Assignment in Generalized Set Partitioning

Assignment 1 in 42134

13 September 2010

This is the first assignment in Advanced Topics in Operations Research (42134). The assignment must be handed in on an individual basis. Deadline is 15:00 on the 14 October 2010.

1 The Meetings Planning Problem

In a company the ten directors is going to attend nine different meetings. These meetings consider different aspects in the important discussions relating to the new company strategy that at the end of the year must be approved by the share holders. Each meeting needs the attendance of some (but not all) of the directors as can be seen from the table below:

Meetings	Needs presence of director
M_1	1, 4, 6
M_2	2, 5, 7, 10
M_3	7, 8
M_4	2, 3, 8, 9
M_5	1, 6, 8
M_6	2, 3, 10
M_7	3, 4, 5
M_8	1, 9
M_9	5, 7, 8

Each meeting requires half a day and meetings that do not require any common directors can be held in parallel. In this instance of the problem the number of rooms is not a problem, there are amble meeting facilities available in the company headquarter.

1.1 Questions

1. Formulate the problem of finding the maximum number of these meetings that can be scheduled in one day. It should be formulated as a set packing problem.

2. Using the model solve the problem of scheduling the meetings. What is the optimal solution?

3. Discuss a strategy to find the smallest total time period necessary for holding all the meetings, using this set packing model more than once if necessary.

4. Describe how the general problem of minimizing the number of meeting days required can be described using column generation (branch-and-price), that is, describe the master problem, the pricing problem and the branching strategy.

2 Air Traffic Control

The air traffic control system manages all aircraft that fly the civil airspace, designs control sectors, manages flows between different airports, and ensures separation between aircraft during their flight, takeoffs and landings. In the US alone around 26,000 people are engaged in some phase of the air traffic control.

This exercise focuses on the part of air traffic control called traffic management adviser (TMA). Each airport has its own TMA. The TMA is basically responsible for directing incoming flights to the designated runways (part of this is also to assign a runway to an aircraft) and to guide leaving aircraft out of the airspace of the airport and hand it over to the regional or national air traffic control.

In the simplified world of this assignment a set of incoming aircraft is given (we do not consider the leaving aircraft in this assignment). We now need to assign the aircraft to one of the available runways in order to minimize the accumulated delay of all aircraft, and at the same time sequence the aircraft.

For each aircraft we have an estimated time of arrival (ETA) to each of the runways. This time is the earliest time the aircraft can get to the runway.

In addition, for safety reasons a separation rule is enforced for all aircraft landing on the same runway. In the real world there could also be separation rules between runways but we will ignore that in this assignment. The separation rule is based on maintaining a minimum separation time between a leading and a trailing aircraft. Aircraft are classified as being: heavy (H), large (L) or small (S). The separation rule basically defines a given amount of time that has to elapse from the landing of the leading aircraft to the landing of the trailing aircraft.

When an aircraft is scheduled to a runway, a delay is defined as the difference between the scheduled time of arrival (STA) and the earliest time of arrival (ETA) among the runways for that flight.

A specific instance of this problem for 7 aircraft approaching a 3-runway airport could look like:

$$\begin{bmatrix} 10 & 11 & 10 \\ 9 & 11 & 12 \\ 7 & 9 & 8 \\ 10 & 13 & 11 \\ 9 & 12 & 10 \\ 8 & 7 & 6 \\ 7 & 7 & 7 \end{bmatrix}$$

Each row represents an aircraft and each column a runway. An entry represents the time in minutes it takes the aircraft from its current position to reach the runway, so eg. aircraft 1 could arrive at runway 1 in 10 minutes, at runway 2 in 11 minutes and at runway 3 in 10 minutes. If aircraft 1 is landing on runway 2 the delay is equal to 1 because it could have landed 1 minute earlier by using runway 1 (or 3) instead. If it lands after 13 minutes on

runway 3 (due to a queue of aircraft scheduled for landing at runway 3) consequently the delay is equal to 3 minutes.

The following vector describes the aircraft types:

$$[H, S, H, S, S, L, L]$$

Then we have the separation constraints:

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

The rows here denote the leading airplane, and the columns represent the trailing aircraft. Both dimensions are organized in the order of Heavy, Large and Small aircraft types. A small aircraft following a heavy aircraft, for example, requires 4 minutes.

2.1 Questions

1. Model the problem as a generalized set partitioning problem. Describe the model: What is the natural interpretation of a column? What defines a row? etc. Describe how you would compute the cost of a column.

2. On the course homepage you can find a program that solves the generalized set partition problem. Generate the columns for the case above in a programming language of your own choice. Enter them into the program and solve the problem. State which aircraft are landing on which runway and the value of the objective function.

3. This problem is small enough for a total enumeration of all possibilities is feasible, but if the problem was larger it would be a challenge to solve using such a brute-force approach. In order to extend our approach to larger problems we would use column generation/branch-and-price. Our master problem would be the generalized set partitioning problem. We then need to have a subproblem for generating favorable columns. Given a dual vector α the pricing problem should return a column with negative reduced cost, if it exists and none otherwise. Can we model the subproblem as an Elementary Shortest Path problem with Resource Constraints?

   If yes, describe the set up: what does a node and an arc represent and how do we calculate costs. What resources are necessary? If no, why not. You may choose to assume the existence of a latest landing time for an aircraft if necessary.

3 Program instructions

The program is written in C for the gbar system. It should run there I do not guarantee that it will run anywhere else. Basically it encapsulates the solver CPLEX in a way that is suited for GSPP.

Having unziped the code you need to compile. This is simply done by writing `make`. Thereafter you should have a program called `assign`. The program needs three parameters in the given order:

1. The number of rows.

2. A data file specifying the type of constraint.

3. A data file with the columns.

All rhs are 1. The rows can either be set partitioning constraints ('='), set packing constraints ('$\leq$') or set covering constraints ('$\geq$'). A set partitioning constraint is specified by a s, a set packing constraint by a p and finally a set covering constraint by a c. So if I have 3 rows the first being a set covering constraint and the next two set partitioning constraints it will be specified by the file containing:

```
css
```

White spaces can be added for clarity, so

```
c s s
```

is equivalent to the first data file.

The data file that contains the columns has one column per line. Each line start with the cost (assumed to be integer) and then followed by the matrix coefficients in the order they appear in the column. Remember to finish with a newline after the last column.