GRASP – A speedy introduction

Thomas Stidsen

thst@man..dtu.dk

DTU-Management
Technical University of Denmark

GRASP

- GRASP is an abbreviation for

  Greedy

  Randomized

  Adaptive

  Search

  Procedure.
- It was “invented” by Feo and Resende in 1989.

Overview of GRASP

GRASP consists of 2 phases:

- Greedy Randomized Adaptive phase (generating a solution).
- Local Search (finding a local minimum).

Greedy Randomized Adaptive phase

The first phase consists of 3 parts:

1. Greedy algorithm.
2. Adaptive function.
3. Probabilistic selection.

Greedy algorithm

- A greedy algorithm always makes the choice that looks best at the moment. It makes a locally optimal choice in the hope that this choice will lead to a globally optimal solution.
- Greedy algorithms do **not** always yield optimal solutions (eg. 0-1-knapsack), but in some cases it does (eg. Minimum spanning tree).

Mathematical Model for Knapsack problem

objective: maximize

$$\sum_i c_i \cdot x_i$$

s.t.

$$\sum_i w_i \cdot x_i \leq W$$

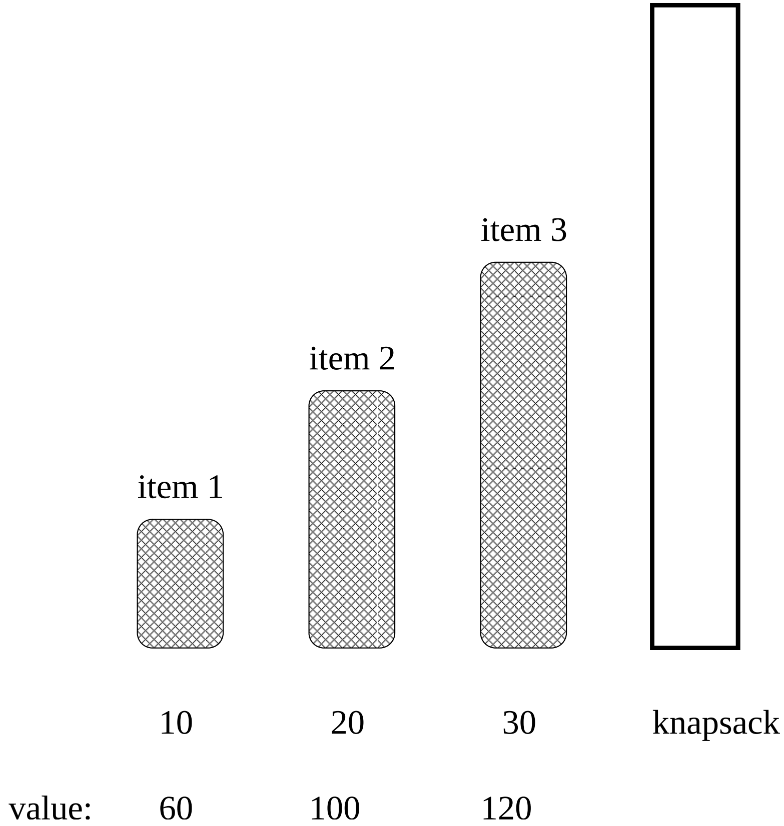
$$x_i \in \{0, 1\}$$

The profit weight ratio:$PF = \frac{c_i}{w_i}$

Greedy algorithm for 0-1 knapsack

procedure greedy_knapsack()
  calculate profit vs. weight ratio
  while the smallest item can still fit in **do**
    Add largest $PF = \frac{c_i}{w_i}$ element
    Update remaining weight
return solution

