TRACKING BOND INDICES IN AN INTEGRATED MARKET AND CREDIT RISK ENVIRONMENT

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Abstract. The management of credit risky assets requires simulation models that integrate the disparate sources of credit and market risk, and suitable optimization models for scenario analysis. In this paper we integrated Monte Carlo simulation models with linear programming penalty models and apply them to the tracking of bond indices. Results show that good tracking performance can be achieved in corporate bond markets by a strategic model, however extra value may be generated with a models that goes down to the tactical level with bond picking decisions. It is also shown that in the context of tracking government bond indices, small corporate bond holdings can lead to superior risk return characteristics. Extensive empirical results with the ex post performance of the model over an thirty month recent period substantiate the conclusions of this paper.

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1. Introduction

Modern Portfolio Theory (for a survey see Constantinides and Malliaris, 1995) was developed after Markowitz (1952) introduced the mean-variance portfolio selection model and the idea of diversification. A number of researchers refined the theory in several directions and developed alternative models, mostly in a single period myopic setting. In reality, however, managing a portfolio of assets under changing market conditions leads to a sequence of buying and selling actions after observing new information. Asset and liability management models have been developed that incorporate the dynamics of the problem and approximate the real world behavior more closely. Especially multi-stage stochastic optimisation models are well suited for this task (see Ziemba and Mulvey, 1998). However, if a number of assumptions are satisfied, such as normality and intertemporal independence of the return distributions, no transaction costs and no cash infusion or withdrawal, the single period model provides a good solution to multi-period asset management problems (see Grauer and Hakansson, 1985).

Fixed-income portfolio management strategies evolved in a similar direction. Immunisation strategies that build portfolios that are hedged against small changes in the current term structure of interest rates date back to Reddington (1952). However these immunisation techniques ignore the true stochastic nature of todays markets. Mulvey and Zenios (1994) argued that bond portfolios should be managed by looking at co-movements of securities, hence applying Markowitz's ideas of diversification to fixed-income portfolio management. Unfortunately, mean-variance analysis is not directly applicable as a number of assumptions underlying the analysis are violated when fixed-income instruments are considered. For example, securities with embedded options (e.g. callable bonds) lead to skewed holding period return distributions, and path-dependent securities (e.g. mortgage-backed securities) violate the assumption of temporally independent returns. However a number of fixed-income portfolio management problems (single- and multi-period) were tackled by integrating simulation and optimisation. Worzel et al. (1994) apply integrated simulation and optimisation models for tracking bond indices in a single-period setting. Zenios et al. (1998) address dynamic multi-period models for fixed-income portfolio management. Consiglio and Zenios (2001) extend the single period formulation to the joint problem of asset allocation and bond picking in international bond markets. A general overview of fixed-income portfolio and asset and liability management problems can be found in Hiller and Schaak (1990).

In this paper we contribute to existing literature on fixed-income portfolio management by taking into account market and credit risk in a portfolio context. Low government bond yields and the reduced liquidity of government debt has attracted investors to corporate products over the last few years. This increased demand in corporate securities and the Russian crises (August 1998), including the "flight to quality" thereafter, led to an increased demand for risk management approaches that integrate disparate sources of risks. Furthermore it is likely that the growing credit derivatives market will impact the practice of fixed-income portfolio and risk management over the next few years. For example, some European telecommunication companies already issue their bonds with embedded credit derivative protection. Gregory-Costello (2000) discusses how this credit protection changes bond markets. As Schönbucher (2000) points out credit derivatives make large and important risks tradeable. They form an important step towards market completion and efficient risk allocation by bridging the market segmentation between corporate loans and bond markets.

Credit risk losses may result from a counterparty default or a decline in the market value due to credit quality downgrading. For a single instrument, credit risk may be decomposed into default risk and the corresponding recovery risk, migration risk, and a security specific risk causing idiosyncratic spread changes. The correlation between migrations and defaults in a portfolio context is very important for both, risk management and derivatives pricing. As a result, the distribution of credit losses can be described by a large chance of small earnings and a very small probability of (extremely) large investment losses. They are non-normal and heavily skewed. Details about credit risk models and management can be found in, e.g., Saunders (1999).

The various risk factors involved and the (un)availability of quality data poses challenges to credit risk modelling. It is still common practice in risk management to treat credit and market risk separately. Furthermore, analytic tractability in VaR calculations can be reached if, for example, market risk is ignored. In general, most methods are purely simulation based and only recently some promising attempts towards optimizing credit risk started; see Mausser and Rosen (1999). In both studies, however, the scenario generation method used as input in the optimization models ignores market risk and considers only credit migrations and defaults sampled from historical transition matrices. However, the actual portfolio risk involved can only be assessed correctly if all risks are considered in an integrated framework (see for example Kijima and Muromachi (2000) and Jarrow and Turnbull (2000)). As a result, simulation methods that incorporate market and credit risk factors in a unified framework combined with suitable portfolio optimisation techniques need to be developed.

A number of researchers have implemented models for the term structure of interest rate to derive possible future realisations of security prices sensitive to interest rate movements. Similarly, we extend credit risk pricing models for scenario generation (see Jobst and Zenios, 2001) in order to apply stochastic optimisation models to solve fixed-income portfolio management problems with credit risk. In this paper we extend our previous work by integrating our simulations into portfolio optimisation. This framework builds the foundation for further studies regarding the integration of more complex securities such as swap products and credit derivatives for integrated asset and liability management.

The rest of the paper is organized as follows. Section 2 describes the problem and the dataset used throughout the paper. Section 3 outlines the scenario generation method and Section 4 develops the optimisation model. Section 5 presents computational results and Section 6 concludes.

2. Problem description and data analysis

We consider the problem of a fixed-income fund manager whose target is to track a certain corporate bond index, such as the Merril Lynch Euro Dollar, the US Domestic Corporate or the US Agency index. Indexed fixed-income funds have gained popularity over the last decade and especially higher yielding fixed-income securities gained popularity lately. We focus on the Merril Lynch Euro Dollar index since it offers good liquidity.

In the numerical studies below we backtest the performance of the models over the 30 months period from January 31, 1999 to July 31, 2001. The growth of an initial investment of $100 is shown in Figure 1. We can note that during the first 17 month the index returns

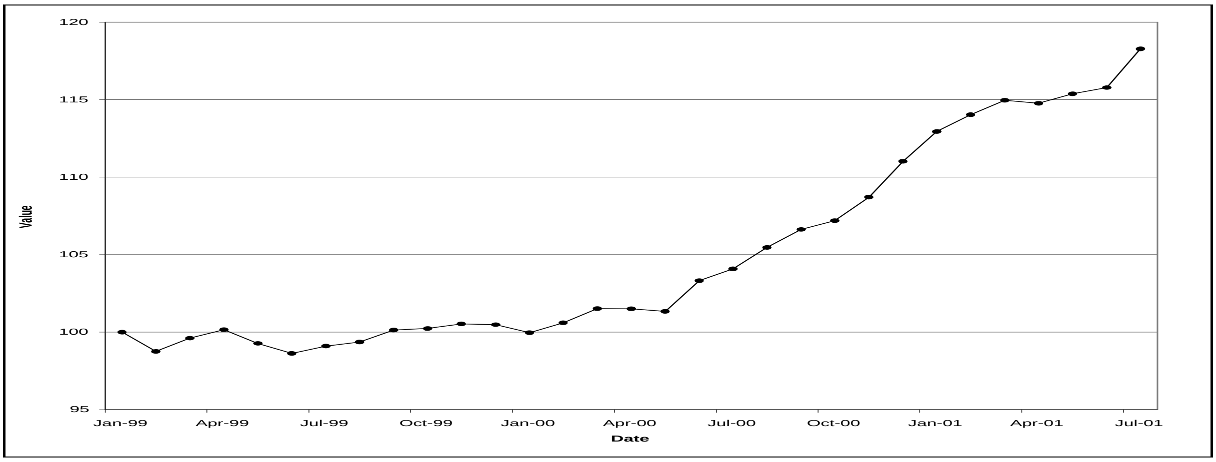

FIGURE 1. Performance of the Merril Lynch Euro Dollar index from January 1999 to July 2001.

are low as this period can be described by rising yields and widening credit spreads. For example, for *Aaa* rated bonds, spreads moved from about 50bp to over 75bp and for *A* rated bonds from about 100bp to 150bp. However, the Eurodollar index performed well compared to other indices (obtained from the Global Bond Index Monitor) over this period (January, 6 1999 to June, 5 2000) as shown in Table 1. The third column of table

Index	Total Return	Price Return
Eurodollar	2.778	-5.939
AAA Agency	1.882	-6.560
Domestic Corp.	-0.807	-10.284
US Treasury	2.099	-6.633

TABLE 1. Global Bond Index Monitor.

1 reports the effect of this rising yields and widening spreads in all rating classes leading to big losses due to decreasing prices. This period was followed by decreasing yields and hence increasing index returns (Figure 1).

Important from an optimisation point of view is the number of bonds in the index. The index contained about 450 securities on January 31, 1997. The number of securities increased to more that 1000 by August 31, 2000 from 665 before March 2000. All bonds issued in the index are investment grade quality and rated *Baa* or higher. On January 31, 1999, approximately 84% of the index value was in bonds rated *Aa* or higher.

3. Scenario generation

Because of the non-normality of the return distributions, the extreme events and the complexity of the risk factors, we apply simulation methods to generate sets of plausible scenarios as the input to our optimisation models. The simulation model is described in detail in Jobst and Zenios (2001) where a number of credit risk pricing models are discussed for simulation and valuation purposes. The holding period returns are not simply forecasted on the basis of information variables, instead they are obtained from scenarios of the underlying risk factors such as interest rates, credit spreads, migrations, defaults and recovery.

3.1. **Models for credit risk.** Two different modelling principles have been developed for modelling credit risky security prices. Black and Scholes (1973) and Merton (1974) introduced *structural* or *firm's value* models where the idea is that default happens when the underlying diffusion process for the value of the firm hits a 'default' boundary. *Intensity* or *reduced* form models assume that the default event is unpredictable and hits the security holders by surprise (Duffie and Singleton, 1999, Lando, 1998, Jarrow and Turnbull, 1995). This approach is not based on any information on the firm's value or its balance sheet. The event of default is described by its probability derived from its instantaneous likelihood, i.e., the hazard rate (intensity).[1] This is often modelled as the intensity λ of a point process where each jump of the process denotes a default event. Frequently, λ is a function of state variables X representing for example interest rates, stock prices, time etc., hence $\lambda(t) = \lambda(X_t)$. Commonly Poisson or Cox processes are applied. If the random intensity is modelled as a finite state space Markov chain representing credit ratings, the model is frequently denoted as rating based model (see Jarrow, Lando and Turnbull, 1997). Our scenario generation method builds on and extends reduced form models as explained below.

