Basic price optimization - Part 2

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42134 Advanced Topics in Operations Research

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Revenue Management Session 04
Outline

• Common price-response functions

• Price response with competition

• The basic price optimization problem
Recap: the price-response function

- The price-response function, \( d(p) \), specifies demand, \( d \), as a function of the price, \( p \).

- Typical price-response curves demonstrate some degree of smooth price response.

- A price-response function is assumed:
  - nonnegative,
  - continuous,
  - differentiable, and
  - downward sloping.

Slope: change in demand divided by change in price,
\[
\delta(p_2, p_1) = \frac{d(p_2) - d(p_1)}{p_2 - p_1}
\]

Price elasticity: percentage change in demand divided by percentage change in price,
\[
\varepsilon(p_1, p_2) = -\frac{d(p_2) - d(p_1)}{p_1} \frac{p_1}{[p_2 - p_1]d(p_1)}
\]
Recap: willingness to pay

• Each potential customer is assumed to have a maximum willingness to pay (w.t.p.) for a given product.

• The number of customers whose w.t.p. is at least $p$ is denoted $d(p)$.

• The w.t.p. distribution function, $w(x)$, across the population has the following properties:
  
  – For any values $0 \leq p_1 < p_2$, $\int_{p_1}^{p_2} w(x) \, dx$ is the fraction of the population with w.t.p. $p \in [p_1, p_2]$.
  
  – For all nonnegative values of $x$, $0 \leq w(x) \leq 1$.

• Recall, $d(p)$ is the number of customers willing to pay at least $p$ for the product.
  
  – The total demand $D = d(0)$ corresponds to the number of customers willing to pay at least zero, i.e. willing to purchase at all.
  
  – Then we can derive the price-response function $d(p)$ from the w.t.p. distribution:

$$d(p) = D \int_{p}^{\infty} w(x) \, dx$$

The partition into components for total demand and willingness to pay is convenient for modeling the market.
The price-response function and the willingness to pay distribution

• The derivative of the price-response function then is:

\[ d'(p) = -Dw(p) \]

which is nonpositive as required by the downward-sloping demand curve property.

• Conversely, we can derive the w.t.p distribution from the price-response function:

\[ w(x) = -d'(x)/d(0) \]
Common price-response functions

- Linear
- Constant elasticity
- Logit
Linear price-response function

- The general formula for the linear price-response function is:

\[
d(p) = D - mp
\]

where \( D > 0 \) and \( m > 0 \). \( D = d(0) \) is the demand as zero price (total demand achievable).

- The satiating price where the demand drops to zero is \( P = D/m \).

- The slope is \(-m\) for \( 0 < p < P \) and 0 for \( p \geq P \).

- The elasticity, \( \varepsilon \), of the linear price-response function is:

\[
\frac{mp}{D - mp}
\]

The elasticity ranges from 0 at \( p = 0 \) and approaches infinity as \( p \) approaches \( P \) and drops to 0 again for \( p > P \).
Linear price-response function

The linear price-response function is convenient but not realistic in a competitive market.

\[ D = d(0) \]

\[ d(p) = D - mp \]

A linear price-response function.
Uniform willingness-to-pay distribution

- A linear price-response function corresponds to a uniform w.t.p. distribution, and vice versa.

Recall how to derive the w.t.p. distribution from the price-response function:

$$w(x) = -\frac{d'(x)}{d(0)}$$
The constant-elasticity price-response function

• Constant-elasticity price-response functions have a point elasticity that is the same at all prices, i.e.

\[
\frac{d'(p)p}{d(p)} = -\epsilon \quad \text{for all } p > 0
\]

• This results in the price-response function

\[
d(p) = C p^{-\epsilon}
\]

where \( C > 0 \) is a parameter chosen such that \( d(1) = C \).

• The slope of the constant-elasticity price-response function is

\[
d'(p) = -C \epsilon p^{-(\epsilon + 1)}
\]

• However, such price-response functions do not meet the usual assumptions for price-response functions as the following examples show...
Examples of constant-elasticity price-response functions

- Constant-elasticity price-response functions are neither finite nor satiating.

- Demand approaches infinity as the price approaches zero, i.e. the total demand $D = d(0) = \infty$.

- Demand does not drop to zero at any price, no matter how high, i.e. there is no satiating price $P$. 