Example: 0-1-Knapsackproblem



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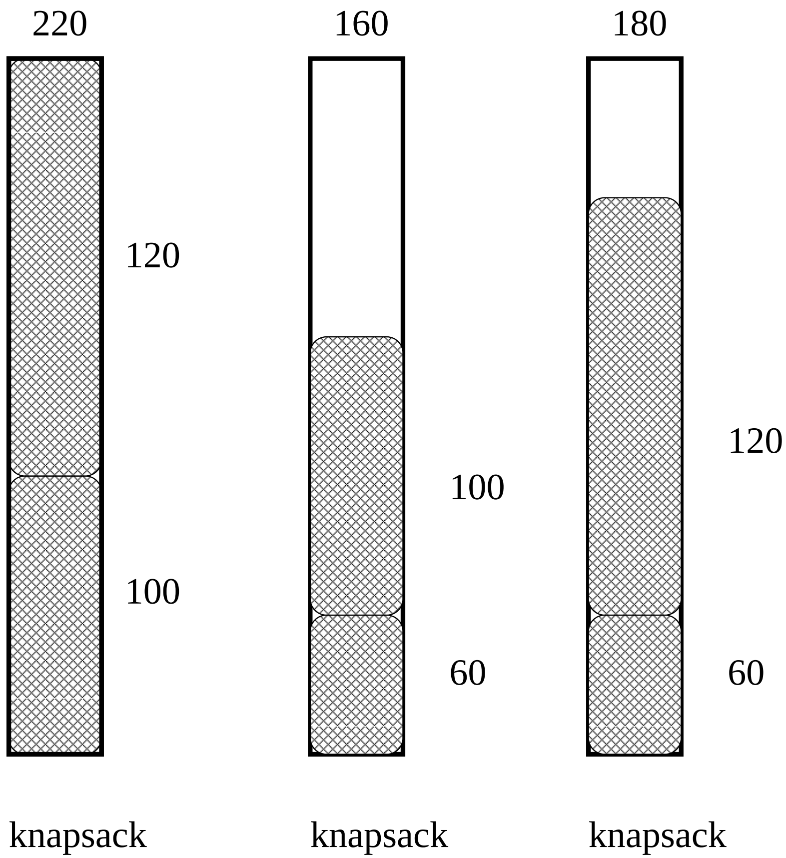

Greedy algorithm for the CLP ?

How can we make a greedy algorithm for the Christmas lunch problem ? Lets remember a couple of facts:

- We want to maximize the interest count.
- A person at a table can only achieve a positive interest from other persons at a table.
- If a table is empty and a person is put there, there is no gain ...

Greedy algorithm for the CLP

So, we will make the greedy algorithm the following way:

```
procedure greedy_clp()
  for each table do
  Place one un-seated person randomly chosen
  while still room at the table do
    Add the person with the largest interest gain
return solution
```

Probabilistic selection

- In the greedy algorithm the selection of the next element to add to the solution is (often !) deterministic.
- The probabilistic selection process selects candidates among which we choose the next element to be added to the presently partial solution. The list of candidates is formally denoted the *Restricted Candidate List* (RCL).

Restricted Candidate List

- Construction:
  - Select the β best elements ($\beta = 1$: purely greedy, $\beta = n$ completely random).
  - Selects the elements that are not more than α% away from the best element. ($\alpha = 0$: purely greedy, $\alpha = \infty$: purely random).
  - Select elements above an absolute threshold of γ.
- Selection:
  - Equally probability.
  - Weighted probability.

Adaptive function

- Modify the function which guides the greedy algorithm based on the element selected for the solution we are constructing.
- The construction is called **dynamic** in contrast to the **static** approach which assigns a score to elements only before starting the construction (example: TSP links).

GRASP main code

procedure grasp()
  CurrentBest = 0
  while $\langle$ more time $\rangle$
    Solution = ConstructSolution()
    NewSolution = LocalSearch(*Solution*)
    if $\langle$ *NewSolution* better than *CurrentBest* $\rangle$ **then**
      CurrentBest = *NewSolution*
return CurrentBest

Greedy Randomized solution construction

procedure ConstructSolution()
    Solution = $\emptyset$
    for $\langle$ solution construction not finished $\rangle$
        CandidateList = MakeCandidateList()
        s = SelectElement(*CandidateList*)
        Solution = *Solution* $\cup\{s\}$
        ChangeGreedyFunction();
return *Solution*

Additions to the GRASP scheme

- Use different adaptive functions during the execution of the algorithm.
- Settings of the Restricted Candidate List dynamic.

