Manpower Planning: Rostering

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Who am I?

- Anders Høeg Dohn, 29 years old.
- Cand. Polyt. in 2006 (from DTU).
  - Applied mathematics with specialization in Operations Research.
  - Master's thesis about a practical crew scheduling problem.
- PhD in Operations Research in 2010 (from DTU).
  - "Rostering and Task Scheduling – Applications in Manpower Planning"
  - Supervisors: Jens Clausen and Jesper Larsen.
- From July 2010:
  - Operations Analyst at Copenhagen Airports A/S.
- Board member and treasurer of the Danish Operations Research Society (DORS)
  - www.dorsnet.dk

Outline of the Remaining Three Lectures

- Rostering
  - Introduction to Manpower Planning
  - What is Rostering?
  - Solving Rostering Problems with Column Generation
- Task Scheduling
  - What is Task Scheduling?
  - Solving Task Scheduling Problems with Column Generation
  - Your Assignment
- Other OR-Problems in Airports
  - "Being an Operations Analyst in an Airport is like being a kid in a candy store"!
  - Questioning session on the assignment

Scope

- During these lectures I will:
  - Go over some of the practical problems encountered in manpower planning.
    - Rostering
    - Task Scheduling
  - Propose models that can be used to solve these problems, i.e. present case studies of these methods.
    - Integer Programming
    - Set Partitioning Formulations
    - Column Generation
    - Branch & Price

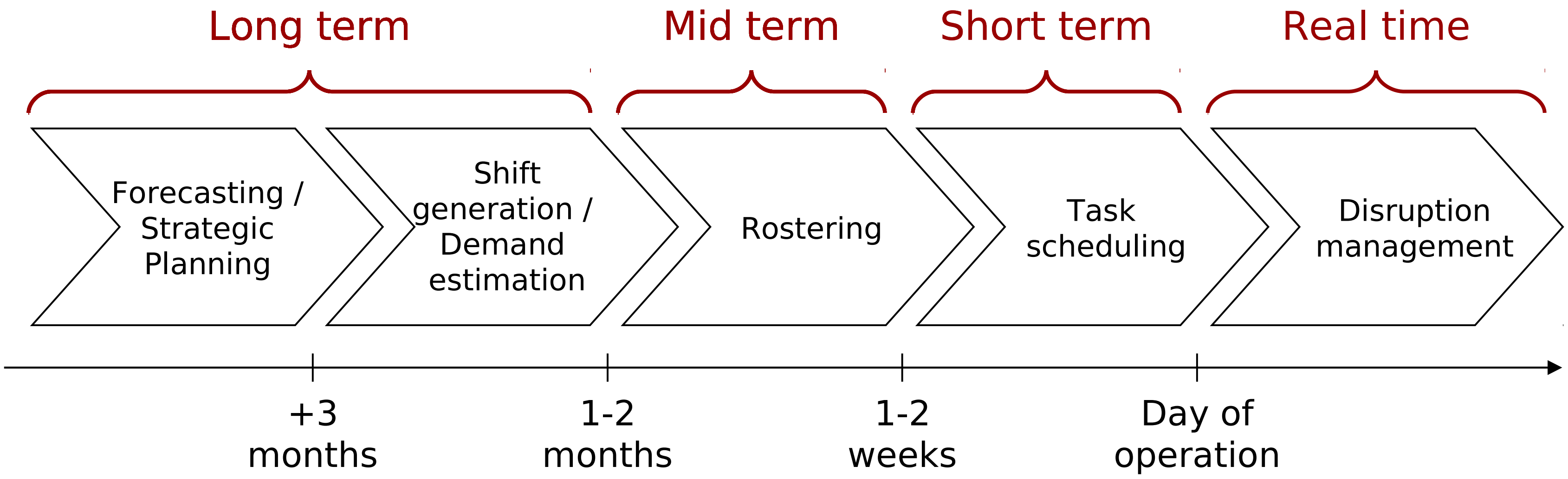
Motivation for using OR in manpower planning

- Large savings of structured manpower planning have been documented in the literature.

- The planning process consists of a number of steps and starts several months before the day of operation.
  - Each step is an interesting operations research problem, where structured forecasting, simulation and optimization may introduce significantly improved results.

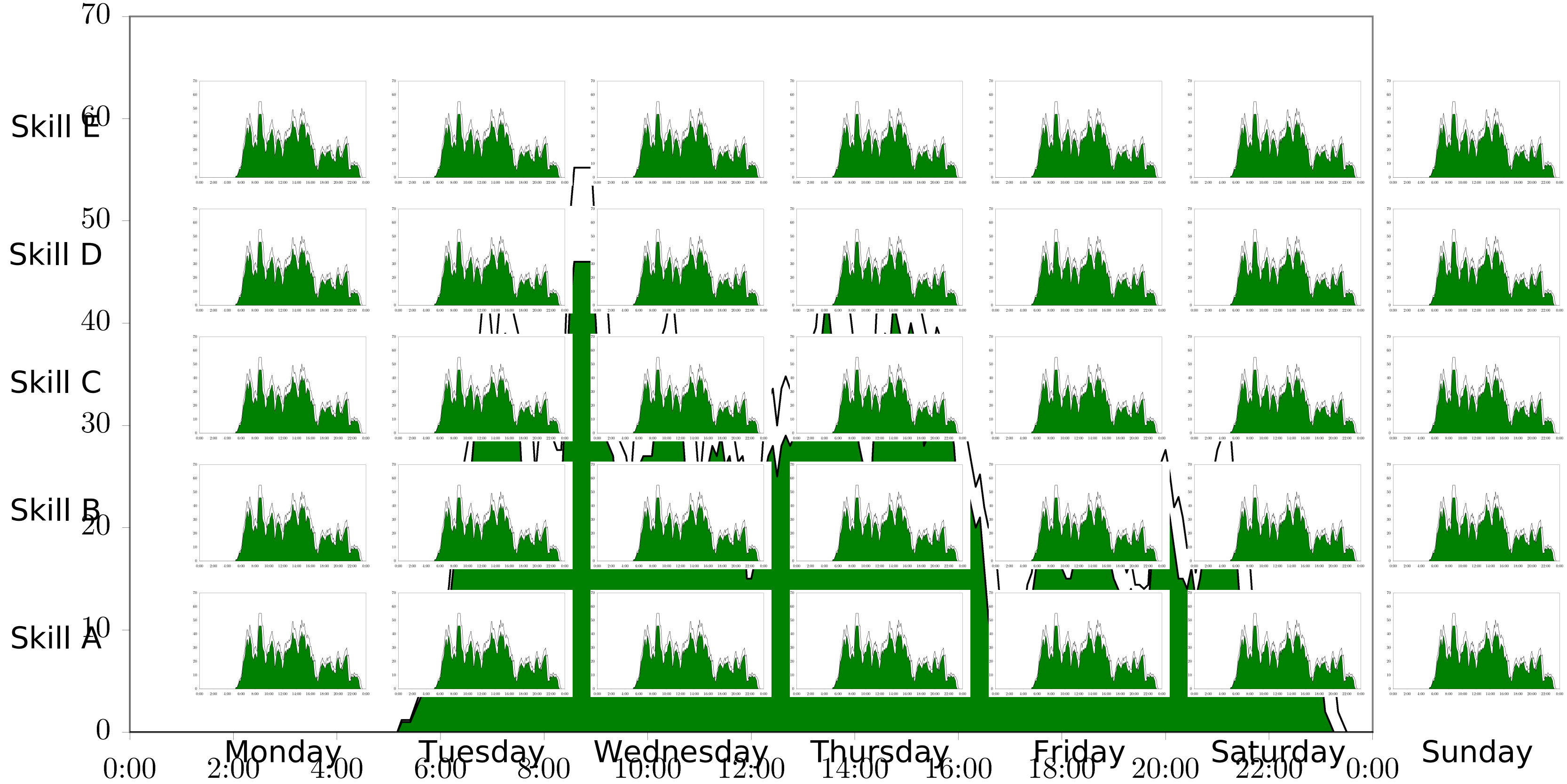
- In many cases, rostering and task scheduling can be modeled and solved as well defined optimization problems.

