Basic price optimization

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42134 Advanced Topics in Operations Research

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Revenue Management Session 03

DTU Management Engineering
Department of Management Engineering

Outline

- The price-response function
- Price response with competition
- Incremental costs
- The basic price optimization problem

Introduction to price optimization

- The basic pricing and revenue optimization problem can be formulated as an optimization problem.
  - The objective is to maximize contribution:
    total revenue minus total incremental cost from sales.

- The key elements of the optimization problem is:
  - the price-response function and
  - the incremental cost of sales.

- In this lecture we will formulate and solve the pricing and revenue optimization problem for a single product in a single market without supply constraints.

- Furthermore, we will discuss some important optimality conditions.

The price-response function

- A fundamental input to any price and revenue optimization (PRO) analysis is the *price-response function* (or *curve*) *d(p)*.

- There is one price-response function associated with each combination of product, market-segment, and channel in the PRO cube.

> *The price-response function, d(p), specifies demand for the product of a single seller as a function of the price, d, offered by that seller.*

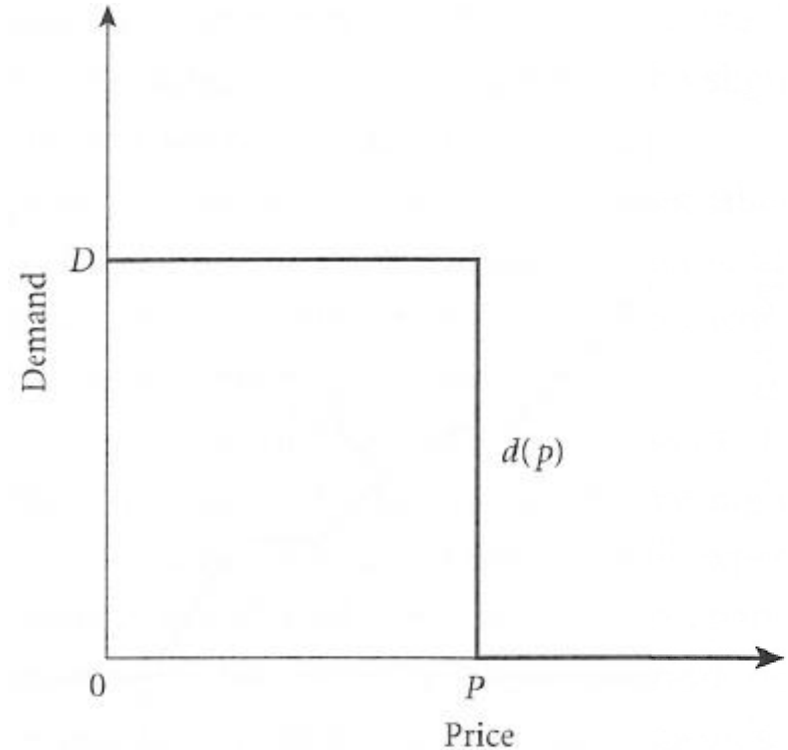
- This constrasts with the concept of a *market demand curve* which specifies how an entire market will respond to changing prices.

- Different firms competing in the same market face different price-response functions.
  - The price-response functions may differ due to many factors, such as the effectiveness of their marketing campaigns, perceived customer differences in quality, product differences, location, etc.

Price-response functions in a perfectly competitive market

- In a perfectly competitive market:
  - The price-response faced by an individual seller is a vertical line at the market price.
  - For higher prices, the demand drops to 0.
  - If he prices below the market price, his demand equals the entire market.

- For example a wheat farmer:
  - If he charges more than the market price, he will sell nothing.
  - If he charges below the market price, the demand will be effectively infinite.



Price-response curve in a perfectly competitive market.

Commodities in a perfectly competitive market

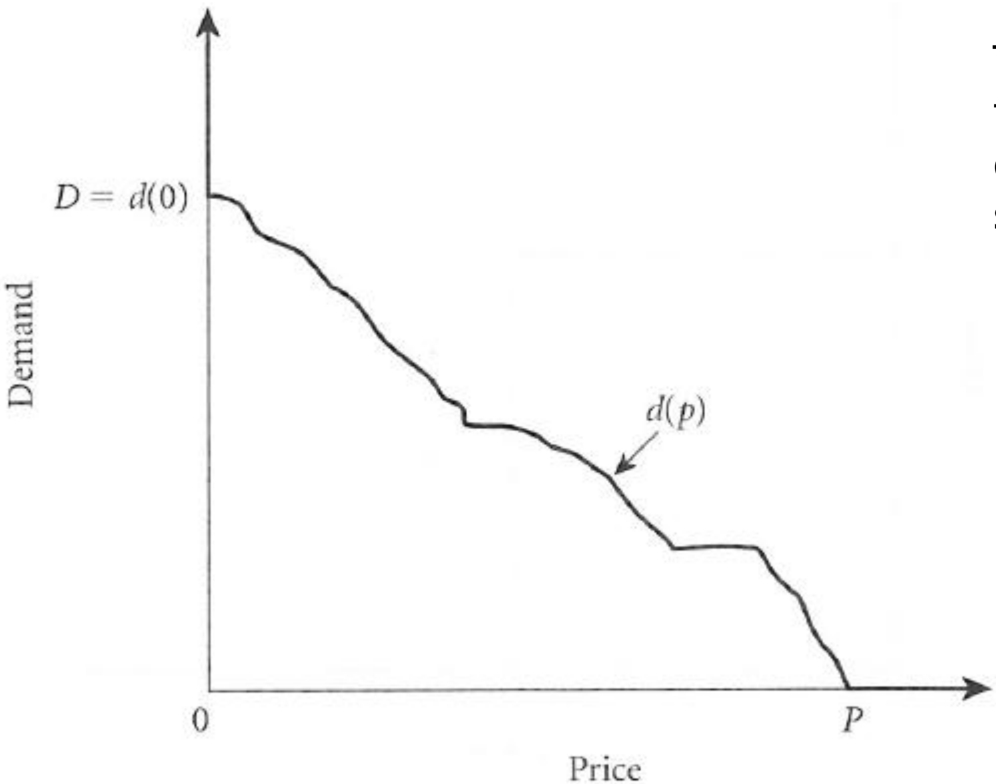
- Commodity producers, such as the wheat farmer, have no pricing decision – the price is set by the operation of the larger market.

- I.e., in a competitive market, each firm only has to worry about how much output it wants to produce. Whatever it produces can only be sold at one price: the going market price.

- Therefore, sellers of true commodities in a perfectly competitive market have no need for pricing and revenue optimization (PRO).

- However, true comodities are surprisingly rare!

Price-response curves in non-competitive markets



Typical price-response curve.

The price-response curves which face most companies demonstrate some degree of smooth price response:

- As the price increases, the demand declines.
- Demand reaches zero at some *satiating price P*.

Properties of the price-response function

- The price-response functions used in PRO analysis are time-dependent.
  - We set prices that will be in place for some finite period of time.
  - The period may be minutes or hours or longer.
  - At the end of each period we have the opportunity to change prices.

- The demand we expect to see at a given price will depend on the length of the time period the price will be in place.
  - I.e. there is no single price-response function without an associated time interval.

- There are many different ways in which product demand might change in response to changing prices but all price-response functions are assumed:
  - nonnegative *($p \geq 0$)*,
  - continuous (no gaps in market response to prices),
  - differentiable (smooth and with well-defined slope at every point), and
  - downward sloping (raising prices decreases demand).

Implies imprecision since using derivatives rather than difference equations.

Measures of price sensitivity

- The two most common measures of price sensitivity are the *slope* and the *elasticity* of the price-response function.
  - The **slope** measures how demand changes in response to a price change and equals the change in demand divided by the difference in prices.
  - The **price elasticity** is defined as the ratio of the percentage change in demand to the percentage change in price.

The slope of price-response functions

- The slope equals the change in demand divided by the change in prices:

$$\delta(p_2, p_1) = [d(p_2) - d(p_1)]/(p_2 - p_1)$$

- Downward sloping: $p_1 > p_2$ implies $d(p_1) \leq d(p_2)$, i.e. $\delta(p_1,p_2) \leq 0$.

- The slope at a single price, p_1, can be computed as the limit of the above equation as p_2 approaches p_1:

$$\delta(p_1) = \lim_{h \to 0}[d(p_1 + h) - d(p_1)]/h = d'(p_1)$$

  where $d'(p_1)$ is the derivative of the price-response function at p_1.

- For *small* price changes we can write: $d(p_2) - d(p_1) \approx \delta(p_1)(p_2 - p_1)$

  I.e. a large slope means that demand is more responsive to prices than a smaller slope.

The price elasticity of price-response functions

- The elasticity equals the percentage change in demand divided by the percentage change in prices:

$$\epsilon(p_1, p_2) = -\frac{100\{[d(p_2) - d(p_1)]/d(p_1)\}}{100\{(p_2 - p_1)/p_1\}}$$

where $\varepsilon(p_1,p_2)$ is the elasticity of a price change from p_1 to p_2.

- This equation can be reduced to:

$$\epsilon(p_1, p_2) = -\frac{[d(p_2) - d(p_1)]p_1}{[p_2 - p_1]d(p_1)}$$

Since downward sloping price-response curve, **ε(p1,p2) ≥ 0**.

EX:

ε = 1.2	ε = 0.8
10 % price increase:	10 % price decrease:
12 % demand decrease.	8 % demand increase.

Point elasticity

- The price elasticity at a single price, p_1, (”point elastiticy at p_1”) can be computed as the limit of the above price elasticity equation as p_2 approaches p_1:

$$\epsilon(p_1) = -d'(p_1)p_1/d(p_1)$$

- I.e. the points elasticity is equal to –1 times the slope of the demand curve times the price divided by the demand.

- The point elasticity is useful as a *local* estimate of the change in demand resulting from a *small* change in price.

- Note that, unlike the slope, the price elasticity is independent of the units in which the price and demand is measured.

Price elasticity in practice

- The term *price elasticity* is often used as a synonym for *price sensitivity*.
  - ”High price elasticity” items have very price sensitive demand, while ”low price elasticity” items have much less price sensitive demand.

- Often, a good with a price elasticity greater than 1 is described as elastic, while one with an elasticity less than 1 is described as inelastic.

- Elasticity is dependent on whether we measure the total market response if all suppliers of a product change their prices or the price-response elasticity for an individual supplier within the market.
  - If all suppliers raise prices, the only alternative for customers is to purchase a substitute product or to go without.
  - If a single supplier raises prices, customers can go to its competitor.