More information

- The website

  `http://www.graspheuristic.org`

  contains an annotated bibliography of GRASP. This is a good starting point for every form of work with the GRASP metaheuristic.
- Object-oriented framework developed by Andreatta, Carvalho and Ribero.

Pros and cons

- PRO: Simple structure (means often easy to implement),
- PRO: If you can construct a greedy algorithm, extension to GRASP is often easy.
- PRO: Can be used for **HUGE** optimization problems.
- CONS: dependent on a “natural” greedy algorithm, no escape from local optimum.
- CONS: may easily re-discover the same solution many times

(Adaptive) Large Neighbourhood Search

A recently developed method is: Large Neighbourhood Search (LNS) or Adaptive Large Neighbourhood Search (ALNS).

- LNS first suggested in 1998 by Shaw: "Using constraint programming and local search methods to solve vehicle routing problems".
- ALNS first suggested in 2006 by Ropke and Pisinger: "An adaptive large neighborhood search heuristic
- Both LNS and ALNS are similar to GRASP and have shown VERY interesting results.

LNS I

The basic idea in LNS is to use DESTROY and REPAIR methods to create new solutions (based on the current solution):

- A destroy method removes a part of the solution ...
- A repair method recreates the whole (feasible) solution.

LNS II

The basic idea in LNS is to use DESTROY and REPAIR methods to create new solutions (based on the current solution):

- Together destroy and repair methods form a neighbourhood, typically a very large neighbourhood.
- The destroy method is often stochastic, i.e. very simply deleting a certain part of the solution.
- The repair method is very often a kind of greedy constructive algorithm.

LNS III : Pseudo code

procedure LNS()
  Generate feasible solution x
  $x^b = x$
  repeat
    $x^t = r(d(x))$
    if accept(x^t,x) **then** $x = x^t$
    if $c(x^t) < c(x^b)$) **then** $x^b = x^t$
  until time limit
return x^b

ALNS I

Adaptive Large Neighbourhood Search is basically an extension of LNS: Instead of having one destroy and one repair method, ALNS allows many different destroy and repair methods:

- In each iteration a destroy and repair methods is chosen probabilistically.
- The chance for being chosen is adaptively chosen so that successful repair and destroy methods are rewarded for their success.

LNS II : Pseudo code

procedure ALNS()
  Generate feasible solution x
  $x^b = x$, $\rho^- = (1, ..., 1)$, $\rho^+ = (1, ..., 1)$
  repeat
    select destroy and repair methods $d \in \Omega^-$
    and $r \in \Omega^+$ using ρ^- and ρ^+
    $x^t = r(d(x))$
    if accept(x^t,x) **then** $x = x^t$
    if $c(x^t) < c(x^b)$) **then** $x^b = x^t$
    update ρ^- and ρ^+
  until time limit
return x^b

How to select destroy and repair

The probability for choosing repair/destroy operator number j is:

$$\phi^{-} = \frac{\rho^{-}}{\sum_{k=1}^{|\Omega^{-}|}}$$

How to adjust ρ^- and ρ^+ I

For every iteration exactly one destroy and one repair method. The used methods (and only them) are given a value $\Psi = MAX(\omega_1, \omega_2, \omega_3, \omega_4)$, where $\omega_1 \geq \omega_2 \geq \omega_3 \geq \omega_4$.

- ω_1: reward if new solution is new global best
- ω_2: reward if new solution is better than the current one
- ω_3: reward if new solution is accepted
- ω_4: "reward" if new solution is rejected

How to adjust ρ^- and ρ^+ II

Finally, the new ρ^- and ρ^+ for the selected destroy and repair methods, ρ_a^- and ρ_a^+:

$$\rho_a^- = \lambda \rho_a^- + (1 - \lambda) \rho_a^-$$

Where $\lambda \in [0, 1]$

LNS and ALNS comments

- Both methods have shown excellent results.
- Both methods can be considered to be generalizations of GRASP (complete destroy and probabilistic repair ...)
- A somewhat similar method "Variable Neighbourhood Search" is described in the book ... but I much prefer the handed out article by Roepke and Pisinger.