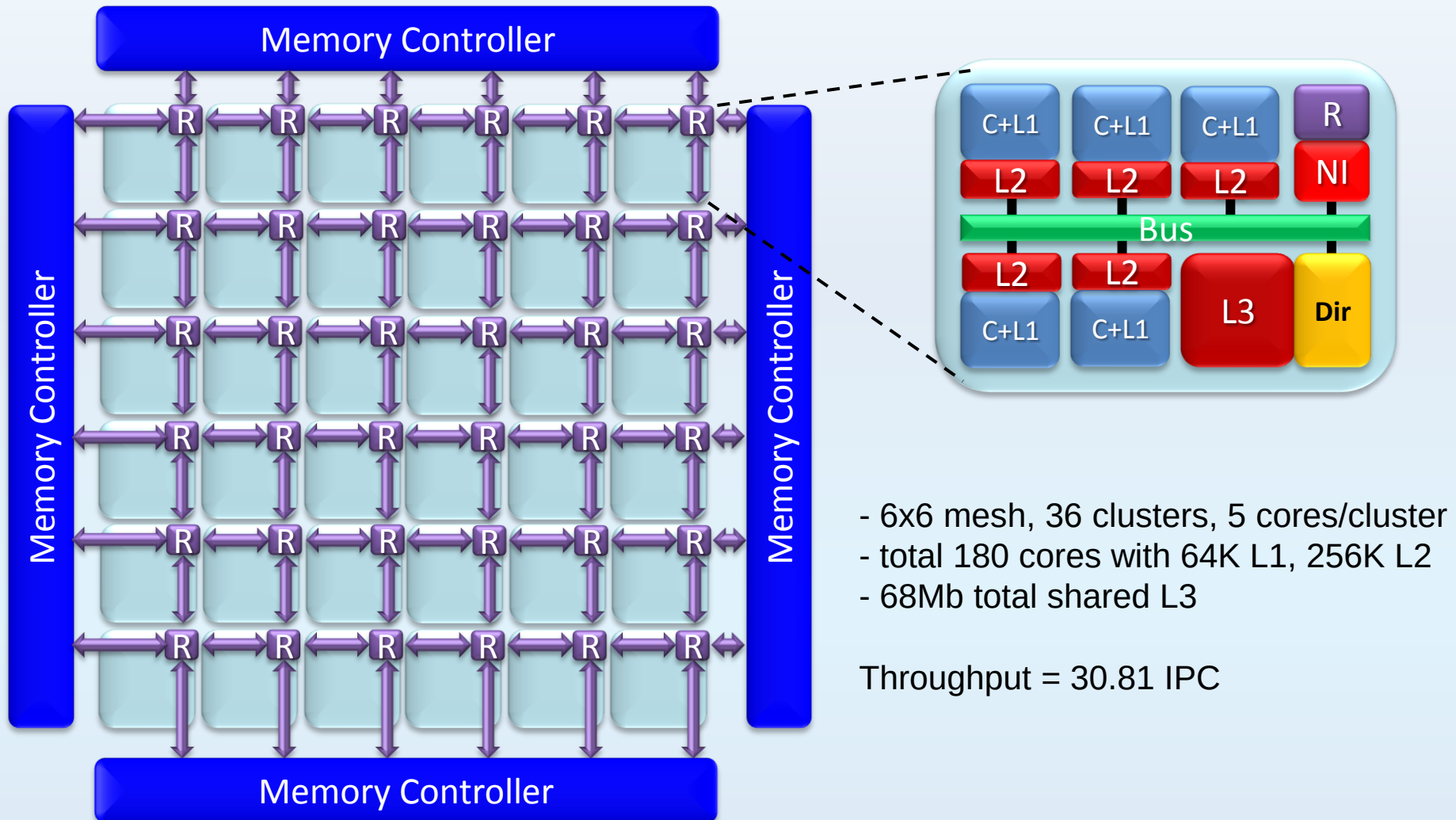
$$p_{L1} = Hit(L1)$$

$$p_{L2} = Miss(L1) \cdot Hit(L2)$$

$$p_{L3} = Miss(L1) \cdot Miss(L2) \cdot Hit(L3)$$

$$p_{MC} = Miss(L1) \cdot Miss(L2) \cdot Miss(L3) \cdot Hit(MC)$$

Best configuration



Runtime: Modeling vs Simulation