- Furthermore, as well as other aspects of price response, elasticity is dependent on the time period under consideration.
  - There may be great difference in price elasticity in the *short run* and in the *long run*...

Price elasticity for different goods

- For most products, *short-run elasticity* is lower than *long-run elasticity* since buyers have more flexibility to adjust to higher prices in the long run.
  - For example, short-run elasticitiy for gasoline has been estimated to be 0.2, while the long-run elasticity has been estimated at 0.7.
  - At first, consumers still need to by gasoline, but in the long term, people will change habits, e.g. buying higher mile-per-gallon cars.

- On the other hand, for many durable goods, such as cars and washing machines, the long-run price elasticity is lower than the short-run elasticity.
  - The reason is that customers initially respond to a price rise by postponing the purchase of a new item.
  - However, they will still purchase at some time in the future, so the long-run effect of the price change is less than the short-run effect.

Examples of price elasticity

Estimated price elasticities for various goods and services

Good	Short-run elasticity	Long-run elasticity
Salt	0.1	—
Airline travel	0.1	2.4
Tires	0.9	1.2
Restaurant meals	2.3	—
Automobiles	1.2	0.2
Chevrolets	4.0	

- **Salt** has a low price elasticity as a respond to *market* price changes (people will by salt even if prices go up) but for an individual seller, the price elasticity would be expected to be high due to competitiveness.
- **Airline tickets** have a large long-term price elasticity since passengers will change their tavel habits if prices stay high.
- **Cars** have a low long-tirm elasticity since initially posponed purchashes will be be realized later in time even though prices stay high.

Price response and willingness to pay

- In reality, the price-response function is not simply given. Demand is the result of each potential customer observing the prices and deciding whether or not to buy a specific product.

- The price-response function specifies how many more of those potential customers would buy if we lowered our price and how many current buyers would not buy if we raised our price.
  - I.e., the price-response function is based on *assumptions* about customer behavior.

- The most important part of models of customer behavior is based on *willingness to pay* (w.t.p).

- The willingness-to-pay approach assumes that each potential customer has a *maximum willingness to pay* (also called a ”reservation price”) for a given product.
  - A customer will purchase if and only if the price is less than his/her maximum w.t.p.

Willingness to pay

- The number of customers whose maximum willingness to pay (w.t.p.) is at least p is denoted $d(p)$.
  - I.e., $d(p)$ is the number of customers who are willing to pay the price p or more for the product.

- Define the function $w(x)$ as the w.t.p. distribution across the population. Then for any values $0 \leq p_1 < p_2$:

$$\int_{p_1}^{p_2} w(x)\, dx$$ is the fraction of the population that has w.t.p. between p_1 and p_2.

- Note that $0 \leq w(x) \leq 1$ for all nonnegative values of x.

The willingness to pay distribution

- Let *D = d(0)*, i.e. the number of customers willing to pay zero or more – i.e. willing to buy the product at all, be the maximum demand achievable. Then we can derive *d(p)* from the w.t.p distribution:

$$d(p) = D \int_{p}^{\infty} w(x)\, dx$$

Recall that d(p) is the number of customers who are willing to pay the price p.

- Note that the price-response function is partitioned into two separate components: the total demand *D* and the w.t.p. distribution *w(x)*.

- Next lecture considers examples of price-response functions and the *basic price optimization problem.*

Simplified airline fare structures and marginal revenue transformation

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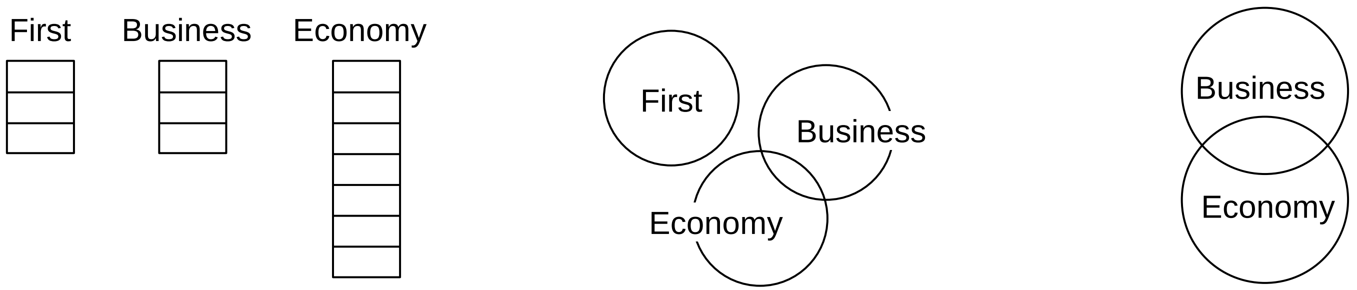
The low cost carrier competition led to simplified fare structures in scheduled airlines

	Past	Transition	Future
	Strong market segmentation	**Weakening of market segmentation**	**Increasing market convergence**
Industry	• Monopoly	• Intense LCC competition • Price transparency	• Consolidation of industry

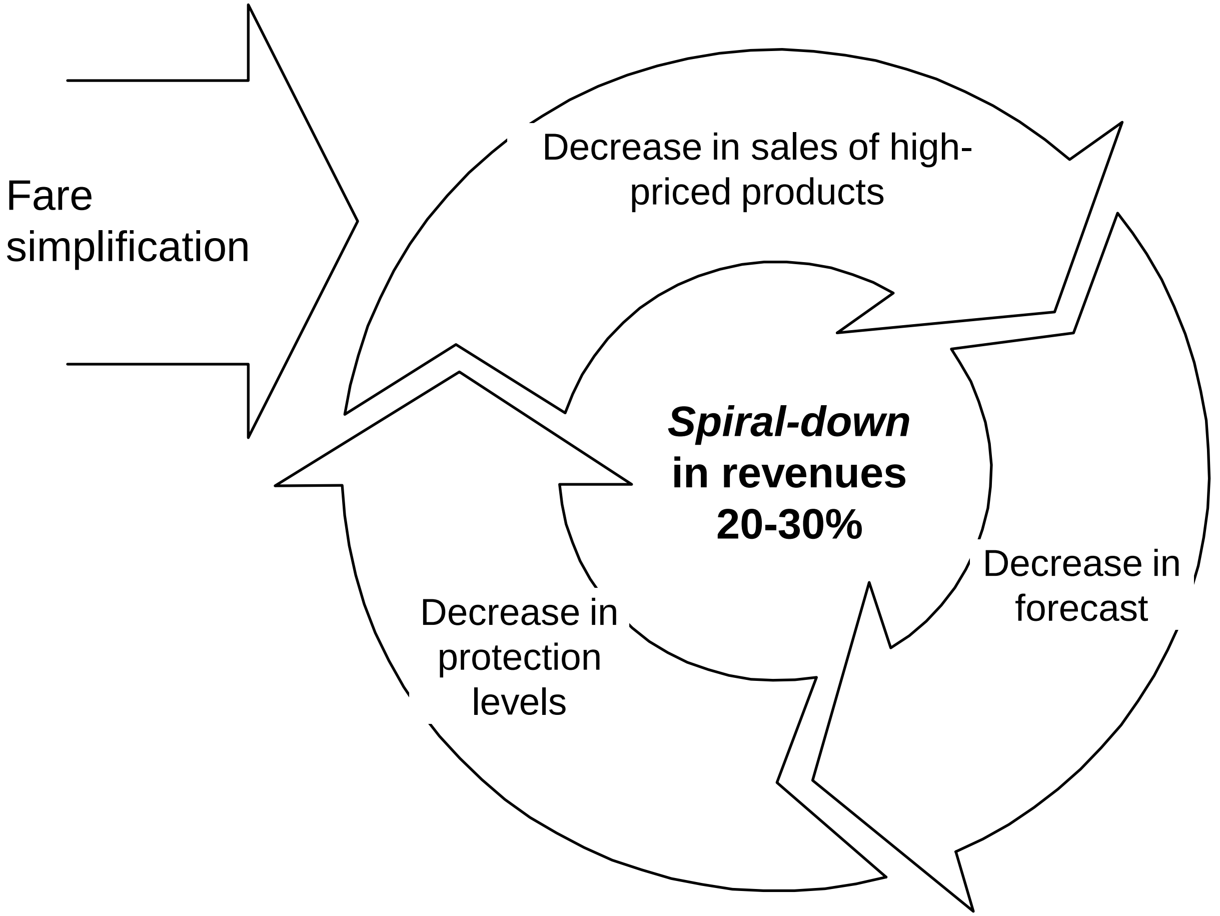
Fare structure



	Past	Transition	Future
Fare structure	• Traditional fare restrictions (AP, RT, SA/SU, min/max) • High fare ratios	• Less-restricted fares • Lower prices	• Stabilization of prices

…simplified fares "*is the most important pricing development in the industry in the past 25 years*" Tretheway (2004)

Without modifications of traditional RM systems fare simplification leads to spiral-down in revenues of 20-30%



The root cause of spiral-down is the break-down of the independent-demand assumption of RM systems

