



Operations Research for Telecommunication

– *Linear Programming and Network Routing*

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Operations Research (OR)

OR: Mathematics for optimal usage of limited resources. Since there are many expensive recourses in the telecommunication industry, there are good possibilities to do OR.



What is a Telecommunication network ???

- There are both wired and wireless networks (and satellite networks), we will only consider wired networks.
- A wired network, consists of a number of switches which are connected by wired connections:
 - ▶ Electrical cables
 - ▶ Optical cables (since 1980)
- The cables are passive whereas the switches actively *routes* the signals through the network.



Circuit switched or Packet switched

A network can be packet switched or circuit switched:

- The Internet is packet switched.
- The (old) telephone network is circuit switched

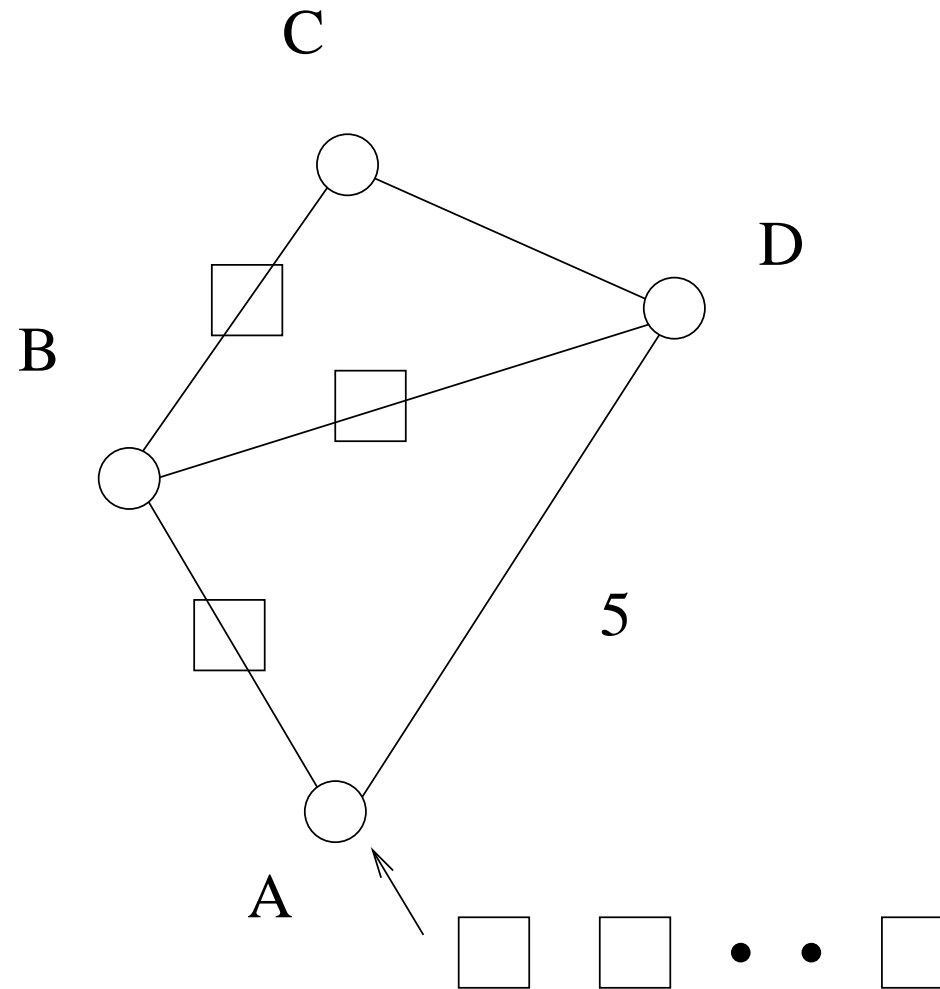


Packet Switched routing

In Packet switched networks, switches (nodes) forwards groups of data around the network. Packets going between the same two nodes may choose different routes. The by far most known network using this approach is the Internet !



Packet Switched routing





OSPF routing

Open Shortest Path First (OSPF): Each switch builds a routing table containing the shortest paths to all other switches in the network.

- The routing decisions are totally distributed (if one half of the planet is bombed, on the other half, the switches will reroute and continue to work)
- The distributed approach makes it difficult to control the network
- What do the switches then use to route the packets ? Assigned weights on the links



Why designed this way ?

The Internet was designed to be **robust**: It was designed to survive a nuclear attack:

- Routers will continuously attempt to update information about the world.
- Then the routers will perform shortest path routing
- Flow control is *distributed* among the switches, which can hence work (somewhat) independently ...
- Routing is *best effort*, i.e. if a packet cannot be forwarded it is simply dropped !



Unfortunately ...

Unfortunately it is not straight-forward to model the packet-switched approach ...