Manpower Planning

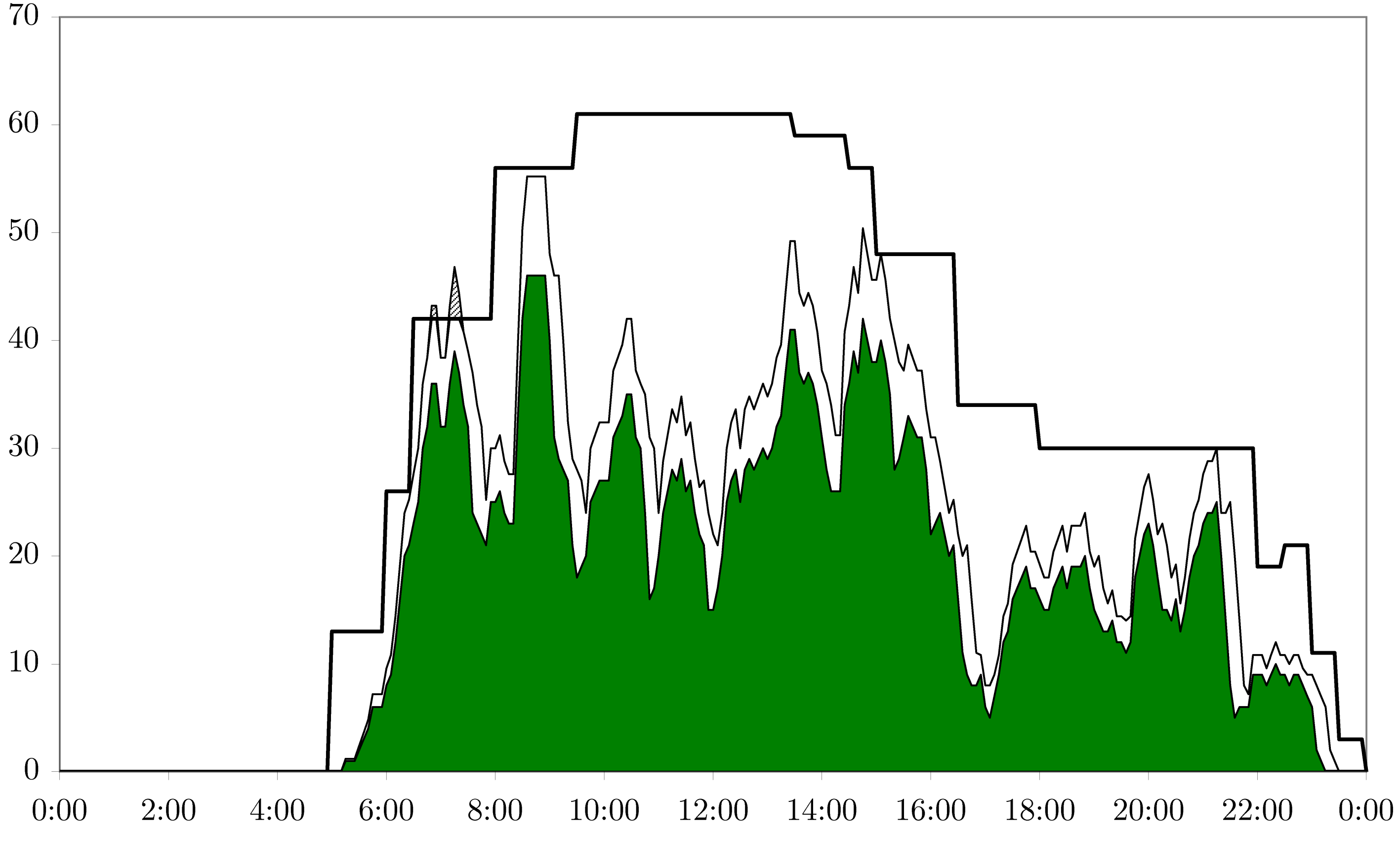


- From: Long term strategic planning
- To: Disruption management on the day of operation

Manpower Planning

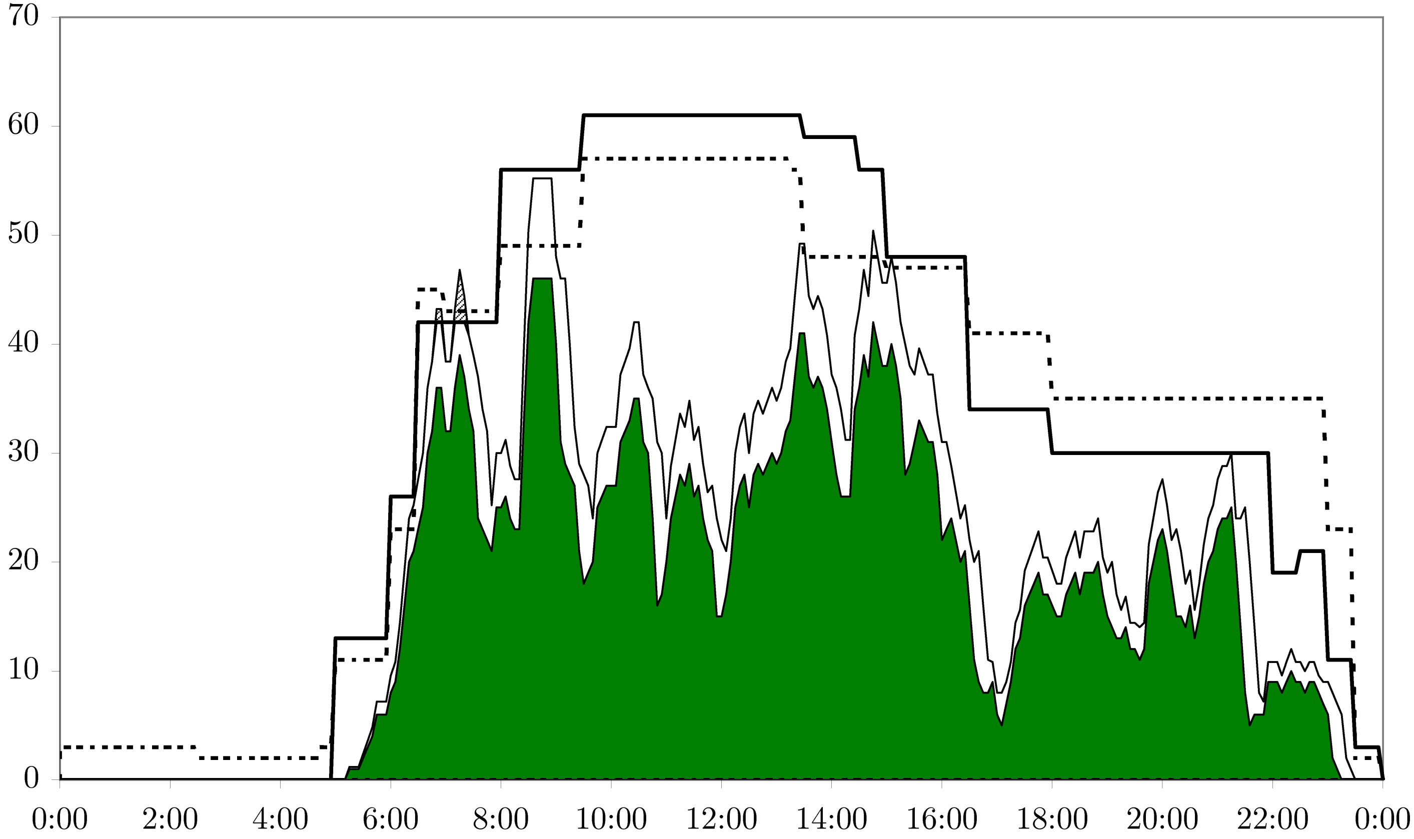


Manpower Planning

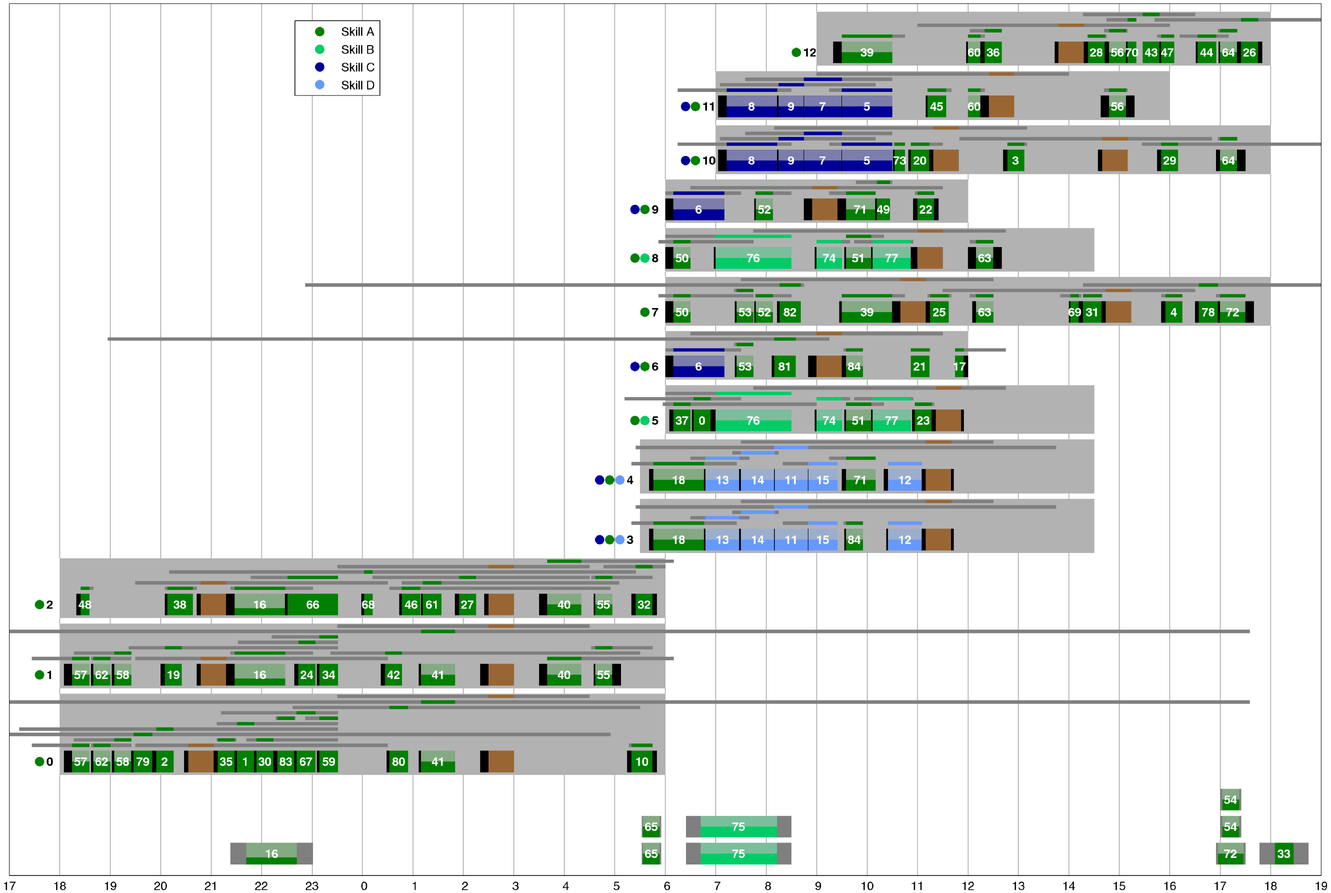


Manpower Planning

N1:	04:45 13:15	**0**
E1:	05:00 13:30	**13**
E2:	06:00 14:30	**13**
E3:	06:30 15:00	**16**
D1:	08:00 16:30	**14**
M1:	09:30 18:00	**5**
L1:	13:30 22:00	**11**
L2:	14:30 23:00	**10**
L3:	15:00 23:30	**8**
N2:	18:00 02:30	**1**
N3:	22:30 07:00	**2**



