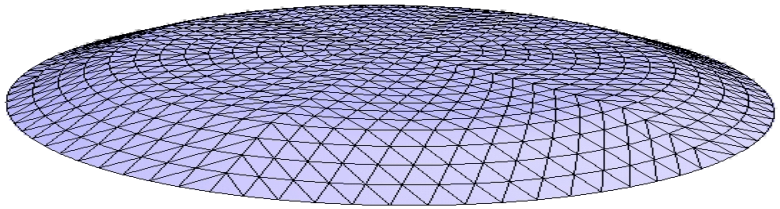

Laplace-Beltrami Eigenstuff Part 3 - Applications

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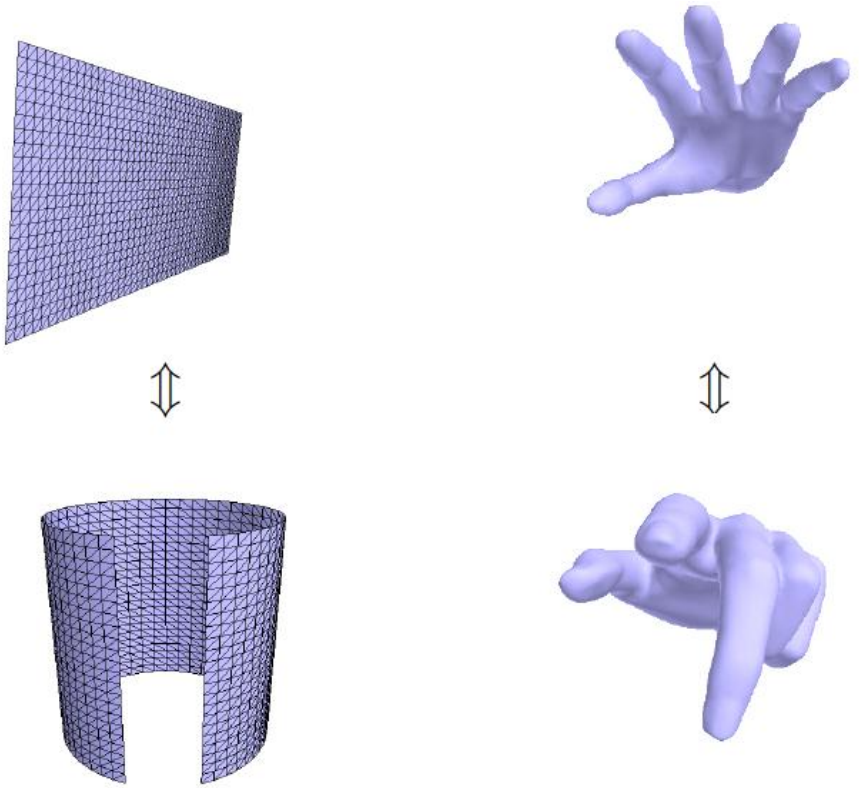
Outline

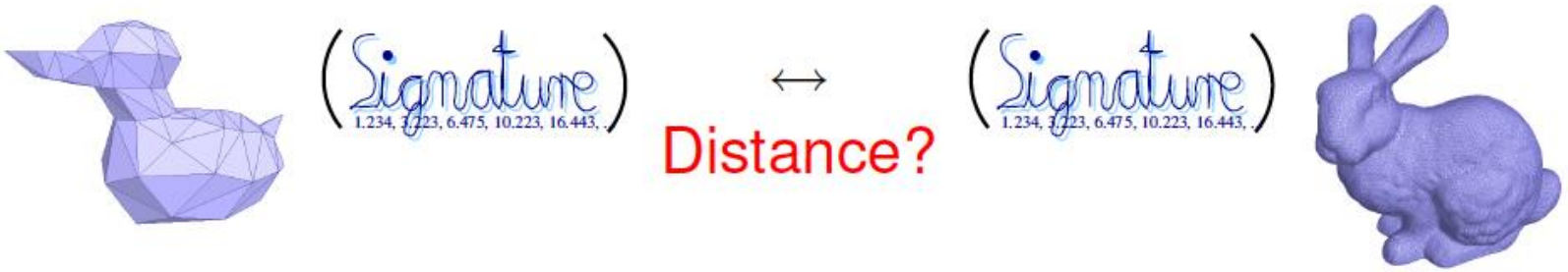
- Shape Analysis Background
- Database Retrieval
- Shape Segmentation
- Subcortical Structures

What is Shape and what is similar?

- Shape should be invariant with respect to:
  - Location (rotation, translation)
  - Size
  - Isometries?



Shape Matching



- Prior alignment, scaling of the objects:
  - normalization, registration
- Computation of a simplified representation
  - Signature, Shape-Descriptor
- Comparison of the signatures
  - distance computation to measure similarity
- Disadvantages of current methods:
  - Over-simplification, missing invariance, complex pre-processing, difficult to compare signatures, support only special representations

New Signature: ShapeDNA [spm05]

We use the (normed) n-dim vector of the **smallest n eigenvalues** $(\lambda_1, \ldots, \lambda_n)$ of the Laplace operator Δ as the signature:

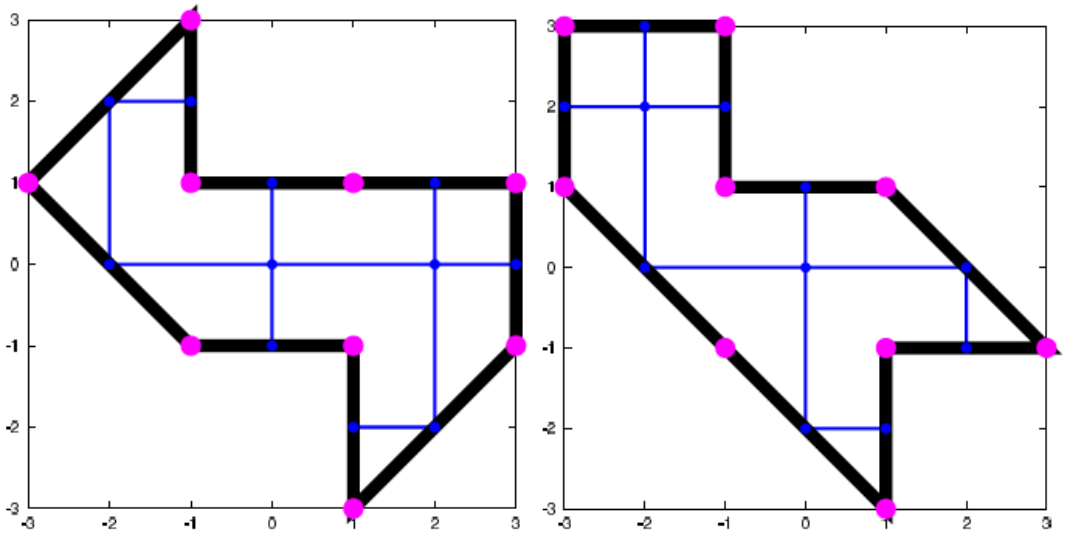
- Isometry invariant $\Rightarrow$ location invariant
- and (where required) scaling invariant
- No registration necessary
- Surfaces & solids (even with cavities)
- Independent of representation
- Simple distance computation of the signature vectors
- Efficient and highly accurate computation with cubic FEM

Can one hear Shape?

Answer

No! Isospectral drums exist (Gordon, Webb, Wolpert - 1992)

- rare
- concave in 2D
- only pairs

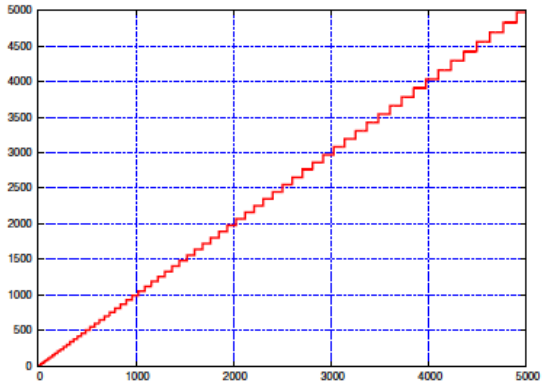


Geometry

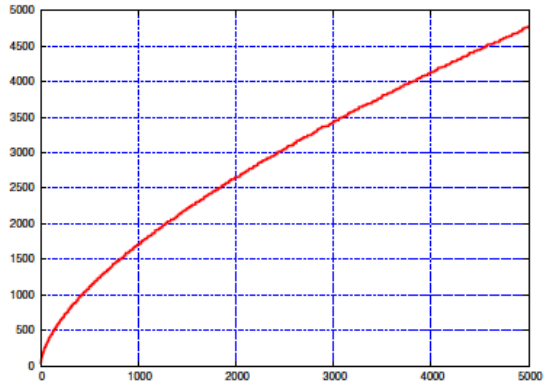
Nevertheless, they share area, boundary length, genus...

Weyl's Theorem

EW 2D-Sphere



EW 3D-Cube



Theorem (Weyl - 1911,1912)