3.2. **Integrated market and credit risk simulations.** The simulation framework of Jobst and Zenios (2001) incorporates stochastic intensity and rating based elements in a manner which is consistent with market risk. In order to simulate an indexed portfolio at future points in time $\bar{T}$ disparate sources of risk are considered simultaneously. In our method we assume that corporate bonds are exposed to changes in *interest rates*, *credit spreads*, *rating migrations*, *default* and uncertainty in *recovery amounts* when in default.

The main components of the scenario generator are models for the uncertainty in interest rates and credit spreads, an intensity based pricing model, a set of observable migration processes described via a finite state space Markov chain, and a model of recovery rates when in default. Interest rates and credit spread processes are modelled via correlated stochastic processes of the form

$$dx(t) = \mu_x(x,t)dt + \sigma_x(x,t)dW_x(t) \qquad (1)$$

[1]This is a measure of propensity of default per unit of time and is widely used in the insurance practice and literature.

where $dW_x(t)$ denotes the standard Brownian motion under the real measure $\mathcal{P}$. The risk neutral dynamics (under the measure $\mathcal{Q}$) can be obtained by introducing a market price of risk. Such a process is considered for the short interest rate, i.e., $x = r$, and for the credit spreads, i.e., $x = s_k$ of each rating class $k = 1, ..., K-1$ where 1 denotes the highest (Aaa) and $K-1$ denotes the lowest pre-default rating Caa-C. Correlation between interest rates and each spread process is captured by correlating the Wiener terms, i.e.

$$dW_r(t)dW_{s_k}(t) = \rho_{rk}dt \tag{2}$$

where $k \in \{1, ..., K-1\}$.

Given these dynamics, the prices for risky zero-coupon bonds can be obtained from the intensity based pricing model (Jobst and Zenios 2001). We denote by $v^{\kappa_t^n}(t, T_n)$ the price of a risky zero-coupon bond n at time t with maturity T_n currently in credit rating $\kappa_t^n \in \{1, 2, ..., K-1\}$. The index superscript n for κ_t^n is dropped when there is no ambiguity. The price of a risky coupon bond is given as

$$P_n^{\kappa_t}(t, T_n) = \sum_{\tau=t}^{T_n} F_n(\tau)v^{\kappa_\tau}(\tau, T_n) \tag{3}$$

where $F_n(t)$ denotes the coupon payments (plus principal at maturity $t = T_n$).

At time $t = 0$ the current rating κ_0^n of each bond n is known and in principle prices can be obtained by taking into consideration the risk-neutral evolution of the interest rate and the κ_0^n-spread process until maturity. However, if we want to obtain future prices we need to simulate interest rates and spreads at the time horizon $\bar{T}$ and the future rating of bond n at time $\bar{T}$, i.e. $\kappa_{\bar{T}}^n$. Given this rating, the zero-coupon bond prices $v^{\kappa_{\bar{T}}^n}(\bar{T}, T_n)$ are obtained according to the intensity based pricing model, taking into consideration the risk-neutral dynamics of the short rate and the $\kappa_{\bar{T}}^n$-spread process from time $\bar{T}$ onwards. Future credit ratings are generated according to a Markov model describing rating transitions, including a transition to the absorbing default state.

The intensity based pricing model was implemented using the tree building method of Schönbucher (1999). This method has the advantage to implement alternative forms of stochastic processes (hence there is not restriction to the Gaussian process and its limitations) and can be implemented and a large large range of credit risky securities can be priced consistent with the current risk free and risky term structures. The rating migrations are simulated from the one year transition matrix published by Moody's. Table 1 reports the average one year transition probabilities (1980 to 1998).

	Aaa	Aa	A	Baa	Ba	B	$Caa-C$	$Default$
Aaa	0.8866	0.1029	0.0102	0.0000	0.0003	0.0000	0.0000	0.0000
Aa	0.0108	0.8870	0.0955	0.0034	0.0015	0.0015	0.0000	0.0003
A	0.0006	0.0288	0.9021	0.0592	0.0074	0.0018	0.0001	0.0001
Baa	0.0005	0.0034	0.0707	0.8524	0.0605	0.0101	0.0008	0.0016
Ba	0.0003	0.0008	0.0056	0.0568	0.8357	0.0808	0.0054	0.0146
B	0.0001	0.0004	0.0017	0.0065	0.0659	0.8270	0.0276	0.0706
$Caa-C$	0.0000	0.0000	0.0066	0.0105	0.0305	0.0611	0.6297	0.2616
$Default$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

TABLE 2. Average 1-year transition matrix (1980 - 1998).

In the event of default, a recovery value consistent with the intensity model recovery assumption or sampled from real world distribution is added to $F_n(t)$ and no future cashflows outstanding are received.

Figure 2 provides an overview of the simulation method. Given input data on the individual securities, term structures on default free and defaultable bonds of different ratings, and the rating transition probabilities, we simulate a set of *economic scenarios* of interest rates and credit spreads for all rating classes. In addition we simulate a set of *credit* scenarios reflecting migrations into different ratings or default. Given that bond n at time $\bar{T}$ is in rating $\kappa^n_{\bar{T}}$ we need to obtain the price of this bond conditioned on the state of the economic scenarios at $\bar{T}$ and according to the evolution of the state variables under the risk neutral measure $\mathcal{Q}$ from $\bar{T}$ until maturity.[2]

Given the current prices of bond n, $P_n^{\kappa_0}(0, T_n)$, and its simulated future price $P_n^{\kappa_{\bar{T}},s}(\bar{T}, T_n)$ under scenario s, it is straightforward to calculate the returns needed in the indexation model (see Worzel et al. 1994).

Figure 3 shows the typical distribution for a portfolio of 16 *Baa* rated bonds from the index on January 31, 1999 at a risk horizon of 8 months. This distribution was generated by considering 250 interest rate and spread scenarios, and 2000 rating and default scenarios under each economic scenario, for a total of half a million scenarios. In the event of default no recovery was assumed. Further results on the price sensitivity of bonds and of a portfolio of assets to the various risk factors can be found in Jobst and Zenios (2001).

In order to keep the optimisation model tractable, we reduce the sample size by dividing the index return distribution at the risk horizon in a number of buckets, and sampling a fixed number of scenarios (typically 2000) out of the original scenarios. Scenarios are sampled out of each bucket with the frequency observed in the original scenario set. This sampling scheme is a simplified variant of stratified sampling. Figure 4 shows the simulated returns of the Eurodollar index over the period from April 1999 to July 2000. The figure shows the 3-month scenarios together with the ex post realized index returns. The scenarios are generated using only information that was available three months before the shown date. The stratified sampled scenarios cover the true index return for every period.

Overall, the simulation model combines elements of reduced-form and rating based models to incorporate a large number of risk factors driving security returns. As such, the simulation model might as well share some of the weaknesses of this type of models. Reduced form models are based on the assumption of unpredictable default events, whereas in reality there is some evidence that defaults may be predicted from market data such as equity returns. Most of this models (such as the KMV approach) are based on the

[2]It is important to note that we do not assume a different intensity or spread process for every issuer, instead we assume that bonds in a given rating class evolve according to the same process. However bonds are priced using the OAS methodology and hence, there is an individual element involved. The main reason for this assumption is that it is difficult to specify a process for each individual issuer as there is not enough data and a number of issuers have only very few bonds outstanding.

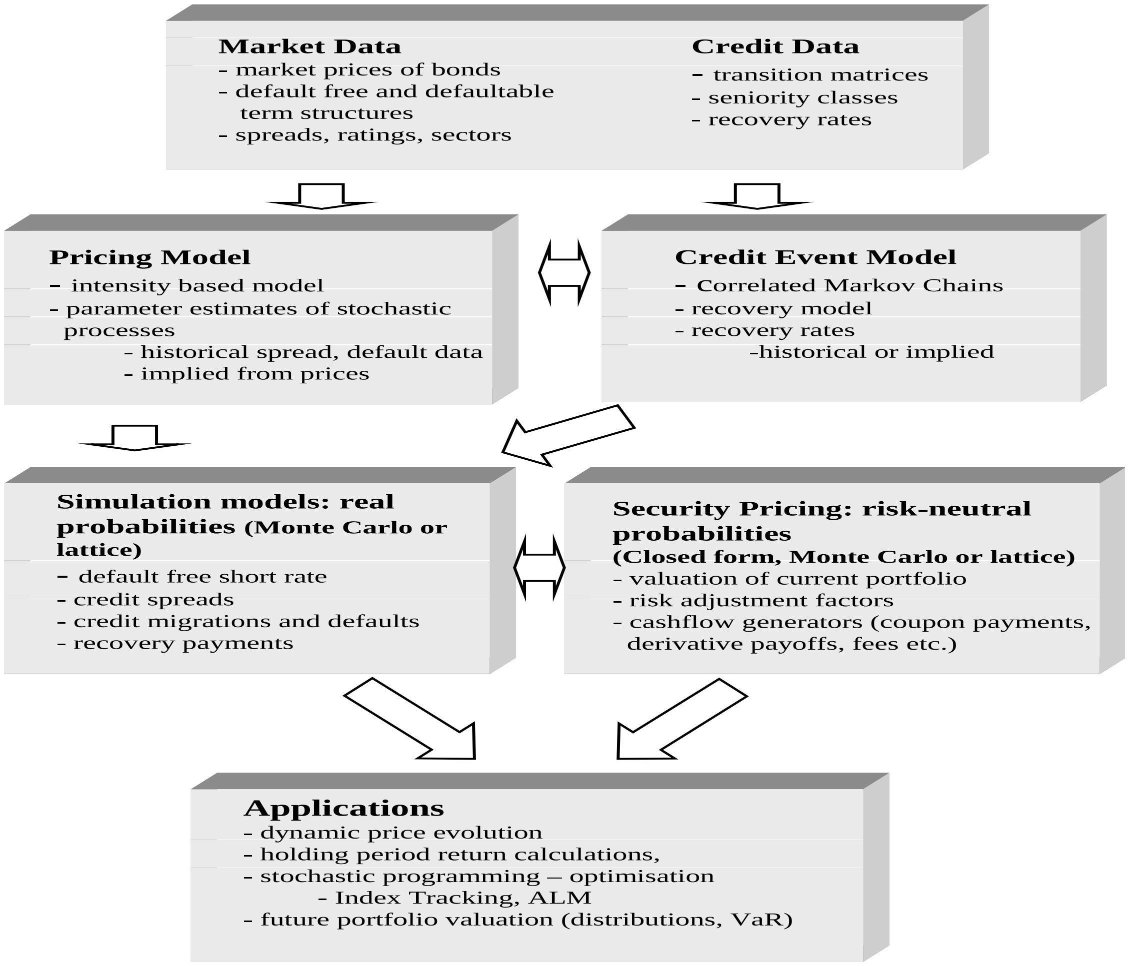

FIGURE 2. An overview of the general simulation framework.

structural (Merton) approach which suffer from drawbacks, too, such as unsatisfactory description of the spreads dynamics.