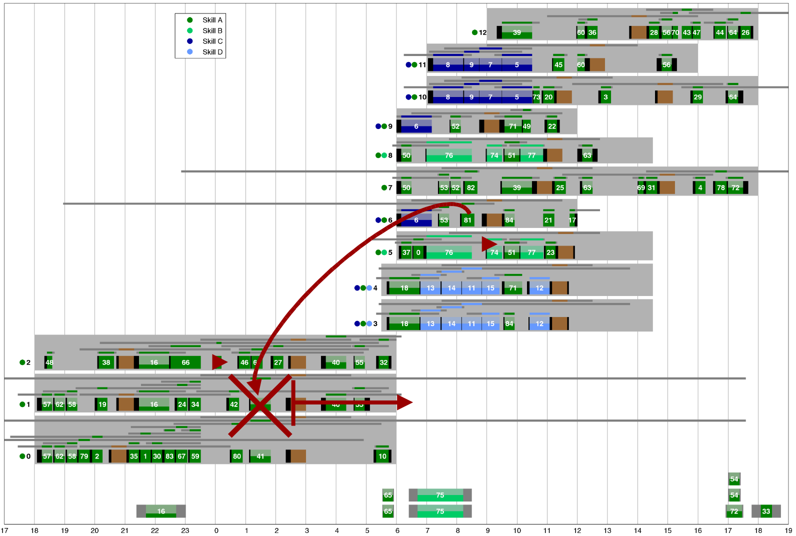
Manpower Planning



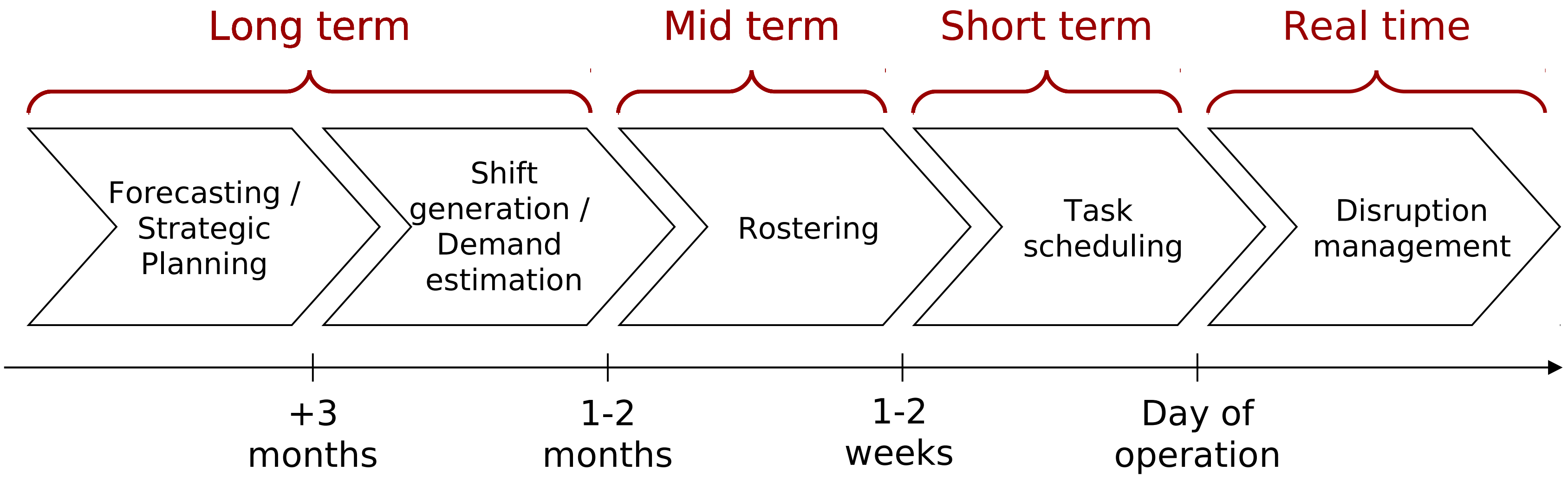
Forecasting / Strategic Planning → Shift generation / Demand estimation → Rostering → **Task scheduling** → Disruption management

Manpower Planning

Employee	Mon 06-aug	Tue 07-aug	Wed 08-aug	Thu 09-aug	Fri 10-aug	Sat 11-aug	Sun 12-aug	Mon 13-aug	Tue 14-aug	Wed 15-aug	Thu 16-aug	Fri 17-aug	Sat 18-aug	Sun 19-aug
1	L3	L3	L2	M1	E2	-	-	-	L3	L3	L3	L2	M1	E2
2	L2	L3	L3	L3	M1	-	-	-	L3	L3	L3	L1	M1	E3
3	L2	L2	L1	D1	E3	-	-	-	L3	L2	L2	L2	M1	E2
4	L2	L2	L1	D1	E3	-	-	-	L3	L2	L2	L2	M1	E2
5	D1	E3	E1	E2	E2	[illegible]	-	-	L2	L1	L2	L3	L1	D1
6	D1	E3	E1	E2	E2	-	-	-	L2	L1	L2	L3	L1	D1
7	L3	L3	M1	E1	E1	-	-	-	L3	L2	L2	L2	L2	L3
8	D1	E2	N1	E1	E3	[illegible]	-	-	L3	L3	L2	L3	M1	E3
9	L3	L3	N2	L2	L1	-	-	[illegible]	L3	L3	L3	L3	M1	E2
10	D1	E3	E3	E3	E3	-	-	-	L3	M1	E2	E3	E3	E3
11	D1	E1	E3	E3	E3	-	-	-	L3	L2	M1	E3	E2	E1
12	D1	E1	E3	E3	E3	-	-	-	L3	L2	M1	E3	E2	E1
13	L2	L2	M1	E1	E3	-	-	-	L2	M1	E3	E3	E1	E1
14	L2	L3	L1	D1	E3	-	-	-	L3	M1	E1	E2	E3	E3
15	L2	L2	N2	L2	L3	-	-	-	L2	L1	L1	D1	E3	E2
16	E1	E1	E1	E3	[illegible]	-	-	E1	E2	E2	E3	E3	E1	-
17	E1	E1	E1	E3	-	-	-	E1	E2	E2	E3	E3	E1	-
18	E1	E1	E1	E3	-	-	-	E1	E2	E2	E3	E3	E1	-
19	E3	E3	E3	E1	-	-	-	L2	L2	L2	M1	E1	E3	-
20	E3	E3	E3	E1	-	-	-	L2	L2	L2	M1	E1	E3	-



Rostering



- Allocate employees to shifts
- Assuming that:
  - Shift demands exist
  - The staff is fixed
  - We don't specify tasks during shifts

Some general characteristics

- Fixed planning period.
- Fixed number of shifts.
- Time norm for each employee.
- Maximum number of days on in a week / on-stretch.
- A minimum rest period after a shift is required.
- Specific shift transitions are not allowed.
- Each employee may have individual preferences.

Demands

			Days																											
Shifts	Skills	Type	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
A		$=$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
D		$\geq$	15	15	15	15	12			16	16	16	16	13			15	15	15	15	12			16	16	16	16	13		
D	2	$\geq$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
D	1	$\geq$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
D	3	$\geq$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
D	4	$\geq$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
L		$=$			2	2	2					2	2	2					2	2	2					2	2	2		
L	0	$\geq$	2	2	1	1	1			2	2	1	1	1			2	2	1	1	1			2	2	1	1	1		
N	0	$=$	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

A Roster

Nurse	Skills	Cost	Days																											
			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
0	0, 1	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
1	0, 2	13	N	-	D	N	-	-	-	D	N	-	N	-	-	-	D	D	D	D	L	-	-	N	-	D	D	D	-	N
2	0, 3	2	-	D	D	L	N	-	-	D	D	D	D	L	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
3	0, 4	2	D	D	D	D	D	-	-	D	D	N	-	D	-	-	L	A	L	D	D	-	-	D	D	-	L	N	-	-
4	0, 1	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
5	0, 2	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
6	0, 3	2	D	D	D	D	D	-	-	D	L	D	D	L	-	-	D	-	D	D	N	-	-	D	D	D	D	D	-	-
7	0, 4	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
8	0, 1	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
9	0, 2	2	-	D	D	D	N	-	-	D	D	D	D	D	-	-	D	D	L	D	D	-	-	D	D	D	D	L	-	-
10	0, 3	15	D	D	D	D	D	-	-	D	D	D	A	D	-	N	-	D	-	D	D	-	-	D	D	D	D	D	-	-
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12	0, 1	2	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	L	-	-	-	L	D	D	N	-	-
13	0, 2	2	D	D	A	D	D	-	-	L	D	D	-	N	-	-	D	D	D	D	D	-	-	D	D	D	D	L	-	-
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26	3	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	A	D	D	-	-
27	4	0	D	D	D	D	D	-	-	D	D	A	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-

A Roster-line

Nurse	Skills	Cost	Days																											
			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
0	0, 1	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
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2	0, 3	2	-	D	D	L	N	-	-	D	D	D	D	L	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
3	0, 4	2	D	D	D	D	D	-	-	D	D	N	-	D	-	-	L	A	L	D	D	-	-	D	D	-	L	N	-	-
4	0, 1	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
5	0, 2	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
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26	3	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	A	D	D	-	-
27	4	0	D	D	D	D	D	-	-	D	D	A	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-