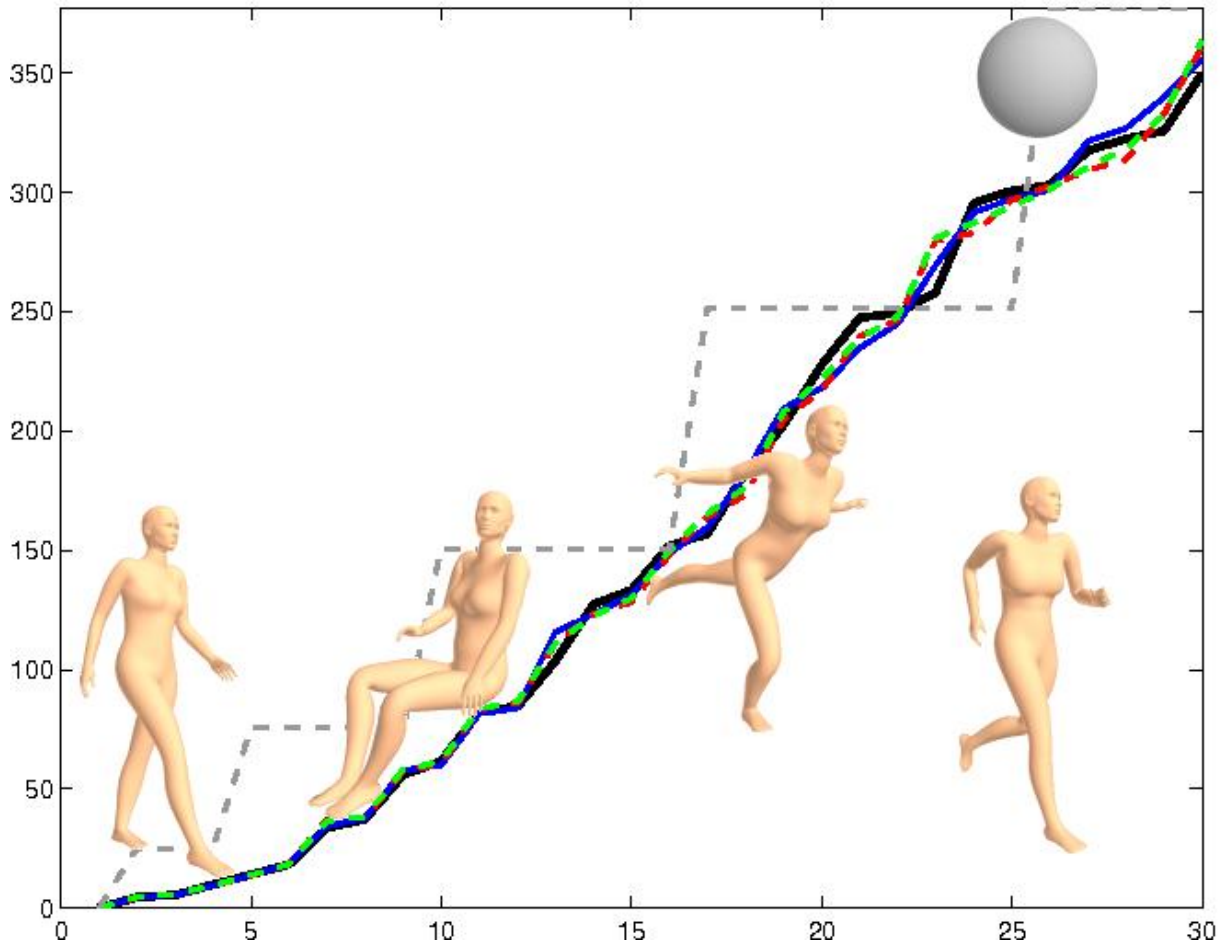
$$\lambda_n \sim \frac{4\pi}{\text{area}(D)}\, n \qquad \text{for } d = 2 \text{ and } n \to \infty$$

$$\lambda_n \sim \left(\frac{6\pi^2}{\text{vol}(D)}\right)^{\frac{2}{3}} n^{\frac{2}{3}} \qquad \text{for } d = 3 \text{ and } n \to \infty.$$

Heat Trace Expension

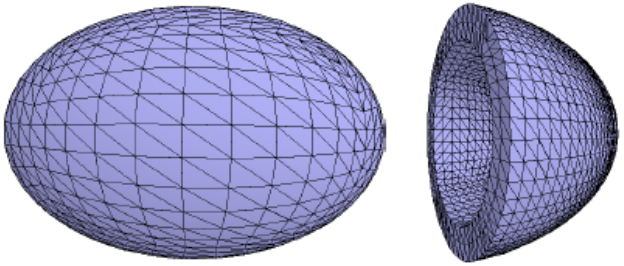
- More Geometric and Toplogical Information:
- Riemannian Volume
- Riemannian Volume of the Boundary
- Euler Characteristic for closed 2D Manifolds
- Number of holes for planar domains
- Possible to extract data numerically from beginning sequence [reuter:06] (500 Eigenvalues)

Isometry Invariance

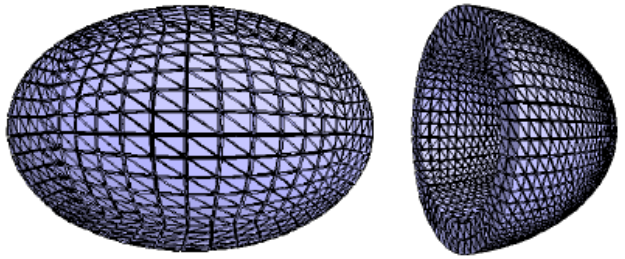


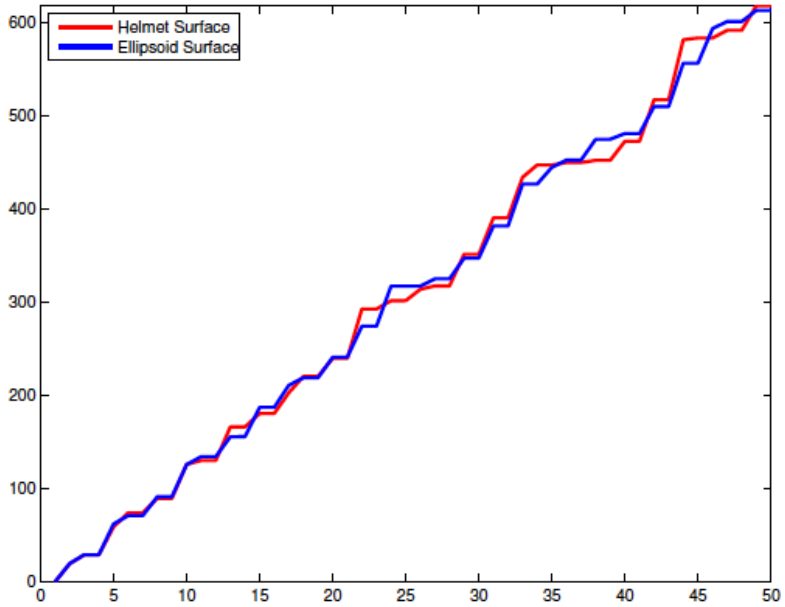
2D near isometry, 3D not

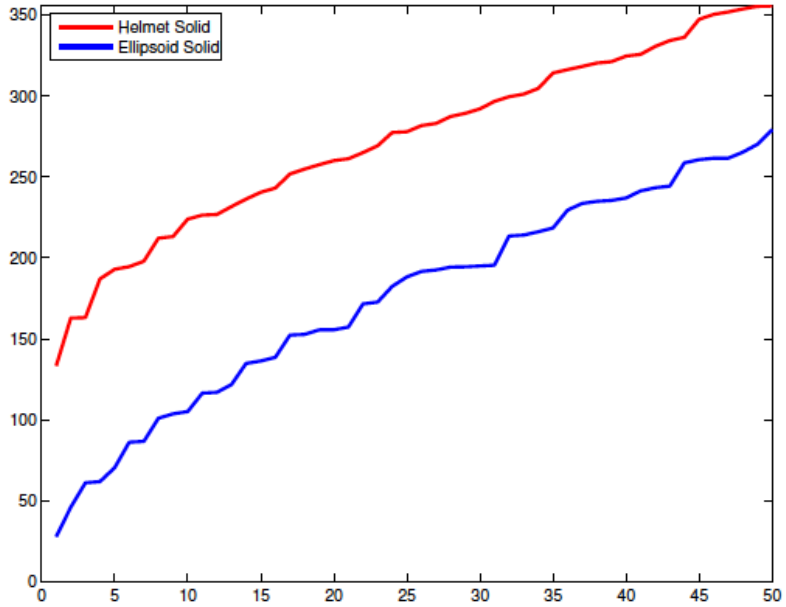
2D Surface ShapeDNA



3D Solid ShapeDNA

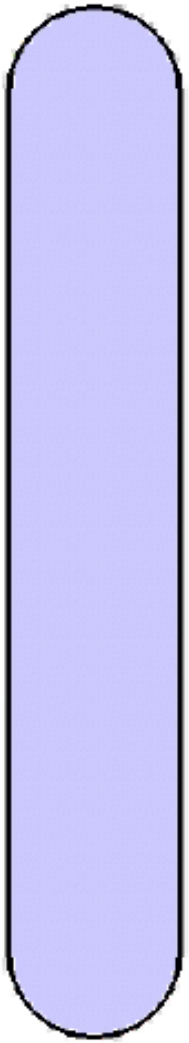


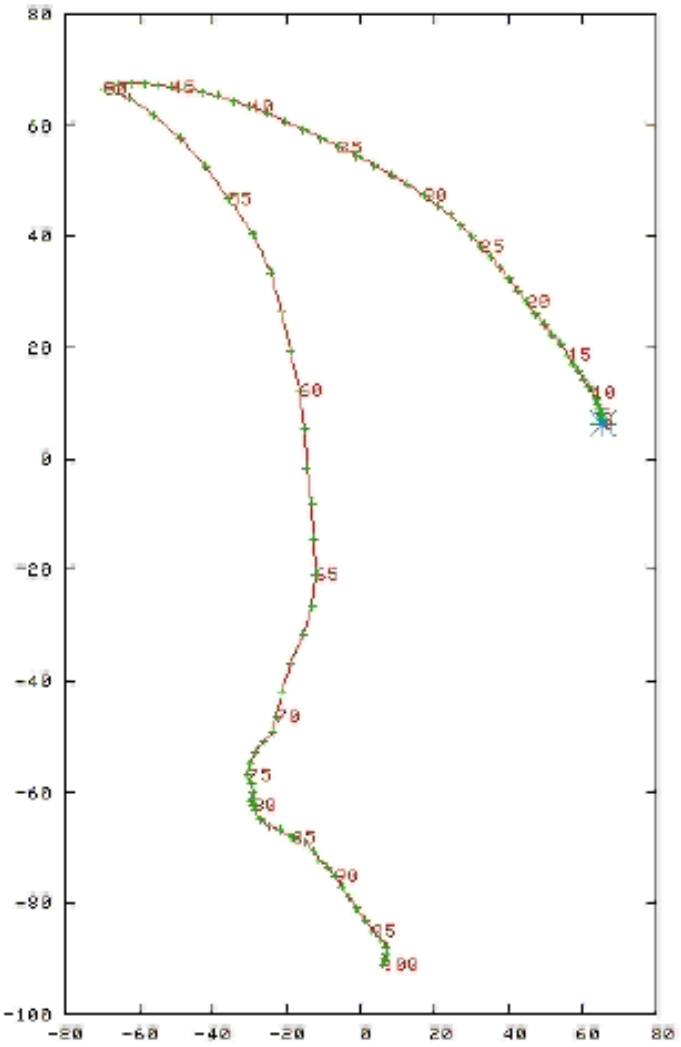




For solid bodies in $\mathbb{R}^3$ isometry is equivalent to congruency.

