Network Protection

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Learning Objectives

After this lecture you will:

- Have learned about different types of network errors
- Have seen a number of different (circuit) protection schemes
- Have seen mathematical models for certain types of protection
- Have learned something about how to analyze the master problem in order to create the correct sub-problem

Circuit Switched Routing

The first telecommunication networks where (surprise) telegraph/telephone networks:

- A fixed line connection (circuit) was established from end to end (sometimes manually ...)
- The circuit exists for a while and takes up capacity on some links and nodes ...
- Quality guarantees exists !!!
- Capacity usage on the network can be controlled ...

Traffic Engineering

Traffic engineering: Given a network use the (limited) recourses as good as possible: (as we saw in the last lecture)

- Max throughput: Multi Commodity Flow problem (MCF)
- Max free capacity on links (MCF)
- Min delay: (Constrained) Shortest path routing (SHP,MCF)
- .... and many many more ...

Network reliability

Many types of failures can occur in networks:

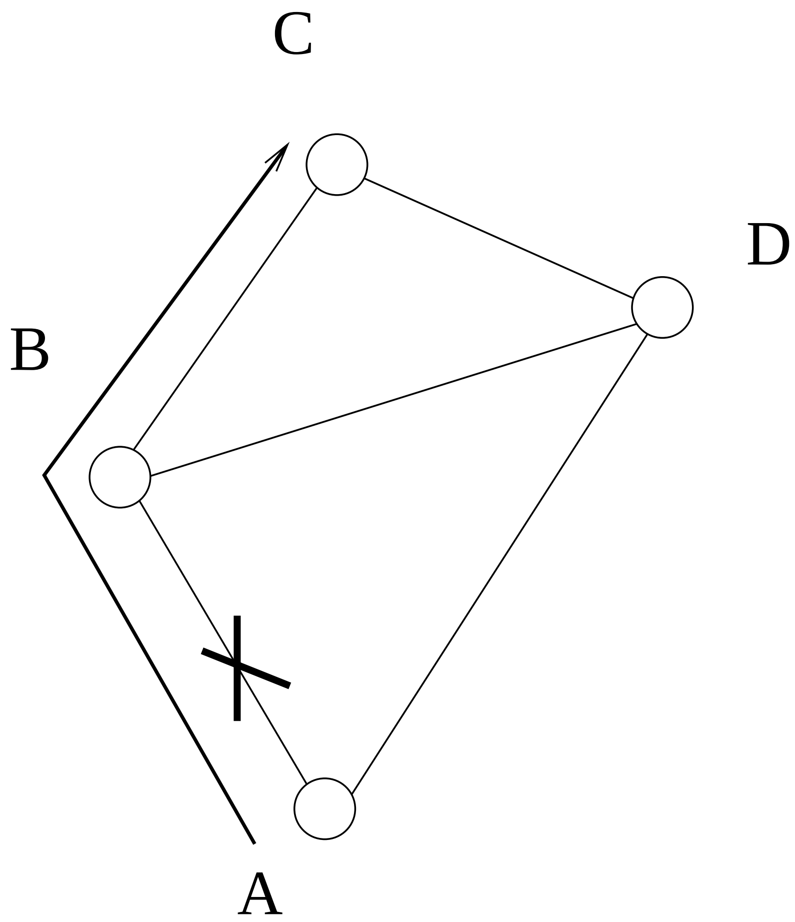
- Electricity failure
- Physical (hardware) node failure
- Node software failure
- Physical link break ...

Network reliability is increasingly important, but here we will only consider cable-breaks