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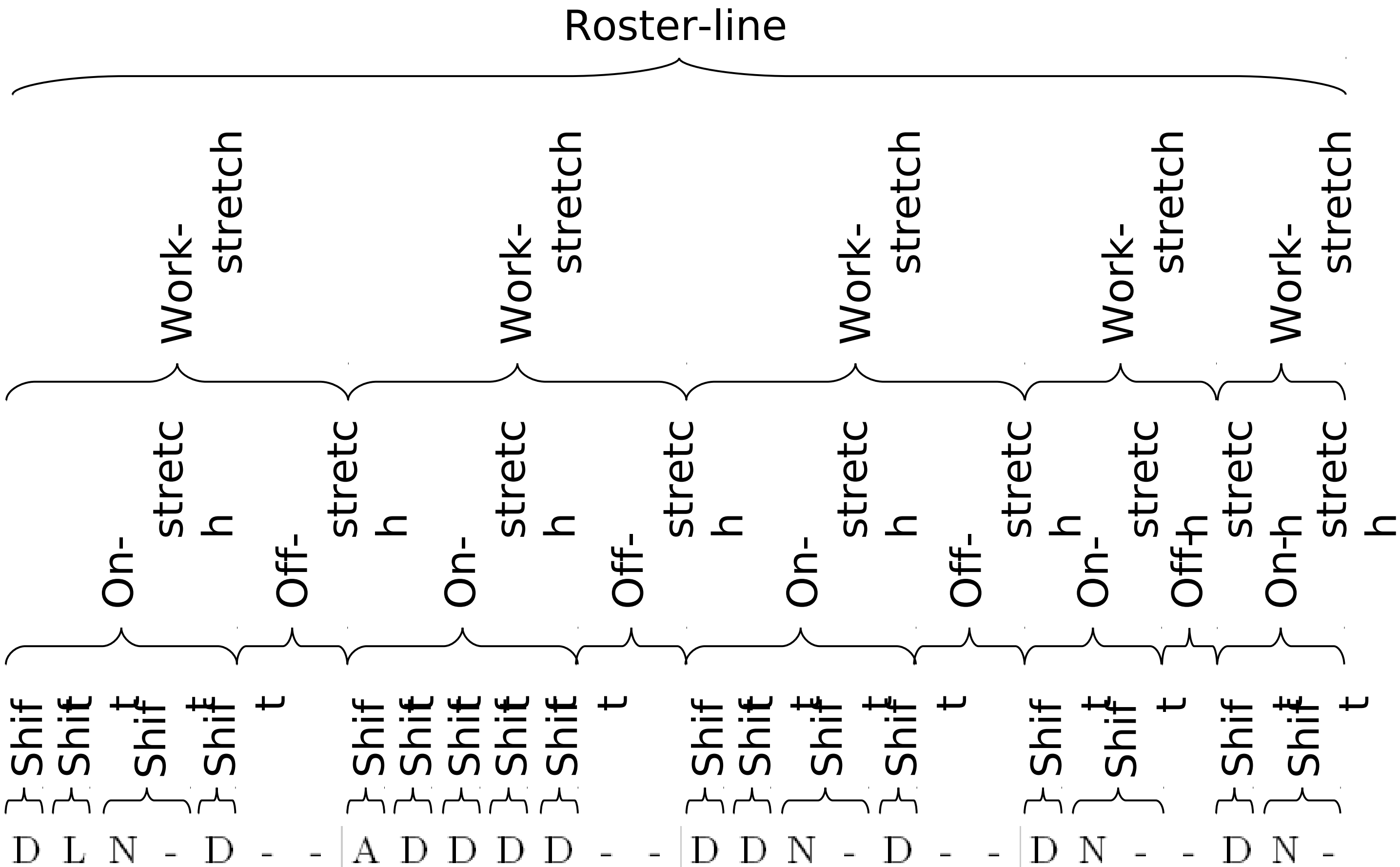
A Roster-line



A generalized set partitioning formulation

- Model the problem as that of choosing the best feasible roster-line for each employee.
  - Meet the demands as well as possible with these roster-lines.
- A multi-objective problem
  - Meet demands
  - Maximize satisfaction
  - Minimize costs
- A subproblem generates the feasible roster-lines.
  - All rules concerning a single employee are handled in the subproblem.

Branch & Price

- Necessary considerations:
  - How do we model and solve the master problem?
  - How do we model and solve the subproblem?
  - How do we ensure integrality in the master problem?

A generalized set partitioning formulation

$\mathcal{E}$	Employees
$\mathcal{D}$	Demands (Shifts)
$\mathcal{R}'_e$	Roster-lines

$$\lambda_e^r = \begin{cases} 1, & \text{if roster-line } r \text{ is chosen for employee } e. \\ 0, & \text{otherwise.} \end{cases}$$

s_d^- = Amount of under-coverage (slack) for demand d.

s_d^+ = Amount of over-coverage (surplus) for demand d.

c_e^r	Cost of roster-line r of employee e.
c_d^-	Cost of under-coverage for demand d.
c_d^+	Cost of over-coverage for demand d.
a_{ed}^r	=1, if demand d is covered by roster-line r of employee e.
b_d	Required number of employees for demand d.

$$\min \sum_{e \in \mathcal{E}} \sum_{r \in \mathcal{R}'_e} c_e^r \lambda_e^r + \sum_{d \in \mathcal{D}} \left(c_d^- s_d^- + c_d^+ s_d^+ \right)$$

$$\sum_{r \in \mathcal{R}'_e} \lambda_e^r = 1 \qquad \forall e \in \mathcal{E}$$

$$\sum_{e \in \mathcal{E}} \sum_{r \in \mathcal{R}'_e} a_{ed}^r \lambda_e^r + s_d^- - s_d^+ = b_d \qquad \forall d \in \mathcal{D}$$

$$\lambda_e^r \in \{0,1\} \qquad \forall e \in \mathcal{E}, \forall r \in \mathcal{R}'_e$$

$$s_d^- \geq 0, s_d^+ \geq 0 \qquad \forall d \in \mathcal{D}$$

A generalized set partitioning formulation

- The number of variables is HUGE.
  - Any legal combination of shifts constitutes a variable.
  - It is impossible to generate all varibles a priori.
  - Therefore, solving the model using column generation is a natural choice.
- Column generation is used directly on the generalized set partitioning formulation.
  - We are not decomposing a compact formulation.
  - The compact model can be “reverse engineered”, but it is not easy and the model usually isn’t very nice, anyway.

Branch & Price

- Necessary considerations:
  - How do we model and solve the master problem?
  - How do we model and solve the subproblem?
  - How do we ensure integrality in the master problem?

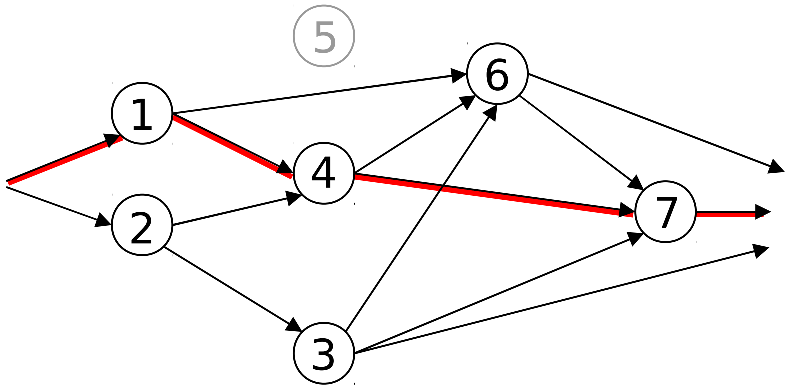
Column generation

Master Problem

Nurse 1	1	1	1	1	0	0	0	=	1	τ_1
Nurse 2	0	0	0	0	1	1	0	=	1	τ_2
Nurse 3	0	0	0	0	0	0	1	=	1	τ_3
Shift 1	1	1	0	0	1	1	0	=	2	π_1
Shift 2	0	0	1	0	0	0	1	=	1	π_2
Shift 3	0	0	1	1	0	0	0	=	1	π_3
Shift 4	0	0	0	0	0	1	1	=	1	π_4
Shift 5	1	0	0	0	1	0	0	=	1	π_5
Shift 6	0	1	1	1	0	0	0	=	1	π_6
Shift 7	0	1	0	0	0	0	1	=	1	π_7