Continuous Shape Dependence

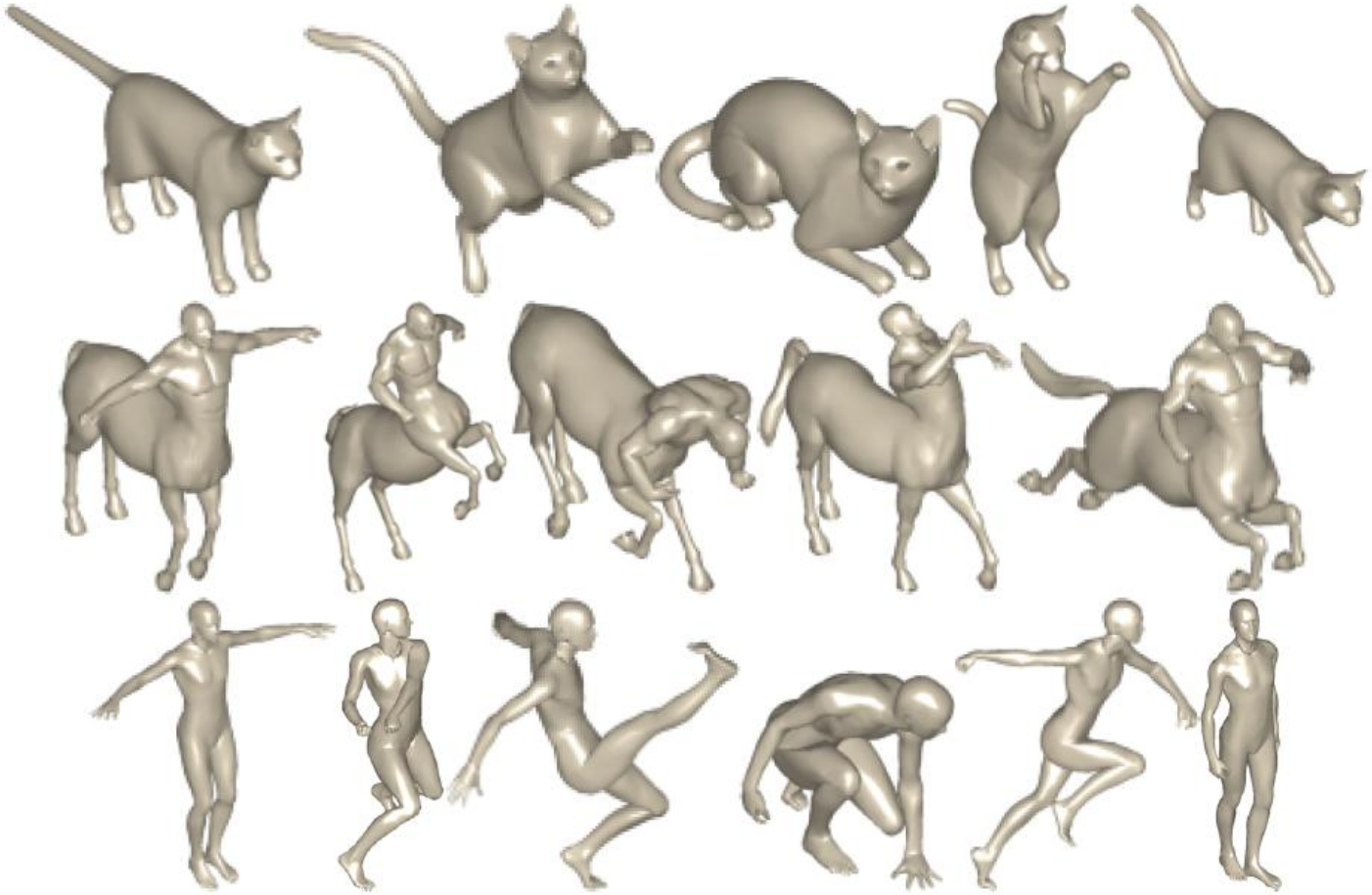




Database Retrieval

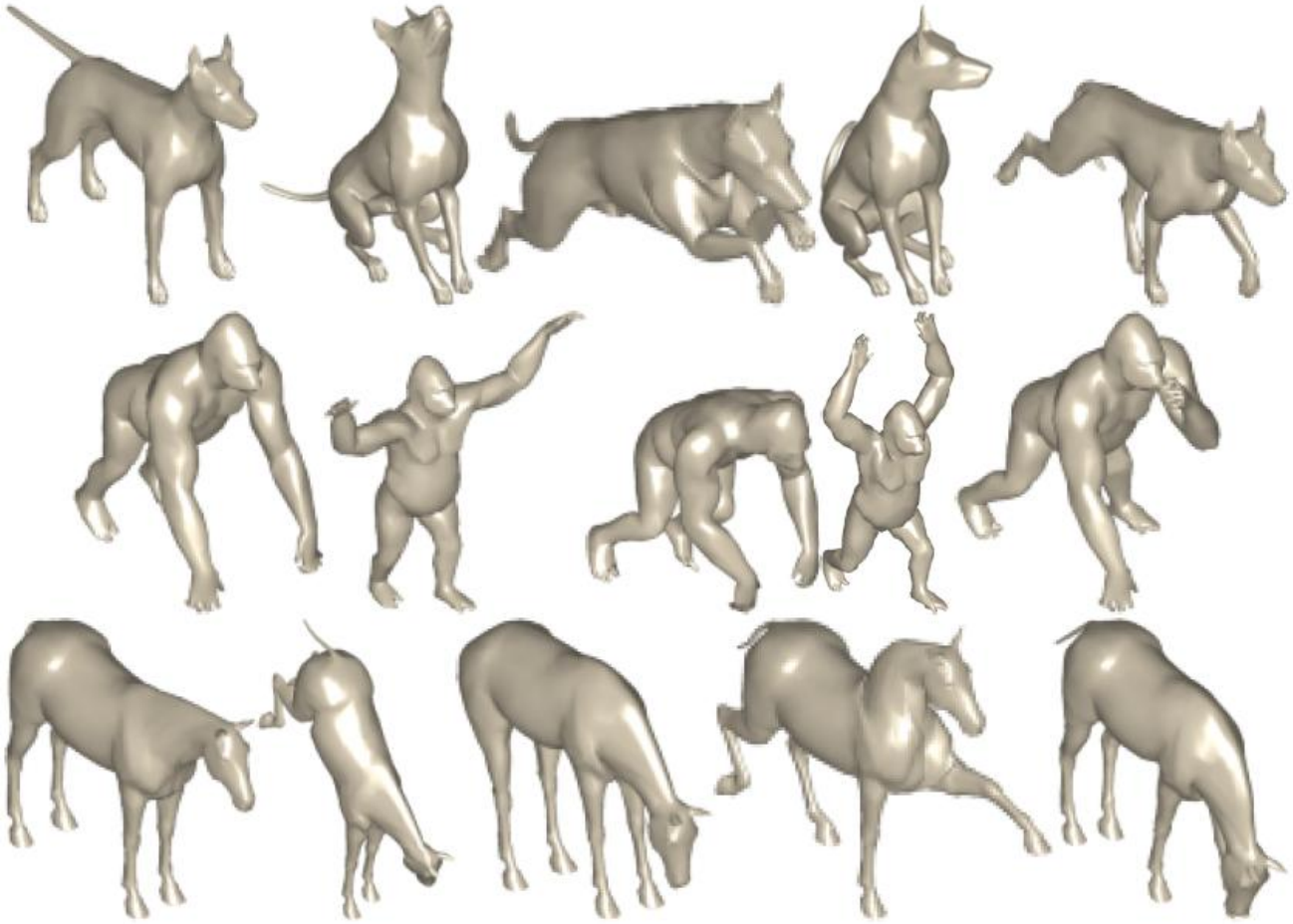
- 1. Computation of the first n Eigenvalues (Shape-DNA)
- 2. Normalization
  - a) Surface area normalized
  - b) Volume normalized
- 3. Distance computation of the Shape-DNA (n-dim vector)
  - a) Euclidean distance (!)
  - b) Another p-norm
  - c) Hausdorff distance
  - d) Correlation . . .

Nonrigid Shape Database (148)



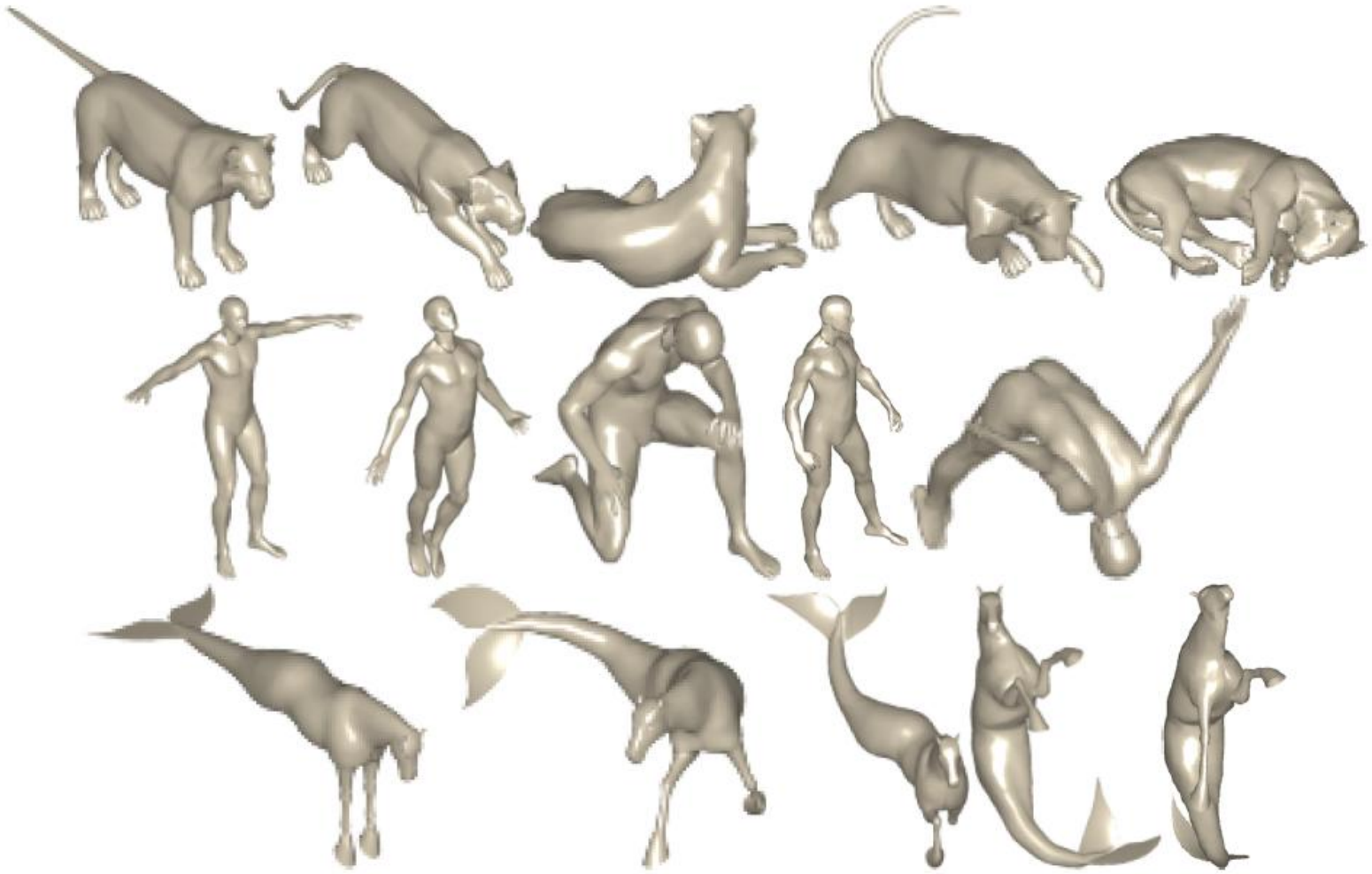
Courtesy of Bronstein, Bronstein, Kimmel, 2006