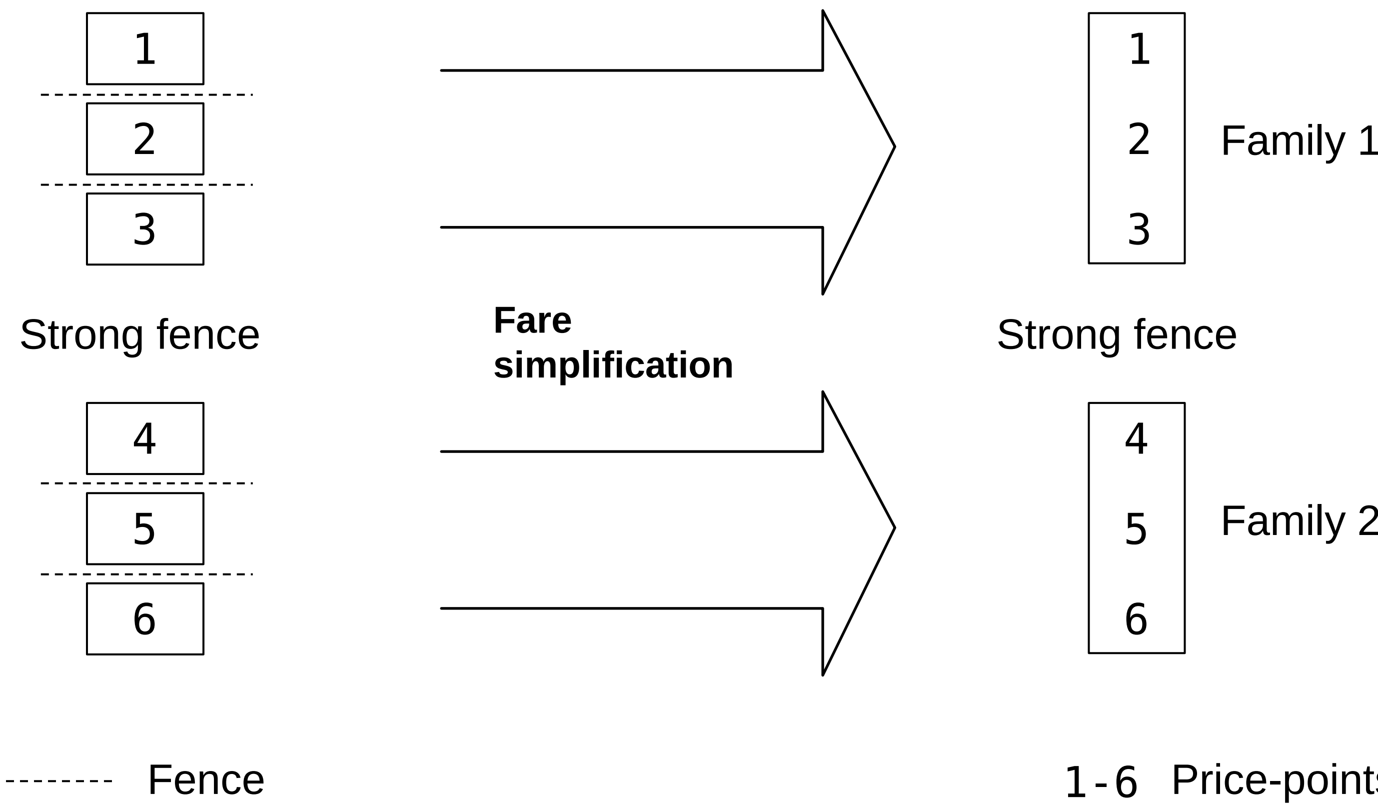
The fare simplification groups fares with similar restrictions into *fare families*

Fare classes

Independent demand model

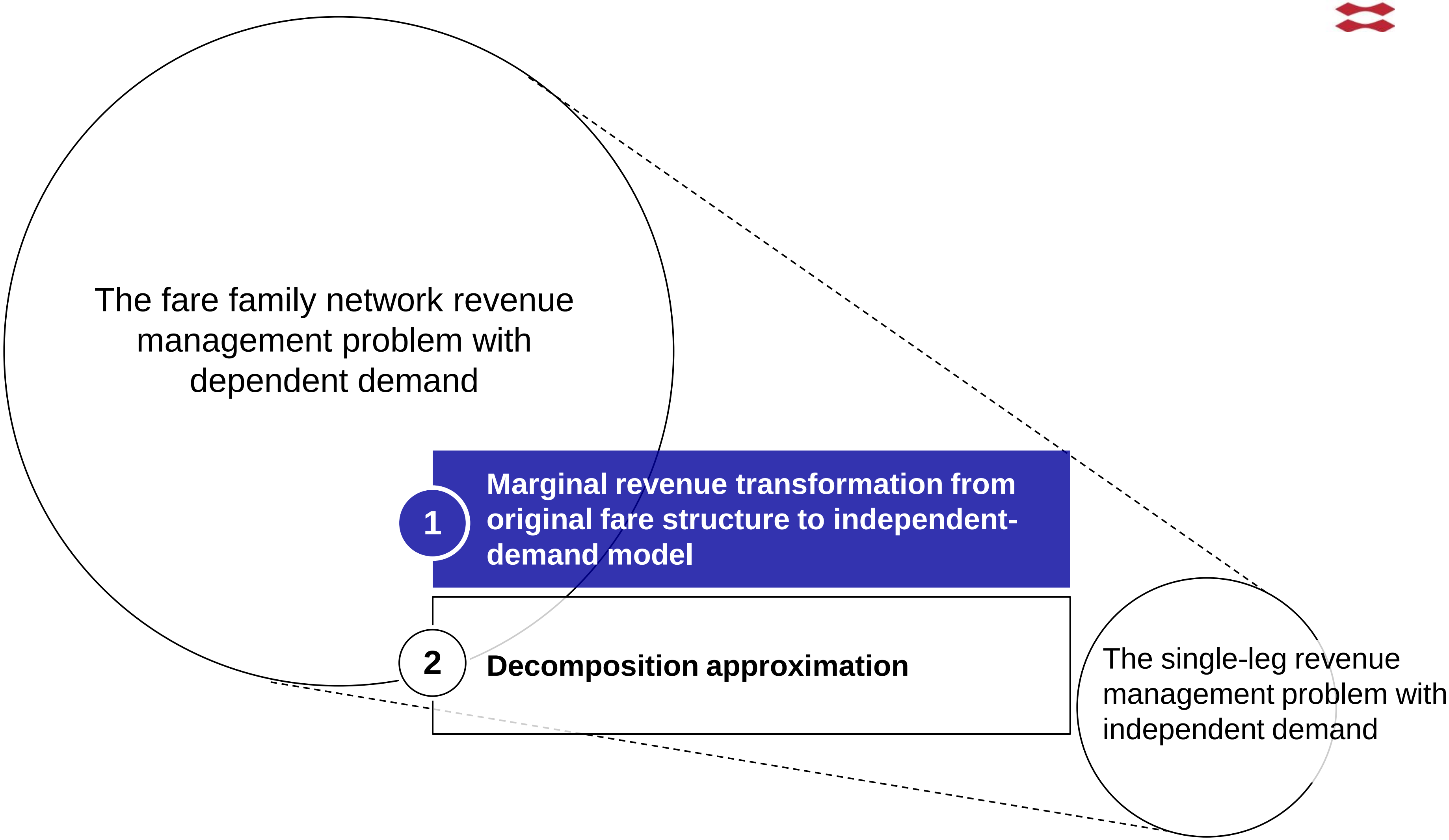
Fare families

Lowest-open-fare demand model



How do we optimize the revenue of the fare families?

What we need to solve…



…vs. what we can solve

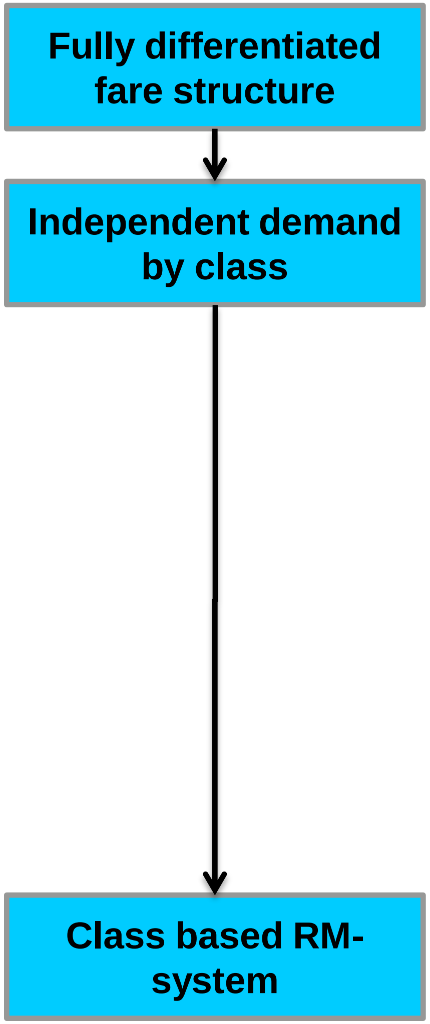
Can Existing RM Systems be Saved?*

- Marginal revenue transformation (Fiig et al. 2009)
  - The authors present a marginal revenue transformation that transforms any fare structure (with any set of restrictions) into an independent demand model.
  - This allows all the traditional RM methods (that was invented assuming independent demand) to be used unchanged.
  - The standard availability control methods can be used unchanged provided that the efficient frontier is nested (or approximately nested).

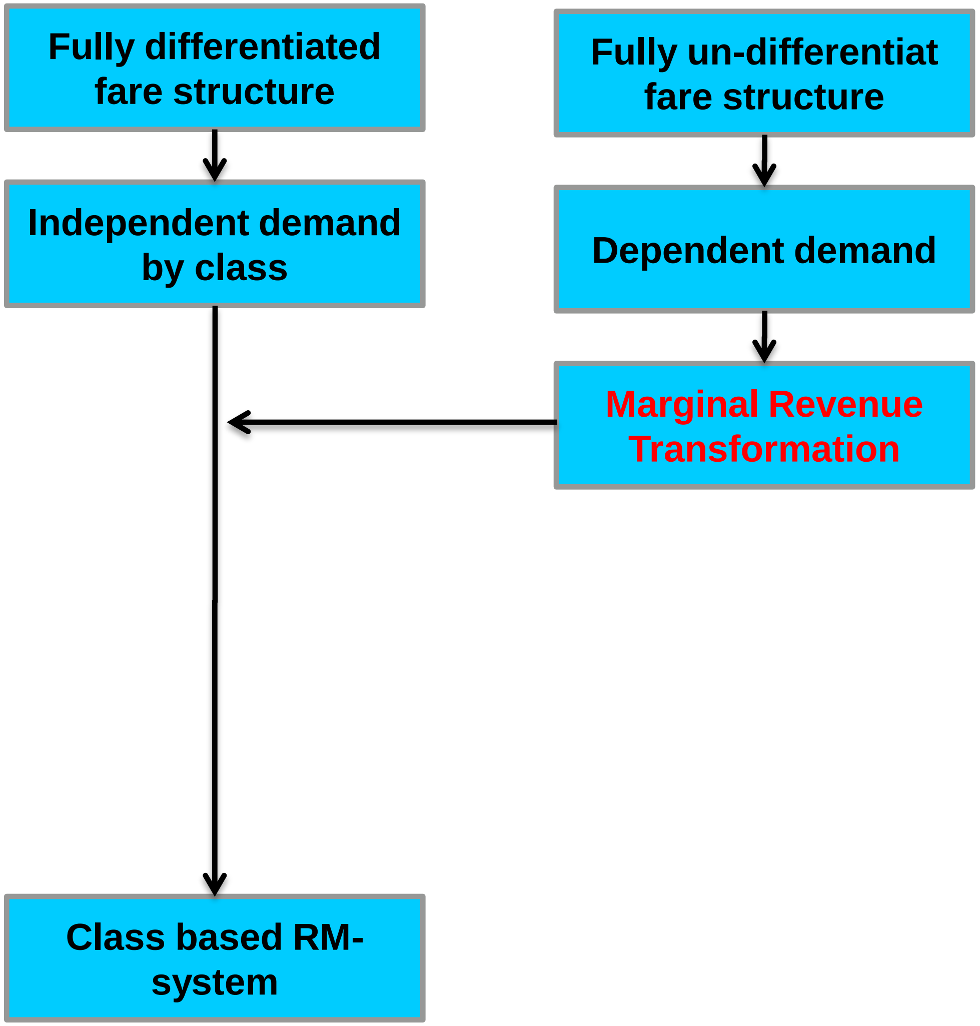
- Previous work has discussed methods to avoid spiral down and optimize simplified fares.
  - Sell-up models in Leg based EMSR, Belobaba and Weatherford (1996)
  - Hybrid Forecasting of Price vs. Product Demand, Boyd, Kallesen (2005)
  - DAVN-MR (Network optimization, mix of fully un-restricted and fully restricted), Fiig et al (2005), Isler et al (2005).
  - Fare Adjustment Methods with Hybrid forecasting, PROS, PODS research.
  - Revenue Management with customer choice models, Talluri and van Ryzin (2004), Gallego et al. (2007).