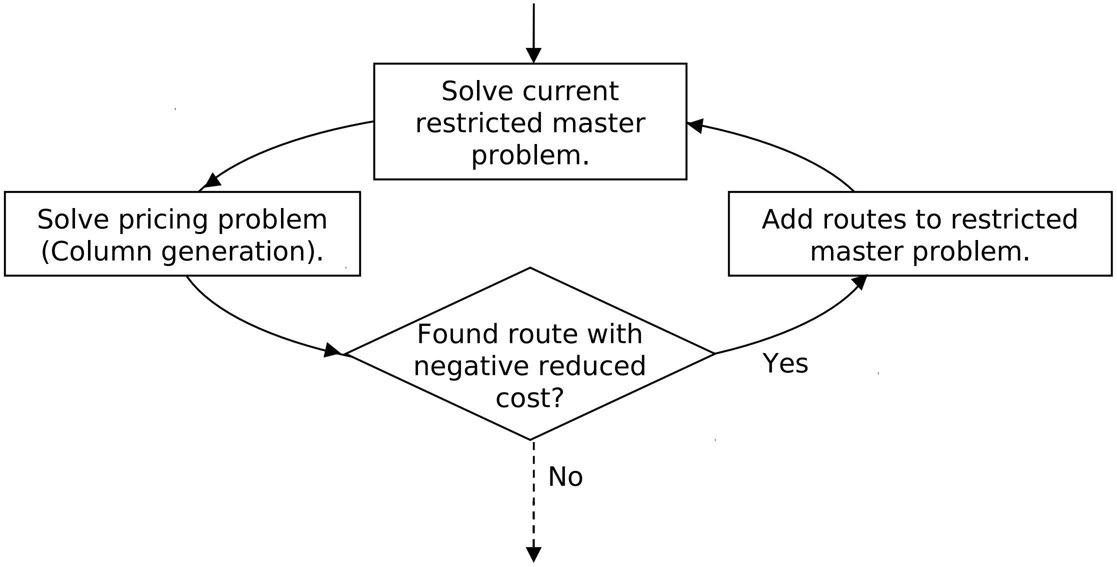
Subproblem

$$\min c_e^r - \sum_{d \in D} \pi_d x_d - \tau_e$$



1 0 0 1 0 0 1

Column Generation – Overview



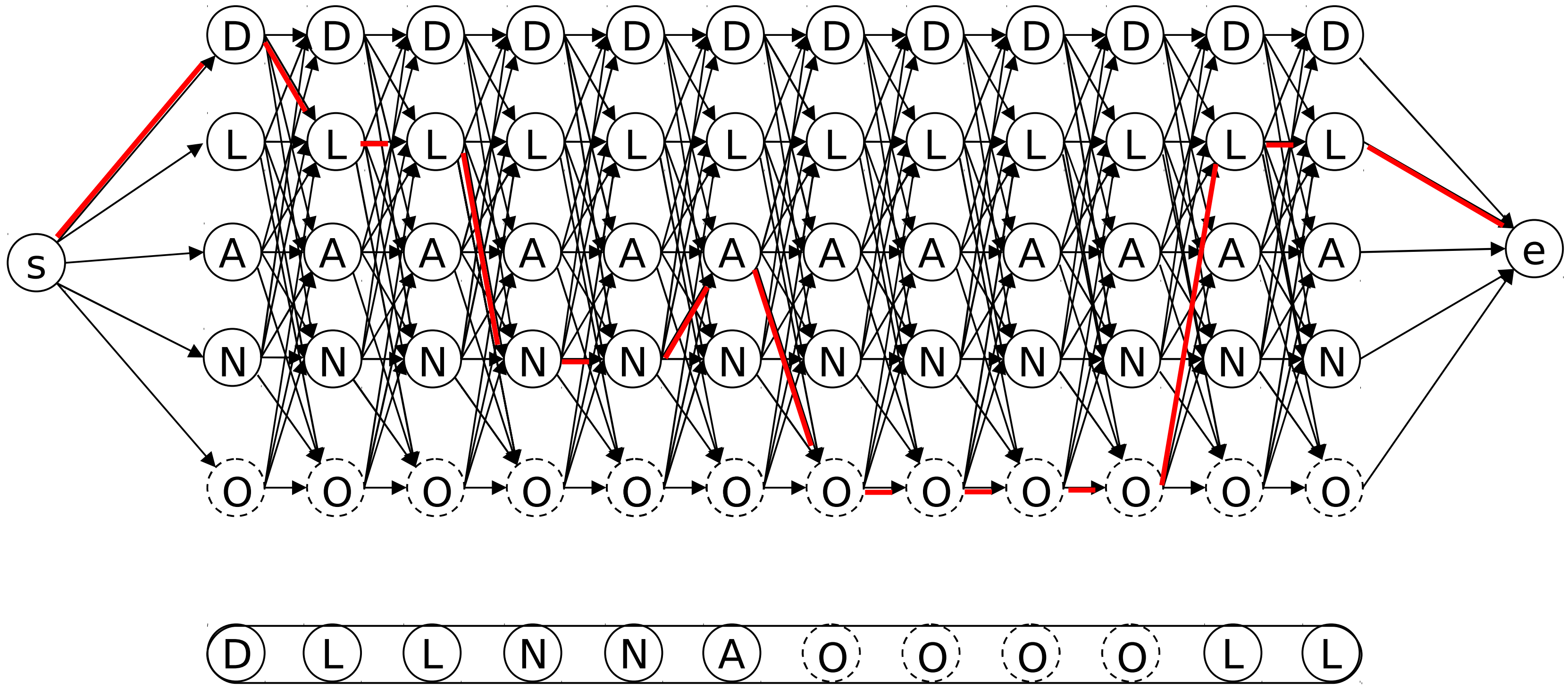
Column generation

- Solve an LP-relaxation of the problem.
- One subproblem for each employee
  - The subproblems can be solved independently, in parallel or sequentially.
  - To terminate column generation, all subproblems must be unable to return new columns.
  - The subproblem can be solved as a Resource Constrained Shortest Path Problem.

Dynamic Programming

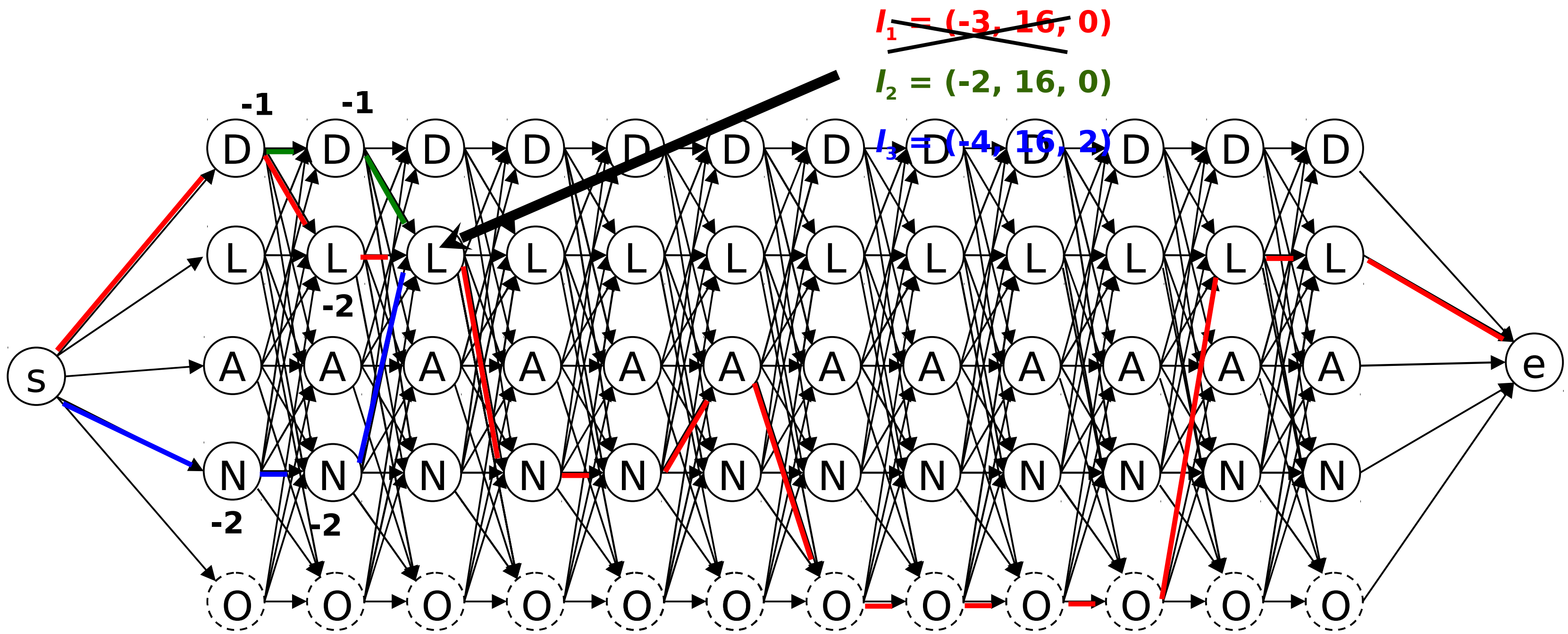
- Resources
  - Anything that can affect feasibility or cost of a label must be considered when dominating labels.
  - Such measures are represented by *resources*.
- Domination
  - For each resource, we must specify what a “better” value is.
  - Domination must consider both feasibility and cost.
  - Sometimes, it is not better to have neither a smaller nor a larger value. Then domination only applies if values are equal.
  - All resources must allow domination (and cost must be lower) for one label to dominate another.

Dynamic Programming



Dynamic Programming

***I* = (*cost, hours, nightshifts*)**



- Assume that we have two resources: number of worked hours and number of night shifts.
- Assume that both D, L and N have a length of 8 hours.
- Is it better to have a smaller/larger number of hours?

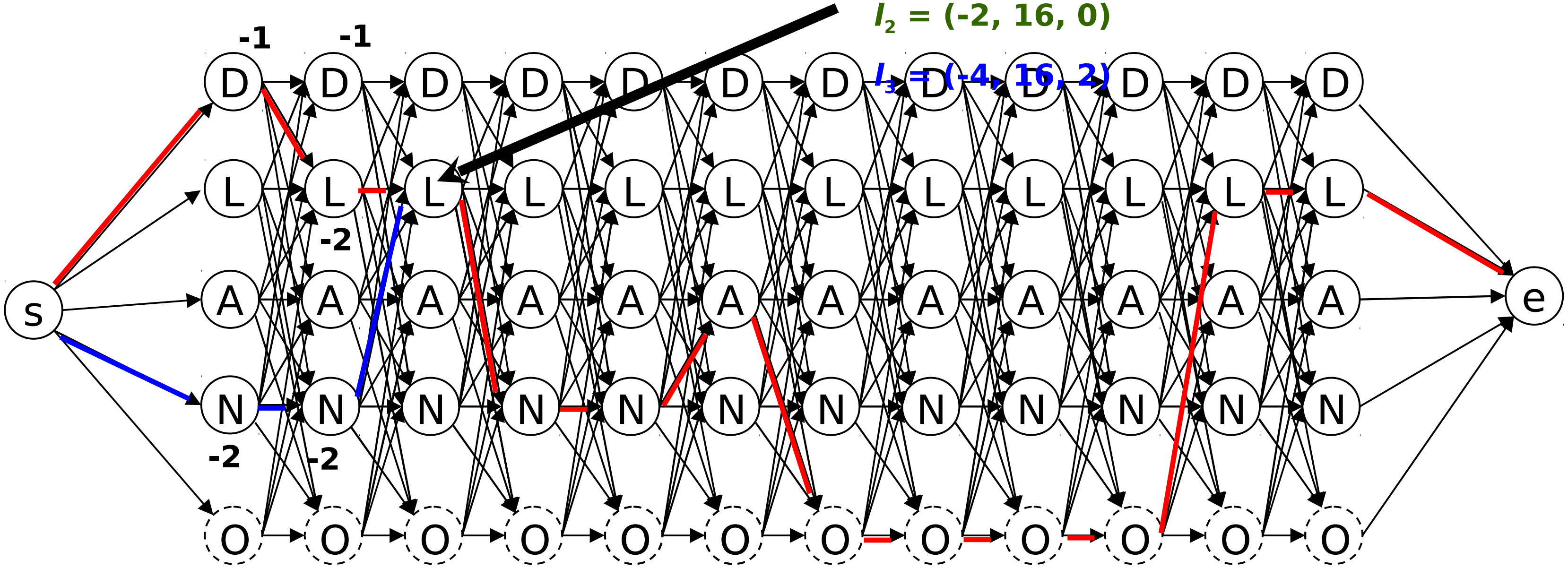
Dynamic Programming

***I* = (*cost, hours, nightshifts*)**

~~I_1 = (-3, 16, 0)~~

I_2 = (-2, 16, 0)

I_3 = (-4, 16, 2)



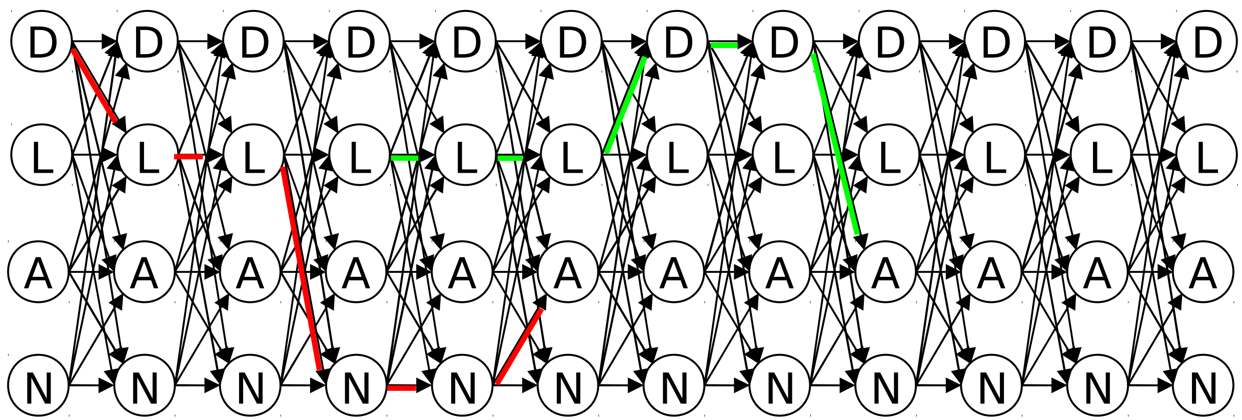
- I_3 does not dominate I_1 because it has a larger number of night shifts.
  - Assuming that lower is better.
  - Why is lower better? Must be defined in the individual case for each of the resources.
    - Due to feasibility
    - Due to costs which depend on resource values
  - What if it is the other way around?