Nonrigid Shape Database (148)



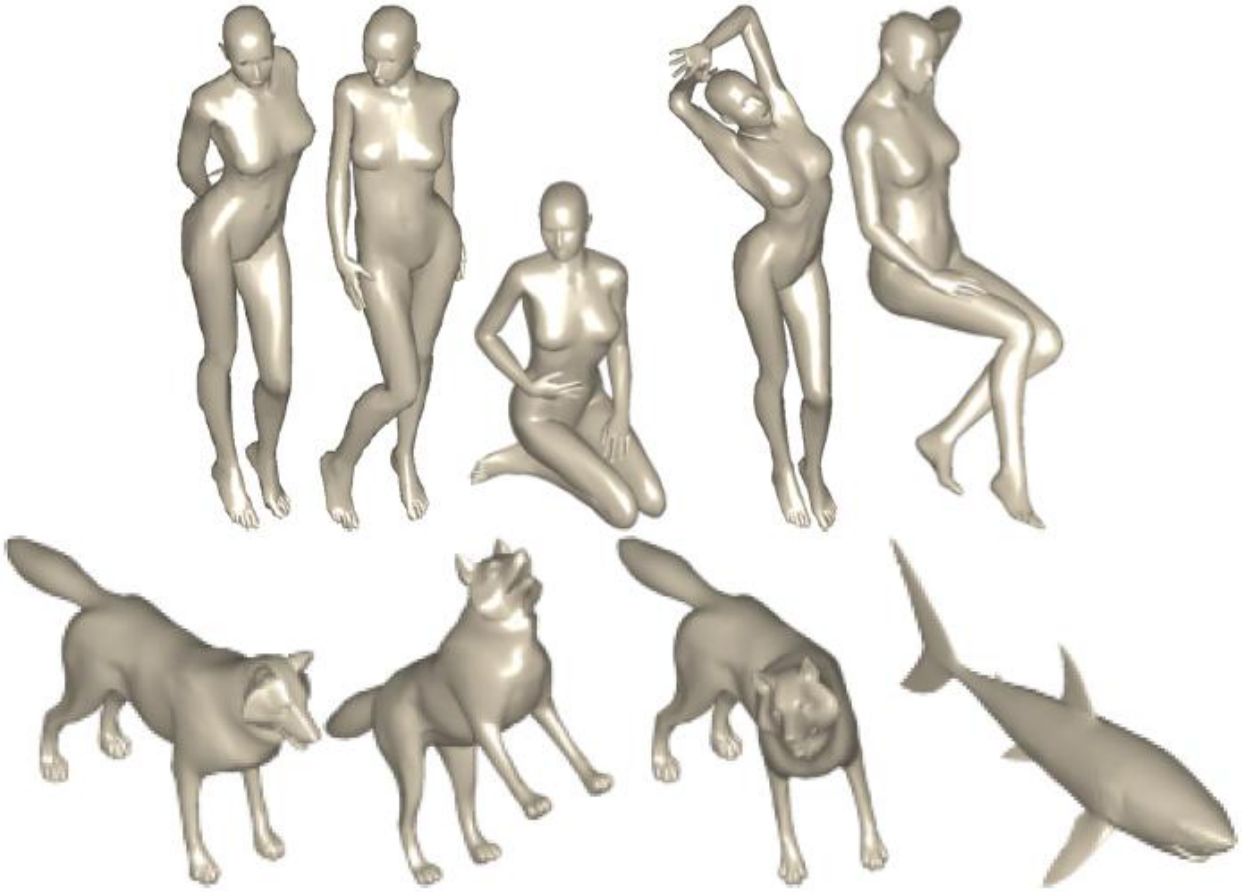
Courtesy of Bronstein, Bronstein, Kimmel, 2006

Nonrigid Shape Database (148)



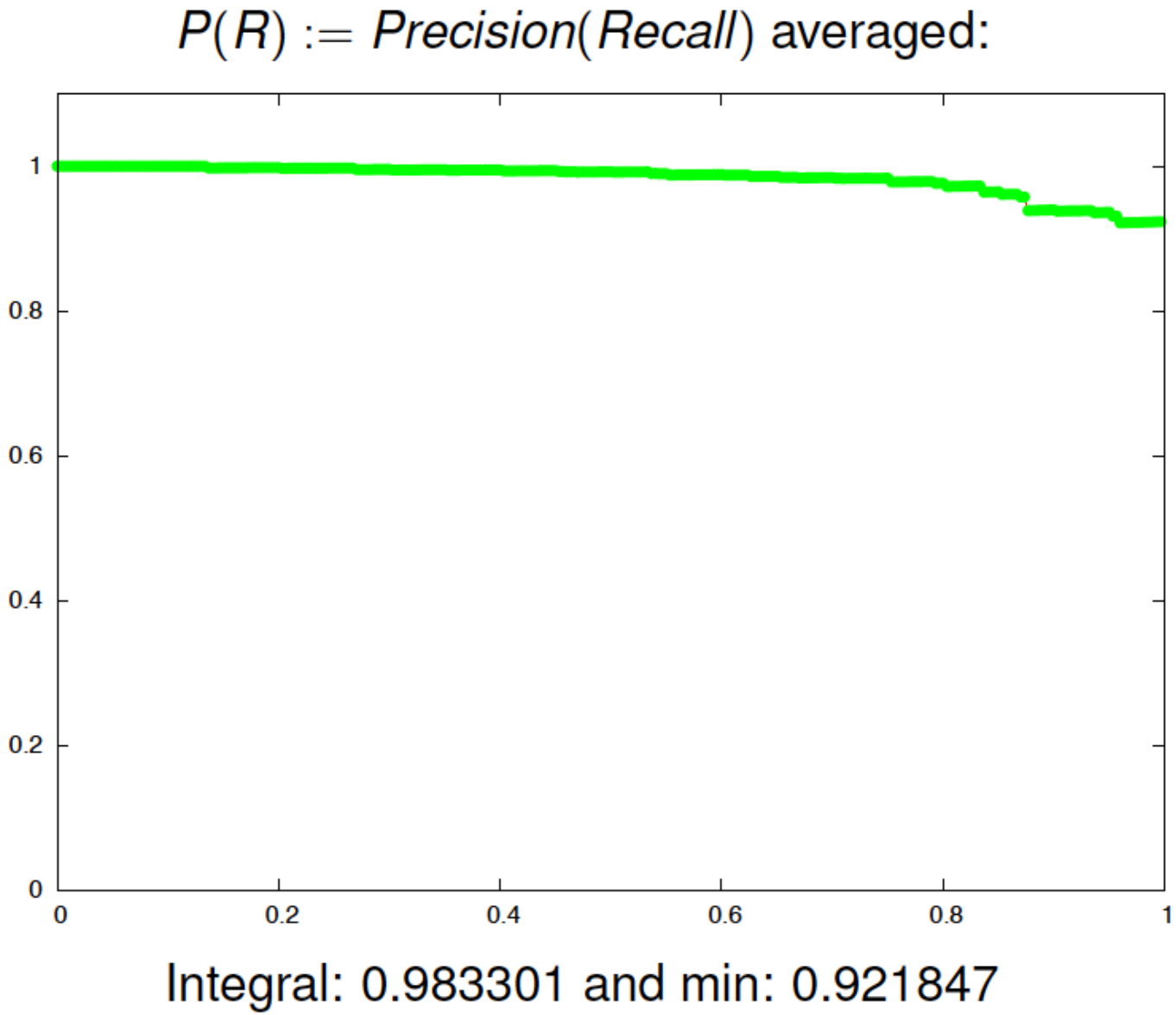
Courtesy of Bronstein, Bronstein, Kimmel, 2006

Nonrigid Shape Database (148)