Constant-elasticity price-response functions with different elasticities.
Properties of the constant-elasticity price-function

• The revenue, \( R \), equals price times demand, i.e.

\[
R(p) = p \cdot d(p) = pCp^{-\varepsilon} = Cp^{(1-\varepsilon)}
\]

• The slope of the revenue function then is

\[
R'(p) = (1 - \varepsilon)Cp^{-\varepsilon} = (1 - \varepsilon) \cdot d(p)
\]

• Since \( d(p) > 0 \), the direction of the slope is determined by \((1 - \varepsilon)\):
  - If \( \varepsilon < 1 \) (inelastic demand), then \( R'(p) > 0 \), i.e. **revenue increases** with **increasing prices**.
  - If \( \varepsilon > 1 \) (elastic demand), then \( R'(p) < 0 \), i.e. **revenue increases** with **decreasing prices**.
  - If \( \varepsilon = 1 \) (elastic demand), then \( R'(p) = C \) for any price, i.e. **revenue is constant** and **independent** of **price changes**.
Revenue and price elasticity

\[ \epsilon > 1: \text{Revenue increases with decreasing price} \]

\[ \epsilon < 1: \text{Revenue increases with increasing price} \]

\[ \epsilon = 1: \text{Constant revenue} \]

Clearly, the constant-elasticity price-response function is unrealistic as a global price-response function.

Revenue as a function of price for the constant-elasticity price-response function.
Limitations of the linear and constant-elasticity price-response functions

• Neither the linear nor the constant-elasticity price-response function is very realistic as a global model.
  – They can, however, be useful as local estimators of demand as a function of the price.

• The limitation of these two models can be seen by considering their corresponding willingness-to-pay distributions.
  – The w.t.p. distribution for a linear price-response function is uniformly distributed between 0 and some maximum value.
  – The w.t.p. distribution for a constant-elasticity price-response function is highly concentrated around 0 and drops steadily as price increases.

• Neither of these functions seem to represent customer behavior realistically.
Market price and changes in price elasticity

- So how would we expect customers to react to changing prices?
  - In general, price elasticity is **high around the market price** $p^m$ and low for smaller or larger values of $p$.

![S-shaped price-response function.](image_url)

- Small changes in demand when prices are low, i.e. $\varepsilon$ is low.
- Dramatic changes in demand when prices are near the market price, i.e. $\varepsilon$ is high.
- Small changes in demand when prices are high, i.e. $\varepsilon$ is low.
The logit price-response function

• The logit price-response function captures the property that small price changes around some market price $p^m$ will lead to substantial shifts in demand whereas demand is less sensitive to price changes if they are much lower or higher than the competitors’ prices.

• Therefore, the logit price-response function is a more suitable way to estimate changes in demand as a function of price changes in a competitive market. Such a price-response function can be defined as

$$d(p) = \frac{Ce^{-(a+bp)}}{1 + e^{-(a+bp)}}$$

• Broadly speaking, $C > 0$ indicates the size of the overall market and $b > 0$ specifies price sensitivity. Usually we also have that $a > 0$.

• The price-sensitivity curve is steepest at $p^m = -(a/b)$ which can be considered to be approximately the market price.
Examples of logit price-response functions

As $b$ grows larger, the market approaches perfect competition.

Logit price-response functions with $C = 20,000$ and $p^m = 13,000$. 

Properties of a logit price-response function

- The w.t.p. distribution of a logit price-response function follows a bell-shaped curve, known as the \textit{logistic distribution} similar to the normal distribution.

\[ p^m = -(a/b). \]

This corresponds to the point where the slope of the price-response function is steepest.

W.t.p. distribution of a logit price-response functions with \( C = 20,000 \) and \( b = 0.0005 \).
Price response with competition

- Competition is a very important fact of life in any market of interest and most managers consider competition as the primary influence on their pricing.

- There are three different levels at which competition might be included in pricing and revenue optimization (PRO):
  1. Incorporating competition in the price-response function.
  2. Explicitly modeling consumer choice.
  3. Trying to anticipate competitive reaction.

- The first action is easiest whereas the third is the most difficult and most sophisticated attempt to incorporate competition.
Incorporating competition in the price-response function

- In many cases, competitors’ prices are not known at the time of optimizing the company’s prices.

- Retailers generally have a good idea of what their major competitors are charging but they usually lack time or resources to investigate competition daily or weekly.