Hence, building simulation models for credit risky securities requires tradeoffs and depends heavily on the application of the model. In the current study, we focus entirely on investment grade, high quality securities contained in the Eurodollar index. Based on the intensity models and the option-adjusted spread methodology, a main source of risk we try to capture is spread risk, whereas the migrations and defaults are based on historical rating matrices. This approach can be justified by recent results of Kiesel et al. 2001, where it is reported that for high-quality debt, most risk stems from spread changes. This is significant, as most (commercial) credit models focus on defaults and rating migrations assuming no spread risk. However, the other sources of risk should not be ignored as extreme events happen and we want to build portfolios that are well diversified with respect to the different risks involved.

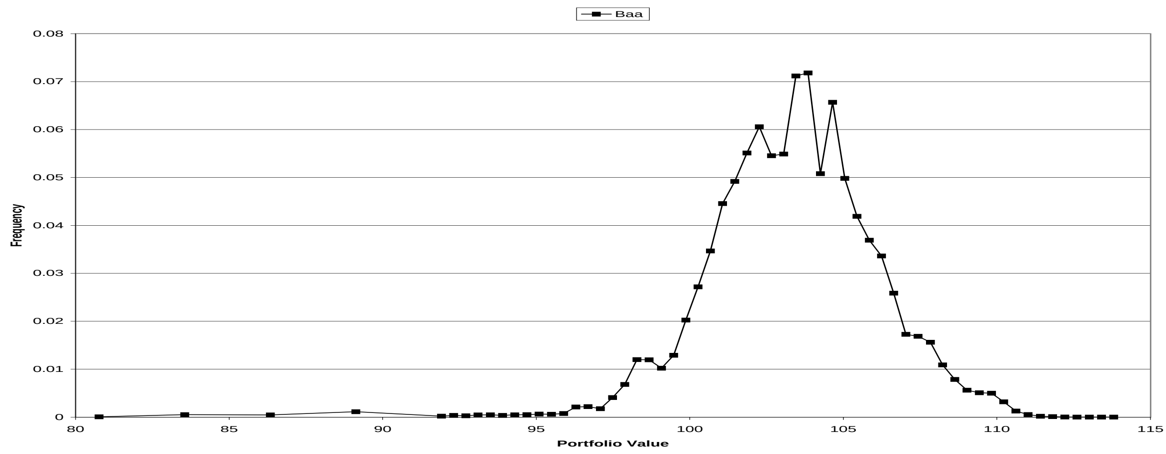

FIGURE 3. Distribution of a *Baa* rated bond portfolio values at the end of the 8 months risk horizon under 250 economic and 2000 rating scenarios.

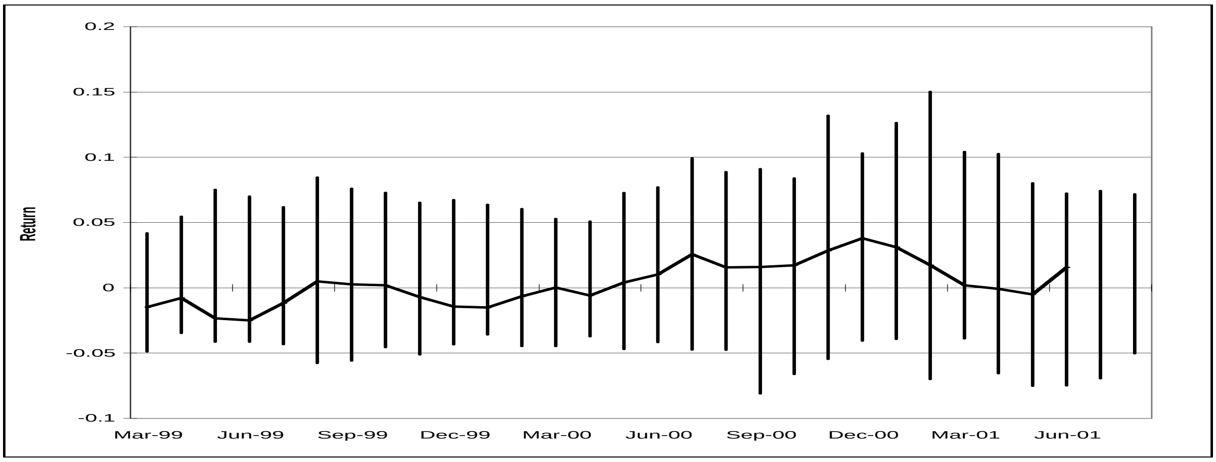

FIGURE 4. Scenario generation for the Merril Lynch Euro Dollar index through time. At each timepoint only information available 3 month prior is used to generate the scenarios.

However, investment and risk management models for very low quality debt may require a different treatment. There, the risk of migrations and defaults may dominate the spread risk and hence, a detailed modelling of these individual defaults and even default correlations may be of paramount importance. The simulation model described above can be extended along this lines. One possibility is to follow the ordered probit approach developed in the Creditmetrics methodology of JP Morgan (1997) which allows to incorporate dependencies between distributions of transitions of different exposures. Another approach focusing on correlated defaults is developed in Schönbucher and Schubert (2001), where a default dependency structure is specified by the Copula of default times and combined with individual intensity based models for each obligor.

4. The optimisation models

We now formulate the optimisation models to address different problems facing a portfolio manager of corporate bonds. Given an index and all bonds therein, a model is required that picks a subset of bonds to achieve a good tracking performance. From a strategic point of view the manager is interested in determining the weight to put into general asset classes. An asset class may consist of all bonds with a certain rating, or in a certain industrial sector, or with a given maturity range, or combinations of such characteristics. The actual bond picking decisions follow the broad asset allocation, and credit analysts bring in expert knowledge about specific companies at that point. For very large indices or multi-country applications, the asset allocation decision may be used to reduce the problem size and the subsequent bond-picking problems may be solved via optimization models on the reduced set as well.

We apply a tracking model, which penalizes downside risk only, see Worzel et al. (1994), which is a variant of the mean absolute deviation model (see Konno and Yamazaki (1991)). If under any single given scenario the portfolio return is less then the index return under that scenario, an infinite penalty is imposed. However, we cannot eliminate all downside risk and we have to accept a small level ϵ of under-performance. Figure 5 illustrates the concept of the tracking penalty function compared to the mean absolute deviation penalty function. We point out that when we apply this penalty function, our tracking portfolio

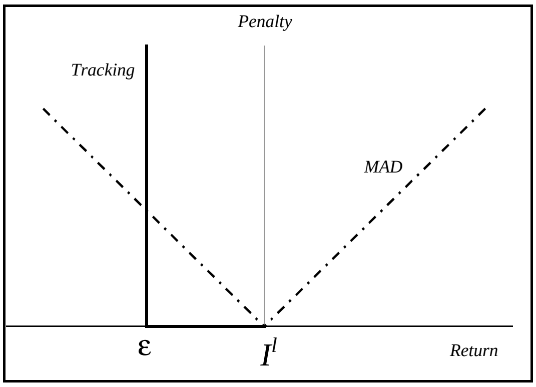

Figure 5. Symmetric penalty function of MAD model and the asymmetric penalty function for the tracking model.

is restricted to stay within ϵ of the index in its downside movements, however we do not limit the upside potential. Mimicking the upside potential, too, may be of interest for some fund managers, and mimicking portfolios (in both, upside and downside) can be achieved by applying symmetric penalty functions with an infinite penalty on the up- and downside. For related work on indexation models in an international setting see Consiglio and Zenios (2001), and for the use of optimisation models for credit risk management see Mausser and Rosen (1999).

In our empirical models we limit only the downside. This penalty function is well suited for problems with credit risk when we deal with likely scenarios of small positive returns and rare situation of high investment losses. Under scenarios of extreme events of price jumps due to downgrading and defaults, the resulting investment losses are very high. Usually the index contains hundreds or thousands of instruments and none of the instruments dominates the composition. If one of the bonds defaults this will not lead to a extreme drop in the index value. This implies that the composition of the tracking portfolio should not consist of investments in only few instruments as in the event of default the underperformance will eliminate most of the portfolio value. This penalty function and the nature of credit risk simulations result into portfolios that are well diversified in terms of the number of bonds and the holding in each bond.

The following notation is used in defining the models. The corporate bond index is defined by the set U of bonds denoted by $i = 1, 2, \ldots, m$. Each bond has attached a given rating, denoted by $k = 1, 2, \ldots, K-1$. The set of all bonds in rating k is denoted by U^k. Similarly, each bond can belong to a certain asset class such as tall, short, caucasian, denoted by $c = 1, \ldots, C$. The set of all class c-bonds is denoted by U^c. $\Omega = \{1, 2, \ldots, N\}$ is the index set for our scenarios where each scenario occurs with probability p_l. With these definitions we can outline the optimisation models.

First we solve the *bond picking* problem that chooses a subset of bonds out of the universe U. We then outline a two step procedure to dealing with a very large universe of bonds or when dealing with sub-indices such as an *Aaa*-utility index containing either all bonds from $U^{utility}$ or from a different source from alternative markets. Finally an *integrated indexation* model is given. This model, in addition to choosing the right bonds from the universe, controls the portfolio composition with respect to the asset class structure, i.e., the holding in each class U^k. The detailed operational model is outlined in the Appendix, where real life constraints, transactions costs and the decision variables required for rebalancing existing portfolios are included.

4.1. **Bond-picking model.** The model is stated as follows:

$$\text{(4)} \qquad \text{Maximize}_h \quad \sum_{l=1}^{N} p_l \sum_{i=1}^{m} x_i r_i^l$$

$$s.t.$$

$$\text{(5)} \qquad \sum_{i=1}^{m} x_i r_i^l \geq I^l - \epsilon, \qquad \text{for all } l \in \Omega,$$

$$\text{(6)} \qquad \sum_{i=1}^{m} x_i = 1,$$

$$\text{(7)} \qquad x_i \geq 0, \qquad \text{for all } i \in U.$$

This model maximizes the portfolio return, defined by $\bar{R}_P = \sum_{l \in \Omega} p_l R_P^l$ where $R_P^l = \sum_{i \in U} x_i r_i^l$ denotes the return of the tracking portfolio under scenario l. x_i denotes the decision variables, i.e. the holding in bond i, r_i^l denotes the holding period return of bond

i under scenario l. The index return under scenario l is given as

$$I^l = \sum_{i=1}^{m} \beta_i r_i^l, \qquad \text{for all } l \in \Omega, \tag{8}$$

where β_i denotes the holding of bond i in the index and is given as input data.