*Source: Thomas Fiig, Chief Scientist, Scandinavian Airlines.

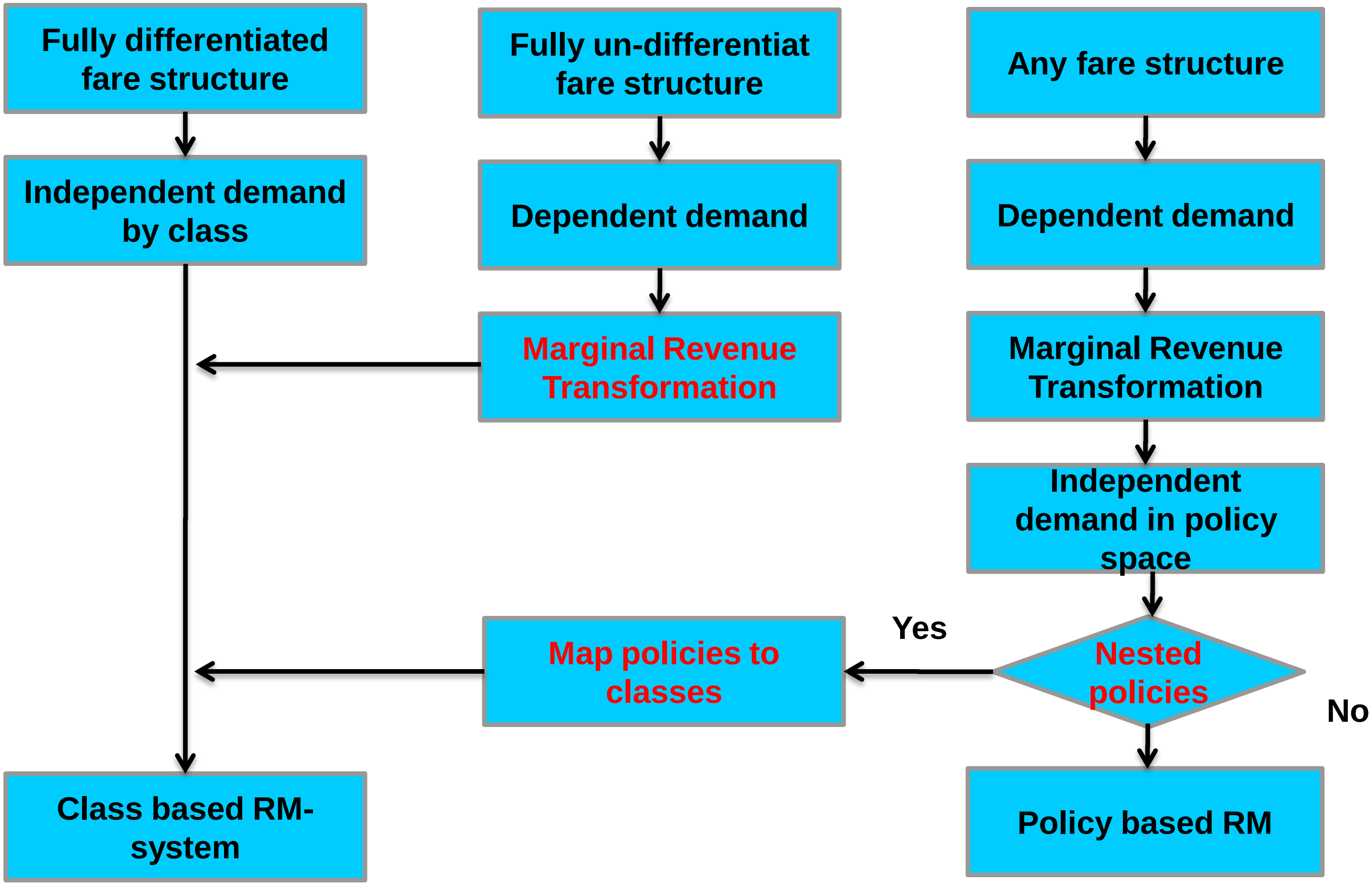
Overview


Fully differentiated fare structure
Independent demand by class
Class based RM-system

Overview


Fully differentiated fare structure
Independent demand by class
Fully un-differentiat fare structure
Dependent demand
Marginal Revenue Transformation
Class based RM-system

Overview



Fully differentiated fare structure
Independent demand by class
Fully un-differentiat fare structure
Dependent demand
Marginal Revenue Transformation
Any fare structure
Dependent demand
Marginal Revenue Transformation
Independent demand in policy space
Nested policies
Yes
No
Map policies to classes
Class based RM-system
Policy based RM

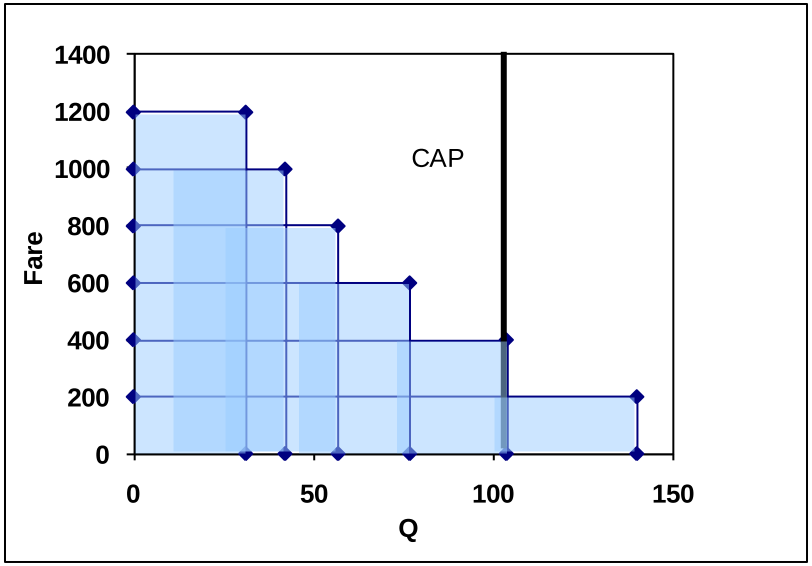
Optimization: Fully differentiated

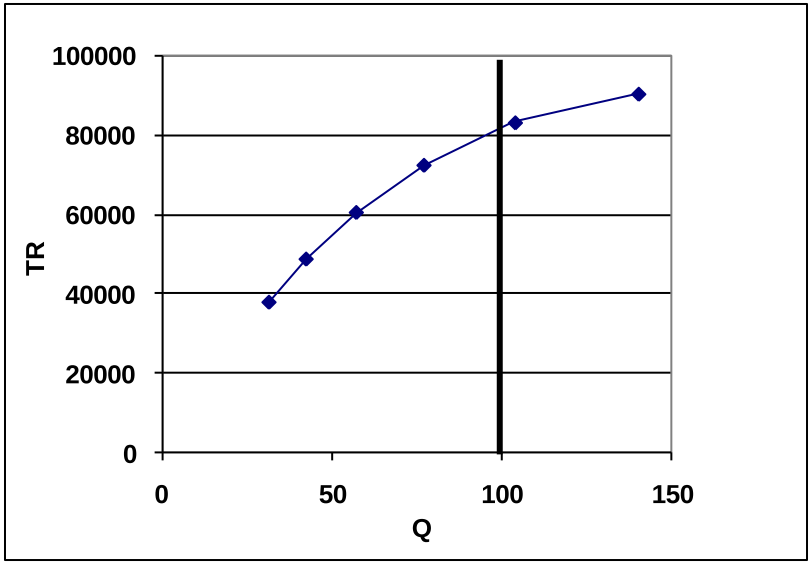
- Deterministic Demand
- Single Leg

$$Q_k = \sum_{j=1}^{k} d_j$$

$$TR_k = \sum_{j=1}^{k} f_j d_j$$

fi	di	Qi	Fully differentiated TRi	MRi
$1.200	31,2	31,2	$37.486	$1.200
$1.000	10,9	42,2	$48.415	$1.000
$800	14,8	56,9	$60.217	$800
$600	19,9	76,8	$72.165	$600
$400	26,9	103,7	$82.918	$400
$200	36,3	140,0	$90.175	$200





Marginal Revenue (Intuitive derivation)