Utilizing the roster-line structure

- Combine shifts into on-stretches
- Combine on-stretches and off-stretches into work-stretches
- Combine work-stretches into roster-lines

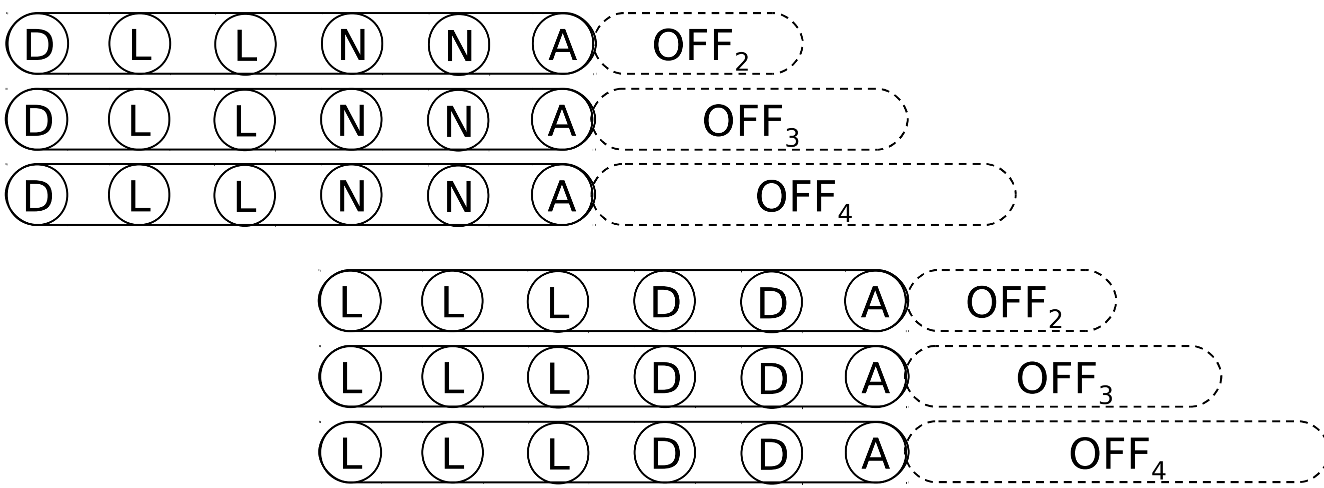
- Use dynamic programming to find only the non-dominated entities in each step.

Utilizing the roster-line structure



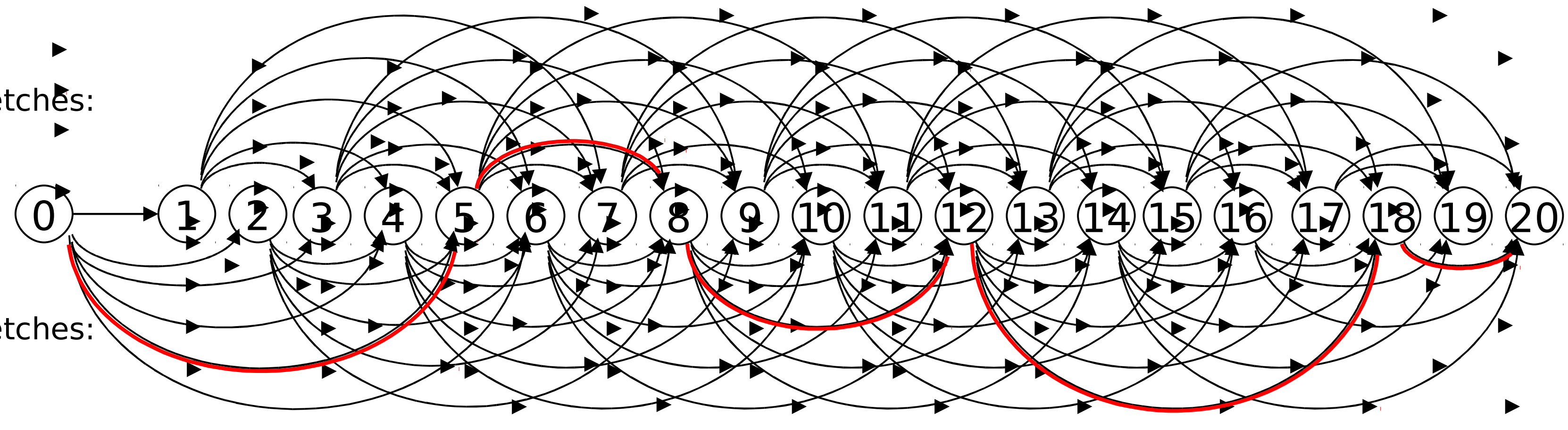
Work-stretch 1:
Work-stretch 2:
Work-stretch 3:

Work-stretch 4:
Work-stretch 5:
Work-stretch 6:



Utilizing the roster-line structure

Work-stretches:



Work-stretches:

An optimal LP-solution

Nurse	x	Cost	Days																											
			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
0	0.378191	2	D	D	D	D	D	-	-	D	L	D	D	D	-	-	-	L	L	D	N	-	-	D	N	-	D	D	-	-
0	0.621809	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
1	0.966728	2	D	L	D	D	D	-	-	D	L	-	D	N	-	-	N	-	L	D	D	-	-	D	D	D	D	D	-	-
1	0.0332715	5	D	D	N	-	L	-	-	D	D	D	D	D	-	-	A	A	D	A	L	-	-	N	-	N	-	-	-	N
2	0.843555	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
2	0.0764406	2	D	D	D	D	A	-	-	L	D	D	A	D	-	-	D	D	D	D	L	-	-	D	A	D	-	N	-	-
2	0.0800048	2	N	-	N	-	L	-	-	L	D	-	D	N	-	-	L	D	L	D	D	-	-	D	D	L	D	D	-	-
3	0.474511	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
3	0.23604	2	L	D	D	L	D	-	-	-	N	-	A	N	-	-	D	L	D	D	D	-	-	D	D	D	D	D	-	-
3	0.18799	5	N	-	D	D	D	-	-	A	D	N	-	D	-	-	D	D	L	D	D	-	-	N	-	D	-	D	-	N
3	0.0410925	2	-	L	D	D	N	-	-	L	D	D	D	D	-	-	A	A	D	D	D	-	-	D	D	L	N	-	-	-
3	0.0603669	12	N	-	N	-	A	-	-	N	-	L	D	D	-	-	A	D	D	D	D	-	-	D	D	-	D	D	N	-
4	0.260215	5	D	N	-	N	-	-	-	D	D	D	D	A	-	-	D	N	-	D	D	-	-	D	D	L	D	-	-	N
4	0.071949	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
4	0.451775	13	D	N	-	N	-	-	-	D	D	D	D	L	-	-	D	D	N	-	D	-	-	N	-	N	-	D	-	N
4	0.216061	2	D	D	L	D	D	-	-	A	D	L	D	D	-	-	D	D	D	D	A	-	-	L	N	-	-	N	-	-
5	0.767695	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
5	0.168765	5	D	D	D	D	D	-	-	D	D	D	N	-	-	-	L	A	D	D	D	-	-	-	N	-	N	-	-	N
5	0.0635403	2	D	D	L	L	D	-	-	A	L	D	D	A	-	-	D	D	D	D	D	-	-	N	-	-	A	N	-	-
6	1	4	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	-	D	-	D	D	N	-
7	0.125366	2	A	D	D	D	D	-	-	D	D	D	D	L	-	-	L	D	-	D	N	-	-	D	A	D	A	D	-	-
7	0.397964	2	D	D	D	D	L	-	-	D	D	L	A	D	-	-	D	D	D	D	A	-	-	D	D	-	A	N	-	-
7	0.0576867	2	D	D	-	D	N	-	-	L	D	D	L	A	-	-	D	D	D	D	D	-	-	L	D	D	N	-	-	-
7	0.418983	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
8	0.422656	5	D	D	D	L	D	-	-	D	N	-	N	-	-	-	D	D	D	D	D	-	-	-	D	N	-	A	-	N
8	0.577344	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
9	0.0772873	13	D	D	A	N	-	-	-	D	D	N	-	D	-	-	N	-	A	D	D	-	-	N	-	N	-	L	-	N
9	0.722712	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
9	0.128979	5	D	D	L	N	-	-	-	D	N	-	N	-	-	-	D	D	D	D	D	-	-	D	A	-	D	D	-	N
9	0.0710214	2	A	D	D	D	A	-	-	L	D	L	N	-	-	-	A	-	D	D	N	-	-	L	D	D	D	D	-	-
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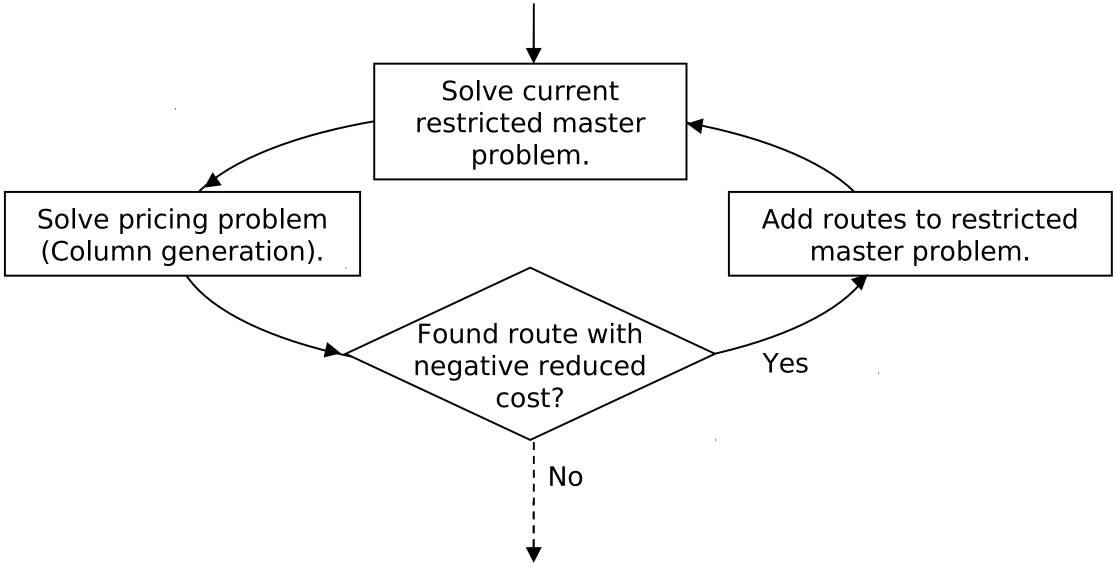
10	0.0220762	2	D	D	A	L	D	-	-	D	D	D	L	D	-	-	A	N	-	D	D	-	-	-	L	D	D	N	-	-
10	0.174596	2	D	N	-	D	A	-	-	D	D	D	D	D	-	-	L	L	D	D	D	-	-	D	D	-	L	N	-	-
10	0.501441	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
11	0.753929	2	D	-	D	D	N	-	-	D	D	D	D	L	-	-	N	-	D	D	L	-	-	L	D	D	L	D	-	-
11	0.246071	5	A	D	N	-	D	-	-	D	D	D	D	D	-	-	L	D	D	D	D	-	-	N	-	-	N	-	-	N
12	0.947893	14	D	D	D	D	D	-	-	D	D	D	D	-	N	-	D	D	-	D	D	-	-	D	D	D	D	D	-	-
12	0.0229911	13	N	-	N	-	D	-	-	D	N	-	D	L	-	-	D	A	N	-	D	-	-	A	D	N	-	D	-	N
12	0.0291159	4	A	A	N	-	D	-	-	D	D	L	D	D	-	-	D	D	L	N	-	-	-	-	-	D	L	A	N	-
13	0.230343	12	D	A	D	D	D	-	-	D	N	-	L	D	-	-	A	N	-	D	D	-	-	D	D	N	-	-	N	-
13	0.069953	2	A	L	D	D	D	-	-	D	A	N	-	D	-	-	D	A	D	-	N	-	-	L	D	L	D	D	-	-
13	0.179915	12	N	-	D	N	-	-	-	D	N	-	D	D	-	-	D	L	D	A	D	-	-	D	-	D	D	D	N	-

