Background and introduction to Revenue Management

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42134 Advanced Topics in Operations Research

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Revenue Management Session 01
Outline

• Introduction
• Historical background and context
• The financial impact of pricing and revenue optimization
Introduction

Pricing refers to how companies should set and adjust their prices in order to maximize profitability:

- Pricing decisions are commonplace
- Pricing decisions can be complex
- Pricing decisions are critical determinants of profitability

Despite this, pricing decisions are often badly managed!

Very few companies have the right prices in place:

- For all products
- To all customers
- Through all channels
- All the time

...this is the goal of pricing and revenue optimization
Introduction

• Pricing and revenue optimization is a *tactical* function, i.e. prices need to change rapidly and often.

• Pricing and revenue optimization provides guidance on how prices should change.

• The goal of *strategic pricing* is to establish a general position within a marketplace.

• Strategic pricing worries about how a product should in general be priced relative to the market.

• Pricing and revenue optimization determines the prices for tomorrow and next week.

• Strategic pricing sets the constraints within pricing and revenue optimization operates.
Introduction

• One of the distinguishing characteristics of pricing and revenue optimization is its use of analytical techniques derived from operations research and management science.

• The analytical approach to setting prices is rather new.

• One of the first applications of the approach was the development of revenue management systems by passenger airlines in the 1980s.

• Since then other industries have adopted similar techniques, including:
  – Automotive
  – Retail
  – Telecommunications
  – Financial services
  – Manufacturing
Introduction

• A number of software vendors provide “price optimization” or “demand management” or “revenue management” solutions focused on one or more industries, e.g.:

• Pricing and revenue optimization is becoming a core competency within many different companies.
Historical background and context

- For most of history philosophers took for granted that goods had an intrinsic (indre) value.
- A fair price reflects the intrinsic value.
- Charging in excess of the fair price was condemned as “averice” (grådighed) and often prohibited by law.
- The problem of pricing came into existence with the emergence of modern market economies in the West in the 17th and 18th centuries.
- Now prices were allowed to move more freely, creating the “tulipomania” in the Dutch republic in the 1630s and the “South Sea Bubble” in England in 1720... and the real estate bubbles of the 2000s.
- For the first time large number of people could amass fortunes – and loose them – by buying goods and selling them on the market.
Historical background and context

• The questions arose:
  – What are prices exactly?
  – Where do they come from?
  – What determines the right price?
  – When is a price fair?
  – When should the government intervene in pricing?

• The modern field of economics arose, at least in part, in response to these questions.
Historical background and context

• Possibly the greatest insight of classical economics is that the price of any good at any time in an ideal capitalistic economy is not based on intrinsic “value” but rather on the interplay of supply and demand.

• The law of supply and demand is a major intellectual breakthrough – on par with Newtonian mechanics and Darwin’s theory of evolution.

• The price of a good or service is determined by the interaction of people willing to sell the good with the willingness of others to buy the good – that’s all there is to it!
Historical background and context

• The following factors do not enter directly into the pricing equation:
  – intrinsic “value”,
  – cost,
  – labor content.

• However, these factors do enter indirectly, because:
  – Sellers would not last long selling goods below cost.
  – Buyers acceptance of prices are based on the “value” they place on the item.

• Sometimes prices are below cost because the goods are perishable (“sell it or smell it”) or to attract new customers.

• According to modern economics there is no normative “right price” for a good or service.
Historical background and context

• Classical economics solved the problem of origin of price but it raised as many questions as it answered.

• If prices were not tied to fundamental values why do they show any stability at all?

• Classical economics explained this primarily based on the concept of perfectly competitive markets:
  – There are many firms, each with an insubstantial market share.
  – They produce a homogeneous product using identical production processes.
  – They possess perfect information.
  – There is free entry to the industry.
  – If a firm raises its prices it looses all of its markets to its competitors.
  – Thus firms are price takers.
Historical background and context

• There are no pricing decisions in perfectly competitive markets – prices are determined by the iron law of the market.

• The economic definition of a commodity good is that its price is “set by the market”.

• Many financial instruments, such as stocks and bonds, satisfy the economic definition of a commodity.

• Certain other physical goods – grain, crude oil, and some bulk chemicals – also come very close to being commodities.

• In these markets there is simply no need for pricing and revenue optimization – the market truly sets the price.
Historical background and context

• Much of the real world is messier – prices vary all over the place, sometimes in ways that seem irrational.

