

7. Continuous location-allocation

7.1. One-dimensional problem

7.2. Euclidean distance

7.3. Rectangular distance

7.4. Alternate location-allocation heuristics

Henrik Juel

OR, IMM, DTU

7. Continuous location-allocation – p. 1/9

Continuous location-allocation

Locate m facilities and determine which facility serves each of the n customers

J_i = set of customers served by facility $i \quad \forall i$

$$\min \sum_i \sum_{j \in J_i} w_j \ell(\mathbf{x}(J_i) - \mathbf{a}_j)$$

s.t. $(J_1, \dots, J_m)$ is a partition of $\{1, 2, \dots, n\}$

where $\mathbf{x}(J_i)$ is the optimal location for the single facility problem

$$\min \sum_{j \in J_i} w_j \ell(\mathbf{x} - \mathbf{a}_j)$$

7. Continuous location-allocation – p. 2/9

7.1. One-dimensional problem

Example 7.1 (with doubled weights) $m = 3$

$$\begin{array}{c|cccccc} a_j & 1. & 2. & 3. & 5. & 7. & 8. & 10. \\ w_j & 4 & 2 & 5 & 3 & 5 & 8 & 6 \end{array}$$

$f_1(s)$ = min. cost of serving customers $1, \dots, s$
with 1 facility

$$\begin{array}{c|ccccc} s & 1 & 2 & 3 & 4 & 5 \\ f_1(s) & 0 & 2 & 9 & 16 & 36 \end{array}$$

7. Continuous location-allocation – p. 3/9

Example 7.1 continued

$f_2(s)$ = min. cost of serving customers $1, \dots, s$
with 2 facilities

$$f_2(s) = \min_t \{f_1(t) + \text{min. cost of serv. } t+1, \dots, s \text{ w. 1 fac.}\}$$

s	2	3	4	5	6
$t = 1$	0	2	8	26	44
$t = 2$		2	$2 + 6$	$2 + 20$	$2 + 34$
$t = 3$			9	$9 + 6$	$9 + 14$
$t = 4$				16	$16 + 5$
$t = 5$					36
$f_2(s)$	0	2	8	15	21

7. Continuous location-allocation – p. 4/9

Example 7.1 continued

$$f_3(7) = \min\{0 + 51, 2 + 26, 8 + 17, 15 + 12, 21 + 0\} = 21$$

Optimal partition for $m = 3$:

$$1234|56|7 \quad x_i^* : 3., 8., 10.$$

m		1	2	3	4	5	6	7
$f_m(7)$		86	33	21	13	7	2	0

Exercises

1. If m is a decision variable, how would you find m^*
2. Do problem 7.1 page 164

7.2 Euclidean distance, $m = 2$

Customers in J_1^* and J_2^* are separated by a line

Exercise: Why?

Consider all such allocations and pick the best one

$$(n(n-1)/2 \ll 2^{n-1} - 1)$$

7.3. Rectangular distance

Each x_i^* is some intersection point in the rectangular hull

Figure 7.8

Exercise: Why?

Denote these points by $\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_k, \dots$ and define decision variables:

$$z_k = 1 \text{ if a facility is located at } \mathbf{p}_k, \text{ else } 0 \quad \forall k$$

$$y_{kj} = \text{fraction of } w_j \text{ served by facility at } \mathbf{p}_k \quad \forall k, j$$

Model

$$\begin{aligned} \min. \quad & \sum_k \sum_j w_j \ell_1(\mathbf{p}_k - \mathbf{a}_j) y_{kj} \\ \text{s.t.} \quad & \sum_k y_{kj} = 1 \quad \forall j \\ & y_{kj} \leq z_k \quad \forall k, j \\ & \sum_k z_k = m \\ & z_k \in \{0, 1\} \quad \forall k \\ & y_{kj} \geq 0 \quad \forall k, j \end{aligned}$$

The LP relaxation of this version of the m -median problem usually yields integer z_k 's, making branch and bound unnecessary

Exercise: Why not replace $y_{kj} \leq z_k \quad \forall k, j$ by

$$\sum_j y_{kj} \leq n z_k \quad \forall k ?$$

7.4. ALA heuristics

0. Choose an allocation
1. Given the allocation, find the best locations
2. Given the locations, find the best allocation;
3. If the allocation has changed, go to step 1,
else stop

Different starting points

Perturbations