- It can be done, but the models becomes very complex ...
- Some routing can be optimized (Thorup et al)



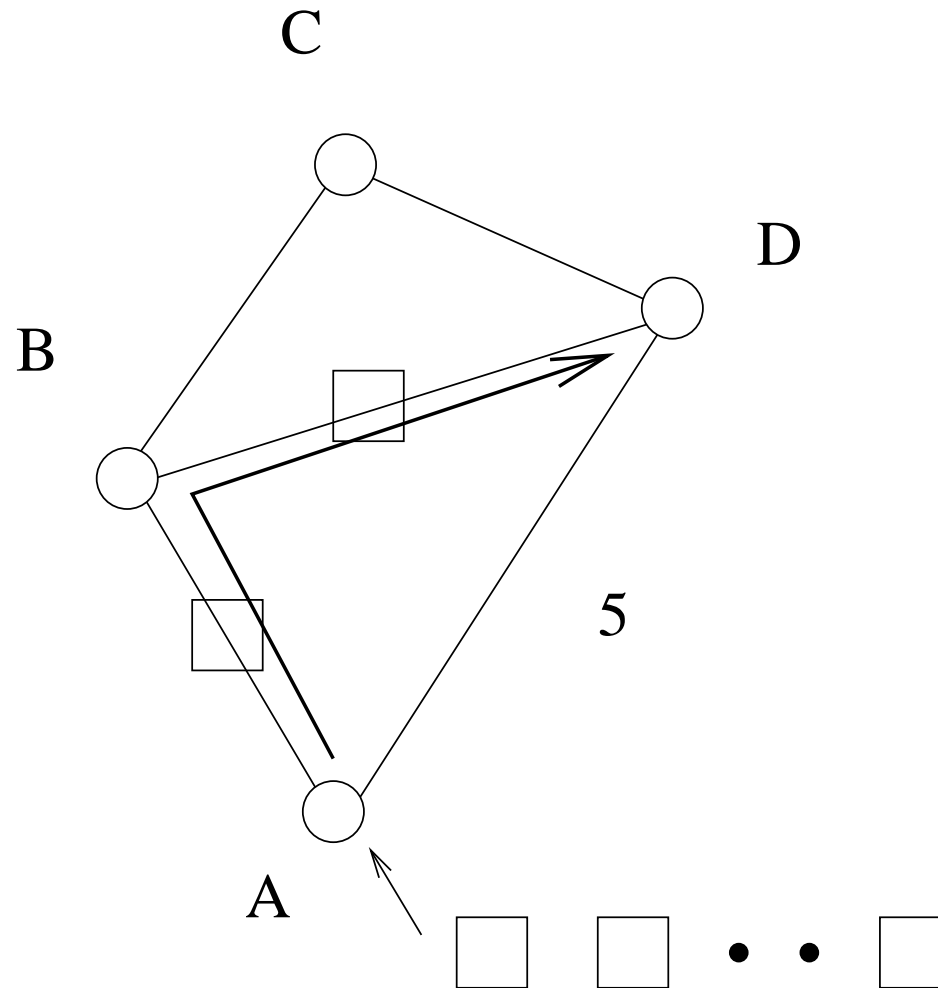
The circuit switched approach ?

First the obvious question: If the packet switched approach is in so widespread use, why deal at all with the old circuit switched approach at all ???

- Actually, most of the Internet (almost all) *are* using circuit switched networks ! The packets starts as packets but at later (in lower layers) routed on circuits
- The telecommunication companies love the control that the specific paths gives them.



Circuit Switched routing





So what is the planning problem ?

Given a network: Use it and expand it as economical as possible. Try to attract new revenue by getting new customers.

- Operational: Use the existing resources (the current network) as efficiently as possible. Set the prices correctly, and *route* the connections (circuits) as efficiently as possible.
- Strategic: Where to expand next ... (lecture in two weeks).



Operational

Assume you are the director of a telecommunication company:

- Your company are in the business of selling fixed broad band connections between different places in your network (e.g. different offices of companies).
- What are the critical issues for your company (assuming that you HAVE a network) ?



So what is the planning problem ?

How will you *utilize* your network ?

- How will you price a connection (e.g. how will you charge possible customers) ? (And is this connected to the routing ???)
- How will you route through the network ?
 - ▶ If you have lots of room in the network ?
 - ▶ If there starting to be bottlenecks ?
 - ▶ If it is congested ?