• It is hardly a secret not only that prices vary between sellers but that a single seller will often sell the same product to different customers for different prices!

• The tools that pricers use today are far more likely to be drawn from the field of statistics or operations research than from economics.

• Marketing science began to emerge in the 1960s, dealing with the quantitative analysis of marketing initiatives, including pricing.

• However, despite the achievements of marketing science there remains a gap between marketing science models and their use in practice.

• One of the possible reasons for the gap between marketing science theory and its applications to real pricing is that the pricing decisions are becoming increasingly *tactical* and *operational* in nature.
Historical background and context

• Companies increasingly need to make pricing decisions more and more rapidly in order to respond to:
  – competitive actions,
  – market changes,
  – or their own inventory situation.

• Companies no longer have the luxury to perform market analyses or extended spreadsheet studies every time a pricing change needs to be considered.

• Now the premium is on speed.
Historical background and context

- The interest in developing tools to enable better pricing and revenue optimization (PRO) decisions has been driven by four trends:
  - The success of revenue management in the airline industry provides an example of how PRO can increase profitability.
  - The widespread adoption of enterprise resource planning (ERP) and customer relationship management (CRM) software systems provides a wealth of new information.
  - The rise of e-commerce makes it necessary to manage and update prices in a fast-moving, highly transparent, online environment.
  - The success of supply chain management proved that analytical software systems can drive real business improvements.
The financial impact of pricing and revenue optimization

• The most compelling reason for a company to improve its pricing and revenue optimization capabilities is to make more money.

• In a 1999 McKinsey study it was concluded that a 1% improvement in profit would on average result in an improvement in operating profit (EBIT) of 11.1%.

• By contrast, 1% improvements in variable cost, volume, and fixed cost would produce operating improvements of 7.8%, 3.3%, and 2.3%.

• Similar results were obtained from a 1999 A.T. Kearney study.

• Passenger airlines typically claim between 8% and 11% benefit from their use of RM systems, which is consistent with the two cited studies.

• Furthermore, the focus on cost improvements in the 1990s means that incremental cost improvements will be more expensive than incremental improvements in pricing and pricing management.
Single-resource capacity control

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• Introduction
• Types of controls
• Displacement cost
• Static models
• Assumptions
• Littlewoods two-class model
• n-class models
• Computational approaches
Introduction to Airline Revenue Management

‘Selling the right seats to the right customers at the right prices and the right time’ (American Airlines 1987)

...squeezing as many dollars as possible out of the customers

‘Integrated control and management of price and capacity (availability) in a way that maximizes company profitability
History of Airline Revenue Management

- RM was ‘invented’ by major US carriers after airline deregulation in the late 1970’s to compete with new low cost carriers

- Matching of low prices was not an alternative because of higher cost structure

- American Airline’s ‘super saver fares’ (1975) have been first capacity controlled discounted fares

- RM allowed the carriers to protect their high-yield sector while simultaneously competing with new airlines in the low-yield sector

- From art to science: By now, there are sophisticated RM tools and no airline can survive without some form of RM

- Other industries followed - hotel, car rental, cruise lines etc.

Source: Pölt (2002)
Characteristics of Revenue Management

- A relatively fixed total capacity
- Demand can be segmented into classes
- Inventory is perishable
- The product can be sold well in advance
- There can be substantial fluctuations in demand
- Variable costs are much less than fixed costs
Advantage of Revenue Management

• Passengers are very heterogeneous in terms of their needs and willingness to pay
• A single product and price does not maximize revenue

\[ \text{revenue} = \text{price} \times \min \{\text{demand}, \text{capacity}\} \]

Source: Pölt (2002)
Revenue Management system

In large systems: 5000 transactions per second in peak time

Source: Boyd, PROS
Introduction to single-resource capacity control

• Definition: Optimally allocating capacity of a single resource (e.g. one flight leg) to different classes of demand
  (Multiple-resource/network: Two connecting flights)
• Part of quantity-based RM: How much to sell to whom
  (price-based RM: How to price to various customer groups or how to vary prices over time)
• Many quantity-based RM problems are network RM problems but still solved as a collection of single-resource problems
• Important to study single-resource RM models
Assumptions

• Capacity is sold in n distinct classes that require the same resource (Airline: Different discount levels with differentiated sale conditions and restrictions)
• Perfect market segmentation: One customer segment for each class
• Units of capacity are homogeneous
• Customers demand a single unit of capacity

The central problem:

How to optimally allocate the capacity of the resource to the various classes

Allocation must be done under uncertainty about the demand, i.e. demand is stochastic
Types of controls #1

- **Definition**: Different mechanisms for controlling availability of capacity

- **Booking limits** $b_j$: Limit the amount of capacity sold to a class
  - Partitioned booking limits: Divides the available capacity into separate blocks
  - Nested booking limits: Capacity for different classes overlap hierarchical
    - higher-ranked classes have access to all capacity reserved for lower classes
  - Most reservation systems use nested limits

- **Protection levels** $y_j$: Amount of capacity to reserve (protect) for a class
  - Partitioned protection levels = Partitioned booking limits
  - Nested booking limits ($j$) = Capacity - Nested protection levels ($j-1$):

  $$b_j = C' - y_{j-1}, \quad j = 2, \ldots, n,$$

- **Standard nesting**: Request for class $j$ -> Capacity is consumed for class $j$
- **Theft nesting**: Request for class $j$ -> Capacity is consumed for $j$ AND for $j+1, j+2 \ldots n$

- **Bid prices**: Threshold prices
  - A request is accepted if its revenue exceeds the threshold price
Types of controls #2

Figure 2.1. The relationship between booking limits \( b_j \), protection levels \( y_j \), and b.d prices \( \pi(x) \).
Displacement cost

- Mathematics of optimal capacity controls can become complex but the logic is simple

- First, capacity should be allocated to a request if and only if its revenue is greater than the value of the capacity required to satisfy it

- Second, value of capacity measured by its expected displacement cost also called opportunity cost

- Displacement cost: Expected loss in future revenue from using the capacity now rather than reserving it for future use

- Captured by using a value function $V(x)$ that measures optimal expected revenue as a function of remaining capacity

- $\text{Displacement cost} = V(x) - V(x-1)$
Static model

Given:

• Fare for each fare class
• Distribution of total demand-to-come by class (demand assumed independent)

Determine:

• Optimal nested booking limits
Assumptions of the static model

1. Demand arrives in non-overlapping intervals
2. Demands for different classes are independent random variables
3. Demand for a given class does not depend on the capacity controls
4. Details of control and demand process is ignored, e.g. that demand arrives sequentially. Fortunately the optimal control is not sensitive to this assumption
5. No group bookings or they can be partially accepted
Littlewoods two-class model #1

- Two classes with associated prices $p_1 > p_2$
- The capacity is $C$
- No cancellations or overbooking
- Demand for class $j$ is denoted $D_j$ and its distribution is $F_j(\cdot)$, e.g. normal distribution
- The problem is to decide how much class 2 demand to accept
- The expected marginal value of reserving the $x^{th}$ unit for class 1:
  \[ p_1 P(D_1 \geq x). \]
- Accept a class 2 request as long as its price exceeds the marginal value:
  \[ p_2 \geq p_1 P(D_1 \geq x). \]
  Note that right-hand side decrease in $x$
Littlewoods two-class model #2

- Optimal protection level satisfies:

\[ p_2 < p_1 P(D_1 \geq y_1^*) \quad \text{and} \quad p_2 \geq p_1 P(D_1 \geq y_1^* + 1). \]

- If \( F_1(x) \) is continuous:

\[ p_2 = p_1 P(D_1 > y_1^*), \quad \text{equivalently,} \quad y_1^* = F_1^{-1}(1 - \frac{p_2}{p_1}), \]

Note that Microsoft Excel has an implementation of the inverse normal distribution \texttt{NORMINV}.
n-class models #1

- The two-class model can be generalized to $n > 2$ classes
- Demand arrive in $n$ stages in increasing order of their revenue values, i.e. from lowest price to highest price
- Dynamic programming formulation in the stages (classes) with remaining capacity $x$ being state variable
- At each stage the following occurs:
  1. Realization of demand $D_j$
  2. Decide on a quantity $u \leq x$ to accept (optimal control $u^*$)
  3. Revenue $p_j u$ is collected and proceed to stage $j-1$ with remaining capacity $x - u$
- Let $V_j(x)$ denote the value function at the start of stage $j$
- The value $u$ is chosen to maximize the current stage $j$ revenue plus revenue to go, or:

$$p_j u + V_{j-1}(x - u),$$

subject to the constraint $0 \leq u \leq \min\{D_j, x\}$
**n-class models #2**

- The value function is the expected value of the optimization:

\[
V_j(x) = E \left[ \max_{0 \leq u \leq \min\{D_j, x\}} \{p_j u + V_{j-1}(x-u)\} \right],
\]

with boundary conditions

\[
V_0(x) = 0, \quad x = 0, 1, \ldots, C.
\]

- Expected marginal value at stage \( j \) \( \Delta V_j(x) \equiv V_j(x) - V_j(x-1) \).