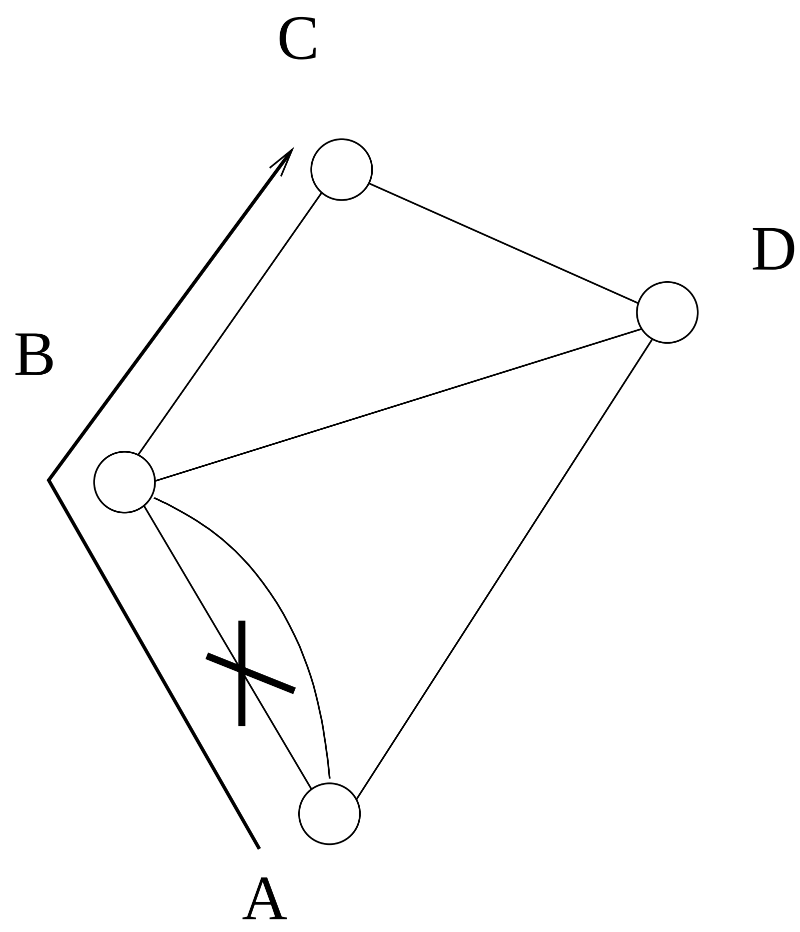
Recovery approaches

- Repair the failed component ... (can take a significant time)
- Establish an alternative route (when the problem arises): Restoration
- Use pre-established routes: Protection
- Use active signal ...: Active protection