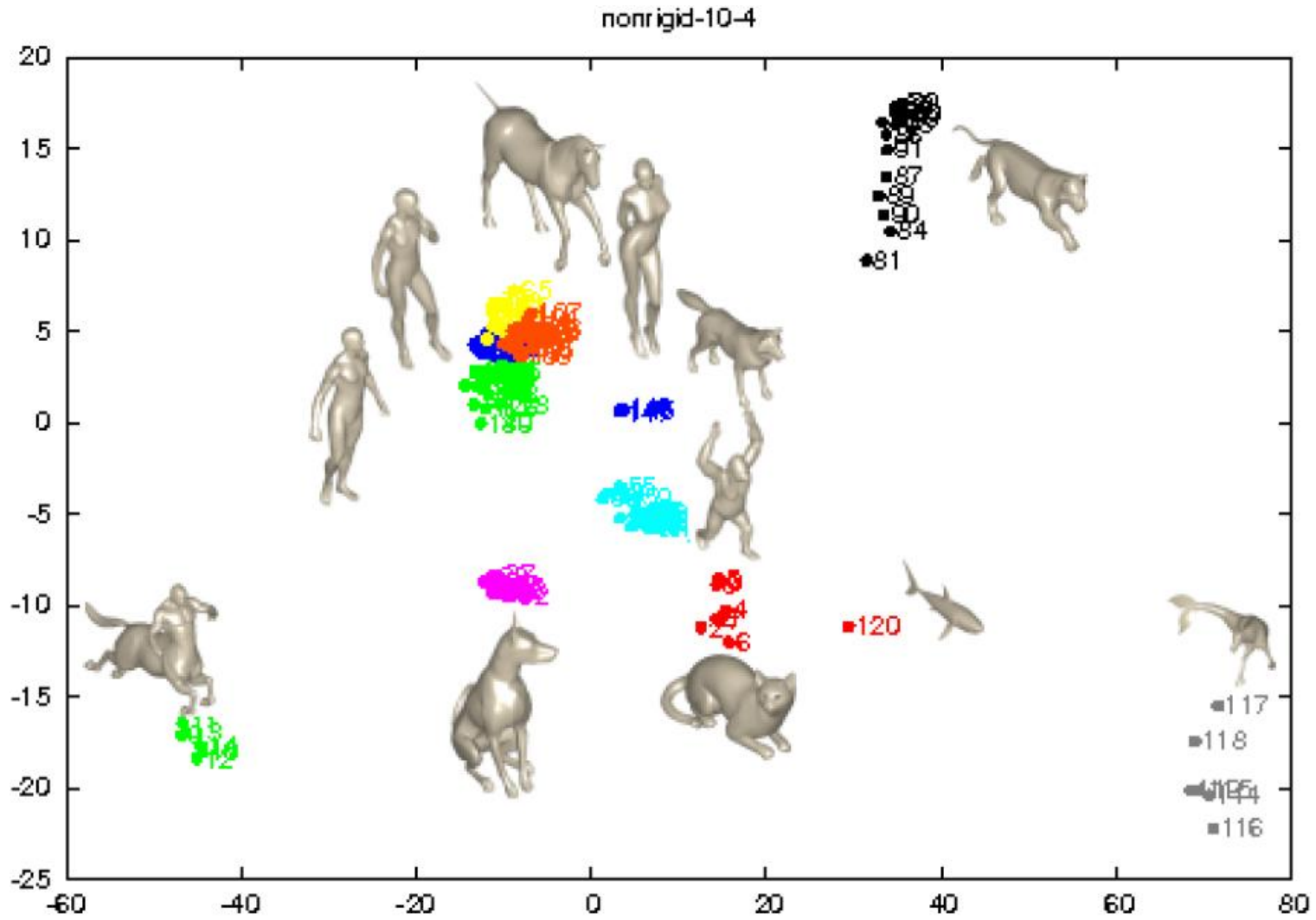


Courtesy of Bronstein, Bronstein, Kimmel, 2006

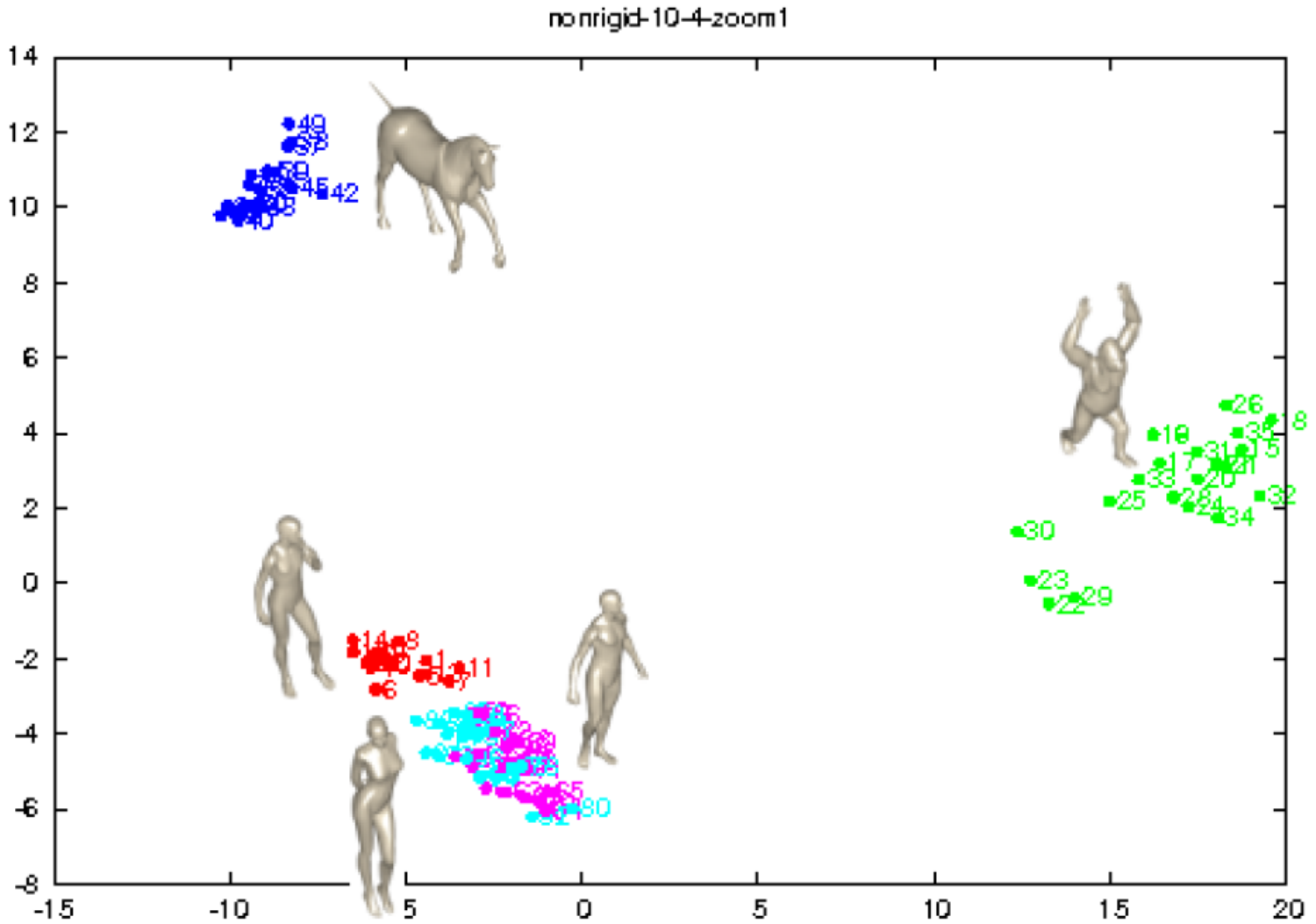
Nonrigid Shape Database (148)