4.2. **Asset-allocation model and bond picking.** The asset allocation model can be derived from the model above by substituting the holdings in bond i, x_i, by holdings in asset class c, x_c. The holding period returns for asset class c, under scenario l can either be obtained by modelling synthetic bonds that mimic the asset class characteristics or, if scenarios for all bonds in the universe U are available, by calculating

$$I_c^l = \sum_{i \in U^c} \beta_i r_i^l, \qquad \text{for all } c = 1, \ldots, C, \tag{9}$$

where β_i are the weights. In a second step we may solve for bond picking by maximizing the expected return of each asset class subject to constraint (5) on asset class index return I_c^l and subject to the holding in the asset class, which is the optimal solution of the asset allocation model.

4.3. **Integrated indexation model.** The bond picking and asset allocation models can be integrated in a common framework. The goal is to pick a subset of bonds out of the universes with some constraints controlling the holding in each asset class. This is done by extending the bond-picking model of the previous section by additional constraints on the sector or asset class holdings. The target holding h_c^t in each asset class can be the class-holding of the index, or obtained from the solution of an asset allocation model, or given by some expert knowledge. This constraint can be relaxed such that our portfolio should be within a certain range to the target holding $[h_c^t - \delta_c^-, h_c^t + \delta_c^+]$. These additional constraints are modelled as

$$\sum_{i \in U^c} x_i \geq h_c^t - \delta_c^-, \qquad \text{for all } c = 1, \ldots, C, \tag{10}$$

$$\sum_{i \in U^c} x_i \leq h_c^t + \delta_c^+, \qquad \text{for all } c = 1, \ldots, C. \tag{11}$$

These constraints allow portfolio managers to further diversify in case a certain sector (such as, for example, the telecommunication sector) hits a crises.

5. Empirical results

We apply the optimisation models to the problem of tracking the Merril Lynch Euro Dollar index and analyse the performance of the resulting portfolios. The tracking portfolios of course perform well, by definition, if the scenarios reflect the real market behavior. However from a practical and validation point of view we are interested in the performance of the model in real life. Therefore, we study the ex post performance once the uncertainty is revealed. Furthermore we study the effect of incorporating corporate bonds in the bond universe when tracking government bond indices, especially we focus on the US Treasury index.

We start our experiments on January 31, 1999, and generate 3-month holding period return scenarios for all bonds with maturities up to ten years and for each credit rating class using the simulation framework of section 3. The required pricing and scenario generation is done consistently with the default free and risky term structures available on January 31, 1999, and the model is calibrated to information available up to that day only. The tracking portfolio optimisation model is then used to select a portfolio. We then move the clock one month forward at which point (February 1999) we know the bond returns and index performance and can therefore calculate ex post performance statistics for the tracking portfolio. Using now the updated information available on February 28, 1999, we repeat the simulation, optimisation, and performance analysis. This process is repeated until July 31, 2001. The first 17 month can be characterized by rising yields and widening spreads and was a difficult period for bond portfolio managers. Furthermore the period had some significant events from a credit risk point of view. For example, the Malaysian government imposed exchange controls in November 1998 and although exchange rate risk is not explicitly present as all bonds are issued in USD, extreme price movements followed. Eleven bonds in the index were hit by a massive price drop, however they recovered afterwards, see the example in Figure 6.[3] From a technical point of view we delete all bonds with optional payoffs such as callable or putable bonds and focus on standard coupon bearing bonds, only.

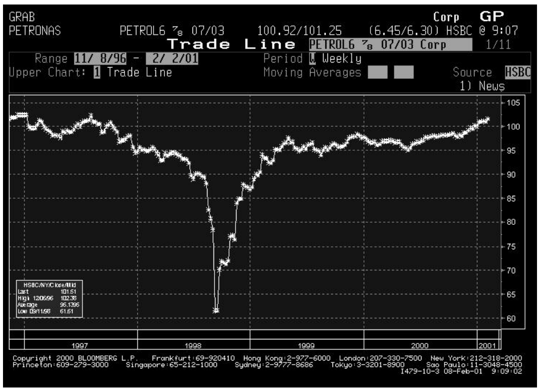

FIGURE 6. Baa rated bond prices during period Nov. 96 to Feb. 2000.

In this section we apply the model of Section 4.1 to the universe of bonds in the index and analyse its performance ex post. Transaction costs are considered for all trades during the backtesting period . We assume the same transaction cost for all bonds within a rating

[3]However unfortunately our database does not contain all detailed information about the events causing a security to drop out of the index, which could be due to default, maturity or in most cases restructuring of the debt and reissuing in form of a different bond. In this respect, we add constraints to our optimization models ensuring that we don't have holdings in those securities dropping out where information is lacking. However the number of bonds in question is very small (approximately one to two bonds per timeperiod) and defaults would not affect the index significantly due to the large number of bonds contained and the low exposure to a single bond or issuer.

class: 5 bp for Aaa, 10 bp for Aa, 20 bp for A and 40 bp for Baa. We are also interested in the structure of the tracking portfolio with respect to their holdings in certain rating classes or sectors as compared to the index portfolio. We also analyse the sensitivity of the tracking portfolios to different risk factors. Subsection 5.4 discusses the application of the asset allocation model, and the model of section 4.3 with imposed bounds on the asset class holdings, with target asset class holding derived in two different ways. First we impose bounds to mimic the underlying index, and second we impose the optimal holdings from the asset allocation model (section 4.2) as bounds to the bond-picking model. Finally we benchmark the tracking models against randomly chosen portfolios (subsection 5.5).

5.1. **Tracking the Eurodollar Index by bond picking.** We backtest the performance of the simulation bond picking model over the 30-month period January 1999 - July 2001. At each time step (i.e. monthly) we generate 250 interest rate and credit spread scenarios for all rating classes $k = 1, 2, \ldots, K-1$, and 2000 rating migration and default scenarios for each bond. Total loss is assumed in the event of default which is consistent with the pricing models when calibrated to credit spread term structures. We therefore obtain half million scenarios of return scenarios for the index portfolio. Given the simulation distribution, we apply a stratified sampling scheme to select 1500 scenarios as an input to the optimisation problem. The optimisation models are implemented using the mathematical modelling language MPL Optimax and solved using FortMP (Ellison et al. 1999).

Figure 7 shows the growth of 100 USD invested in the index on January 31, 1999 and in the optimal portfolio obtained with the bond picking tracking model as well as the annualized tracking errors for every month observed over this period.

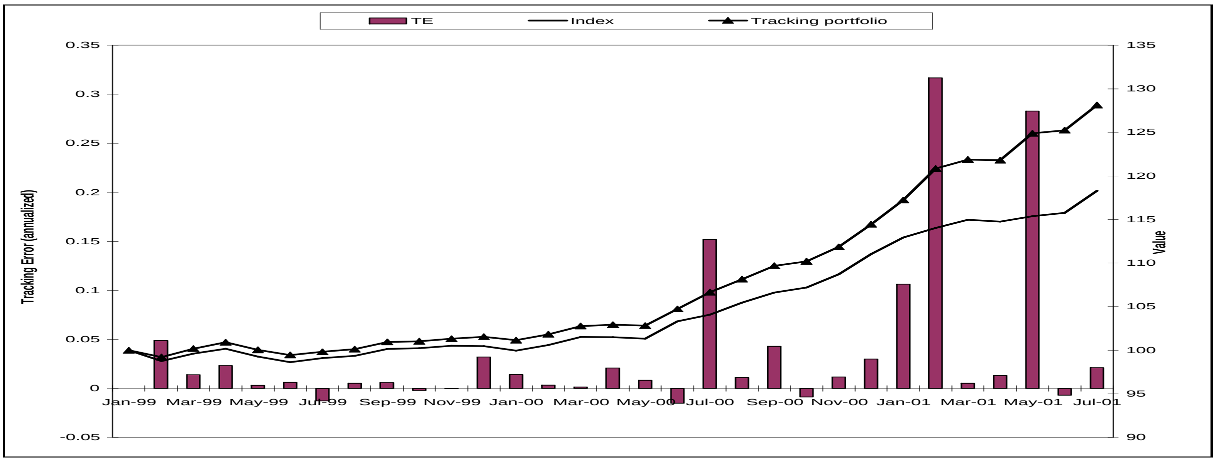

FIGURE 7. Performance of the bond picking tracking model vs the Merril Lynch Euro Dollar index.

Given the realized portfolio and index returns during this 30-month period we calculate the historical Sharpe ratio for the tracking portfolio with respect to the index returns as 0.497; this is a encouraging statistic. The tracking errors are small on average and, as expected, the model underperforms by small amounts only in five months.

Figure 8 shows the quality rating structure of the tacking portfolio compared to the index holding. This figure is a snapshot from January 31, 1999 and shows that we have an overweight in Aa rating in the optimal tacking portfolio, however A and Baa holdings are very similar. Figure 9 compares the portfolio holdings in each sector to the corresponding

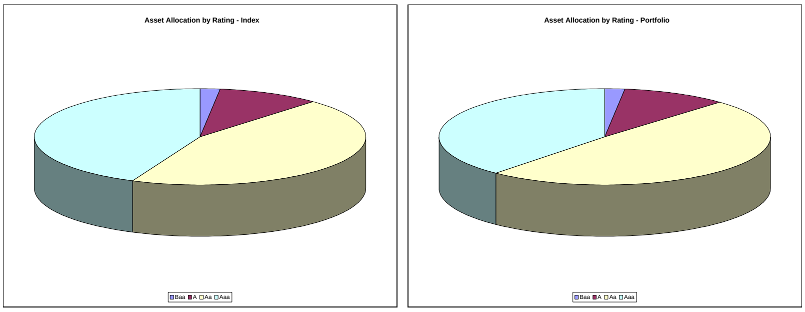

FIGURE 8. Structure of the optimal tracking portfolio on January 31, 1999 with respect to the holdings in each rating class.

index holdings. We observe that the structure of the portfolio is quite different form the

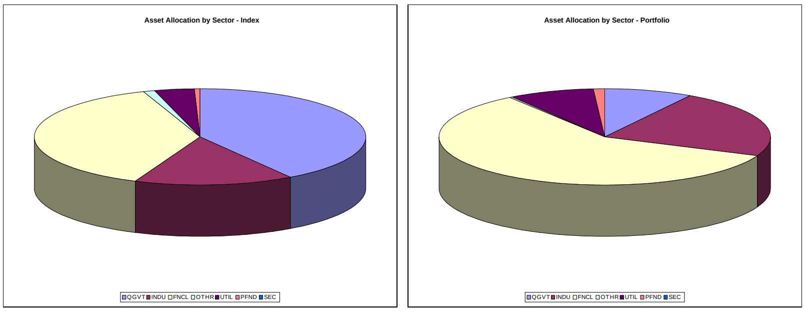

FIGURE 9. Structure of the optimal tracking portfolio on January 31, 1999 with respect to the holdings in each sector.

index structure with respect to sector holdings. In general, we observe an overweight in Aa financial securities.