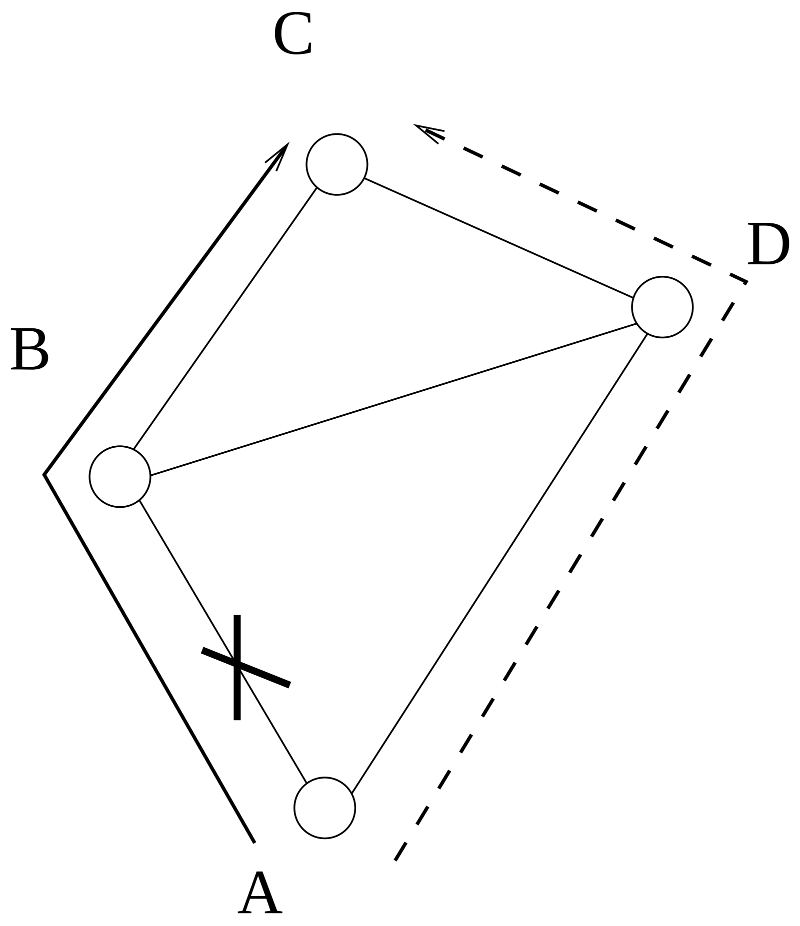
Failure



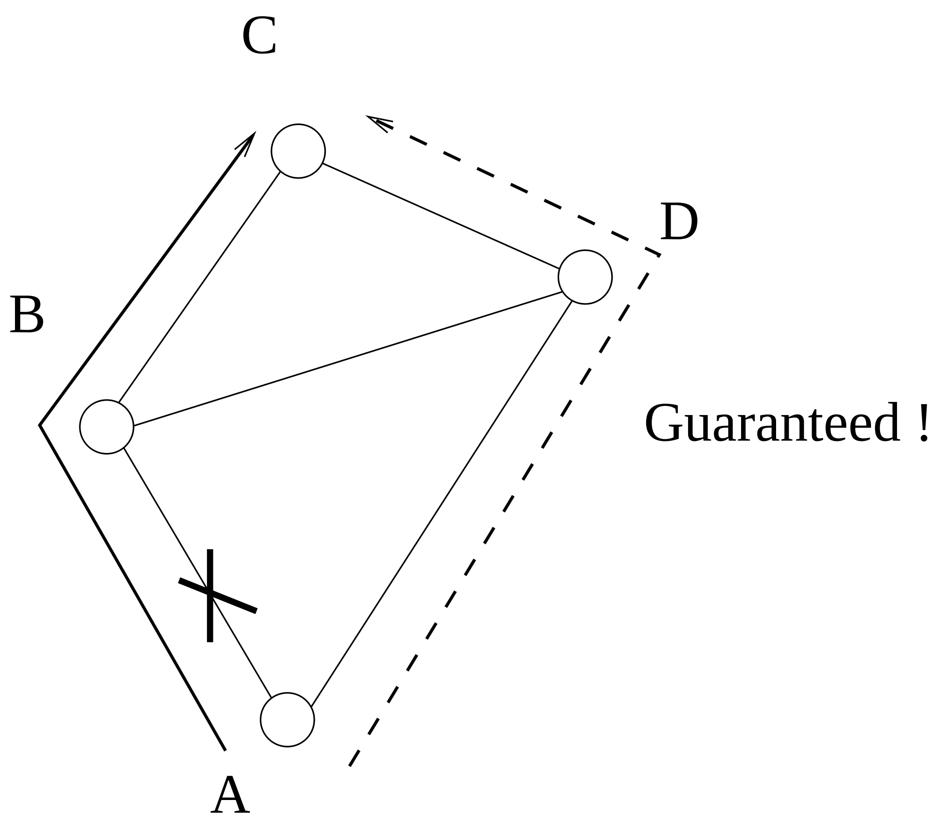
Repair



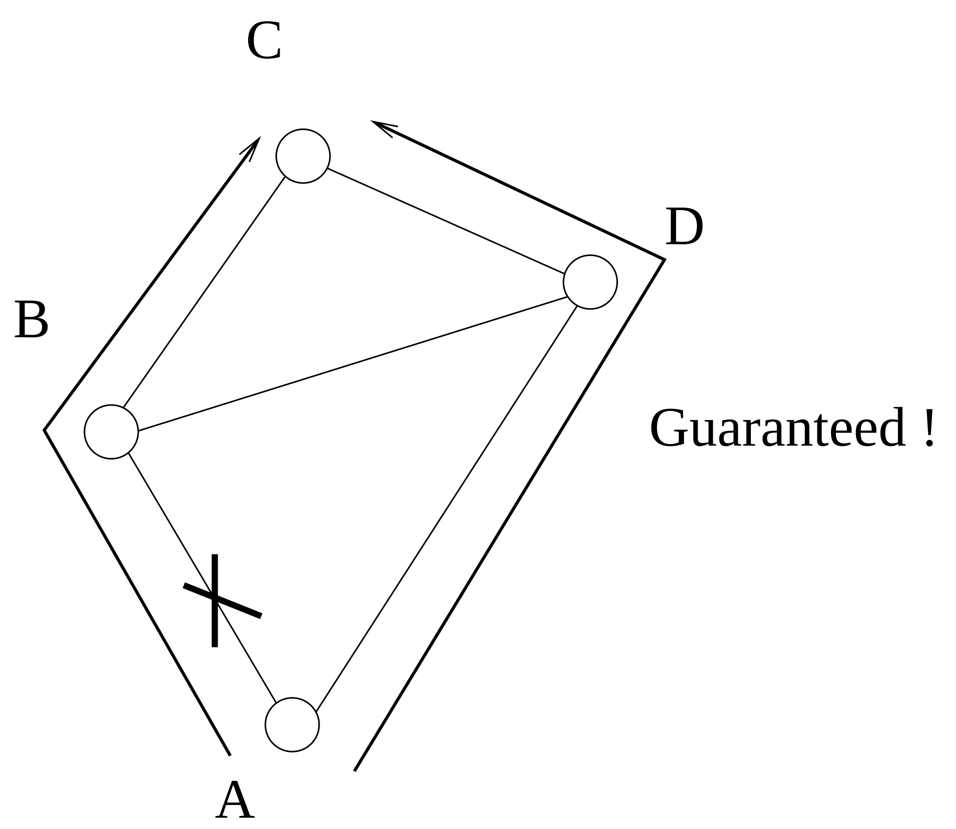
Reroute



Protect



Active protect



Recovery Time

The time it takes before communication has been restored is critical: If connection is lost for more than a few seconds, phone calls will stop, people will stop watching TV and (worst): Higher layer protection mechanisms will start to reroute ...

Recovery Time / Cost trade-off

There is a very basic cost vs. recovery time trade-off: A fast recovery *cost* capacity (and possibly expensive switch technology).

Repair/Restoration/Protection/Active Protection

Extra capacity required:

- Repair: None (we will not consider further)
- Restoration: Some (but since recovery cannot be guaranteed, we will not consider it any further).
- Protection: Require some more network capacity (50 % to 100 %) extra capacity
- Active Protection: A lot (more than 200 % extra capacity).

Because of the increased capacity requirements we will often only protect against one component failure.

Two different approaches to protection

There are two very different approaches which can be attempted to achieve (fast) re-routing:

- Establish *network protecting components* in the network
- Plan protection when routing

Ring protection of a network

Assume you have a network. You can now establish *self protecting rings* in the network !