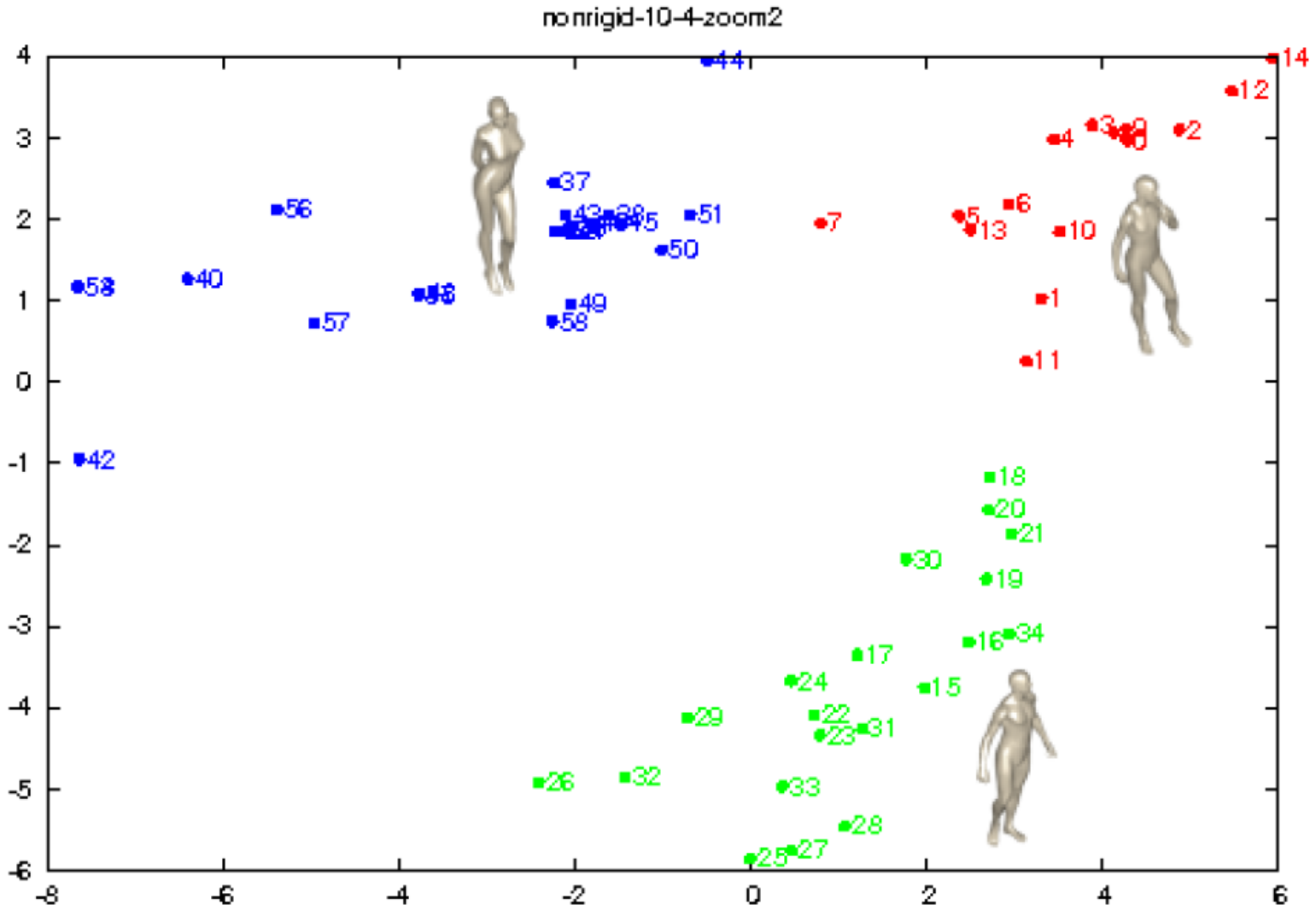
Nonrigid DB – MDS Plot



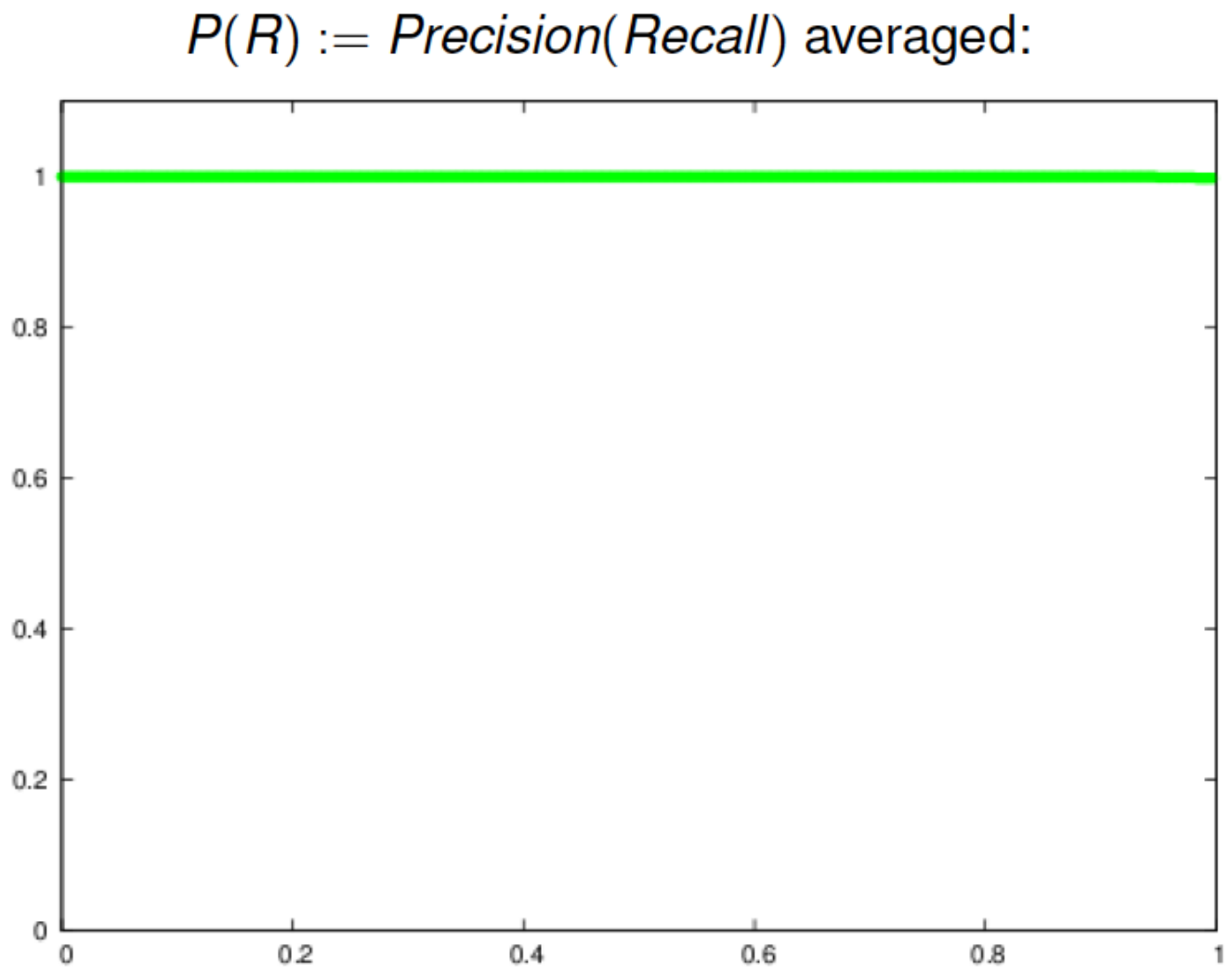
Nonrigid DB – Zoom 1



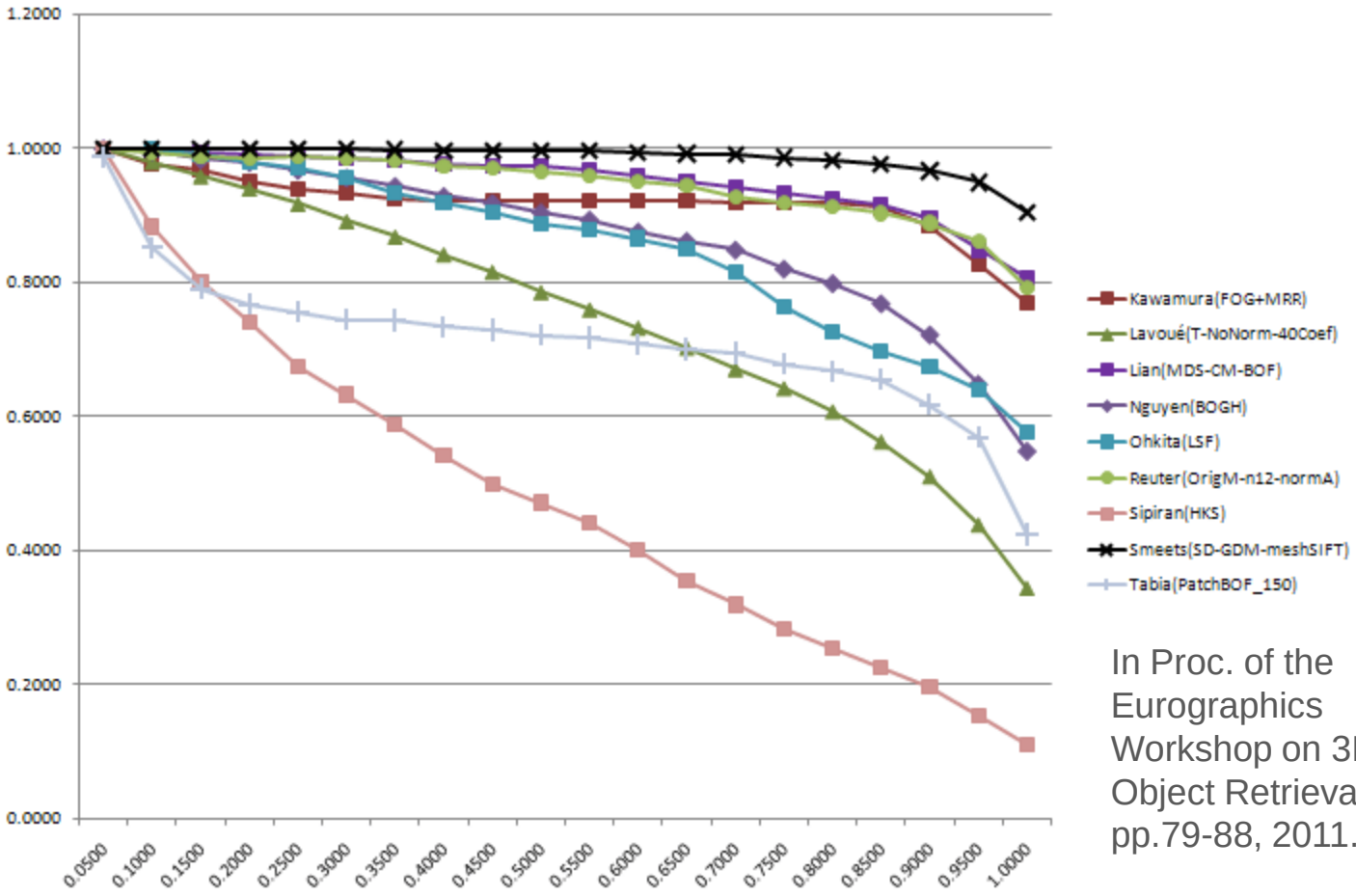
Nonrigid DB – Zoom 1



Nonrigid DB


$P(R) := Precision(Recall)$ averaged:
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
Integral: 0.999947 and min: 0.99829

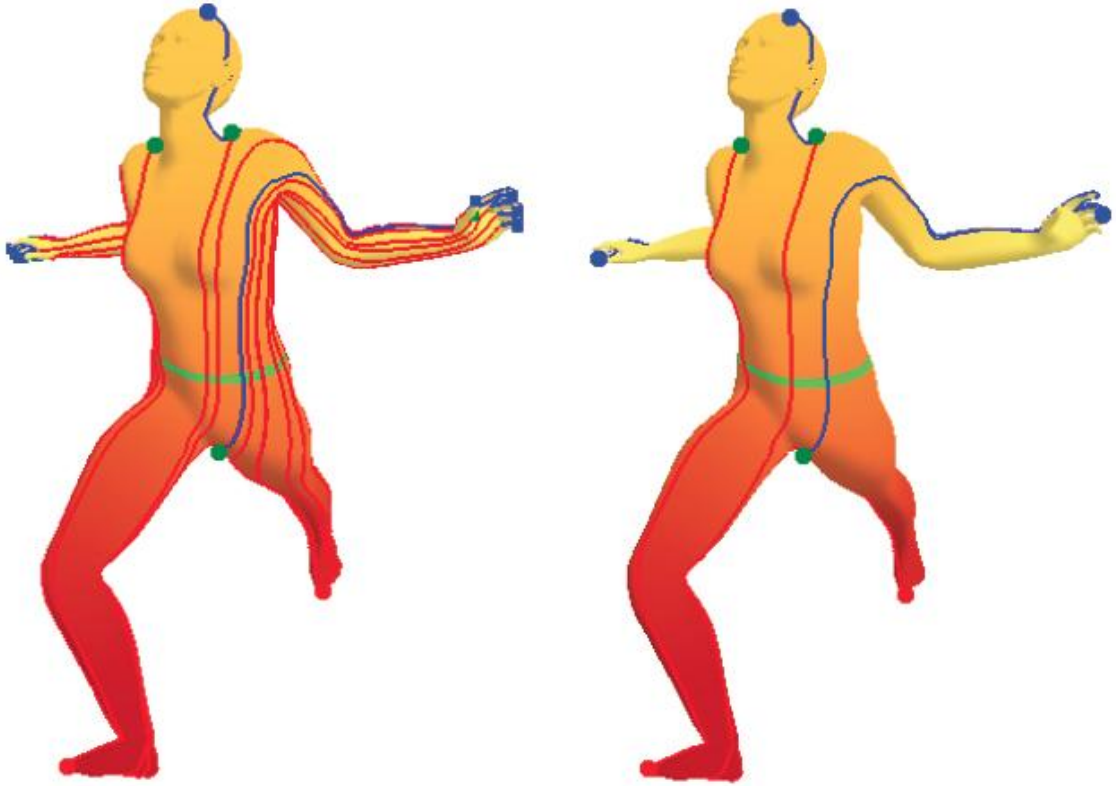
Shape Retrieval Contest 11 Non Rigid Track



In Proc. of the Eurographics Workshop on 3D Object Retrieval, pp.79-88, 2011.