5.2. **Sensitivity to alternative risk factors.** Modelling credit risk is complex due to the diverse risk factors driving security prices and portfolio performances. There is an interest in how these risk factors affect portfolio performance. Bucay and Rosen (1999) study the impact of alternative transition matrices on the portfolio composition. However they do not show any results on ex post model performance and the secenario generation did not include interest rate and spread stochasticity. In this section, we study the impact of recovery rates, rating and default scenarios, and economic scenarios on portfolio performance.

5.2.1. *Alternative recovery assumptions.* In section 5.1 we generate 250 economic and 2000 rating scenarios, assuming zero recovery rates. In the event of default the price of the defaulted bond drops to zero. In this section we sample the same interest rate, credit spread, rating and default scenarios, but in the event of default different assumptions about recovery rates were made as zero recovery is unrealistic in practice.

Figure 10 shows the backtesting performance when we assume the average historical price (see Table 3) after default by original bond rating as reported in Altmann (1999). Similarly, Figure 11 shows the backtesting results when a random recovery rate is

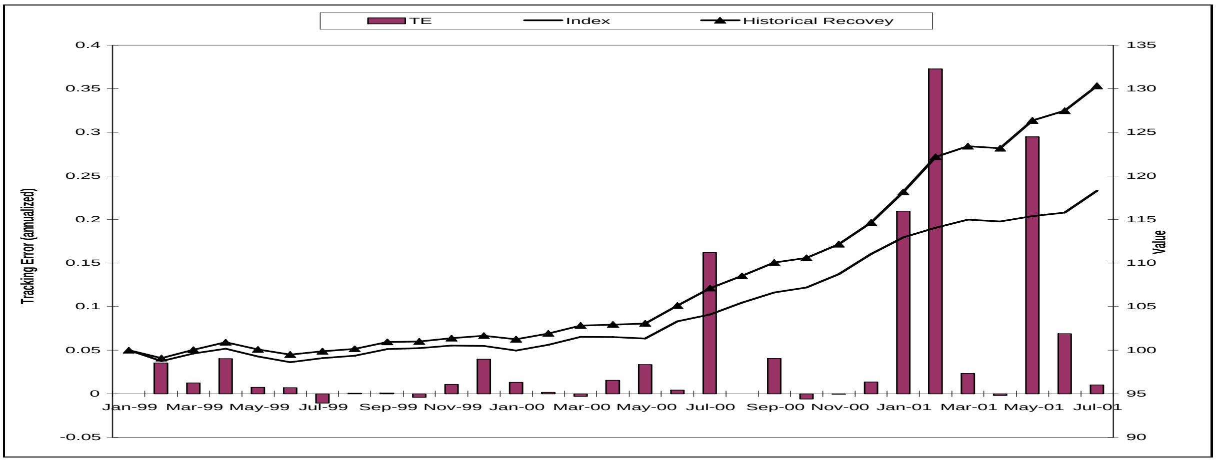

FIGURE 10. Performance of the tracking portfolios built with different recovery models vs the index. At default, a historical recovery rate is assumed in the scenario generation. Bars denote tracking errors.

Original rating	Average price	Original rating	Average price
Aaa	68.34	Ba	39.25
Aa	59.09	B	37.89
A	60.63	Caa-C	38.23
Baa	49.05		

TABLE 3. Average price after default for different (initial) bond rating, 1971-1998.

assumed, we sampled uniformly from the interval $[0, 1]$ for simplicity.[4] Both figures show a similar performance with respect to the number, the timeperiods and the magnitude of underperformance in comparison to the the zero recovery model of section 5.1. However we observe some differences which are indicated by the Sharpe ratios of 0.530 for the historical recovery rates as opposed to 0.538 for the random recovery rates.

Also the performance of the models is very similar, it seems intuitive that a model with positive recovery chooses a more aggressive portfolio due to the smaller losses in the default event. This is reflected in the overall value of the portfolio at the end of the period. The higher Sharpe ratios indicated that taking the risk works in our favour. However as defaults or downgrade would lead to high losses, credit risk portfolio management is also concerned about diversification with respect to the number of bonds and exposure

[4]In practice, a beta distribution is used frequently.

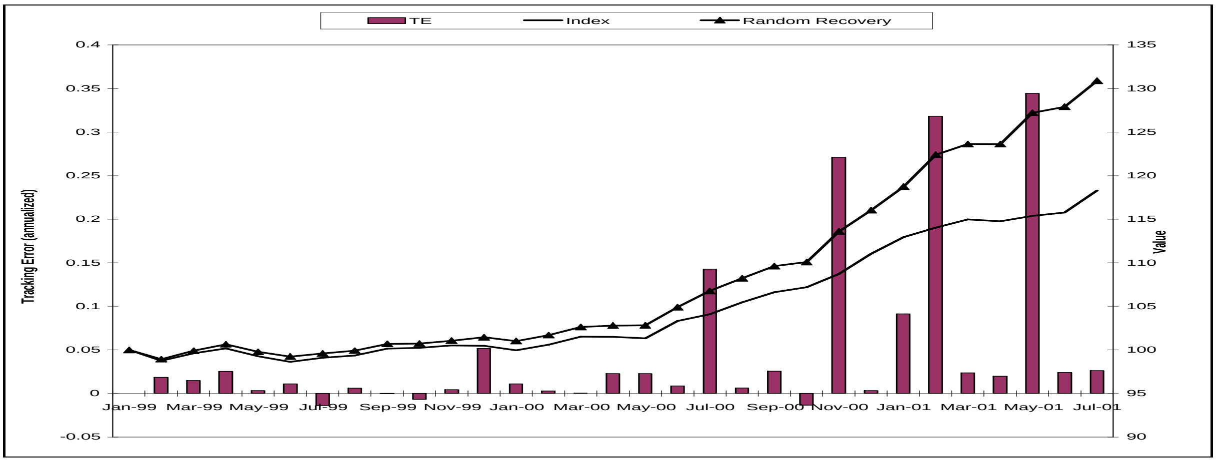

FIGURE 11. Performance of the tracking model vs the index and corresponding tracking errors. At default, a random recovery rate is assumed in the underlying scenario generation.

to a single issuer, in the portfolios (name concentration). In this respect, we analyse the different strategies and find no significant differences, the portfolios contain approximately 40 to 100 bonds in the portfolio throughout the backtesting period.

5.2.2. *Impact of economic scenarios.* We now turn to the effects of the sampling of economic scenarios. In this example we generate the 2000 rating and default scenarios under a single economic scenarios. Choosing one economic scenario can be interpreted as assuming zero (or a very low) volatility in interest rates and credit spreads. Assuming zero volatility is nonsensical for fixed income portfolio management problems, however, as pointed out in the introduction, the separation of market and credit risk is still practiced and many models focus on the rating and default forecasts, only. Figure 12 shows the growth of our initial investment in a portfolio that is chosen ignoring uncertainty in interest rates and credit spreads. The figure shows that uncertainty in interest rates and spreads has a significant impact on the performance of the tracking portfolio, and the Sharpe ratio drops to -0.23. This result provides strong support to the proponents of enterprise wide risk management and the integration of market and credit risk. Furthermore it complements the results in Kiesel et al. 2001 that spread risk is crucially important, not only for Value-at-Risk analysis but also for risk and portfolio management, when dealing with high quality portfolios.

5.2.3. *Impact of rating and default scenarios.* Finally we switch off rating and default scenarios in our generation and focus entirely on economic scenarios. We can expect, that our optimization chooses a high risky strategy investing heavily in low quality bonds and those bonds that appear cheap (with respect to their OAS). This expectation stems from the fact that low quality bonds offer on average higher yield (or spread) and hence by not taking the potential downside risk into consideration, this strategy can be seen as introducing arbitrage opportunities to the models. Figure 13 shows the backtesting performance if we switch off the rating and default simulations, thus assuming that all

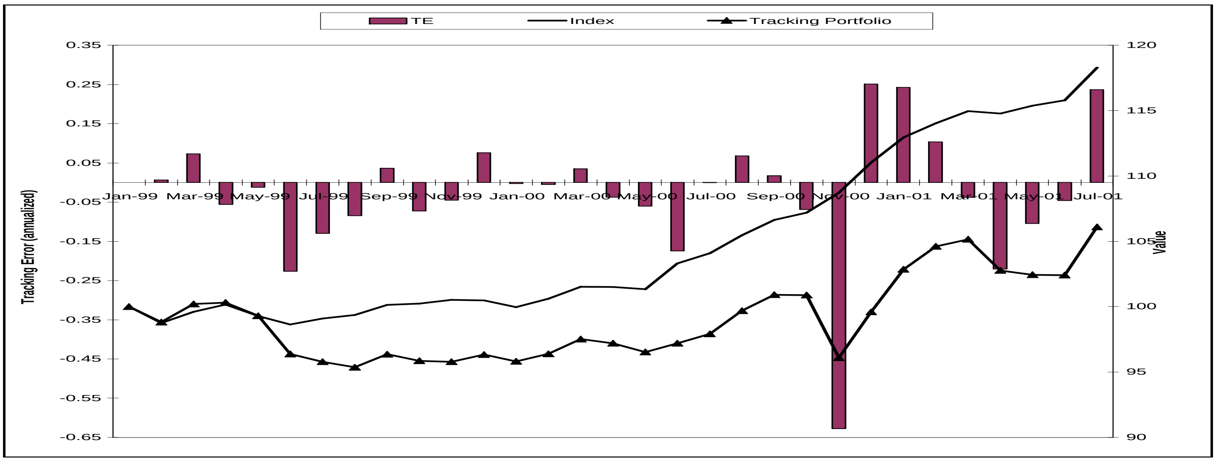

FIGURE 12. Performance of the tracking model vs the index and corresponding tracking errors when the scenario generation does not include uncertainty in interest rates and credit spreads.

bonds stay in their current rating class. However there is still uncertainty reflected in the short interest rate and credit spread scenarios. We sample exactly the same 250 economic scenarios, price all bonds according to their current rating, and calculate the holding period returns. The figure shows a very good performance, both with respect to the final