- Each ring has a certain capacity (i.e. it takes up a certain amount of capacity on *all* the links in the ring
- It can protect all the links with at most the assigned capacity

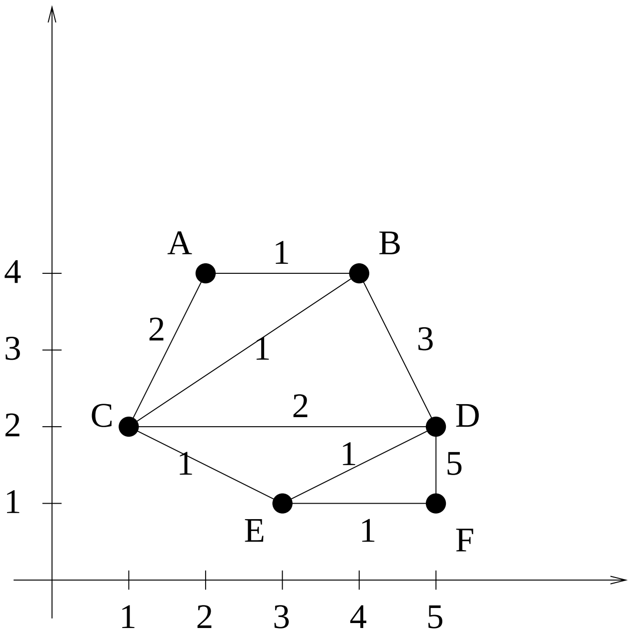
Ring protection: The planning problem

What are the issues when protecting a network using rings:

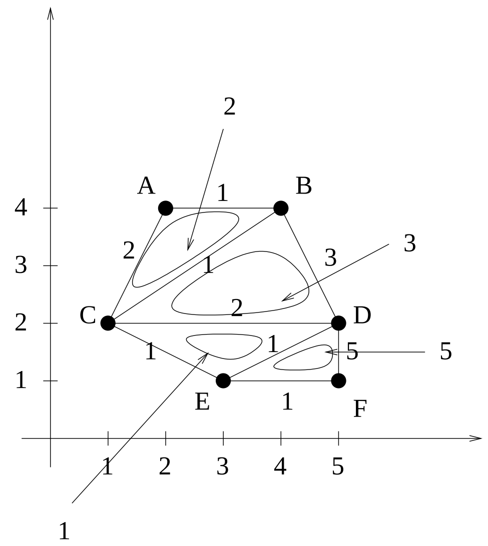
- Select *which* rings to use (how many different rings exists ?)
- Select the *capacity* for the used rings

Your assignment

Given a (routed network or fore-casted network demand):



A possible solution



BUT IS IT THE OPTIMAL SOLUTION ??? Assuming unit costs, this configuration costs: 6 + 9 + 3 + 15=33, the network cost is 17, hence the overcapacity is 33+17/17=2.94

Your job

Since there are exponentially many possible rings in a network, you will apply column generation to solve the optimization problem:

- You do *not* need to perform the Dantzig-Wolfe decomposition (just write the master problem with an exponential number of columns, directly).
- Describe the sub-problem intuitively. You don not need to present a MIP model.

Your job II

Since there are exponentially many possible rings in a network, you will apply column generation to solve the optimization problem:

- Solve the master problem using CPLEX, OPL or GAMS as you like
- Solve the sub-problem manually
- The cost for each unit of capacity is simply the euclidean distance of the link.

Pro's and Con's with networking protecting com

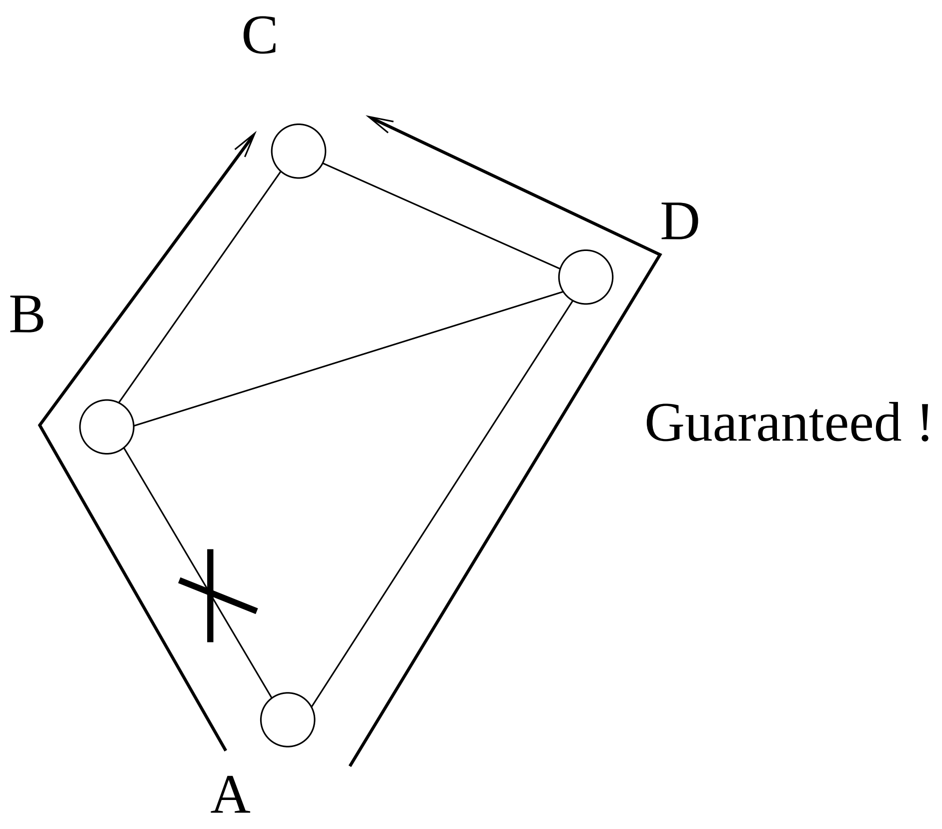
- Pro: Simple fast recovery (and the equipment exists today !)
- Pro: The network may be able to sustain several cable-cuts
- Con: You need a forecast

Active Protection

The classical approach: 1+1 protection

- Send the signal along two link (node) disjoint paths from end to end
- Requires more than 100% extra capacity and most often more than 135%
- Very fast protection, only end node needs to react ...