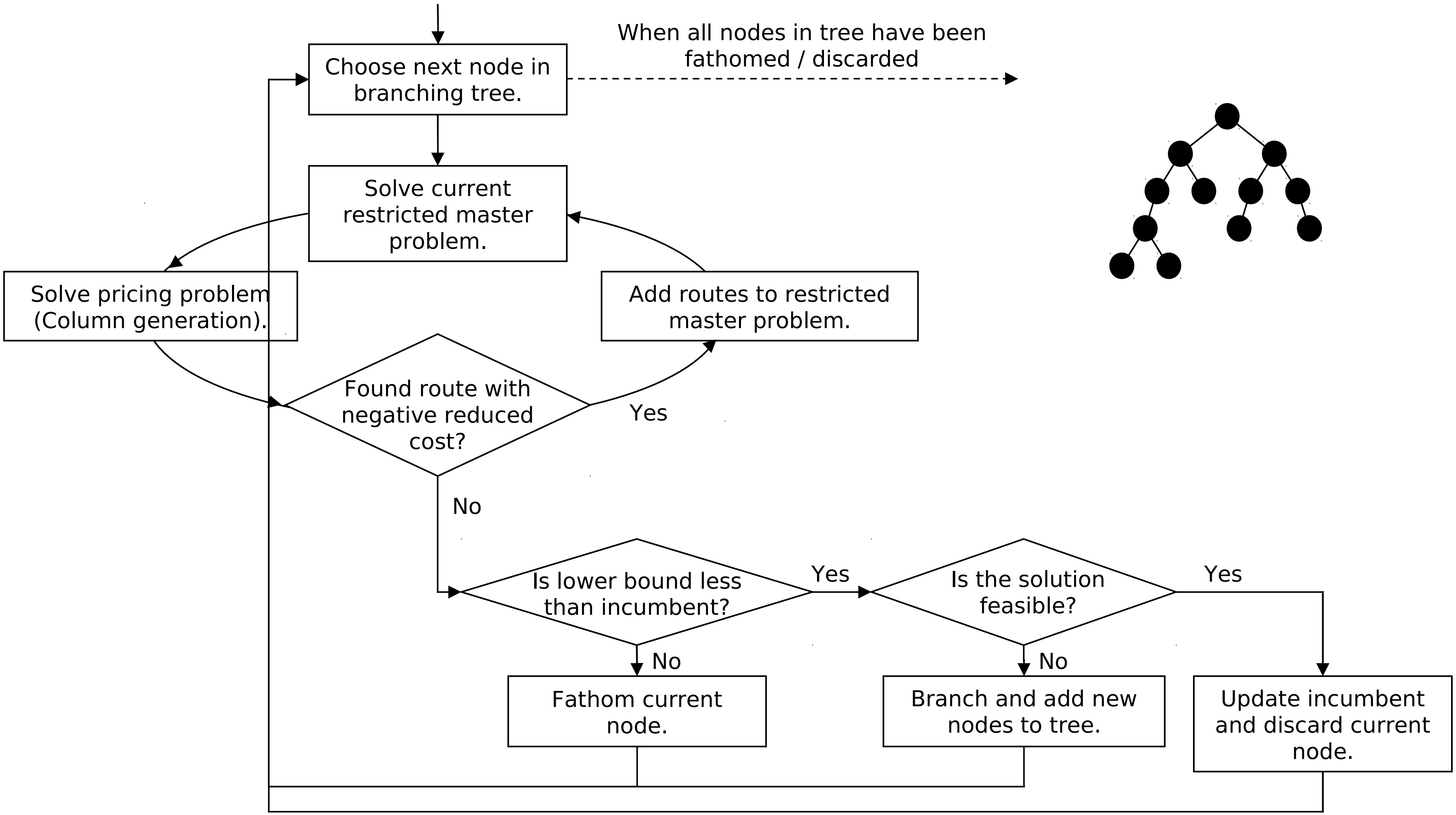
Branch & Price

- Necessary considerations:
  - How do we model and solve the master problem?
  - How do we model and solve the subproblem?
  - How do we ensure integrality in the master problem?

Branch & Price – Overview



Branch & Price – Overview



Branching

- Variable branching is usually not a possibility in column generation.
  - How do you prohibit a variable from reappearing?
- Branch on a fractional shift allocation.
- When all shift allocations are integer the full master problem solution is integer.
  - As the constraint matrix for each employee is a perfect matrix.
  - For more on this: Follow the course “42122 The Set Partitioning Optimization Model and its Application in Practical Scheduling Problems”.

Branching

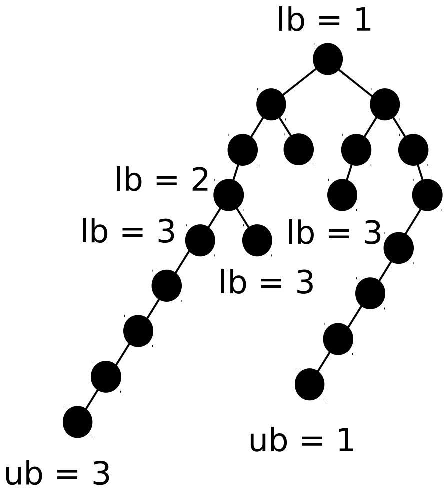
Nurse	x	Cost	Days																											
			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
0	0.378191	2	D	D	D	D	D	-	-	D	L	D	D	D	-	-	-	L	L	D	N	-	-	D	N	-	D	D	-	-
0	0.621809	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
1	0.966728	2	D	L	D	D	D	-	-	D	L	-	D	N	-	-	N	-	L	D	D	-	-	D	D	D	D	D	-	-
1	0.0332715	5	D	D	N	-	L	-	-	D	D	D	D	D	-	-	A	A	D	A	L	-	-	N	-	N	-	-	-	N
2	0.843555	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
2	0.0764406	2	D	D	D	D	A	-	-	L	D	D	A	D	-	-	D	D	D	D	L	-	-	D	A	D	-	N	-	-
2	0.0800048	2	N	-	N	-	L	-	-	L	D	-	D	N	-	-	L	D	L	D	D	-	-	D	D	L	D	D	-	-
3	0.474511	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
3	0.23604	2	L	D	D	L	D	-	-	-	N	-	A	N	-	-	D	L	D	D	D	-	-	D	D	D	D	D	-	-
3	0.18799	5	N	-	D	D	D	-	-	A	D	N	-	D	-	-	D	D	L	D	D	-	-	N	-	D	-	D	-	N
3	0.0410925	2	-	L	D	D	N	-	-	L	D	D	D	D	-	-	A	A	D	D	D	-	-	D	D	L	N	-	-	-
3	0.0603669	12	N	-	N	-	A	-	-	N	-	L	D	D	-	-	A	D	D	D	D	-	-	D	D	-	D	D	N	-
4	0.260215	5	D	N	-	N	-	-	-	D	D	D	D	A	-	-	D	N	-	D	D	-	-	D	D	L	D	-	-	N
4	0.071949	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
4	0.451775	13	D	N	-	N	-	-	-	D	D	D	D	L	-	-	D	D	N	-	D	-	-	N	-	N	-	D	-	N
4	0.216061	2	D	D	L	D	D	-	-	A	D	L	D	D	-	-	D	D	D	D	A	-	-	L	N	-	-	N	-	-
5	0.767695	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
5	0.168765	5	D	D	D	D	D	-	-	D	D	D	N	-	-	-	L	A	D	D	D	-	-	-	N	-	N	-	-	N
5	0.0635403	2	D	D	L	L	D	-	-	A	L	D	D	A	-	-	D	D	D	D	D	-	-	N	-	-	A	N	-	-
6	1	4	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	-	D	-	D	D	N	-
7	0.125366	2	A	D	D	D	D	-	-	D	D	D	D	L	-	-	L	D	-	D	N	-	-	D	A	D	A	D	-	-
7	0.397964	2	D	D	D	D	L	-	-	D	D	L	A	D	-	-	D	D	D	D	A	-	-	D	D	-	A	N	-	-
7	0.0576867	2	D	D	-	D	N	-	-	L	D	D	L	A	-	-	D	D	D	D	D	-	-	L	D	D	N	-	-	-
7	0.418983	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
8	0.422656	5	D	D	D	L	D	-	-	D	N	-	N	-	-	-	D	D	D	D	D	-	-	-	D	N	-	A	-	N
8	0.577344	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
9	0.0772873	13	D	D	A	N	-	-	-	D	D	N	-	D	-	-	N	-	A	D	D	-	-	N	-	N	-	L	-	N
9	0.722712	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
9	0.128979	5	D	D	L	N	-	-	-	D	N	-	N	-	-	-	D	D	D	D	D	-	-	D	A	-	D	D	-	N
9	0.0710214	2	A	D	D	D	A	-	-	L	D	L	N	-	-	-	A	-	D	D	N	-	-	L	D	D	D	D	-	-
10	0.301887	2	N	-	-	D	N	-	-	L	D	L	L	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
10	0.0220762	2	D	D	A	L	D	-	-	D	D	D	L	D	-	-	A	N	-	D	D	-	-	-	L	D	D	N	-	-
10	0.174596	2	D	N	-	D	A	-	-	D	D	D	D	D	-	-	L	L	D	D	D	-	-	D	D	-	L	N	-	-
10	0.501441	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
11	0.753929	2	D	-	D	D	N	-	-	D	D	D	D	L	-	-	N	-	D	D	L	-	-	L	D	D	L	D	-	-
11	0.246071	5	A	D	N	-	D	-	-	D	D	D	D	D	-	-	L	D	D	D	D	-	-	N	-	-	N	-	-	N
12	0.947893	14	D	D	D	D	D	-	-	D	D	D	D	-	N	-	D	D	-	D	D	-	-	D	D	D	D	D	-	-
12	0.0229911	13	N	-	N	-	D	-	-	D	N	-	D	L	-	-	D	A	N	-	D	-	-	A	D	N	-	D	-	N
12	0.0291159	4	A	A	N	-	D	-	-	D	D	L	D	D	-	-	D	D	L	N	-	-	-	-	-	D	L	A	N	-
13	0.230343	12	D	A	D	D	D	-	-	D	N	-	L	D	-	-	A	N	-	D	D	-	-	D	D	N	-	-	N	-
13	0.069953	2	A	L	D	D	D	-	-	D	A	N	-	D	-	-	D	A	D	-	N	-	-	L	D	L	D	D	-	-
13	0.179915	12	N	-	D	N	-	-	-	D	N	-	D	D	-	-	D	L	D	A	D	-	-	D	-	D	D	D	N	-