-Fully un-differentiated,
- Single Leg

$$Q_k = \sum_{j=1}^{k} d_j$$

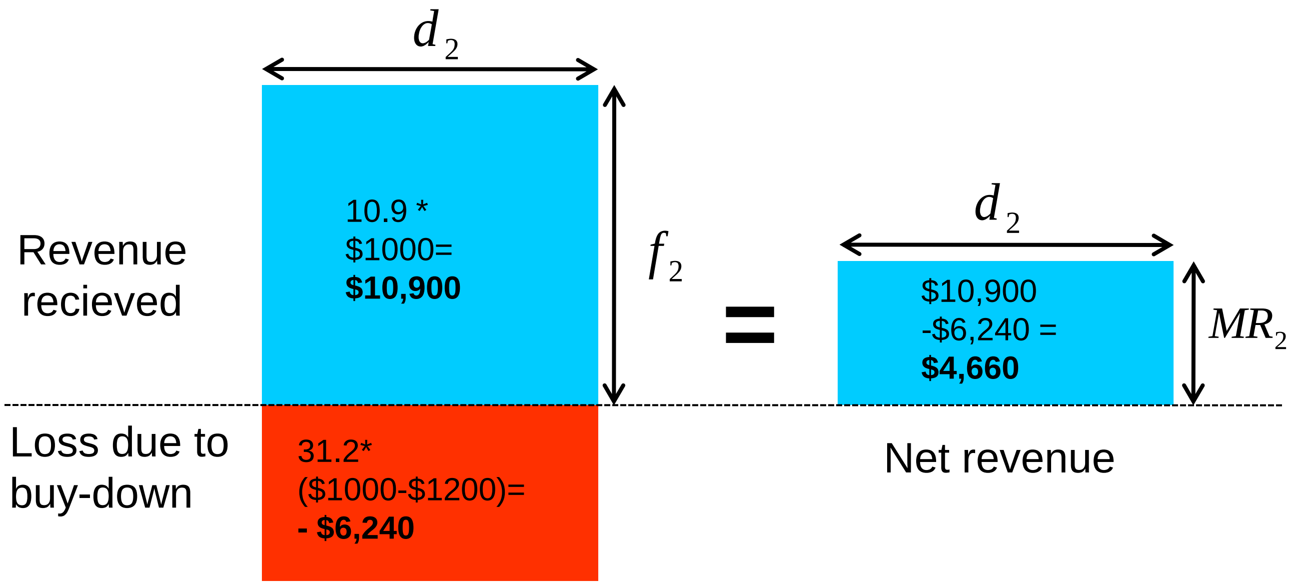
$$TR_k = f_k Q_k$$

			Fully un-differentiated	
fi	di	Qi	TRi	MRi
$1.200	31,2	31,2	$37.486	$1.200
$1.000	10,9	42,2	$42.167	$428
$800	14,8	56,9	$45.536	$228
$600	19,9	76,8	$46.100	$28
$400	26,9	103,7	$41.486	-$172
$200	36,3	140,0	$28.000	-$372

$$MR_2 = \frac{TR_2 - TR_1}{Q_2 - Q_1}$$

$$= \frac{\$42{,}167 - \$37{,}486}{42{,}2 - 31.2}$$

$$= \$428$$



$$MR_2 = \frac{\$4{,}660}{10.9} = \$428$$

Optimization: Fully un-differentiated

- Deterministic Demand
- Single Leg

$$Q_k = \sum_{j=1}^{k} d_j$$

$$TR_k = f_k Q_k$$

fi	di	Qi	Fully un-differentiated TRi	MRi
$1.200	31,2	31,2	$37.486	$1.200
$1.000	10,9	42,2	$42.167	$428
$800	14,8	56,9	$45.536	$228
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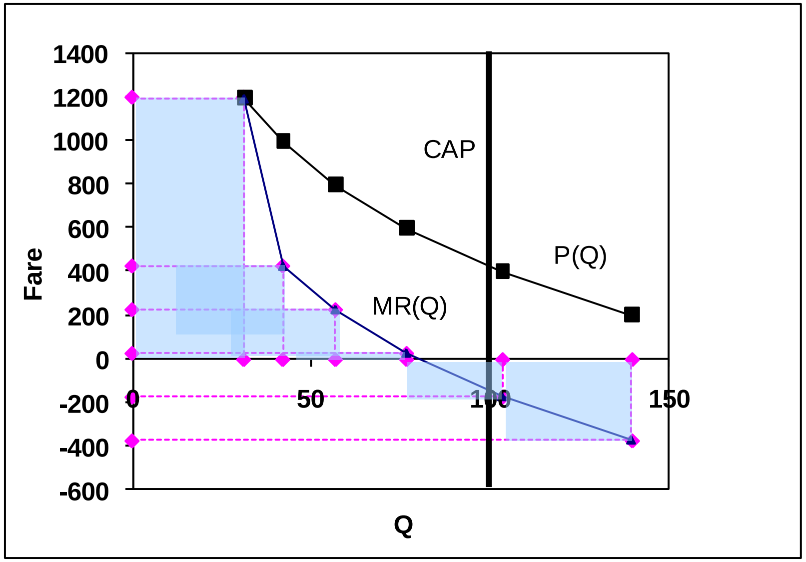
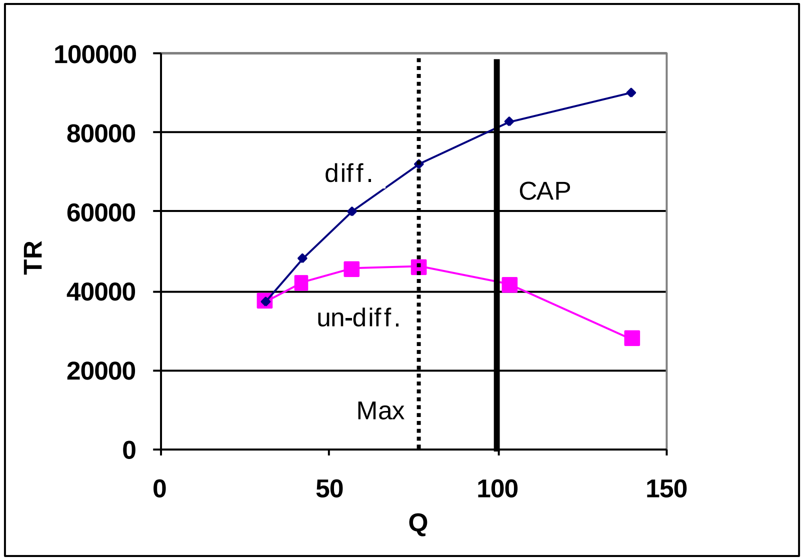
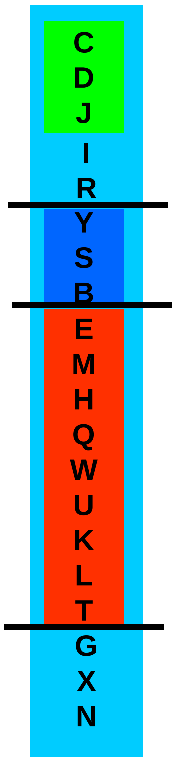




Definition of policies



Policies: the set of fare products S that the airline chooses to have open.

n classes gives potentially 2^n policies. Examples could be:

All classes closed {},

All classes in economy open {E,…,T},

Only classes E,H, and K open: {E,H,K}.

Fare families: {Y,S; E,M,H,Q}

Nested policies:

$$S_k \subset S_l, k < l$$

Examples

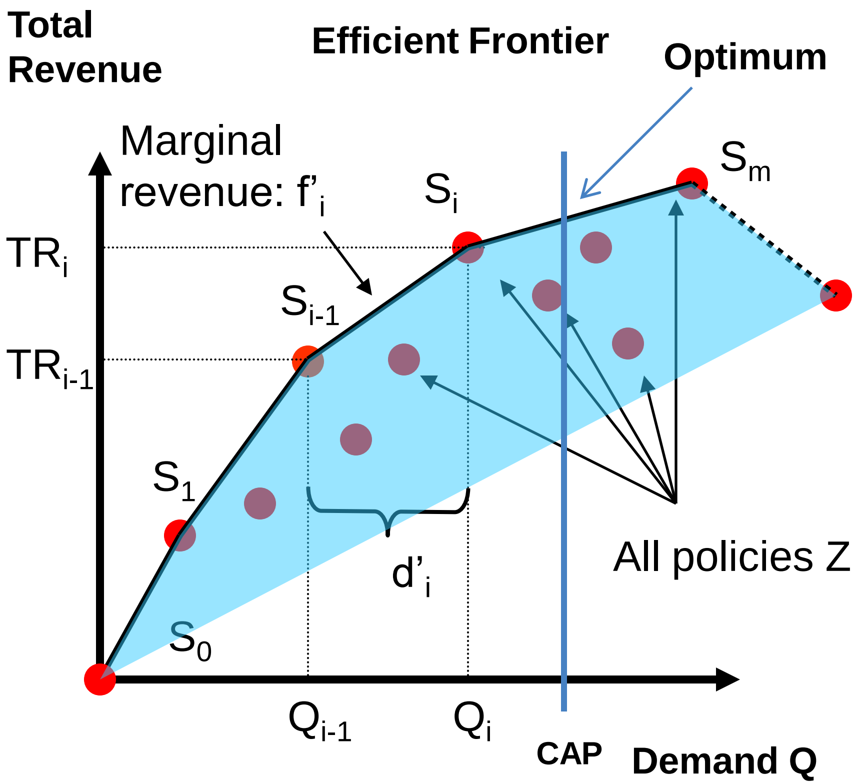
Nested in economy: {},{E},{E,M},...,{E,M,...,L}

Non-nested in economy: {},{E},{E,H},...,{E,M,...,L}

Optimization: General Formulation

- Arbitrary fare structure
- Deterministic Demand
- Single Leg

Fare products	$f_j,\ j=1,\dots,n$
Policy $Z \subseteq N$ (any set of open classes)	$\{\},\{1\},\{1,3\},\dots$
Demand	$d_j(Z)$
Accumulated Dem.	$Q(Z)=\sum_{j\in Z} d_j(Z)$
Total Revenue	$TR(Z)=\sum_{j\in Z} d_j(Z)\,f_j$
Objective	$\max TR(Z)$ $s.t.\quad Q(Z)\le cap$



Marginal Revenue Transformation

Policies on the convex hull → **Independent demand**

Policy	Dem.	TR
S_1	Q_1	TR_1
S_2	Q_2	TR_2
...	...	
S_m	Q_m	TR_m

Partition Dem.	Adj. Fare
$d_1' = Q_1$	$f_1' = f_1$
$d_2' = Q_2 - Q_1$	$f_2' = (TR_2 - TR_1)/d_2'$
...	...
$d_m' = Q_m - Q_{m-1}$	$f_m' = (TR_m - TR_{m-1})/d_m'$