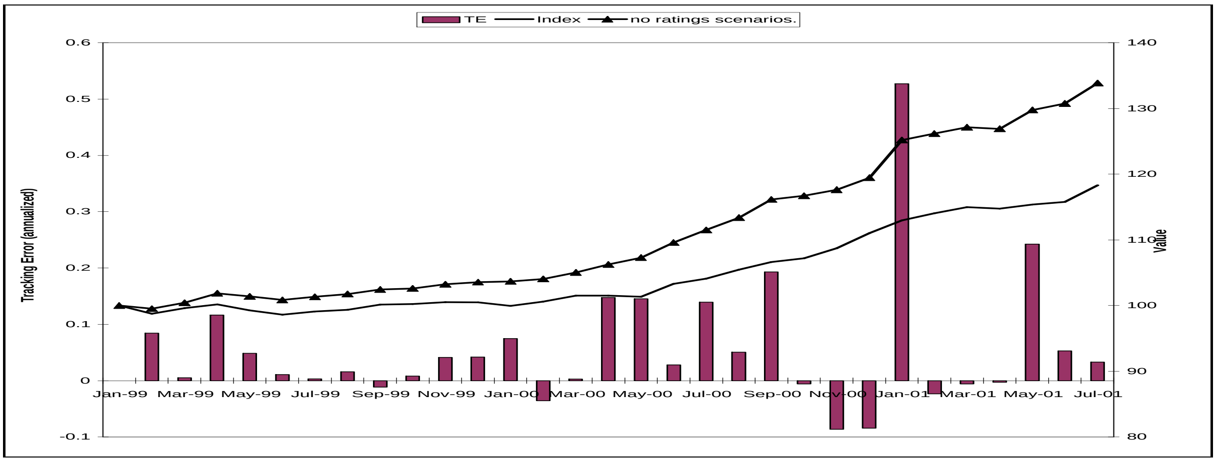

FIGURE 13. Performance of the tracking model vs the index and corresponding tracking errors. The scenario generation does not include rating and default simulations.

value and tracking errors with a Sharpe ratio of 0.560. This result indicates that our model is doing well in terms of picking the right bonds in a stochastic interest rate spread environment, when there is no downside risk. However if we take a close look at the portfolio holdings, we notice that there is a high concentration in individual exposures. All portfolios throughout the backtesting period hold only between 4 and 10 bonds, only. Hence the portfolios are badly diversified and heavily hit if a rating downgrade or a more catastrophic default happens (also if they are very low probability events).

On the other hand, the model might be useful in practice when the actual investment decision is very often taken in jointly with credit analysts who focus on different sectors and companies (balance sheet, economic perspective etc.). In this situation, the model, due to its good bond picking performance, can be used as a tool suggesting a number of bonds from the index or from a subsector of the index. In general, however, the model introduces bias in the decisions as the potential downside risk is ignored, leading to bad diverisification with respect to the exposures to a single bond.

5.3. **Tracking the Eurodollar index by asset allocation.** We focus now on the tracking problem from an asset allocation viewpoint. Instead of choosing individual securities, we seek the optimal asset allocation among asset classes. This asset class can be a credit rating, an industrial sector, a maturity range or a combination of these. Scenarios for asset class holding period returns r_c^l are generated as outlined in section 4.2. We calculate the holding period returns of the asset classes based on the average return of all securities in a given rating class c for every scenario l, i.e.

$$r_c^l = \sum_{i \in U^c} \beta_i r_i^l, \qquad \text{for all } c = 1, \ldots, C, \tag{12}$$

where β_i is the weight if bond i in the asset class. In the first example, asset classes are the different credit rating categories, whereby the set U^c is given as $U^c = \{Aaa, Aa, A, Baa\}$. In the ex post analysis we calculate the realized holding period returns for our asset classes based on the corresponding returns of the underlying bonds. Figure 14 shows the tracking errors of the asset allocation model. In addition we show the tracking errors when an asset class is a combination of industrial ratings and maturity ranges, where we consider 1-3, 3-5, 5-7 and 7-10 years as the ranges. The portfolio tracks the index very closely, however we do not seem to accumulate any extra value. The reason is that the optimal portfolios are almost identical to the index structure, and hence the portfolio growth is almost identical to the index growth. This exercise was repeated for different asset class definitions such

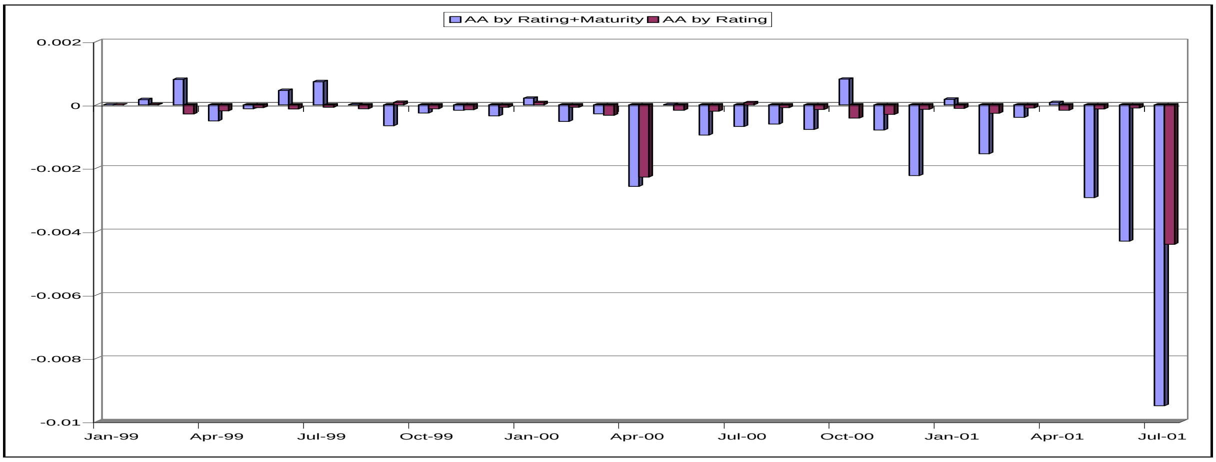

FIGURE 14. Performance of the asset allocation model compared to the bond picking model of the previous section.

asset industrial sectors, ratings and sectors, rating and maturity, and sector and maturity combinations. In all cases, the ex post analysis does not give significantly different results from those shown in Figure 14, that is, the tracking performance is close but no extra value is obtained. The resulting asset allocation is very similar to the holdings in the index.

5.4. **Bond picking under asset or risk class constraints.**

5.4.1. *Imposing bounds based on the index composition.* We now impose constraints on the asset class holdings to the bond picking model such that the portfolio mimics the index with respect to the holding in each asset class in every month throughout the backtesting period. First we match the holdings in each rating category and in a second exercise we ensure that the tracking portfolios mimic the index maturity structure. Maturities are divided in 1-3, 3-5, 5-7 and 7-10 year ranges. The random recovery model was chosen in the event of default in any of the simulations.

Figures 15 and 16 show the ex post performance of portfolios constructed with this model. Constraining the overall holding in certain rating categories leads a very similar performance to the unconstraint model, indicating that the overall portfolio structure is similar to the index and the deviations do not impact the performance significantly, resulting in a historical Sharpe ratio of 0.541. Constraining the portfolio composition with respect to index rating and maturities impacts the performance more significantly. The corresponding tracking errors are smaller and less volatile resulting in a increased Sharpe ratio of 0.644. This experiment demonstrates that constraining the asset class holdings controls the portfolio structure and hence imposes control on the tracking performance, too. The ex post results show that volatility is decreased while the models still has some flexibility to pick certain assets and is able to cumulate extra value from this decision. In sum-

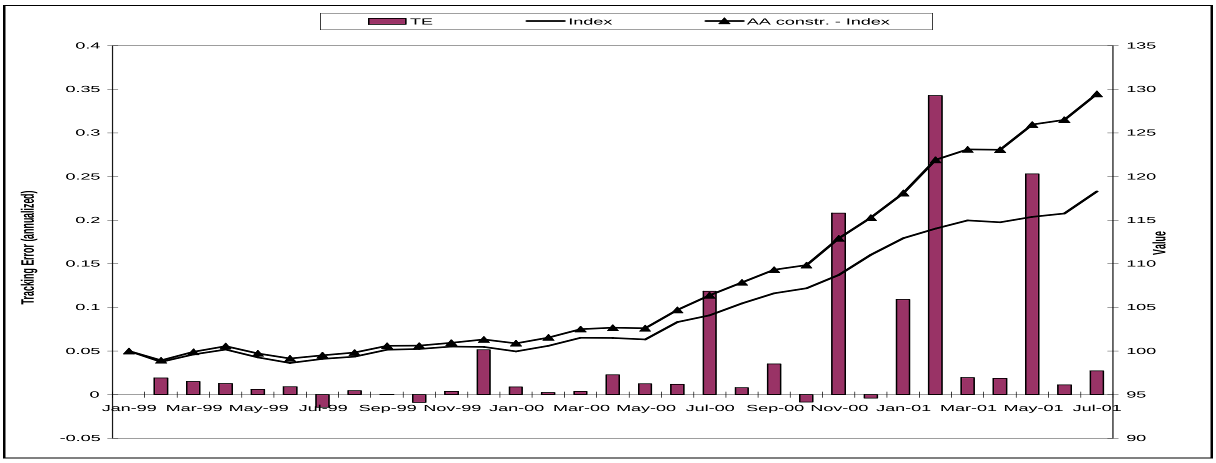

FIGURE 15. Performance of the integrated model when the bond picking model is constrained to index rating category holdings.

mary imposing bounds on the exposure of the tracking portfolios to certain asset classes affects the optimal decision and, hence, the performance of the model. If we are able to choose the bounds with foresight, i.e. knowing the spread movements over the next tree

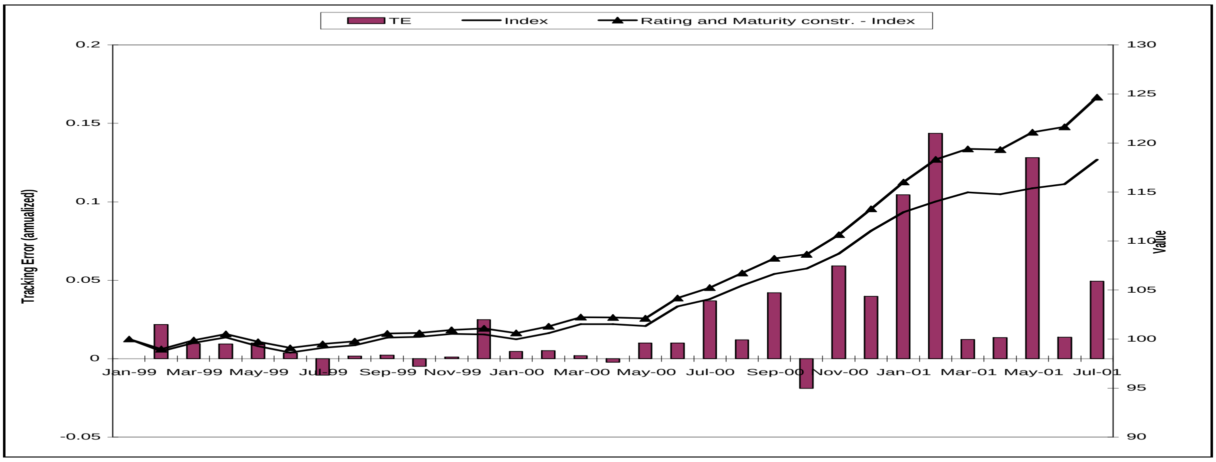

FIGURE 16. Performance of the integrated model when the bond picking model is constrained to index rating category and maturity holdings.

month for example, or to guess them correctly, this bounds may even improve the model performance ex post. In our experiments, choosing the bound according to the exposure of the index to the corresponding risk classes, we observe a good tracking performance in each case. However, with respect to the final portfolio value, no extra value was generated compared to the pure bond picking model.