1+1



How can we find the correct routes ?

$$Min: \sum_{kl}\sum_{ij} c_{ij}\cdot D_{kl}\cdot x_{ij}^{kl}$$

$$s.t. \sum_j x_{ij}^{kl} - \sum_j x_{ji}^{kl} = \begin{cases} 1 & i=k \\ -1 & i=l \\ 0 & \end{cases}$$

$$\sum_j y_{ij}^{kl} - \sum_j y_{ji}^{kl} = \begin{cases} 1 & i=k \\ -1 & i=l \\ 0 & \end{cases}$$

$$x_{ij}^{kl} + x_{ji}^{kl} + y_{ij}^{kl} + y_{ji}^{kl} \leq 1 \quad \forall ij, kl$$

$$\sum_{kl}(x_{ij}^{kl} + x_{ji}^{kl} + y_{ij}^{kl} + y_{ji}^{kl})\cdot D_{kl} \leq CAP_{\{ij\}} \quad \forall \{ij\}$$

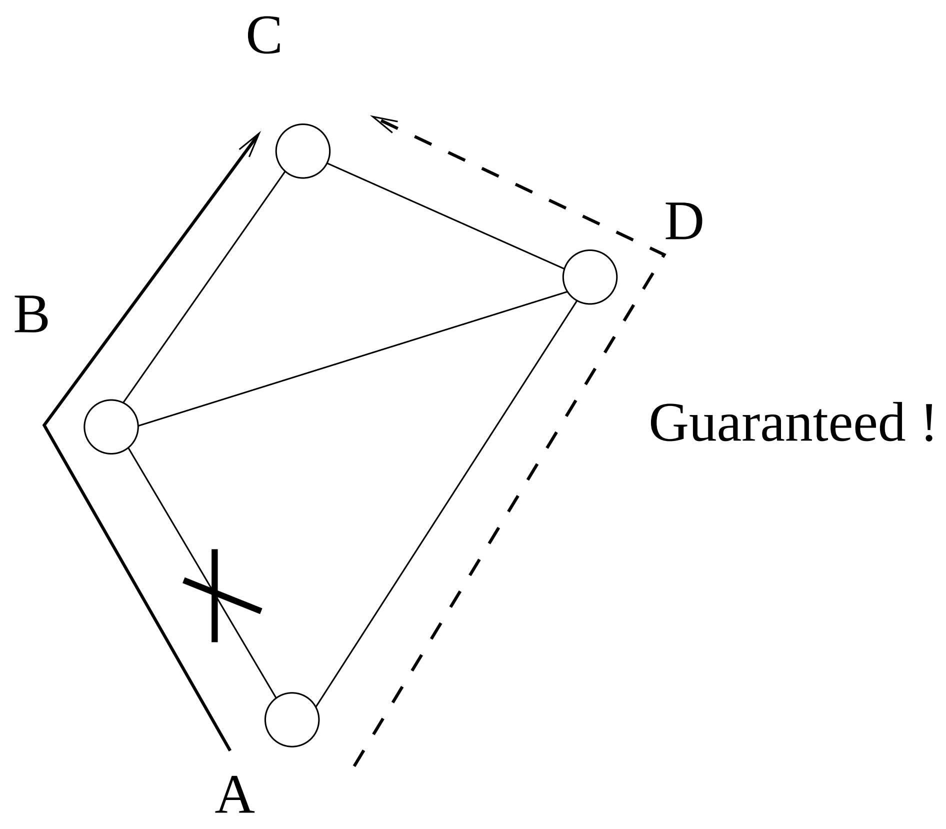
How can we find the correct routes ?

Suurballe algorithm .... polynomial ....

This is not so easy ...

What I am talking now, is actually new research: "Optimal Routing with Failure Independent Path Protection", Stidsen et. al., accepted to Networks journal, dec. 2008 (not printed yet).

Single Backup Path Protection



Single Backup Path Protection

- Instead of having two active paths we have:
  - A primary path
  - A backup path, which is *only* used when the primary path breaks down

The parts of the problem

- The variables are a *pairs* of paths, a primary and a backup path: λ_p^k for all demands k
- The capacity variables are: θ_a
- We have incidence matrices: $PRI_{p,a}^k$ for the primary paths and $BAC_{p,a}^k$ for the backup paths
- $SWITCH\_ON_{p,s}^k$ an incidence for whether the backup path is needed or not

The Master problem

Min:

$$\sum_{a \in A} c_a \cdot \theta_a$$

s.t.:

$$\sum_{p \in P_k} \lambda_p^k \quad = \quad d_k \qquad \forall\, k$$

$$\sum_{k \in K} \sum_{p \in P_k} PRI_{p,a}^k \cdot \lambda_p^k +$$

$$\sum_{k \in K} \sum_{p \in P_k} SWITCH\_ON_{p,s}^k \cdot BAC_{p,a}^k \cdot \lambda_p^k \quad \leq \quad \theta_a \qquad \forall\, a, s : a \notin$$

$$\lambda_p^k, \theta_a \in R_+$$

Exponential paths ...