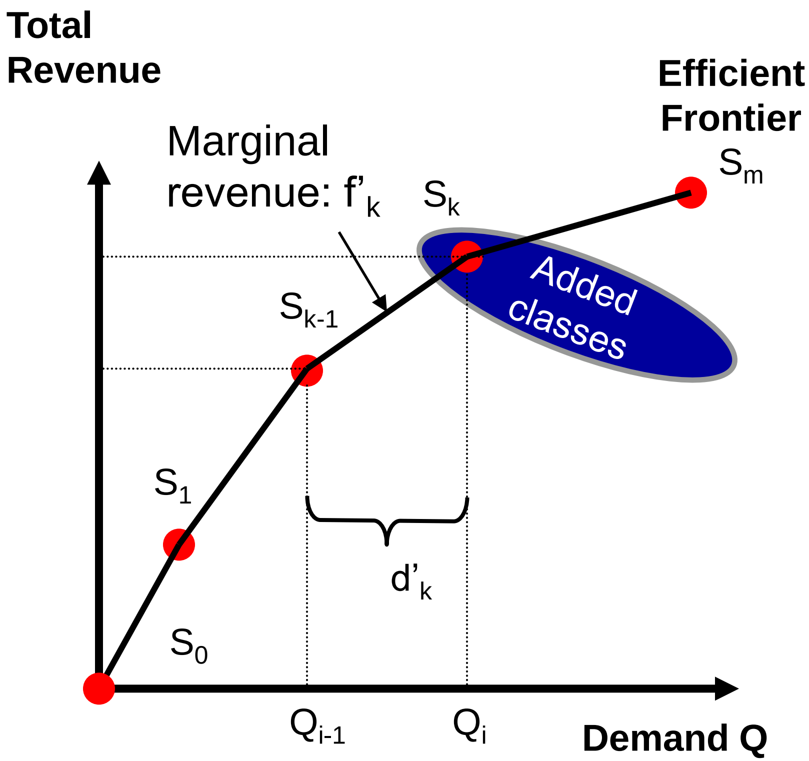
Marginal Revenue Transformation Theorem

- The transformed policies are independent.
- Optimization using the original fare structure and the marginal revenue transformed in policy space gives identical results.

Mapping nested policies to Classes

Many choice models have the desirable property that the policies are nested on the efficient frontier.
For nested policies we can assign demand and adj. fares back to the original classes and continue reusing class based RM-systems.

Mapping from policies to classes	
Newly added classes $S_k \setminus S_{k-1}$	
Partitioned demand	Split demand d'_k any way between newly added classes
Adj fare	Assign the adjusted fare f'_k to all newly added classes.



Applications: Fully differentiated demand

Assume fare class independence.
(the fare products are adequately differentiated, such that demand for a particular fare product will only purchase that fare product)

Acc. demand $Q_k = \sum_{j=1,\ldots,k} d_j$	Total Revenue $TR_k = \sum_{j=1,\ldots,k} f_j d_j$
Partitioned demand $d_k' = Q_k - Q_{k-1} = \sum_{j=1,\ldots,k} d_j - \sum_{j=1,\ldots,k-1} d_j = d_k$	Adjusted fare $f_k' = \dfrac{TR_k - TR_{k-1}}{Q_k - Q_{k-1}} = \dfrac{\sum_{j=1,\ldots,k} d_j f_j - \sum_{j=1,\ldots,k-1} d_j f_j}{d_k} = f_k$

Thus demand and fares are unchanged by the MR transformation.

Denote the unadjusted fare: $f_k^{'prod}$

Applications: Fully un-differentiated demand

Passengers will only buy the lowest available fare (demand for all other fare products except the lowest becomes zero)	
Acc. demand $Q_k = Q_n \, psup_k$	Total Revenue $TR_k = Q_k \, f_k$
Partitioned demand $d_k' = Q_k - Q_{k-1} = Q_n \left(psup_k - psup_{k-1} \right)$	Adjusted fare $f_k' = \frac{TR_k - TR_{k-1}}{Q_k - Q_{k-1}} = \frac{f_k Q_k - f_{k-1} Q_{k-1}}{Q_k - Q_{k-1}} = \frac{f_k \, psup_k - f_{k-1} \, psup_{k-1}}{psup_k - psup_{k-1}}$
Denote the partitioned demand: $d_k^{'price}$	Denote the adjusted fare: $f_k^{'price}$

Applications: Fully un-differentiated demand (exponential sell-up, equal spaced fare grid)

Passengers will only buy the lowest available fare	
Acc. demand $Q_k = Q_n \exp(-\beta(f_k - f_n))$	Total Revenue $TR_k = Q_k f_k$
Partitioned demand $d_k^{'} = Q_n \exp(\beta f_n) \times \left(\exp(-\beta f_k) - \exp(-\beta f_{k-1})\right)$	Adjusted fare $f_k^{'} = f_k - f_M$, where $f_M = \Delta \cdot \frac{\exp(-\beta \cdot \Delta)}{1 - \exp(-\beta \cdot \Delta)}$
f_M is called the fare modifier	

Applications: Hybrid demand

The fare class demands are decomposed into contributions from both differentiated (product-oriented) and un-differentiated (price-oriented) demand

Acc. demand $Q_k = \sum_{j=1,\ldots,k} d_j^{prod} + d_k^{price}$	Total Revenue $TR_k = \sum_{j=1,\ldots,k} d_j^{prod} f_j + d_k^{price} f_k$
Partitioned demand $d_k' = Q_k - Q_{k-1} = d_k^{prod} + d_k^{price}$	Adjusted fare $f_k' = \dfrac{TR_k - TR_{k-1}}{Q_k - Q_{k-1}} = r_k f'^{prod}_k + (1 - r_k) f'^{price}_k$ where $r_k = \dfrac{d_k^{prod}}{d_k^{prod} + d_k^{price}}$

The adjusted fare in the hybrid case equals a demand-weighted average of:
– the unadjusted fare for the product-oriented demand
– the adjusted fare for the price-oriented demand.

Extension to Stochastic models

- Derivation using DP

Fare products	$f_j,\ j = 1,\dots,n$
Policy $Z \subseteq N$ (any set of open classes)	$\{\ \},\{1\},\{1,3\},\dots,\{2,4\},\{1,2,4\},\dots$
Arrival rate	λ
Prob. of booking	$p_j(Z)$
Accumulated Dem.	$Q(Z) = \sum_{j \in Z} p_j(Z)$
Total Revenue	$TR(Z) = \sum_{j \in Z} p_j(Z) f_j$
Objective (Bellman recursion formula)	$J_{t-1}(x) = \max_{Z \subseteq N} \left\{ \begin{array}{l} \lambda \cdot \left[TR(Z) + Q(Z) \cdot J_t(x-1) \right] + \\ \left[1 - \lambda \cdot Q(Z) \right] \cdot J_t(x) \end{array} \right\}$

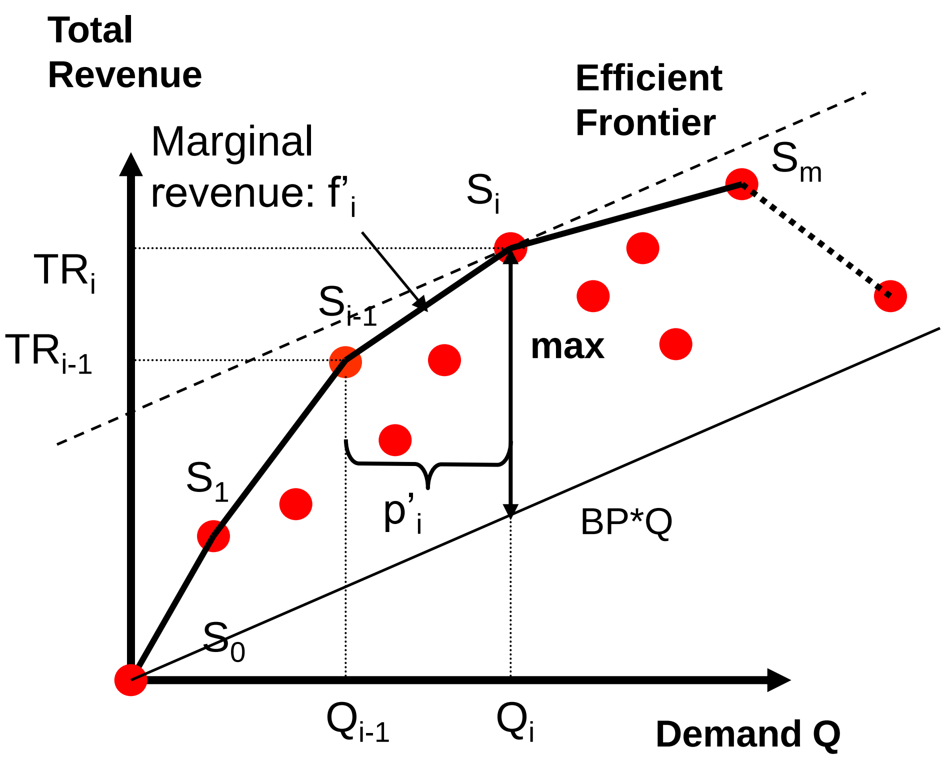
Extension to Stochastic models

- Derivation using DP

Bidprice vector	$BP_t(x) = J_t(x) - J_t(x-1)$
Bellman recursion eq.	$J_{t-1}(x) = J_t(x) + \lambda \cdot \max_{Z \subseteq N} \left(TR(Z) - BP_t(x) \cdot Q(Z) \right)$

Independent demand model

Partitioned demand	$p_k^{'} = Q(S_k) - Q(S_{k-1})$
Adj. fare.	$f_k^{'} = \dfrac{TR(S_k) - TR(S_{k-1})}{Q(S_k) - Q(S_{k-1})}$



Recover the Marginal Revenue Transformation:

Using the transformed choice model (primed demand and fares) in an independent demand DP instead of the original choice model DP, the Bellman equation will produce the same bid-prices.

Applications: EMSRb-MR

Cook book constructing EMSRb-MR (How to construct XXX-MR)

1. Determine the policies on the efficient frontier
2. Apply the marginal revenue transformation to both demands and fares.
3. Map policies back to classes
4. Apply EMSRb in the normal fashion using the transformed demands and fares.