- So how is it possible to include competition in the price optimization?
  - Price-response functions are based on history so they already include “typical” competitive pricing. (It would actually be impossible to estimate price-response without competition.)
  - If competitors behave similarly in the future, the price-response function will be a fair representation of market response.
  - If the market changes the company needs to adjust the price-response function and re-optimize the prices.
Consumer-choice modeling

• If we have access to current competitive prices, we are better able to predict customer response to our own prices.
  – Such information is increasingly available, especially in online markets.

• Consumer-choice modeling is based on the situation in which a number of competitors provide similar products to some population of customers.
  – Each competitor sets a price for the product.
  – Each customer has a willingness to pay for each product, corresponding to the “brand value” associated with the product.
  – The surplus for each customer and each product is defined as the difference between the customer’s w.t.p. and the price.
  – Each customer then purchases the product with the highest positive surplus, corresponding to the “best buy” of the given brand.
  – If none of the products have a positive surplus for a customer, he/she will not purchase at all.
Example of customer-choice

- A customer can choose between five models of stereo amplifiers.
  - The models are priced differently and the customer has different willingness to pay for each model.
  - This results in a different surplus value for each of the models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Price</th>
<th>Willingness to pay</th>
<th>Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Audio Two</td>
<td>$400</td>
<td>$250</td>
<td>−$150</td>
</tr>
<tr>
<td>Koshiba</td>
<td>$650</td>
<td>$950</td>
<td>$300</td>
</tr>
<tr>
<td>Takata and Fuji</td>
<td>$500</td>
<td>$550</td>
<td>$50</td>
</tr>
<tr>
<td>Soundmaster</td>
<td>$750</td>
<td>$950</td>
<td>$200</td>
</tr>
<tr>
<td>Cacophonia</td>
<td>$1,200</td>
<td>$1,300</td>
<td>$100</td>
</tr>
</tbody>
</table>

- This customer will purchase the Koshiba amplifier since it provides the highest surplus, i.e. difference between w.t.p. and price.
  - Note that neither the cheapest model (Audio Two) nor the customer’s favorite (Cacophonia) is purchased.
Market share

• Assume that we have a fixed population of potential buyers, each with a positive surplus for at least one product. Then, the market share obtained by a particular alternative \( i \), \( \mu_i \), is

\[
\mu_i = \text{Fraction of buyers for whom } w.t.p.(i) - p_i > w.t.p.(j) - p_j \text{ for all } j.
\]

• Let \( p = (p_1, p_2, \ldots, p_n) \) be the vector of prices for the alternatives.
• Then, a market-share function determines \( \mu_i = f_i(p) \) for all \( i \).

• A market-share function has the following characteristics.
  1. The market share of each alternative is between 0 and 1: \( 0 \leq f_i(p) \leq 1 \).
  2. Every buyer chooses some alternative: \( \sum_{i=1}^{n} f_i(p) = 1 \).
  3. Increasing price \( p_i \) decreases the market share for product \( i \) but increases the market share for other products: \( \frac{\partial f_i(p)}{\partial p_i} < 0 \), \( \frac{\partial f_i(p)}{\partial p_j} > 0 \) for \( i \neq j \).

Given the total demand \( D \), the demand for product \( i \) is:

\[
d_i(p) = D\mu_i = Df_i(p)
\]
Anticipating competitive response

- Since we take competitive prices into account in setting our prices, we should anticipate that our competitors will take our price into account when setting theirs.
  - If we drop a price, some competitors will probably match and thereby erase much of the demand we had predicted by the price decrease.

- Attempts to predict competitive response and incorporate it into current pricing decisions falls within *game theory*.
  - Such approaches apply to strategic (long-term) pricing but are less suitable for tactical (medium-term) pricing and revenue optimization.

- There does not appear to be a single pricing and revenue optimization (PRO) system that explicitly attempts to forecast competitive response using game theory.

- However, many of the (small) price adjustments in PRO are not likely to trigger any explicit response from the competition, so effort may be better used elsewhere, at least in many industries.
Incremental costs

- Calculating costs may seem much easier than estimating demand, price-response, and competitive reactions.
  - However, retrieving the company’s own costs is often not that simple!

- PRO decisions are based on the *incremental cost* of a customer commitment:

> The incremental cost of a customer commitment is the difference between the total costs a company would experience if it makes the commitment and the total cost it would experience if it doesn’t.
The basic price optimization problem

- The difference between the price of a product sold and its incremental cost is called its margin.
- The sum of the margins of all products sold during a time period is called the total contribution.
- When a supplier is selling a single product at price $p$, the total contribution will be
  \[ m(p) = (p - c) d(p) \]
  where $c$ is incremental cost for the product.