- We want to generate the path possibilities on the fly !
- In this way we (in principle) only need a (tiny) subset of the path-pairs, $|K| + \frac{1}{2} \cdot |A| \cdot |A|$
- We will start with dummy path pairs which are guaranteed to be too expensive to be selected in the final solution ...
- ... we simply select **all** links in all failure situations.
- How can we select improving path pairs ?
- **And more fundamentally: What is the price of SBPP path-pairs ?**

The Reformulated Master problem

Min:

$$\sum_{a \in A} c_a \cdot \theta_a$$

s.t.:

$$\sum_{p \in P_k} \lambda_p^k \geq d_k \quad \forall\, k$$

$$\theta_a - \sum_{k \in K} \sum_{p \in P_k} PRI_{p,a}^k \cdot \lambda_p^k -$$

$$\sum_{k \in K} \sum_{p \in P_k} SWITCH\_ON_{p,s}^k \cdot BAC_{p,a}^k \cdot \lambda_p^k \geq 0 \quad \forall\, a, s : a \notin$$

$$\lambda_p^k, \theta_a \in R_+$$

The prices

The sign of the dual variables comes directly because the LP formulation now is “sensible” (Sensible Odd Bizar (SOB) sense [**?**])

- $\alpha_k \geq 0$: The (highest) price we are currently paying for satisfying demand k with a disjoint path pair
- $\beta_a^s \geq 0$: The price the *backup* path has to pay for using arc a in failure situation s .... what is this ???
- β_a^s **is the forcer price**

Path pair price I

We need to calculate the *reduced cost* of the path pairs:

- $c_{reduced} = c_{original} - \sum_i \pi_i a_i$
- In our case it consists of:

  $c_{reduced} = c_{original} - c_{demand} - c_{primary} - c_{backup}$

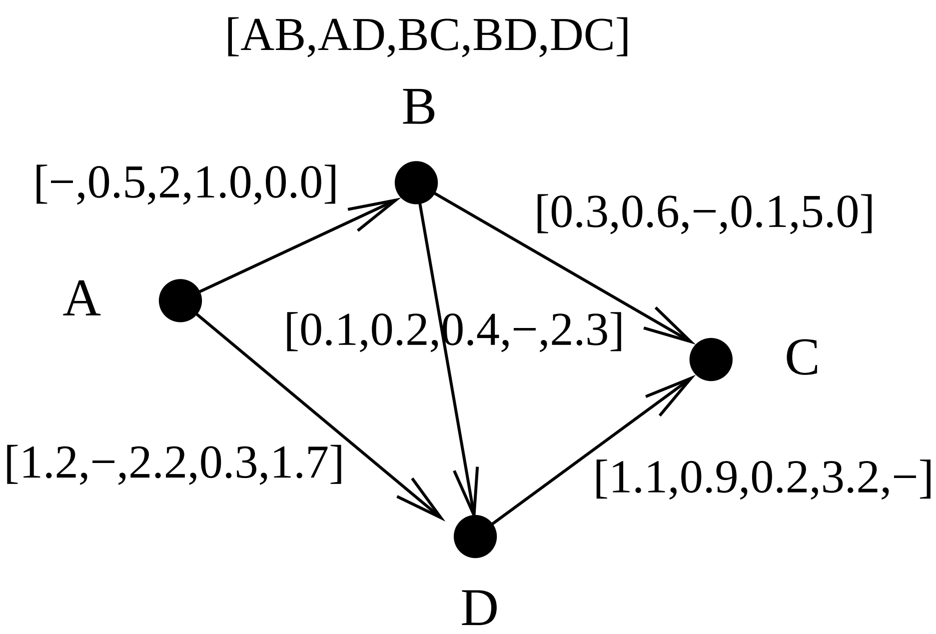
  - $c_{original} = 0$, there are no direct costs !
  - $c_{demand_k} = \alpha_k$ dual variables only participate once

Path pair price II

- $c_{primary,k,p \in P_k} = \sum_a PRI^k_{p,a}(\sum_s \beta^s_a)$
- $c_{backup,k} = \sum_a \sum_s SWITCH\_ON^k_{p,s} \cdot BAC^k_{p,a} \cdot \beta^s_a$
- Notice we for a given demand k only need to consider the cost of the path-pair since the reward α_k is constant.