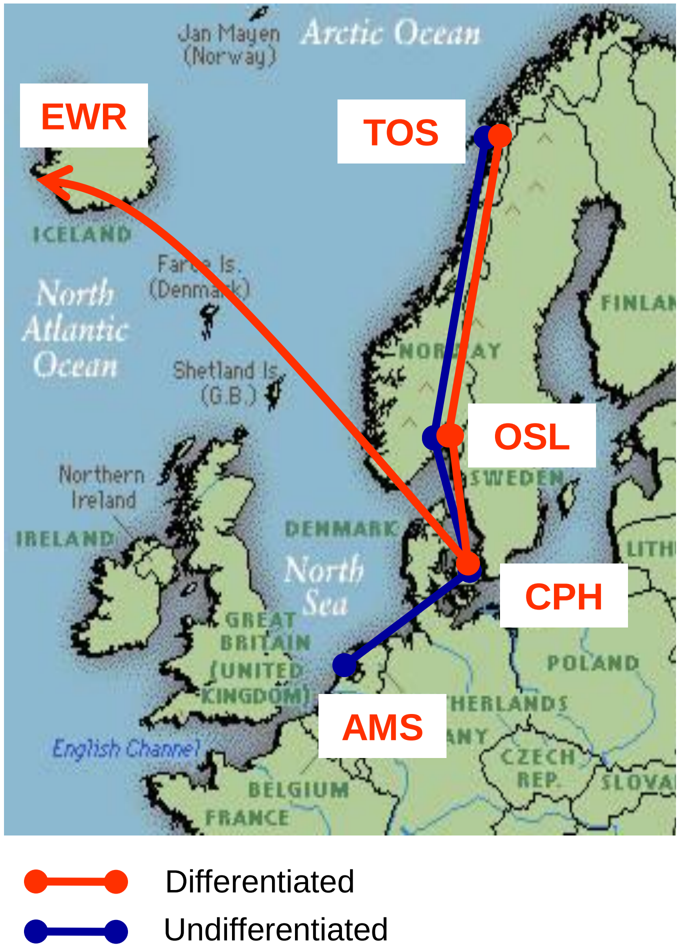
Partitioned demand	Adjusted fare	Protection Level	Booking Limit
$d_k^{'} \sim N(\mu_k, \sigma_k^2)$	$f_k^{'}$	$\pi_k^{'} = \mu_{1,k} + \sigma_{1,k}\Phi^{-1}\left(1 - \frac{f_{k+1}^{'}}{\overline{f_{1,k}^{'}}}\right)$	$BL_k^{'} = cap - \pi_k^{'}$

Fare Product	Fare Value	Mean Demand	Standard Deviation	EMSRb Limits	Adjusted Fares (MR)	EMSRb-MR Limits
1	$ 1,200	31.2	11.2	**100**	$ 1,200	**100**
2	$ 1,000	10.9	6.6	**80**	$ 428	**65**
3	$ 800	14.8	7.7	**65**	$ 228	**48**
4	$ 600	19.9	8.9	**46**	$ 28	**16**
5	$ 400	26.9	10.4	**20**	$ (172)	**0**
6	$ 200	36.3	12.0	**0**	$ (372)	**0**

EMSRb-MR applied to the un-restricted fare structure example.

Applications: DAVN-MR

- Follow Cook Book



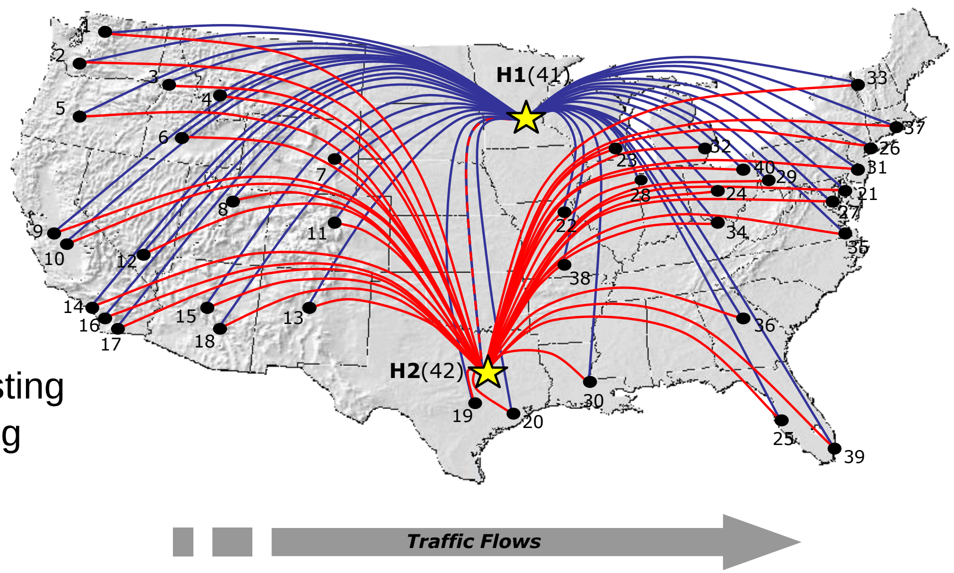
DAVN-MR constructed to handled a mix of fully differentiated and undifferentiated fare structures.

- Assuming exponential sell-up and equally spaced fares for simplicity.
- The fare modifier is calculated individually by path.

Adj fare	$adj\ f = f' - \sum DC = f - f_M - \sum DC$
Differentiated fare products	The fare modifier $f_M = 0$ since path are not affected by risk of buy-down.
Un-differentiated fare products	Mapped to lower buckets since $f_M > 0$ Thus fares $f < f_M$ are closed regardless of remaining capacity. Thus avoiding spiral down.

PODS Simulations

- PODS network D
  - 2 airlines. AL1and AL2
  - 20 cities east/west. 2 hubs
  - 126 legs in 3 banks
  - 482 markets. 1446 paths.
- Sell-up parameters
  - Input Frat5 sell-up.
- Forecasting
  - Standard path/fare class forecasting
  - Hybrid path/fare class forecasting
- Fare structure
  - 6 fare classes
  - Unrestricted & Semi-restricted
- RM methods
  - Standard DAVN (std. forecast, no fare adj.) (Baseline)
  - Hybrid DAVN (hybrid forecast. No fare adj.)
  - Full DAVN-MR (hybrid forecasting and fare adj.)
- Competitive Scenarios
  - Monopoly and Competition



PODS Simulations

- Fare structure

- A un-differentiated structure
- A semi-differentiated structure

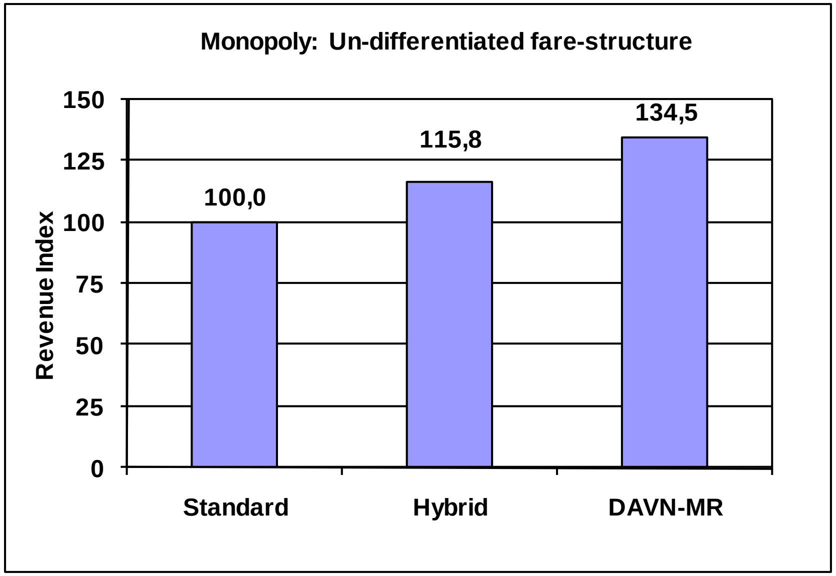
UNDIFFERENTIATED

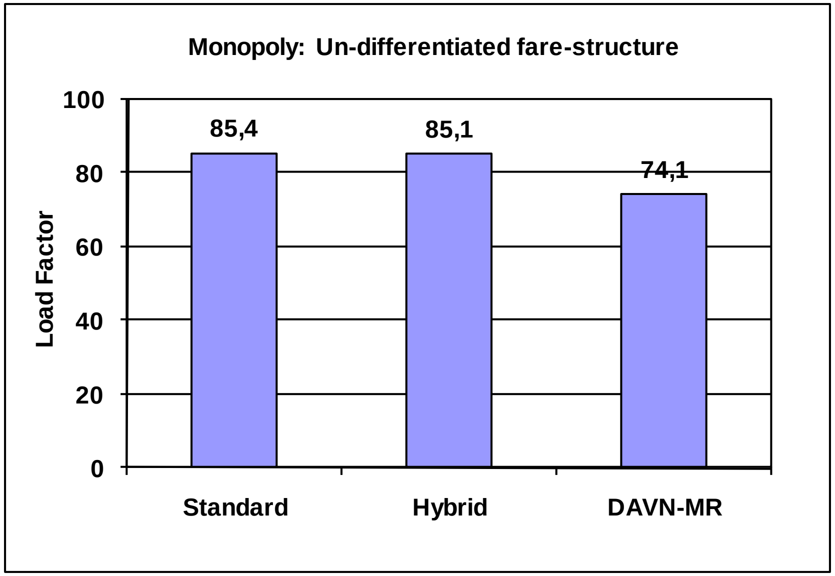
FARE CLASS	AP	Min Stay	Cancel Fee	Non Refund
1	0	NO	NO	NO
2	0	NO	NO	NO
3	0	NO	NO	NO
4	0	NO	NO	NO
5	0	NO	NO	NO
6	0	NO	NO	NO

SEMI-DIFFERENTIATED

FARE CLASS	AP	Min Stay	Cancel Fee	Non Refund
1	0	NO	NO	NO
2	0	NO	YES	NO
3	0	NO	YES	YES
4	0	NO	YES	YES
5	0	NO	YES	YES
6	0	NO	YES	YES

PODS Simulations
-Monopoly Un-differentiated





- Hybrid forecasting leads to 16% gain compared to standard due to reduced spiral down.
- Full DAVN-MR (hybrid forecasting + fare adjustment) adds an additional 18% gain.
- The effect comes from closing lower inefficient classes, which leads to lower LF.