- The optimal control can be expressed in terms of optimal protection levels:

\[
y^*_j \equiv \max\{x : p_{j+1} < \Delta V_j(x)\}, \quad j = 1, \ldots, n-1.
\]

- The optimal control at stage \( j+1 \) is then

\[
u^*(j+1, x, D_{j+1}) = \min\{(x - y^*_j)^+, D_{j+1}\},
\]
n-class models #3

\[ \Delta V_j(x) = \pi_{j+1}(x) \]

\[ P_{j+1} \]

\[ 0 \rightarrow y_j^* \rightarrow b_{j+1}^* \rightarrow C \]

Reject class \( j+1 \)  
Accept class \( j+1 \)

*Figure 2.2.* The optimal protection level \( y_j^* \) in the static model.
Computational approaches: Dynamic Programming

- The optimal nested allocations can be computed using dynamic programming.

- Substituting (2.5) into (2.3) gives the recursion

\[ V_j(x) = E \left[ p_j \min \{ D_j, (x - y_{j-1}^*)^+ \} + V_{j-1}(x - \min \{ D_j, (x - y_{j-1}^*)^+ \}) \right] , \]

\[ y_0^* = 0. \]

- The DP procedure is started from \( j = 1 \) and working backward to \( n \).
Computational approaches: Heuristics

• When the airline industry started using RM in the 1970s, applying heuristics were the only way to find the optimal controls

• Theory of optimal solution approaches was developed more than 10 years later

• Routines for finding optimal controls exist but heuristics are easier to implement and faster to run

• EMSR-a and EMSR-b (Expected Marginal Seat Revenue – versions a and b) produce results that are very close to optimum

• EMSR-b is the most widely used solution approach in the industry
Computational approaches: EMSR-a

The idea of EMSR-a is to apply Littlewood’s rule to pairs of fair classes and add the protection levels

For each stage $j$, we find the protection level $y_j$ in the following way

- We consider class $j+1$ with demand $D_{j+1}$ and price $p_{j+1}$
- For all future classes $k = j, j-1, ..., 1$, we apply Littlewood’s rule:

$$P(D_k > y_k^{j+1}) = \frac{p_{j+1}}{p_k}.$$

- After finding the individual protection levels $y_k^{j+1}$ for all future classes $k$, these are added to compute the total protection level $y_j$:

$$y_j = \sum_{k=1}^{j} y_k^{j+1},$$
Computational approaches: EMSR-b

EMSR-b takes into account the weakness of EMSR-a, which is ignoring the statistical averaging effect of aggregating demand across classes when only considering pairs of classes $k, j+1$

EMSR-b also approximates the protection levels $y_j$ by considering pairs of classes but instead of adding protection levels, demands are aggregated and all future classes are treated as one
Computational approaches: EMSR-b

For each stage $j$, we find the protection level $y_j$ in the following way

- We consider class $j+1$ with the price $p_{j+1}$
- The aggregated demand $S_j$ for future classes $j, j-1, ..., 1$ is calculated:

$$S_j = \sum_{k=1}^{j} D_k,$$

- and the average revenue $\bar{p}_j$ for future classes $j, j-1, ..., 1$ is computed:

$$\bar{p}_j = \frac{\sum_{k=1}^{j} p_k E[D_k]}{\sum_{k=1}^{j} E[D_k]}.$$  \hspace{1cm} (2.13)

- The protection level $y_j$ is found by applying Littlewood’s rule:

$$P(S_j > y_j) = \frac{p_{j+1}}{\bar{p}_j}.$$  \hspace{1cm} (2.14)
Numerical example

Example 2.3 There are four classes, and demand is assumed to be normally distributed. Table 2.1 shows the demand data and Table 2.2 the protection levels produced by EMSR-a, EMSR-b, and the optimal policy.

Table 2.1. Static single-resource model and protection levels.

<table>
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<tr>
<th>j</th>
<th>p_j</th>
<th>( \mu_j )</th>
<th>( \sigma_j )</th>
<th>OPT</th>
<th>EMSR-a</th>
<th>EMSR-b</th>
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