- The basic price optimization problem is to maximize total contribution:
  \[ \max_p (p - c) d(p) \]
- I.e. the goal is to find the optimal price for the products sold.
The total contribution function

- The total contribution function $m(p)$ is hill-shaped with a single peak, corresponding to the optimal price $p^*$. 

![Graph showing total contribution function with peak at optimal price $p^*$ and negative contribution at low prices.]

- Pricing too low (below incremental cost) implies losing money.
- Pricing high never implies losing money, worst case just driving demand to zero.
Optimality conditions

• Finding the optimal price is an unconstrained optimization problem which can be solved by taking the derivative of \( m(p) \) and set it equal to zero:

\[
m'(p) = d'(p)(p - c) + d(p) = 0
\]

• The optimal price \( p^* \) will then satisfy

\[
d(p^*) = -d'(p^*)(p^* - c)
\]

• Rewriting this we get optimality: marginal revenue = marginal cost

\[
p^* d'(p^*) + d(p^*) = c d'(p^*)
\]

Marginal revenue (derivative of total revenue with respect to price): the additional revenue achievable by a small increase in price. Usually positive for low prices and negative for high prices.

Marginal cost: the additional cost the seller would incur from a small increase in price. Always negative as increase in price leads to lower demand and thereby lower costs.
Increasing total contribution

- If marginal revenue is greater than marginal cost, the supplier can increase total contribution by increasing the price.

- If, on the other hand, marginal revenue is lower than marginal cost, decreasing the price will increase total contributions.

- One way to check the optimality of $p^*$ is to use the marginality test:

  \[\text{A particular price can only be optimal if raising the price by a penny or lowering the price by a penny results in reduced margin contribution.}\]
Optimality and price elasticity

• Rewriting the optimality condition by including price elasticity we get

\[ m'(p) = d(p) \left[ \frac{d''(p)p}{d(p)} + 1 \right] - d'(p)c \]

\[ = d(p)[1 - \epsilon(p)] - d'(p)c \]

Always nonnegative since \( d(p) \) is downward sloping.

• This means that if the point elasticity at our current price is less than 1, we can increase total contribution by increasing price.

• However, typically the price elasticity will increase when prices go up!
Checking the optimal price

• If maximizing total revenue instead of total contribution, the optimization problem is

\[
\max_{p} R(p) = d(p)p
\]

• Maximizing total revenue ignores incremental cost.
  – Incremental cost equals or approaches zero in some service industries such as movie theaters, video rentals, sporting events, etc.
  – Also companies that have already purchased a fixed amount of perishable, nonreplenishable inventory have incremental cost approximately equal to zero.

• Optimality is achieved by

\[
R'(p') = d'(p')p' + d(p') = 0
\]

which yields the optimal price satisfying

\[
-\frac{d'(p')p'}{d(p')} = \varepsilon(p') = 1
\]

The revenue-maximizing price occurs where the price elasticity is equal to 1
Revenue- and contribution maximization

- The optimal price when maximizing revenue is lower than the optimal price when maximizing total contribution.

If combining the two optimization criteria, the optimal price lies between $p'$ and $p^*$. 

Marginal revenue curve is downward sloping. Marginal revenue $= 0$ 

Marginal revenue $= 0$ 

Marginal cost $< 0$ 

Revenue-maximizing price 

Contribution-maximizing price
Summary

• The core problem in pricing and revenue optimization is maximizing the total contribution subject to strategic and/or physical constraints.

• A key input to any price optimization problem is the price-response function that relates price to demand.

• Two common key measures of price sensitivity are the slope and elasticity of the price-response function.

• Price-response functions are often a measure of the number of people whose maximum willingness to pay is greater than a certain price.