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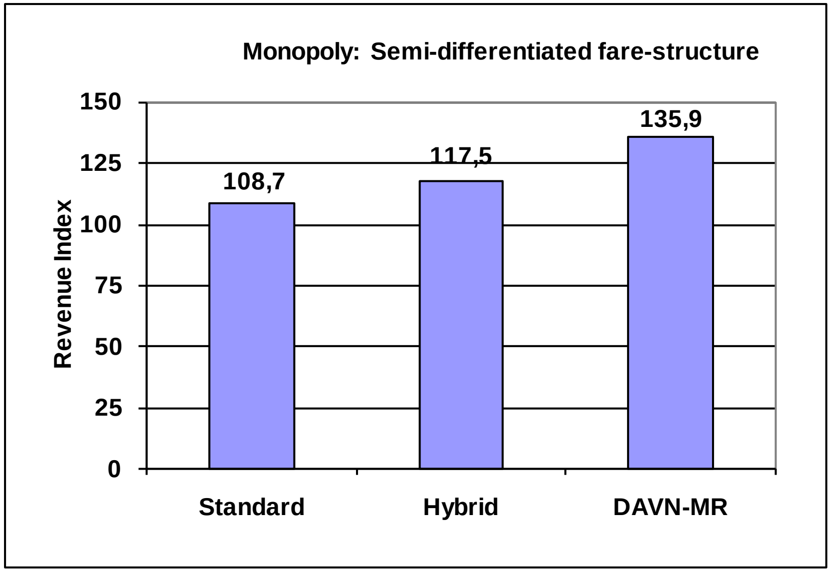
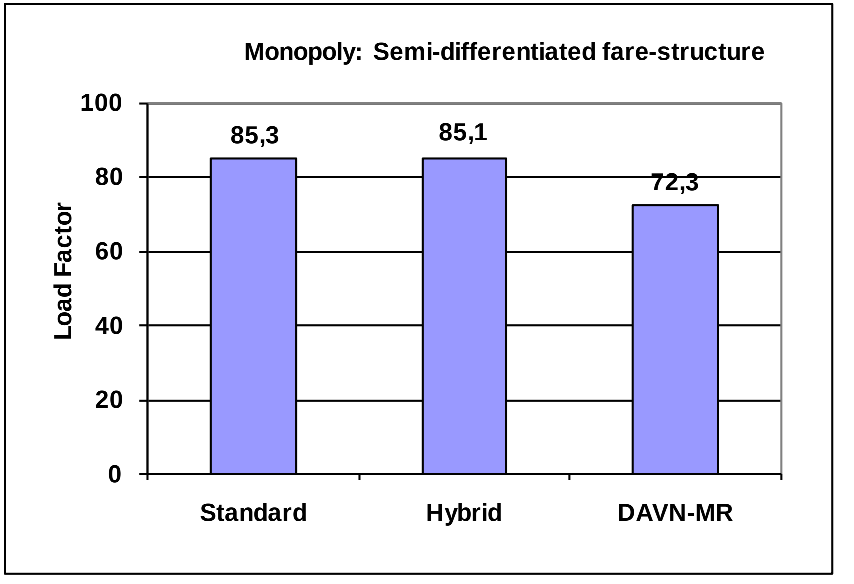
PODS Simulations
-Monopoly Semi-differentiated





- Same overall trend compared to un-differentiated. Slightly less effect due to restrictions.

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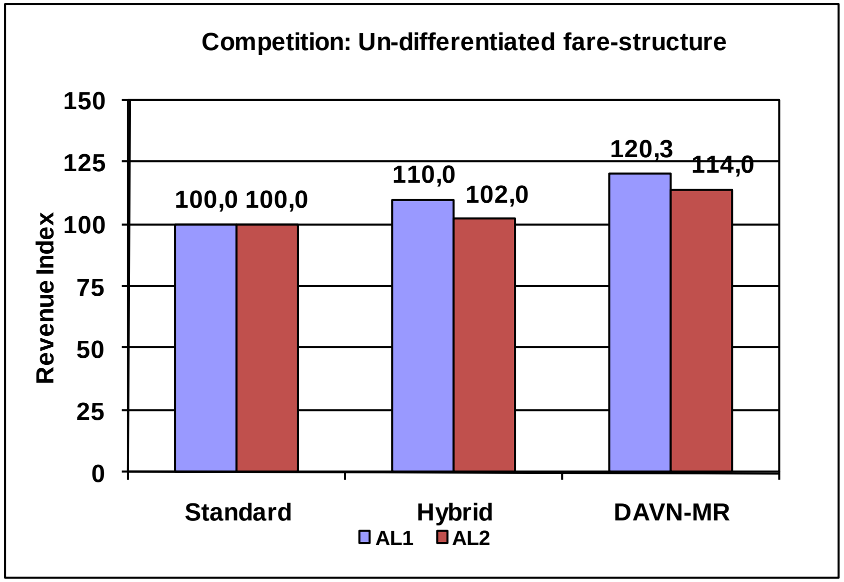
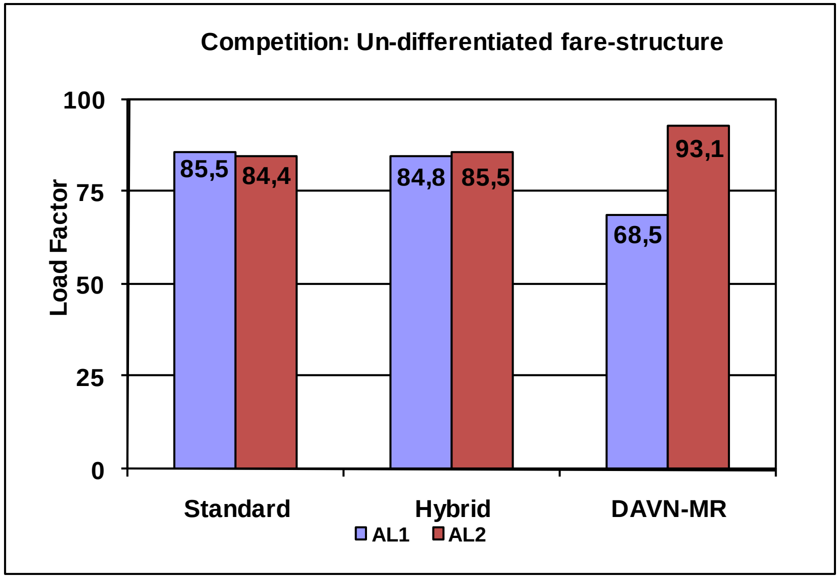
PODS Simulations
-Competition Un-differentiated





- Hybrid forecasting leads to 10% gain compared to standard. Less than monopoly due to competition.
- Full DAVN-MR (hybrid forecasting + fare adjustment) adds an additional 10% gain.
- The effect comes from closing lower inefficient classes, which leads to lower LF.

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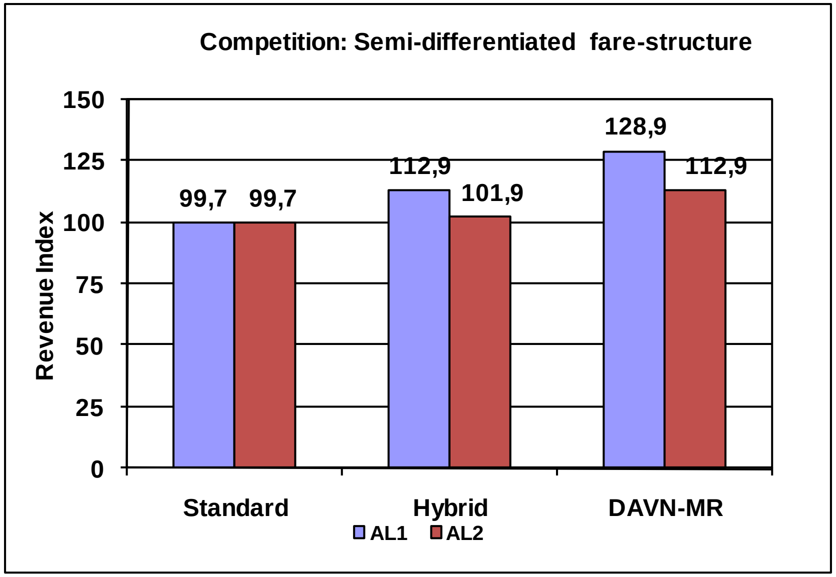
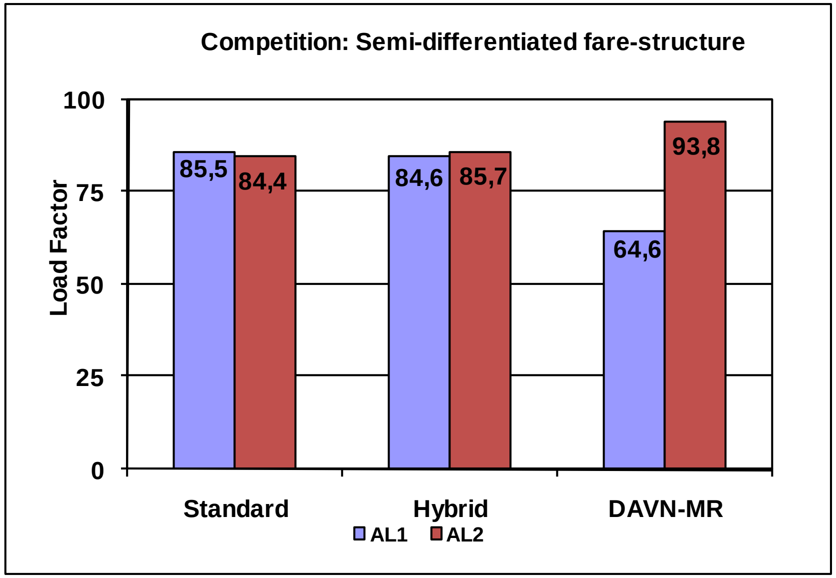
PODS Simulations
-Competition Semi-differentiated





Conclusion

- Marginal revenue transformation transforms a general discrete choice model to an equivalent independent demand model.
- The marginal revenue transformation allows traditional RM systems (that assumed demand independence) to be used continuously.
- The marginal transformation is valid for:
  - Static optimization
  - Dynamic optimization
  - Network optimization (provided the network problem is separable into independent path choice probability).
- If the efficient frontier is nested (or approximately nested), the policies can be remapped back to the original classes allowing the class based control mechanism to be used in the standard way.
- DAVN-MR was tested using PODS for both un-differentiated and semi-differentiated networks. Revenue gains are significant, 10-20 pct point better that hybrid forecasting.