Shape Segmentation

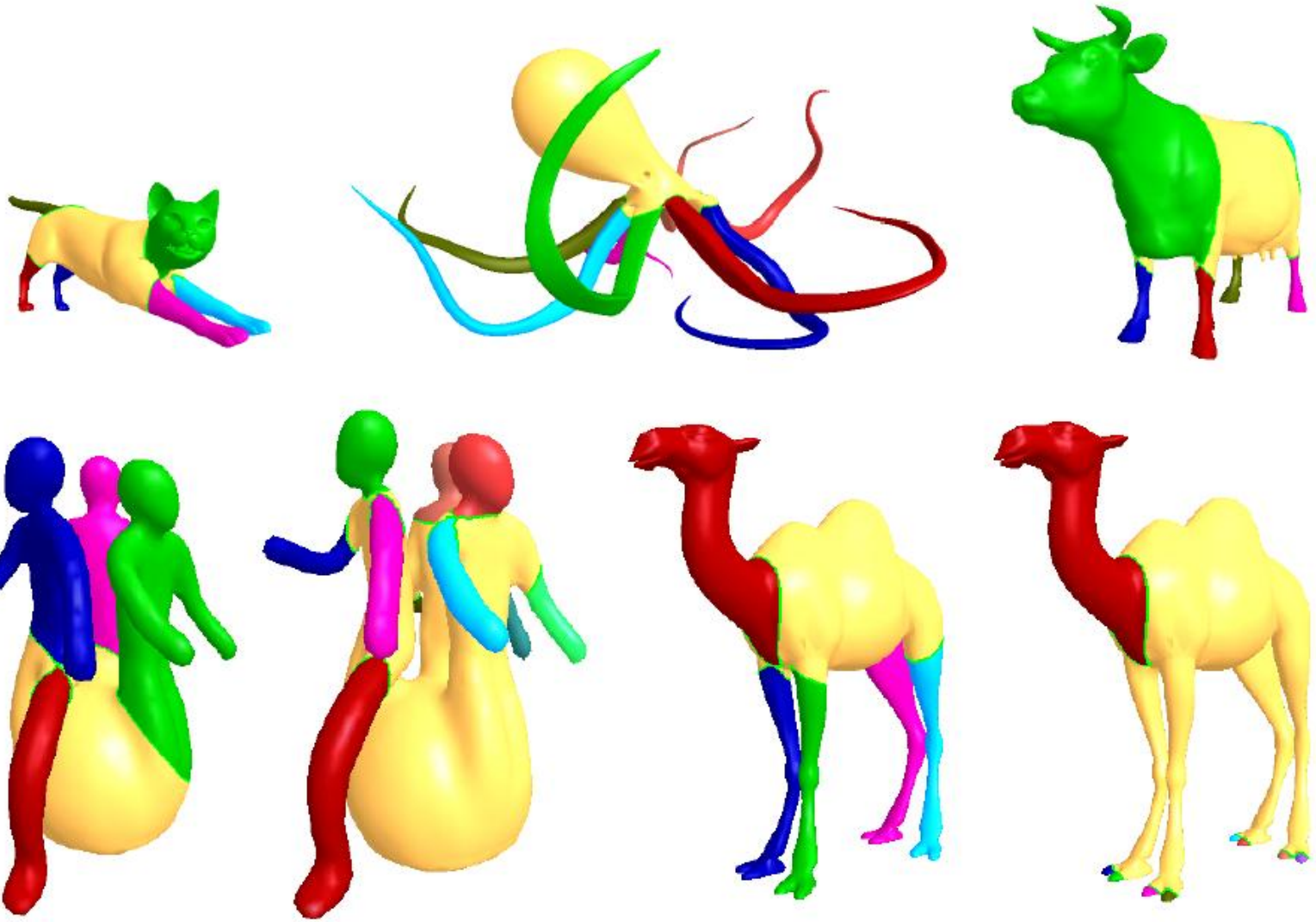
- Morse-Smale Complex of the 1st Eigenfunction
  - Left: full complex Right: simplified (3min,2max,3saddles)



Shape Segmentation

- Segmentation on different ‘persistence’ levels
  - Left: using only the most significant critical points
  - Right: close-up of hand using all (except noise)

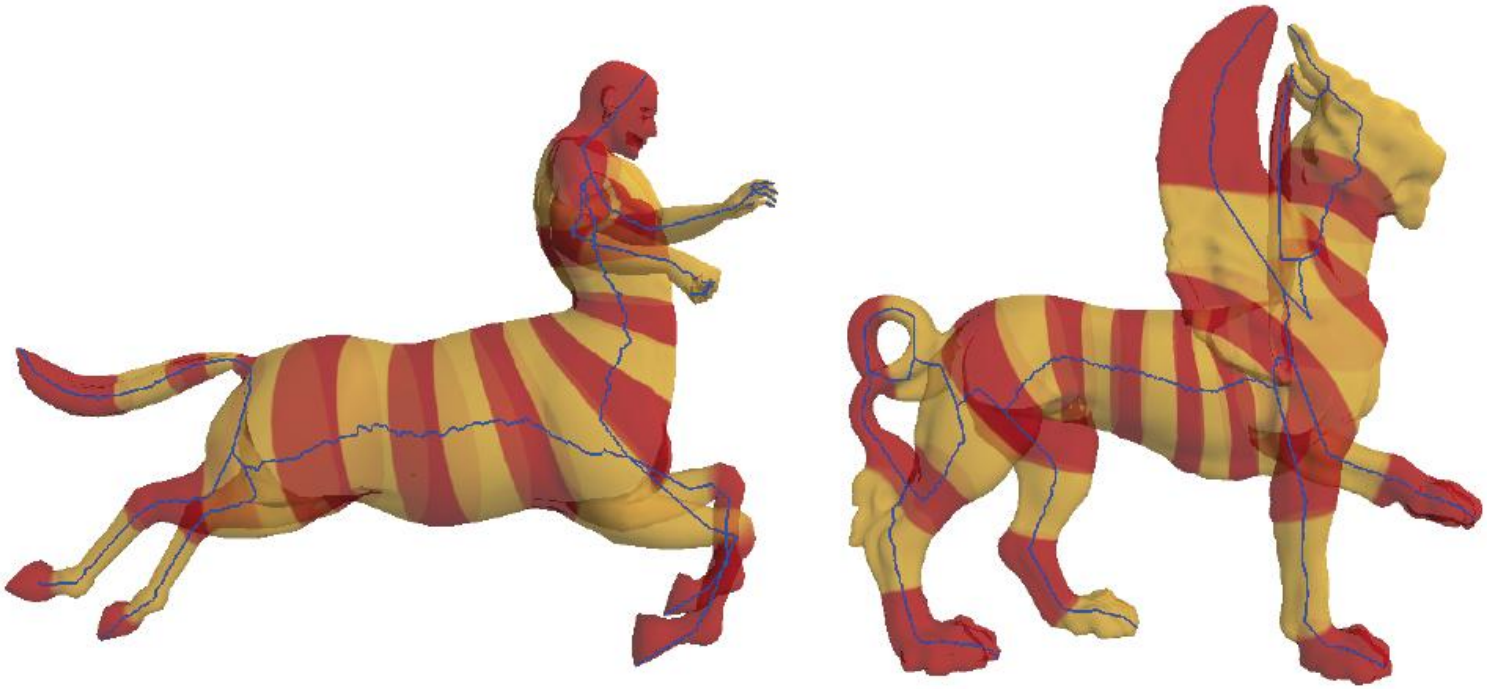
Hierarchical Segmentation



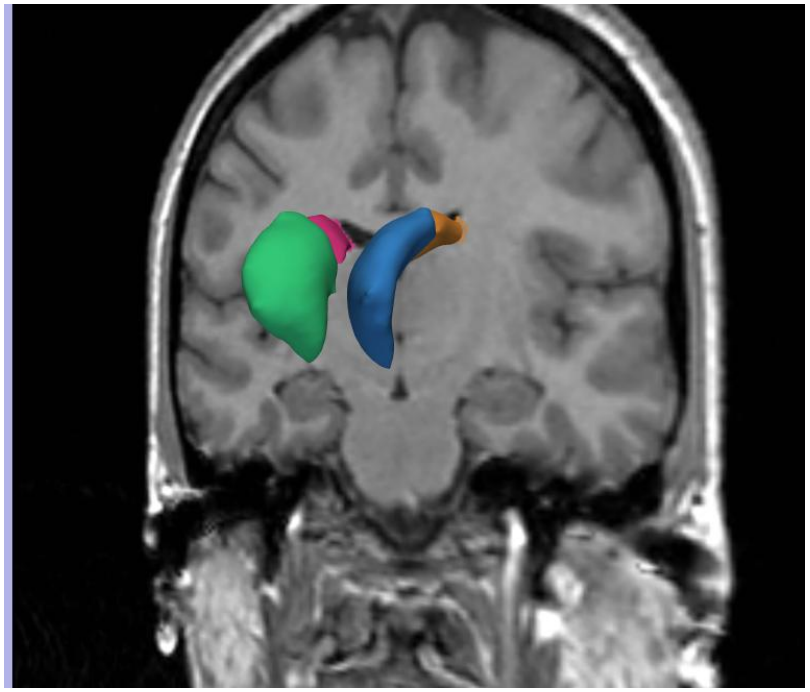
Consistent Segmentation and Registration

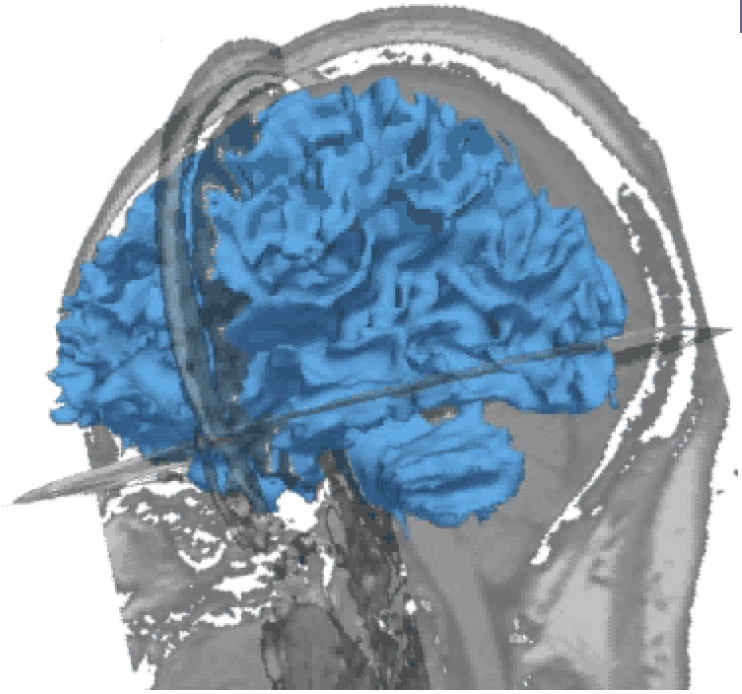
Future Directions

- Dense correspondence: Texture or Marker transfer, Surgical Planning
- Segmentation plus Skeleton: Pose Interpolation, Animation