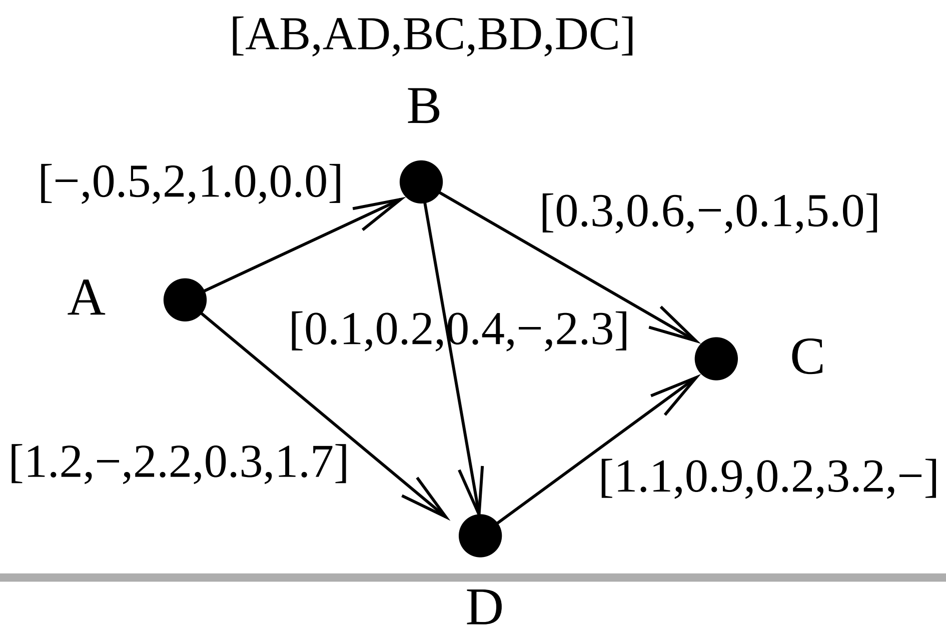
A numerical example

- A very simple example network, with β arrays for each arc (the dash "-") correspond to the arc it self ...



A numerical example

- There are two possible path pairs:
- $[(AB, BC), (AD, DC)] = 3.5+6+3.4+1.3 = 14.2$
- $[(AD, DC), (AB, BC)] = 5.4 + 5.4 + 0.5 + 1.3 = 12.6$ (The winner)



The Path-Pair generation problem I

Min:

$$c^k_{reduced} = \sum_{a \in A} (\sum_{s \in S} \beta^s_a) \cdot x_a \; + \; \sum_{a \in A} \sum_{s \in S} \beta^s_a \cdot z^s_a \; - \; \alpha_k$$

s.t.:

The Path-Pair generation problem II

s.t.:

$$\sum_{a \in \delta_i} x_a - \sum_{a \in \gamma_i} x_a = \begin{cases} 1 & i = s \\ -1 & i = t \ \forall\ i \\ 0 & i \end{cases}$$

$$\sum_{a \in \delta_i} y_a - \sum_{a \in \gamma_i} y_a = \begin{cases} 1 & i = s \\ -1 & i = t \ \forall\ i \\ 0 & i \end{cases}$$

The Path-Pair generation problem II

s.t.:

$$
\begin{aligned}
|F_a| \cdot u_s &\geq \sum_{a \in F_a} x_a \quad \forall\ s \\
|F_a| \cdot v_s &\geq \sum_{a \in F_a} y_a \quad \forall\ s \\
u_s + v_s &\leq 1 \quad \forall\ s \\
z_a^s &\geq u_s + y_a - 1 \quad \forall\ s, a \\
x_a, y_a, u_s, v_s \in \{0, 1\}, & \quad z_{ij,qr} \in [0, 1]
\end{aligned}
$$

Column Generation Algorithm

Initialize $PRI^k_{p,a}$,$BAC^k_{p,a}$
do
  SOLVE SBPP($PRI^k_{p,a}$,$BAC^k_{p,a}$)
  repeat
    SOLVE QCDPP(k, α^k, β^f_a)
  until enough improving path-pairs found: $c^{p,k}_{reduced} < 0$
  add path-pairs
while improving paths found

Test Networks

	Nodes	Edges	Avg. Node Degree	Number of Demands
newyork	16	49	6.12	240
ta1	24	51 (55)	4.58	396
france	25	45	3.6	300
norway	27	51	3.78	702
cost266	37	57	3.08	1332

