

8. Discrete location-allocation

8.2 Single stage, single commodity

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SPLP

The simple plant location problem

Given parameters:

c_{kj} = cost of supplying customer j
from a plant located at site $k \quad \forall k, j$

f_k = cost of locating a plant at site $k \quad \forall k$

Decision variables:

y_{kj} = fraction of customer j 's demand
supplied from a plant located at site $k \quad \forall k, j$

$z_k = 1$ if a plant is located at site k , else 0 $\forall k$

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LP relaxation

$$\begin{array}{ll} \min. & \sum_k \sum_j c_{kj} y_{kj} + \sum_k f_k z_k \\ \text{s.t.} & \sum_k y_{kj} = 1 \quad \forall j \\ & z_k - y_{kj} \geq 0 \quad \forall k, j \\ & z_k \geq 0 \quad \forall k, \quad y_{kj} \geq 0 \quad \forall k, j \end{array}$$

Dual:

$$\begin{array}{ll} \max. & \sum_j v_j \\ \text{s.t.} & \sum_j w_{kj} \leq f_k \quad \forall k \\ & v_j - w_{kj} \leq c_{kj} \quad \forall k, j \\ & w_{kj} \geq 0 \quad \forall k, j \end{array}$$

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DUALOC approach

Find a good dual solution by a dual ascent procedure
(Improve it by a dual adjustment procedure)

Construct an SPLP solution by the complementary
slackness conditions

$$(f_k - \sum_j w_{kj}) z_k = 0 \quad (c_{kj} - v_j + w_{kj}) y_{kj} = 0 \quad \forall k, j$$

Often, the two objective function values are equal,
implying optimality

Exercises

1. Why does equality imply optimality?
2. Find the missing complementary slackness condition

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Dual ascent procedure

Set $v_j = \min_k \{c_{kj}\} \forall j$

Increase each v_j to the next higher c_{kj} (or less)

such that $\sum_j \max\{0, v_j - c_{kj}\} \leq f_k \forall k$

until no further increases are possible

The resulting v_j 's are dual feasible,

along with $w_{kj} = \max\{0, v_j - c_{kj}\} \forall k, j$

Example

f_k	c_{kj}						
7	3	6	8	9	2	4	8
8	4	9	4	2	8	6	4
9	8	4	3	6	9	8	5
7	2	7	7	4	6	6	4
6	5	5	6	8	3	2	3
v_j	5	6	6	7	5	4	4

SPLP solution

Among rows with $\sum_j \max\{0, v_j - c_{kj}\} = f_k$

select a subset K such that for each column

$v_j \geq c_{kj}$ for some $k \in K$

(often K comprises all those rows)

Set $z_k = 1$ for $k \in K$

For each column set $y_{kj} = 1$

for the minimal c_{kj} with $k \in K$

(all other z_k 's and y_{kj} 's are set to 0)

$$z_2 = 1, y_{21} = y_{23} = y_{24} = 1$$

$$z_5 = 1, y_{52} = y_{55} = y_{56} = y_{57} = 1$$

$$\sum_j v_j = 37 = \sum_k \sum_j c_{kj} y_{kj} + \sum_k f_k z_k \Rightarrow \text{optimal!}$$

Failure

The complementary slackness condition

$(z_k - y_{kj})w_{kj} = 0$ fails only when

$z_k = 1, y_{kj} = 0, v_j > c_{kj}$ which happens only when

$v_j > c_{kj}$ for several $k \in K$

Exercise

Solve problem 8.11c page 217