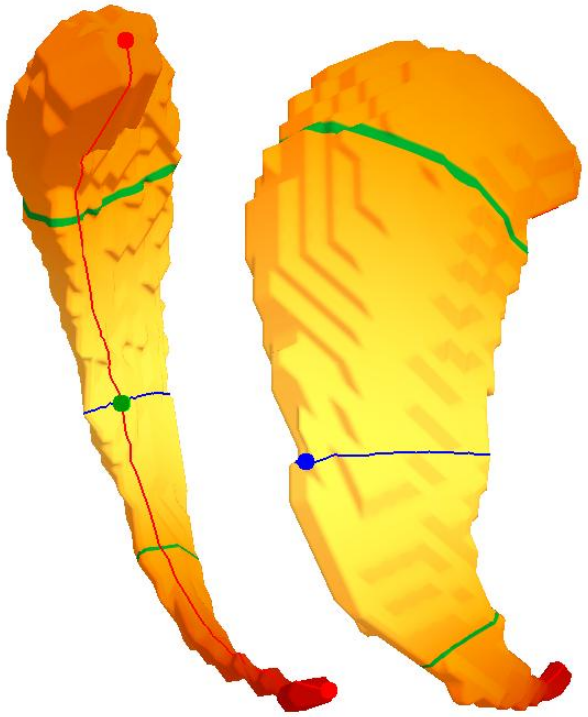
Caudate Nucleus





- Involved in memory function, emotion processing, and learning
  - Psychiatry Neuroimaging Lab (BWH - Martha Shenton)
  - Population: 32 Schizotypal Personality Disorder, 29 NC

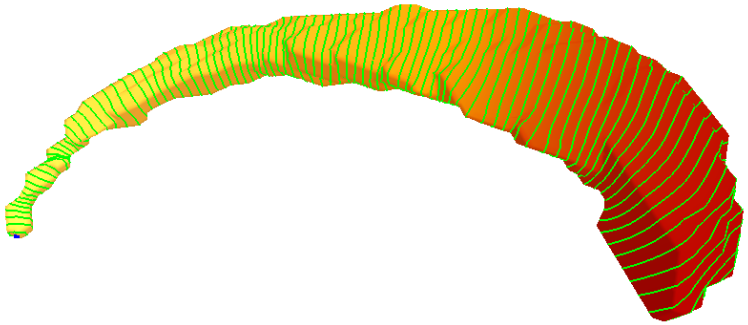
Shape Analysis Caudate

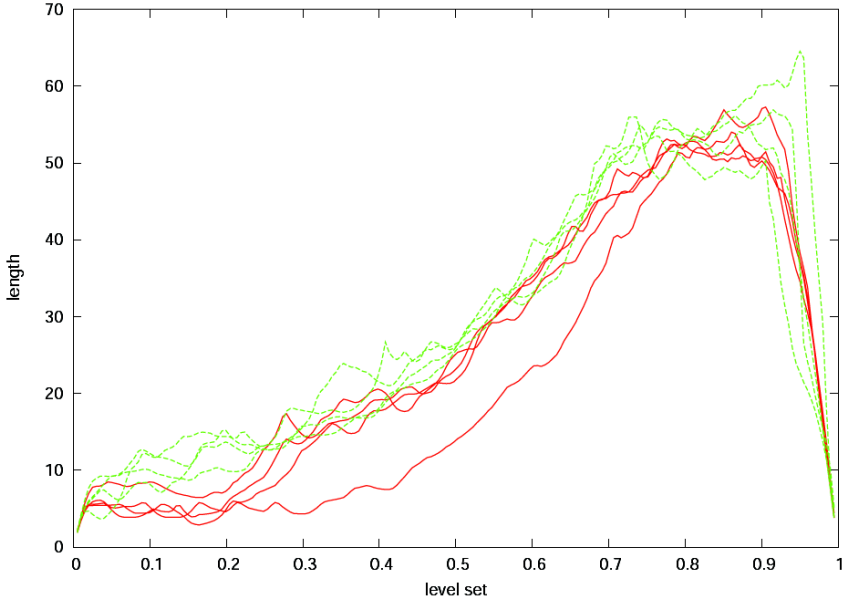


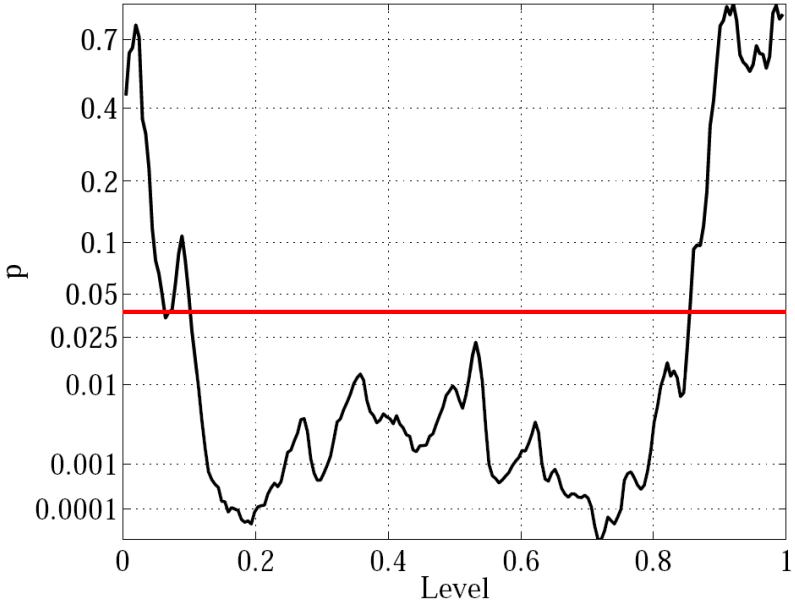
- Eigenfunction (EF): 2
- maxima at tips (red)
- minimum at outer rim (blue, middle)
- saddle at inner rim (green, left),
- integral lines (red and blue curve) run from the saddle to the extrema
- closed green curves denote the zero level sets

- (h) the head circumference (long green curve)
- (w) the waist circumference (blue curve)
- (t) the tail circumference (short green curve)
- (l) the length (red curve).

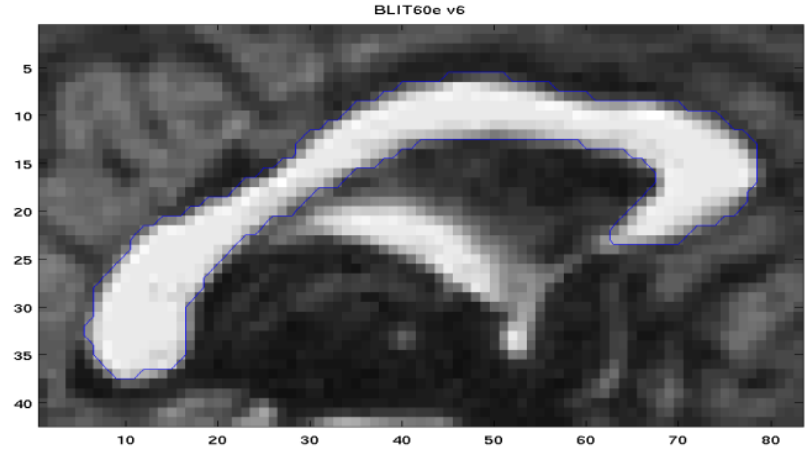
Shape Analysis Caudate

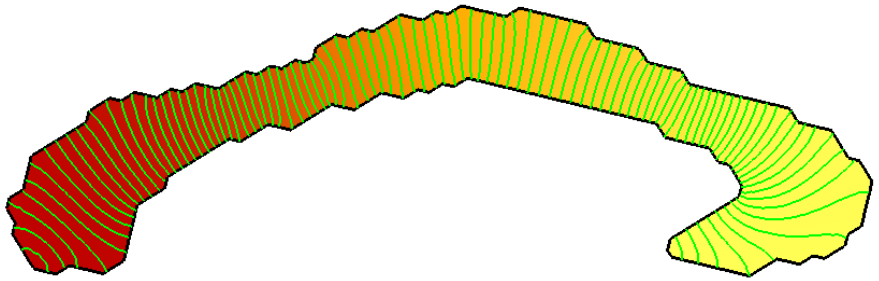


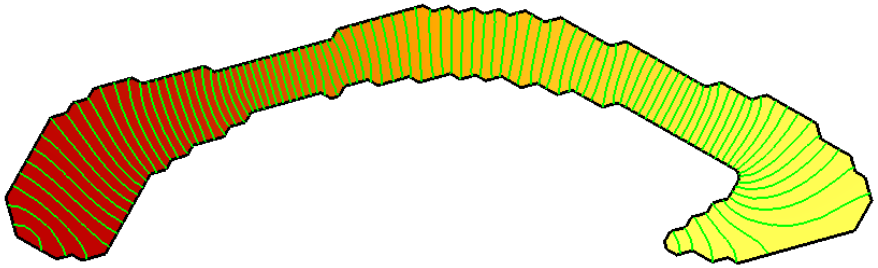


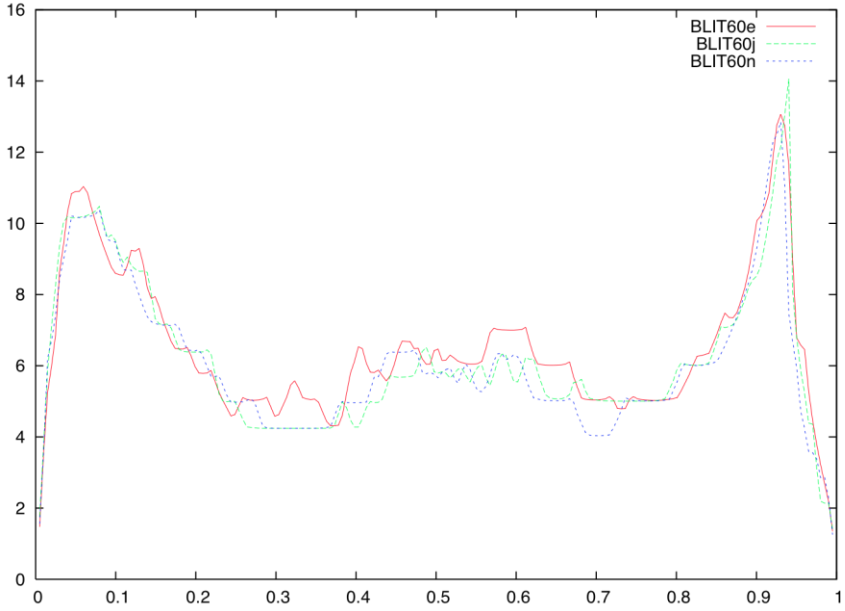


Shape Analysis Corpus Callosum









Thanks

Publications and Software:
http://reuter.mit.edu