Table 1: The tested networks

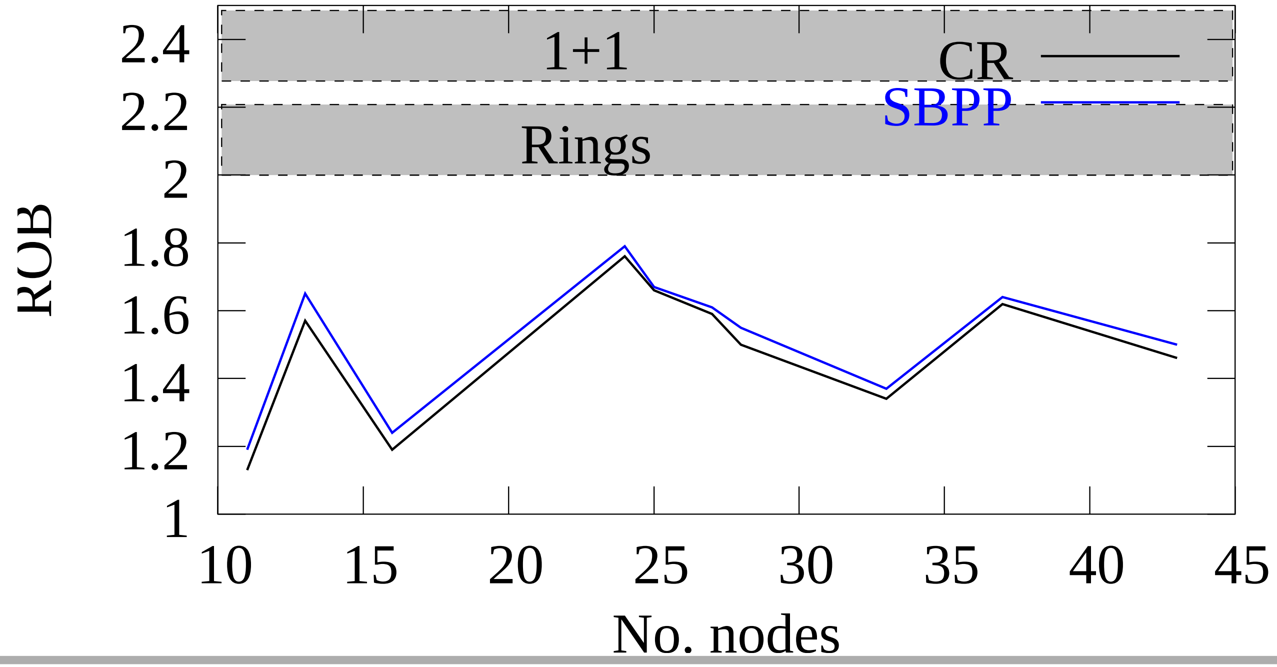
SBPP efficiency

	NF	CR		SBPP	
	Capacity	Abs.	RROB	Abs.	RROB
newyork	412	488.23	1.19	509.9	1.24
ta1	732.99	1287.93	1.76	1308.89	1.79
france	9825	16300	1.66	16456.2	1.67
norway	61.41	97.66	1.59	98.67	1.61
cost266	14587.46	23587.58	1.62	23988.5	1.64
avg			1.56		1.59

Table 2: Network protection requirements for SBPP protection compared to NF and CR [?]

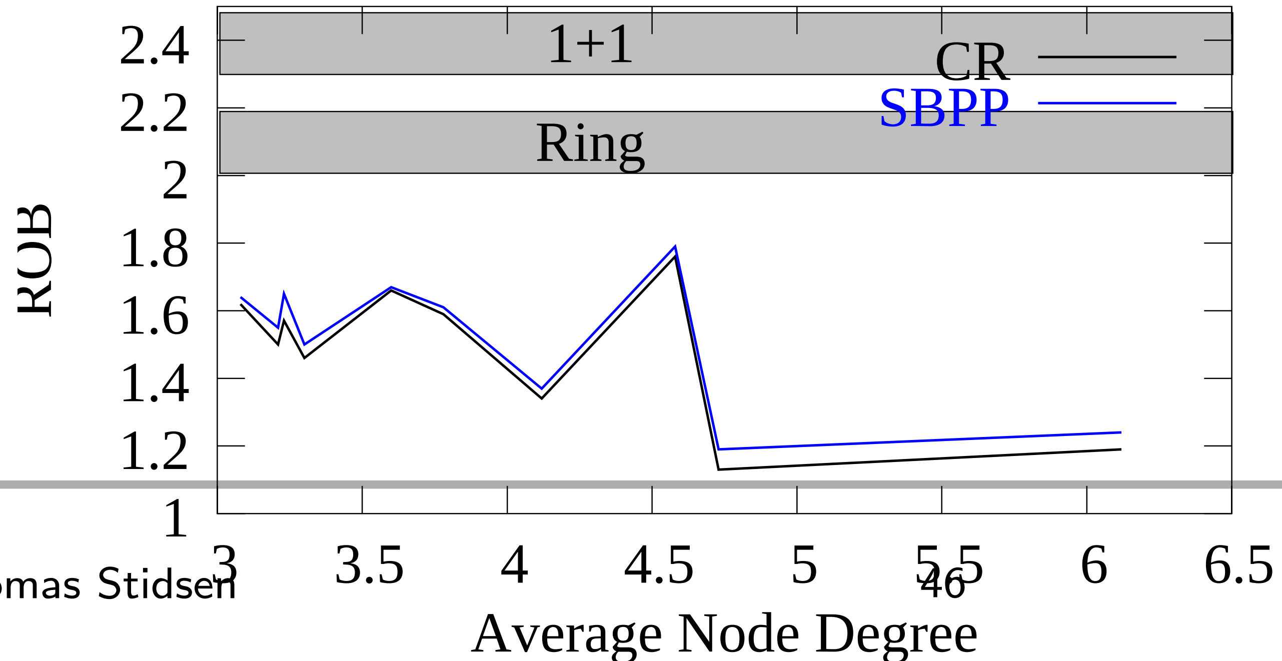
Graphical Setup

- Very little difference !
- But also large architecture dependence
- What about the Average node degree ?



Graphical Setup

- It is difficult to conclude something based on so few examples
- RROB seems to be dropping as the average node degree grows ...
- ... but there are large variations ...



Running time: Brute force version

	It	Total	Master		Sub	
		Sec.	Sec.	%	Sec.	%
newyork	1622	123284.15	3461.59	2.81	119818.33	97.19
ta1	1190	5850.22	840.66	14.37	4998.58	85.44
france	1100	3003.47	722.55	24.06	2271.49	75.63
norway	1593	16612.94	2848.51	17.15	13748.79	82.76
cost266	2079	26075.23	12758.92	48.93	13253.95	50.83

Table 3: Running times in seconds