5.4.2. *Imposing bounds derived from the optimal asset allocation decision.* We repeat the exercise of the previous section, but impose bounds according to the optimal solution of the asset allocation models of section 5.3. As expected the performance of the rating class constrained model is very similar, due to the similarity of the optimal strategy to the index structure (and hence the figure is not shown). The Sharpe ratio increases slightly to 0.572.

The ex post performance of the bond picking model constraint to the optimal rating and maturity holdings[5] is shown in Figure 17. Again, the performance is similar to the index constraint model and the Sharpe ratio increases slightly to 0.677.

5.5. **Summary.** All models that incorporate disparate sources of risk perform well as indicated by their Sharpe ratios and the reported growth and tracking error results. Table 4 summarizes the Sharpe ratios. The most important source of risk appears to be the interest rate and spread volatility which is consitent with the findings in Kiesel et al. 2001. However not taking into consideration the actual downgrading risk and default risk has a huge impact on the porfolio composition and hence riskiness in a credit risky environment. Choosing only a few number of securities for investment, and hence, having huge bets on single names is clearly a very risky strategy as if for example default happens, the actual portfolio is hit very hard and leads to large investment losses. The integrated models lead to less volatile tracking errors and are usefull as they allow us to incorporate expert knowledge about market movements in certain sectors or rating categories without too

[5]With "optimal" we mean the optimal solution of the corresponding bond picking model.

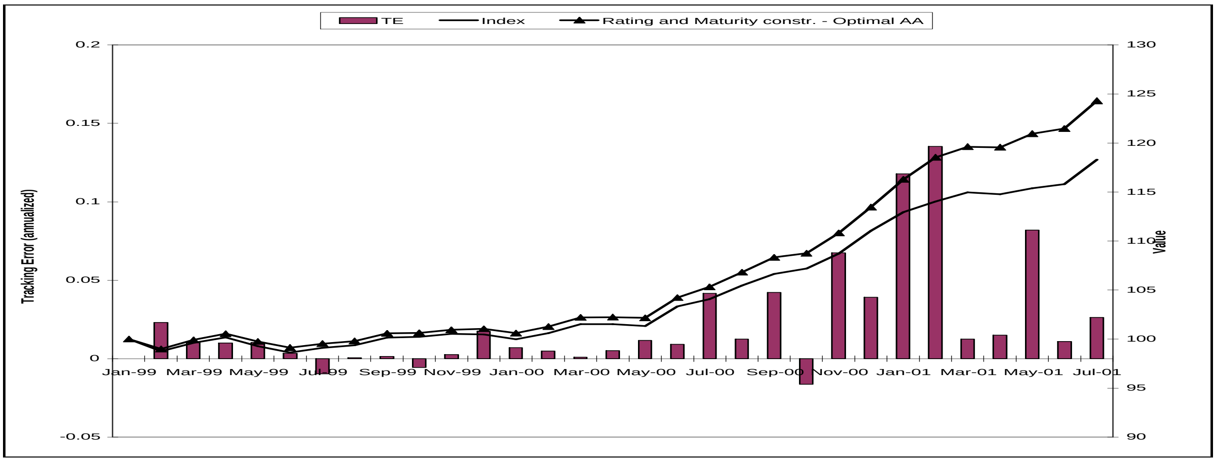

FIGURE 17. Performance of the integrated model when the bond picking model is constraint to rating category and maturity holdings obtained from the optimal solution of the corresponding asset allocation model.

Model	Sharpe ratio	Models with bounds	Sharpe ratio
zero recovery	0.497	by index rating	0.541
random recovery	0.538	by index rating and maturity	0.644
historical recovery	0.530	by optimal rating	0.572
no rating migration	0.560	by optimal rating and maturity	0.677
no economic scenarios	-0.230		

TABLE 4. Overview of Sharpe Ratios

high exposures to single names. In addition, improved forecasting methods for asset class returns or better asset allocation approaches may lead to optimal solutions that can be imposed to the model and generate superior performance.

5.6. **Benchmarking against Random Portfolios.** Using the machinery described above produces good tracking results. However it is interesting to compare the results to more simple approaches. In the spirit of earlier studies we compare our portfolios to randomly selected portfolios. We select a portfolio randomly at the beginning of the backtesting period and assume to hold this portfolio until July 2001. All intermediate cash-flows are assumed to get reinvested at the riskless rate. The same assumption is applied in case a bond matures before the end of the test period. Figure 18 shows the results of the experiment for two different exercises. We plot the average monthly return and the standard deviation of these returns for the index, our tracking portfolios, and 100 randomly selected portfolios. In the first case (left) the portfolios contain between 7 and 15 bonds and in the second example (right), the random selection leads to portfolios between 23 and 44 securities. We can observe in all instances, that the portfolios obtained with the various optimisation models perform well compared to the randomly selected portfolios and to the to Merril Lynch Eurodollar index.

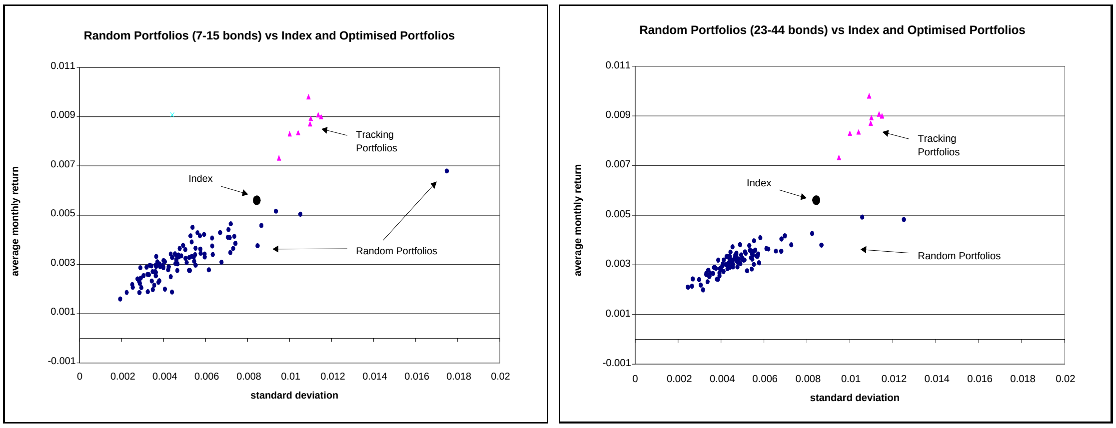

FIGURE 18. Random Portfolios

5.7. **Tracking Government Bond Indices and corporate holdings.** In this section we focus on the problem of tracking a government bond index when the available instruments are government bonds as well as corporate bonds. We focus in particular on the Merril Lynch US Treasury index. Figure **??** show the evolution of the US Treasury index during our backtesting horizon.

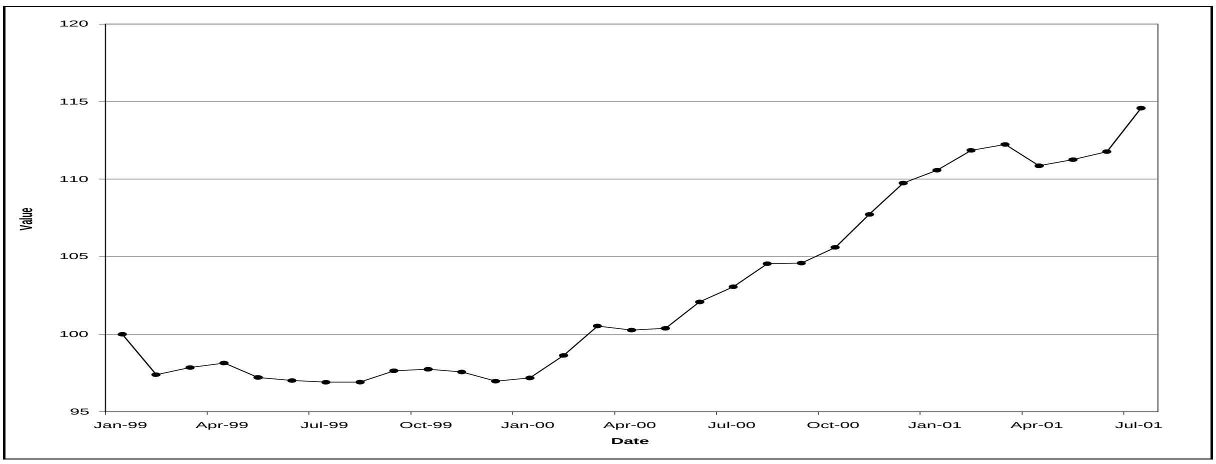

FIGURE 19. US Treasury index evolution

In a first step we try to track this index by using US Treasury securities of maturities of 1 to 30 years, only. From the database, we delete all bonds with optional payoffs such as callable or putable bonds. Scenarios are generated wihtin the same framework, the bonds are priced according to the term structure of the trading date (end of month) and scenarios are sampled accordingly. A bid-ask spread of 5bp is assumed to capture transaction costs. Figure 20 shows the performance of the Tracking experiment throughout the period. Tracking errors are very small and the historical Sharpe ratio is 0.04, however the model does not seem to pick up some extra value from the bond picking.

