Exercise 1, question 1

Exercise 1.

The purpose of this exercise is to give a visual and intuitive understanding of the Fisher discriminant ratio (eq. 12.1 in the book).

1. Consider a two group discriminant analysis with equal group sizes and two independent variables, X_1 and X_2. The centroid for the two groups are

   $\mathbf{x}_{g_1} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\mathbf{x}_{g_2} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$.

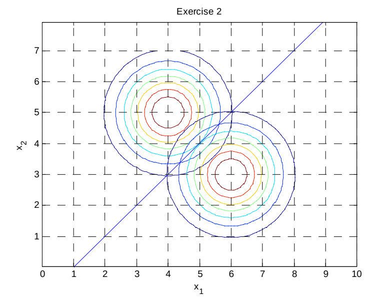
   The pooled within group covariance matrix is

   $\mathbf{C}_w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

   Please explain why a discriminant analysis of this data will return $\mathbf{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$**?**

   As a help plot the two groups on to a piece of paper.

Exercise 1, question 1



Exercise 1, question 2

2. Consider the following discriminant analysis problem. Sixty observations are drawn at random the population; the associated summary statistics are

   $\mathbf{x}_{g_1} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}$ and $\mathbf{x}_{g_2} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$

   $\mathbf{C}_w = \begin{bmatrix} 0.8 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}$

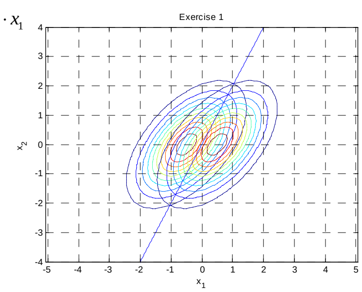
   A new observation, $\mathbf{x}' = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$, is drawn at random from the population.

   Based on the Mahalanobis approach, find the locus of points equidistant from the two group centroids? On what side of the locus is x' located?

Exercise 1, question 2

- Mahalanobis' approach:

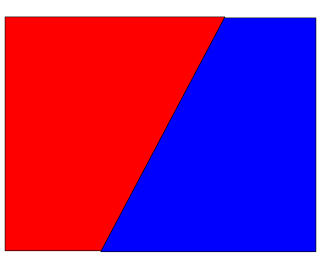
$x_2 = 2 \cdot x_1$



Exercise 1

- Mahalanobis' approach:

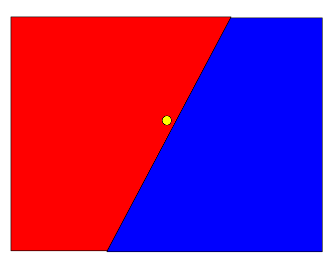
$x_2 = 2 \cdot x_1$



Exercise 1

- Mahalanobis' approach:

$x_2 = 2 \cdot x_1$



Exercise 2

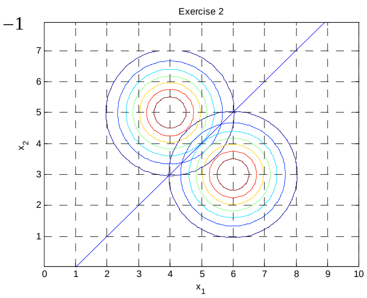
Based on the data in the above Exercise 1, question 1.

Please set up a (linear) classification scheme for new observations sampled from the population. Equal losses and equal a priori probabilities are assumed for the two groups (sec. 12.4.4 or the lecture notes).

Exercise 2

- Bayesian approach:

$x_2 = x_1 - 1$



Exercise 3

Consider a two class discriminant analysis with equal group sizes and two independent variables, X_1 and X_2. The centroid for the two groups are

$$\mathbf{x}_{\pi_1} = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \text{ and } \mathbf{x}_{\pi_2} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}.$$

The within group covariance matrices are

$$\mathbf{C}_{w,1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and. } \mathbf{C}_{w,2} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

The inverse of the covariance-matrices are

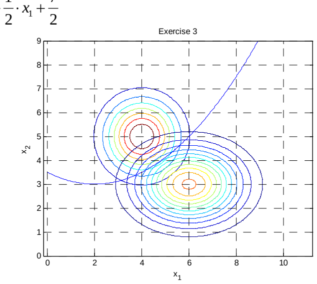
$$\mathbf{C}_{w,1}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{C}_{w,2}^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}.$$

Please set up a (Quadratic) classification scheme for new observations sampled from the population. Equal losses are assumed for each of the groups (sec. 12.4.4 or the lecture notes)

Exercise 3

- Bayesian approach:

$$x_2 = \frac{1}{8} \cdot x_1^2 - \frac{1}{2} \cdot x_1 + \frac{7}{2}$$



Exercise 3

- Verification: Mahalanobis approach

$$x_2 = \frac{1}{8} \cdot x_1^2 - \frac{1}{2} \cdot x_1 + \frac{7}{2}$$