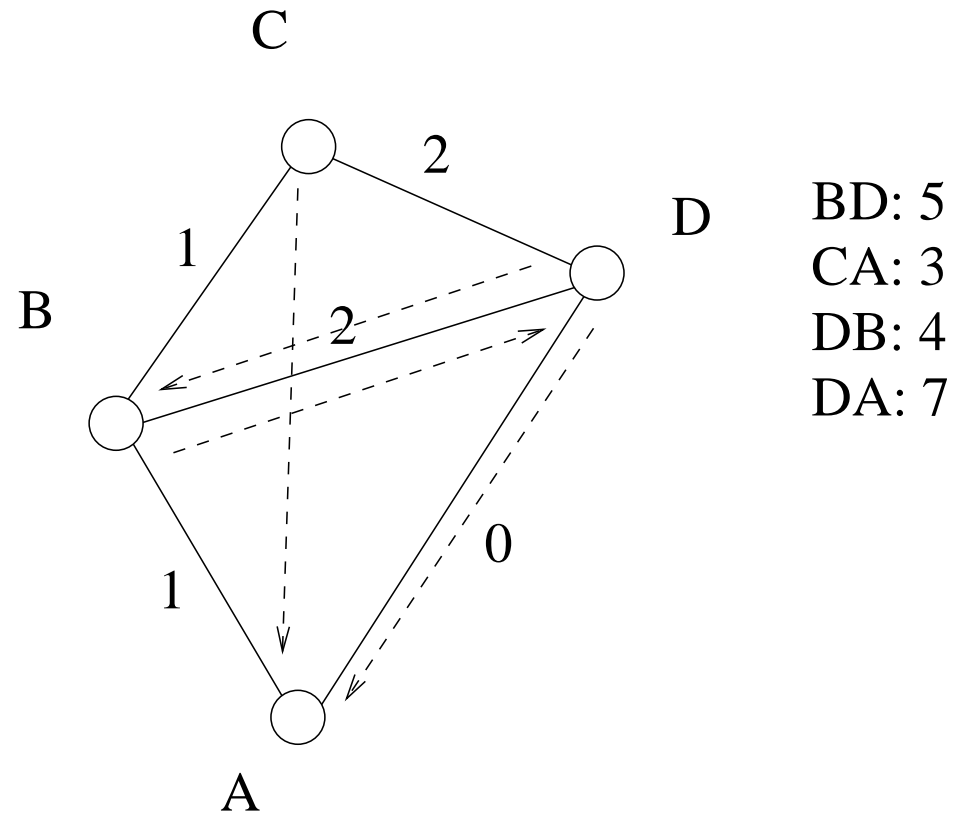
Routing

Assume you are the boss of operations. You have:

- A network
- Links between the nodes, *of limited capacity*
- A number of requests: Customers want to pay a certain price for getting a connection (if you don't accept it your competitor will).



Circuit Switched routing





Multi Commodity Flow

This problem is called the Multi Commodity Flow problem:

- It is the mother of all telecommunication administration problems.
- There are MANY variations of this problem.
- Our problem is to maximize the network revenue



MCF Terms

- $x_{ji}^{kl} \in [0, 1]$: The routing of the flows
- $y_{kl} \in \{0, 1\}$: Do we accept the customer or not
- c_{ij} : User revenue
- $CAP_{\{ij\}}$: Maximal capacity through the link



Multi Commodity Flow Problem (MCF)

Max:

$$\sum_{kl} c_{kl} \cdot y_{kl}$$

s.t.:

$$\sum_j x_{ij}^{kl} - \sum_j x_{ji}^{kl} = \begin{cases} y_{kl} & i = k \\ -y_{kl} & i = l \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{kl} (x_{ij}^{kl} + x_{ji}^{kl}) \cdot D_{kl} \leq CAP_{\{ij\}} \quad \forall \{ij\}$$

$$x_{ij}^{kl} \in \{0, 1\} \quad y_{kl} \in [0, 1]$$



Multi Commodity flow (MCF)

- There are **MANY** different variations
- Typically MCF is a lower bound (there are many other types of constraints).



Solution methods for MCF

- MCF is NP-complete (hence hard)
- Even the continuous case can be difficult because the number of flow variables increase as $O(N^4)$
- (Meta) Heuristics are often used in practice
- What about decomposition methods ?



Possible decomposition methods

- Benders decomposition ? (not used)
- Lagrange relaxation ?
- Dantzig-Wolfe/Column Generation ?



Lagrange relaxation

- Which constraints to relax ?
 - ▶ Capacity constraints: Shortest path sub-problem, hence no bound gain.
 - ▶ Routing Constraints: Multi-knapsack problem, but with many variables
- In practice Lagrange relaxation is not used ...



Dantzig-Wolfe MCF reformulation

The mother of all routing problems

Max:

$$\sum_{kl} c_{kl} \sum_p u_p^{kl}$$

s.t.:

$$\sum_p u_p^{kl} \leq 1 \quad \forall kl$$

$$\sum_{kl} \sum_p A_{p,\{ij\}}^{kl} \cdot D_{kl} \cdot u_p^{kl} \leq CAP_{\{ij\}} \quad \forall \{ij\}$$

$$u_p^{kl} \in \{0, 1\}$$



Path formulation

- This problem is exactly the same as before ...
- We have more variables, i.e. the *paths* (there are exponentially many of them)
- On the other hand, we get a much more detailed control of the actual solution (we love control ...)
- We can apply *column generation* to get fast solution of the LP-bounds



Sub-problem

We need to generate variables with *positive reduced profits* on the fly:

- $\alpha_{kl} \in R^{kl}$: The max sale dual.
- $\beta_{\{ij\}} \in R^{kl}$: The capacity dual.
- $c_{reduced} = c_{kl} - \alpha_{kl} - \sum_{\{ij\}} \beta_{\{ij\}}$
- Notice that we can solve $\{kl\}$ shortest path sub-problems, and compare the cost of the generated path $\sum_{\{ij\}} \beta_{\{ij\}}$ with the fixed terms $c_{kl} - \alpha_{kl}$.