• Logit functions, a reverse S-shaped model, is often appropriate, whereas linear and constant-elasticity functions tend to be unrealistic.
Summary continued

- Optimality conditions for the unconstrained price optimization problem:
  - Marginal revenue equals marginal cost
  - The derivative of total contribution with respect to price is zero
  - The contribution margin ratio is equal to 1 over the price elasticity
Customized pricing

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Revenue Management Session 04
Characterization of list-pricing

• The seller offers a stock of goods to different customer segments through different channels

• Potential customers arrive, observe a price, and decide whether or not to purchase

• Seller needs to decide what price to offer for each product to each customer group through each channel

• Seller monitors sales of his goods and periodically updates the prices and/or availabilities

• The airline seat allocation problem is an example of the list-pricing model
Characterization of customized pricing

• Potential customers approach the seller, one by one

• The seller quotes each one a price

• Often (but not always) each potential buyer wants something different, e.g. different variant or different quantity

• The seller can quote a different price for each request

• Price can be based on knowledge of the individual buyer, products requested, general market conditions, and so on.
Example of customized pricing: Unsecured Consumer Loans

- Unsecured loans are monetary loans that are not secured against the borrower's assets

- Amounts for unsecured consumer loans in the UK ranges from £500 to £20,000

- Typical terms are between one to seven years

- The market has many competitors with more than 20 nationwide providers in the UK
Example of annual percentage rates for a £3000 unsecured consumer loan

<table>
<thead>
<tr>
<th>Provider</th>
<th>APR (%)</th>
<th>Monthly payment (£)</th>
<th>Total interest (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Rock</td>
<td>6.9</td>
<td>92.49</td>
<td>329.79</td>
</tr>
<tr>
<td>Nationwide</td>
<td>7.9</td>
<td>93.87</td>
<td>379.35</td>
</tr>
<tr>
<td>AA Loan</td>
<td>10.9</td>
<td>98.07</td>
<td>530.67</td>
</tr>
<tr>
<td>Egg</td>
<td>10.9</td>
<td>98.07</td>
<td>530.67</td>
</tr>
<tr>
<td>Sainsbury's Bank</td>
<td>11.5</td>
<td>98.93</td>
<td>561.41</td>
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<td>American Express Bank</td>
<td>11.7</td>
<td>99.21</td>
<td>571.69</td>
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<td>MBNA Express Loan</td>
<td>11.9</td>
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<tr>
<td>Lombard Direct</td>
<td>11.9</td>
<td>99.50</td>
<td>581.99</td>
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<td>Woolwich</td>
<td>11.9</td>
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<td>581.99</td>
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<tr>
<td>Virgin</td>
<td>11.9</td>
<td>99.50</td>
<td>581.99</td>
</tr>
<tr>
<td>Alliance and Leicester</td>
<td>12.5</td>
<td>100.36</td>
<td>612.99</td>
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<td>Barclays</td>
<td>12.9</td>
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<td>105.32</td>
<td>791.63</td>
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<tr>
<td>Lloyd's TSB</td>
<td>17.7</td>
<td>108.01</td>
<td>888.22</td>
</tr>
</tbody>
</table>

Sales and pricing process for an unsecured consumer loan

• Step 1 First quote. Customers are asked how much they want to borrow, term, name, age, and time at current address

• The bank quotes an APR and monthly payment. About 14% of the customers drop out (“lost quotes”)

• Step 2 Application. The remainder fills out a more extensive application, including employer, income, expenses, and other loans outstanding.

• The bank assigns a credit score to each applicant. About 50% of the applications are declined by the bank as too risky.

• Step 3 Final quote. The bank can adjust the APR for the accepted applications. About 90% of customers go on to take a loan
Overview of consumer loan sales and pricing process
Decision problems of the bank

• What APR to offer in the initial quote?

• What adjusted APR to offer to customers whose application has been accepted?

• This is a customized-pricing problem because the bank can choose different APRs for different customers based on type of loan, channel, and market segment.
Overview of differences between list pricing and customized pricing

• The key issue in customized pricing is not *if* the customer will purchase but *who* the customer will purchase from, as opposed to list pricing

• In customized pricing the seller can quote a different price to each customer. The price should reflect the best information the seller has at that time

• In customized pricing the seller can track lost business whereas in list pricing the seller can only observe how many units he sold. The lost *bid* information is important for the estimation of the *bid-response* function
Bids and price quotes

- The existence of a bid or price quote is common in customized pricing.

- In business-to-business markets the quote is often a response to a request for proposal (RFP) or request for quote (RFQ) issued by a buyer.

- In business-to-consumer markets, such as unsecured consumer loans, the price quote often occurs after a customer phone call or application.

- In both markets the price quote occurs after the seller knows something about the buyer and what he wants.