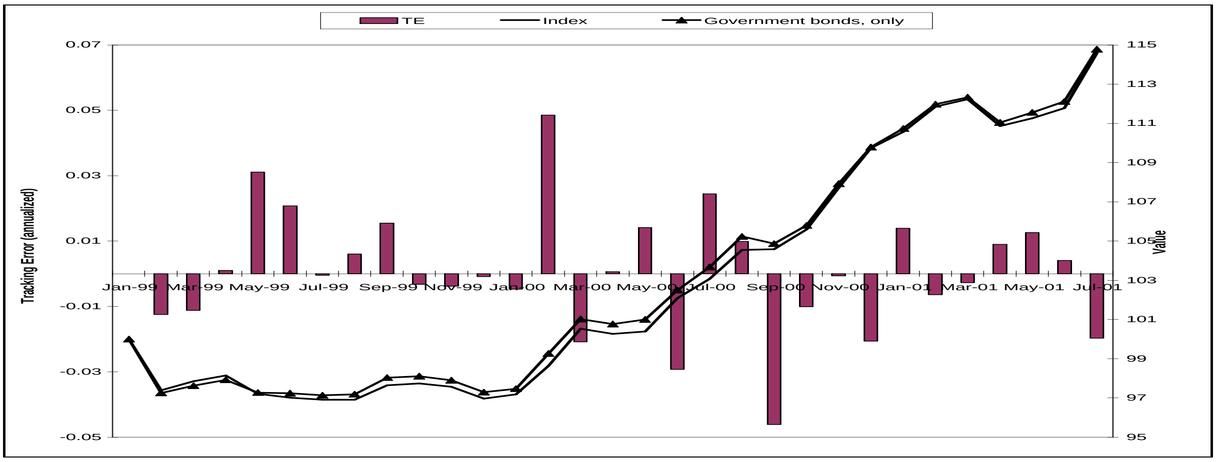

FIGURE 20. Tracking performance of the bond picking model when applied to track a government bond index with treasury securities, only.

As a next experiment, we expand the underlying universe of bonds to include all bonds from the Eurodollar index. We do not expect to increase the overall performance in terms of the final value lots, due to the increased downside risk of corporate bonds and the strict risk measure regarding an infinite penalty in case of underperformance. However the increased bond universe may offer new diversification opportunities which may lead to a different performance. Figure ?? shows the ex post performance when we assume random recovery in the underlying corporate bond scenario generation. We can see lower and

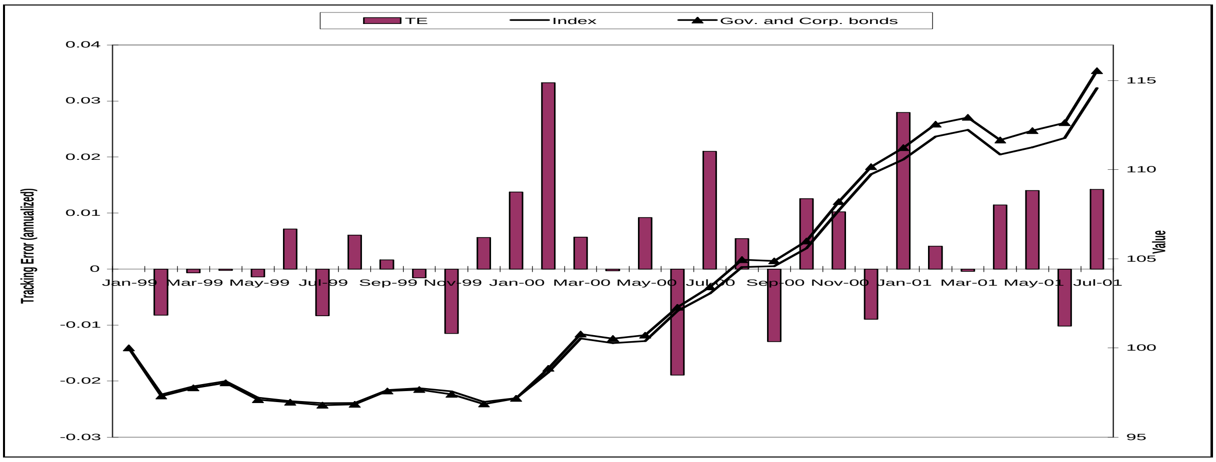

FIGURE 21. Tracking performance of the bond picking model when applied to track a government bond index with treasury securities and corporate bonds.

less volatile tracking errors and hence, including corporate bonds improves the tracking performance. The historical Sharpe ratio increases from 0.04 to 0.31 which is a result of the lower volatility and higher expected return. Taking a look at the portfolio structure reveals that the optimal portfolio composition is as expected. Only a small fraction (approximately 5%) is invested in a number of different corporate bonds (approximately 10). Hence the increased downside risk of corporate bonds is taken into consideration by only investing a small fraction of the value in the risky asset class and within this class, the model invests in a range of corporate bonds in order to minimize the exposure to a single security. In summary, the model works well and does not blindly invest in risky bonds due to the higher expected value these securities offer.

6. Conclusion and future develpments

We developed a framework for simultaneously simulating default-free and defaultable bonds consistent with observed term structures of interest rates and credit spreads. The simulations include disparate sources of risks such as interest rates, credit spreads, credit quality migrations, defaults and the corresponding recovery risk. We use these simulations as an input to a single period stochastic optimisation problem. The combined simulation and optimisation models are applied to track corporate bond indices.

Numerical results show that the models, when applied at a tactical, bond picking level perform very well. At a strategic level, when the main decision is the amount of wealth to invest among asset classes, the tracking performance is good, however no extra value is created. A single model for integrating both approaches is presented and numerical results are discussed. We notice that the main source of risk to capture is interest rate and spread risk, however the inclusion of credit events in the scenario sets is crucial to obtain well diversified portfolios without risky positions in only a few bonds.

The current simulations do not capture correlation between the migration and default events. Capturing this non-independence of extreme events is a current topic of research in both the portfolio simulation and the credit derivatives pricing context. This correlation is more important in the credit framework than in, for example, the equity context. However, the lack of quality historical data poses challenges on the modelling task. In this respect incorporating models that try to predict the credit events (e.g. form equity returns) into the reduced for framework may be promising and required, especially when focusing on high risky (low quality) risk management problems.

From an applications viewpoint, the framework of combined simulation and optimisation strategies can be extended to other securities subject to market and credit risk. As such, the incorporation of interest rate swaps to corporate bond portfolios in the spirit of the asset swap economics is a prime candidate. This would allow investors to limit their exposure to interest rate changes and take a pure view on the credit side of the investment. In an advanced development, portfolios of default swaps, for example, could be optimised and seen as a new investment class, assuming that standardized markets exist.

Finally the extension of the models to multiperiod optimisation begs for study, given the long maturities of corporate bonds and more empirical work is needed regarding different risk measures for portfolio management in a credit risk environment.

Appendix A. The operational models

In this Appendix we present a detailed formulation of the model. Variables and parameters are expressed in terms of face value instead of percentage value of portfolio wealth. This allows us to incorporate realistic features of the portfolio management problems such as portfolio rebalancing, transaction costs for buying and selling, cash infusion or withdrawal and further operational constraints on liquidity and diversification.

In this setup the initial portfolio value V_0 is given as

$$V_0 = c_0 + \sum_{i \in U} z_i^0 P_i^0, \tag{13}$$

where c_0 denotes the initial cash holding, z_i^0 denotes the initial face value holding in security i and P_i^0 the initial price of bond i. P_i^0 is the short notation for $P_i^{\kappa_0^i}(0, T_i)$ used throughout section 3. Similarly, the portfolio value at the end of the holding period under scenario l is given by

$$V^l = v\, r_0^l + \sum_{i \in U} z_i P_i^0 r_i^l, \qquad \text{for all } l \in \Omega, \tag{14}$$

where r_0^l denotes the riskless rate of return under scenario l and v and z_i are decision variables, introduced next. We also have: v: amount invested in cash in the tracking portfolio
z_i: face value holding of security i in the tracking portfolio
x_i: face value purchased of security i
y_i: face value sold of security i

Given these variables and parameters, the face value holding in security i is given as

$$z_i = z_i^0 + x_i - y_i, \qquad \text{for all } i \in U, \tag{15}$$

and the return of the tracking portfolio under scenario l is given by

$$R_P^l = \frac{V^l}{V_0}, \qquad \text{for all } l \in \Omega. \tag{16}$$

The operational downside mean absolute deviation model can be written as

$$\text{(17)} \quad \text{Maximize}_{h,b,s,c} \quad \sum_{l\in\Omega} p^l R_P^l$$

$$s.t.$$

$$\text{(18)} \quad z_i = z_i^0 + x_i - y_i, \qquad \text{for all } i \in U,$$

$$\text{(19)} \quad c_0 + \sum_{i\in U} y_i P_i^0 (1-\zeta_i) = \sum_{i\in U} x_i P_i^0 (1+\xi_i) + v,$$

$$\text{(20)} \quad R_P^l \geq I^l - \epsilon, \qquad \text{for all } l \in \Omega,$$

$$\text{(21)} \quad \sum_{i\in U^c} z_i P_i^0 \geq (h_c^t - \delta_c^-) \left(\sum_{i\in U} z_i P_i^0 + v \right), \qquad \text{for all } c = 1,\ldots,C,$$

$$\text{(22)} \quad \sum_{i\in U^c} z_i P_i^0 \leq (h_c^t + \delta_c^+) \left(\sum_{i\in U} z_i P_i^0 + v \right), \qquad \text{for all } c = 1,\ldots,C,$$

$$\text{(23)} \quad z_i, x_i, y_i, v \geq 0, \qquad \text{for all } i \in U.$$

Parameters ξ_i and ζ_i denote the transaction costs for buying and selling of asset i, respectively. Constraint (18) are the inventory balance constraints for each security i, constraint (19) is the cashflow balance constraint and constraints (20) are the penalty functions for all scenarios. All variables are restricted to be non-negative so that short positions are prohibited (23). Constraints (21) and (22) ensure that the holding in each credit risk or asset class c is within a range $[h_c^t - \delta_c^-, h_c^t + \delta_c^+]$ of the total portfolio value. If, for example $U^c := U^{\text{rating}}$, $\delta_c^- := \delta_c^+ := 0$ and the target holding h_c^t is chosen to be equivalent to the holding of the index in the corresponding asset class, we limit the exposure of our portfolio to hold exactly the same fraction of investment in the different rating categories *Aaa* to *Baa* as the index does. Of course, similar constraints can be added to limit the holdings of each security in the portfolio, however from a practical credit risk portfolio management point of view, constraints (21) and (22) may prove particularly useful.

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