Branch & Price

- Necessary considerations:
  - How do we model and solve the master problem?
  - How do we model and solve the subproblem?
  - How do we ensure integrality in the master problem?
- Other considerations:
  - Master problem
    - Solve it to optimality every time?
    - Use dual stabilization?
  - Subproblem
    - Use heuristics?
    - In what order should the individual subproblems be solved?
  - Branching
    - How do we search the branch-and-bound tree?

Searching the Branch-and-Bound Tree

- Best first
  - Theoretically the most efficient
- Depth first
  - Works very well in practice for rostering problems.
  - Efficient because many nodes have the same lower bound.
  - Often, a solution exists in the bottom of the tree with solution value equal to the lower bound.



Branch & Price

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An optimal solution

2

10

Nurse	Skills	Cost	Days																											
			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
0	0, 1	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
1	0, 2	13	N	-	D	N	-	-	-	D	N	-	N	-	-	-	D	D	D	D	L	-	-	N	-	D	D	D	-	N
2	0, 3	2	-	D	D	L	N	-	-	D	D	D	D	L	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
3	0, 4	2	D	D	D	D	D	-	-	D	D	N	-	D	-	-	L	A	L	D	D	-	-	D	D	-	L	N	-	-
4	0, 1	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
5	0, 2	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
6	0, 3	2	D	D	D	D	D	-	-	D	L	D	D	L	-	-	D	-	D	D	N	-	-	D	D	D	D	D	-	-
7	0, 4	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
8	0, 1	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
9	0, 2	2	-	D	D	D	N	-	-	D	D	D	D	D	-	-	D	D	L	D	D	-	-	D	D	D	D	L	-	-
10	0, 3	15	D	D	D	D	D	-	-	D	D	D	A	D	-	N	-	D	-	D	D	-	-	D	D	D	D	D	-	-
11	0, 4	2	D	D	D	A	L	-	-	D	D	D	D	D	-	-	D	D	D	-	N	-	-	L	D	D	D	D	-	-
12	0, 1	2	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	L	-	-	-	L	D	D	N	-	-
13	0, 2	2	D	D	A	D	D	-	-	L	D	D	-	N	-	-	D	D	D	D	D	-	-	D	D	D	D	L	-	-
14	0, 3	13	D	N	-	L	D	-	-	N	-	D	N	-	-	-	N	-	D	D	D	-	-	N	-	L	N	-	-	N
15	0, 4	12	D	L	N	-	D	-	-	A	D	D	D	D	-	-	D	D	N	-	D	-	-	D	N	-	-	D	N	-
16	0, 1	2	A	D	D	D	L	-	-	L	D	-	D	N	-	-	D	D	D	D	A	-	-	D	D	D	D	D	-	-
17	0, 2	12	D	N	-	D	D	-	-	D	N	-	D	D	-	-	A	D	D	N	-	-	-	D	L	D	D	-	N	-
18	0, 3	28	L	D	N	-	-	-	-	N	-	L	L	-	N	-	D	-	N	-	D	-	-	D	D	N	-	-	-	-
19	0, 4	25	D	A	D	-	-	-	N	-	D	L	-	D	-	-	D	L	-	L	-	-	N	-	D	-	L	D	-	-
20	0, 1	25	-	D	D	D	-	-	N	-	D	-	L	D	-	-	L	L	D	-	-	-	N	-	-	L	D	D	-	-
21	0, 2	39	D	D	-	-	A	N	-	D	-	D	D	D	-	-	-	N	-	A	-	N	-	A	N	-	-	D	-	-
22	0, 3	36	N	-	D	N	-	-	-	D	D	-	-	A	N	-	N	-	-	D	D	-	-	D	D	D	N	-	-	-
23	0, 4	31	-	L	L	-	-	N	-	-	L	N	-	D	-	-	-	N	-	L	-	N	-	L	-	N	-	D	-	-
24	0, 1	21	L	-	L	D	D	-	-	D	A	D	D	-	-	N	-	-	A	N	-	-	-	D	A	-	A	A	-	-
25	2	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-
26	3	0	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-	D	D	A	D	D	-	-
27	4	0	D	D	D	D	D	-	-	D	D	A	D	D	-	-	D	D	D	D	D	-	-	D	D	D	D	D	-	-

10-7 = 3

2

2

2

2

10+10-7 = 13

An optimal solution: slack / surplus

			Days																											
Shifts	Skills	Type	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
A		=	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
D		$\geq$	15	15	15	15	12			16	16	16	16	13			15	15	15	15	12			16	16	16	16	13		
D	2	$\geq$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
D	1	$\geq$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
D	3	$\geq$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
D	4	$\geq$	1	1	1	1	1			1	1	1	1	1			1	1	1	1	1			1	1	1	1	1		
L		=			2	2	2					2	2	2					2	2	2					2	2	2		
L	0	$\geq$	2	2	1	1	1			2	2	1	1	1			2	2	1	1	1			2	2	1	1	1		
N	0	=	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

				Days																											
Shifts	Skills	Type	Cost	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
A		=	0	-	-	-	-	-			-	-	-	-	-			-	-	-	-	-			-	-	-	-	-		
D		$\geq$	0	4	5	5	3	5			4	4	2	2	6			4	3	3	4	5			4	3	2	3	6		
D	1	$\geq$	0	3	5	5	6	4			4	5	4	5	4			4	4	5	4	2			4	3	4	5	4		
D	2	$\geq$	0	4	4	3	4	3			5	3	4	4	4			4	5	4	4	3			4	3	5	5	3		
D	3	$\geq$	0	3	4	4	2	3			4	3	4	2	1			3	2	3	5	5			5	5	3	3	3		
D	4	$\geq$	0	5	3	4	2	3			3	5	2	3	6			4	3	2	2	3			3	4	2	2	5		
L		=	0			-	-	-					-	-	-					-	-	-					-	-	-		
L	0	$\geq$	0	-	-	1	1	1			-	-	1	1	1			-	-	1	1	1			-	-	1	1	1		
N	0	=	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Rotation Patterns

- Some companies use rotation patterns.
  - Because that is how they have always done it.
  - Because they are easier to create and maintain manually.
  - Because they are easier to discuss with unions and managers.
- Rotation patterns are more restrictive than individual roster-lines.
  - Hence, less desirable from an optimization point of view.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Employee 1	D	D	-	-	A	N	-
Employee 2	L	A	L	D	D	-	-
Employee 3	N	-	D	D	L	L	L
Employee 4	-	-	-	L	N	-	-
Total	**D, L, N**	**D, A**	**D, L**	**2 x D, L**	**D, L, A, N**	**L, N**	**L**

Rotation Patterns

- We can no longer create an individual roster-line for each employee.
- Several employees share one rotation.
  - The rotation can be generated by solving the subproblem from before with some modifications.
    - The pattern must have a feasible wrap.
  - Demands are given for each weekday (not for each calendar day).
  - Each rotation contributes with several shifts for each day in the roster.
- Works badly when individual skills are important and when we have varying demands on each individual day.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Employee 1	D	D	-	-	A	N	-
Employee 2	L	A	L	D	D	-	-
Employee 3	N	-	D	D	L	L	L
Employee 4	-	-	-	L	N	-	-
Total	**D, L, N**	**D, A**	**D, L**	**2 x D, L**	**D, L, A, N**	**L, N**	**L**

Summary

- What you should be able to remember from this lecture:
  - Manpower planning consists of several steps
    - Each is an interesting optimization problem in its own right.
  - Rostering
    - The practical problem
    - The set partitioning formulation
    - Characteristics and variations
      - Cost structure
      - Rotations
  - Solution method
    - Column generation
    - Dynamic programming
    - Branching strategy and tree search strategy