- This is in contrast to list pricing where customers find prices by accessing a price list or reading a price tag in a store. Here the prices need to be set before the seller knows anything about the individual potential customer.
Coexistence of list pricing and custom pricing

- Sellers often use list pricing to set a “ceiling”, while most customers are quoted a price at a discount from the list

- The manufacturer’s suggested retail price (MSRP) (in Danish vejledende udsalgspris) is usually an upper limit on the price

- The selling price is determined by some combination of the customer’s ability to negotiate and the seller’s willingness to sell at a lower price
Calculating optimal customized prices

- Definition of the objective function of the customized-pricing problem (maximize):

- Expected contribution at price $p = (\text{Deal contribution at } p) \times (\text{Probability of winning bid at } p)$

- Uncertainty derives from two sources:
  - Competitive uncertainty
  - Preference uncertainty

- If there was no uncertainty the problem would be easy – we would submit the most profitable bid that would win

- No preference uncertainty when buyer must choose lowest-cost bidder
**Single-competitor model**

- Suppose you are an auto manufacturer bidding against a single competitor to sell 50 pickup trucks to a county park district.

- The park district will buy all 50 trucks from a single supplier and is committed by law to pick the lowest bidder.

- Each supplier has been asked to submit a single bid.

- Your production cost per truck is $10,000.

- Based on your past experience your belief about what your competitor will bid can be described by a uniform distribution between $9,000 and $14,000.

- What should you bid to maximize expected profitability?
Single-competitor model

• Call you bid $p$ and the competing bid $q$

• You will win if your bid is less than the competing bid, i.e. $p < q$
  (for simplicity we ignore the possibility of a tie)

• Let $\rho(p)$ be the probability that you would win if you bid a price $p$

• Let $F(x)$ be the probability that the competing bid will be less than $x$

• Then,

$$\rho(p) = 1 - F(p)$$
Uniform probability distribution on a competitor's bid
Probability of winning the bid as a function of price
Bid-response function

- We call $\rho(p)$ the bid-response function of this deal

- For each deal, the bid-response function specifies the probability of winning the deal as a function of our bid

- The bid-response function is the customized-pricing analog of the price-response curve

- Price-response curve: Total expected demand as a function of list price

- Bid-response curve: Probability of winning an individual bid as a function of price bid

- For the example:

  $$\rho(p) = 2.8 - \frac{p}{5000} \quad \text{for} \quad $9,000 \leq p \leq $14,000$$
Results

• The price that maximizes expected profitability can be found by solving

\[ \text{Maximize } 50 \left(2.8 - \frac{p}{5000}\right) (p - 10,000) \]

• Solving for \( p \) gives \( p^* = \$12,000 \)

• At this price the probability of winning the bid is \( 2.8 - \frac{12,000}{5000} = 0.4 \)

• The margin per unit if the deal is won is \( \$12,000 \) - \( \$10,000 \) = \( \$2,000 \)

• Expected total margin is \( 40\% \times 50 \times \$2,000 = \$40,000 \)
Expected profitability as a function of bid price
The customized pricing problem

Maximize $\pi(p) = \rho(p) \ m(p)$

where

$\pi(p)$ = expected contribution at price $p$

$\rho(p)$ = bid-response function

$m(p)$ = deal contribution at price $p$
Bid response

• The most challenging part of optimizing customized prices is determining an appropriate bid-response function

• If we already have a bid-response function, finding the price that maximizes expected contribution is relatively simple

• How do we estimate the bid-response function?
  
  – Bottom-up modeling from our probability distributions on how we believe our competitors will bid and the selection process of the buyer
  
  – Expert judgment, i.e. gather people that have knowledge of the deal to derive the bid-response function
  
  – Statistical estimation based on historic patterns of wins and losses
Minimum information needed to estimate a bid-response function

Table 11.4
Sample bid prices and results

<table>
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Wins = 1 and losses = 0
Linear fit

A

B

Win probability

Price ($)

Win probability

Price ($)
Examples of logit response curves

\[ \rho(p) = \frac{1}{1 + e^{(a+bp)}} \]
Logit bid-response curve fit by maximizing log likelihood
Conclusions

• Estimating the bid-response function is the challenging part of customized pricing

• A variety of approaches can be used, but the logit function is quite popular

• It is easy to incorporate multiple dimensions in the bid-response function using the multivariate logit function

• There exists a range of sophisticated statistical estimation techniques for practical applications belonging to the field of discrete choice methods