

Decomposition Exercise 7

A company needs a number of different jobs to be performed by its staff, in such a way that the utility is maximised. The company has reformulated the problem to a minimization problem by assigning a costs for using the different employees for the different jobs. The company hence wants an assignment of employees to the jobs in such that the costs are minimized. At the same time, certain educational costs needs to be paid for each combination of an employee and a job, because it costs to train the employees for the different jobs. The job training is performed internally in the company, but there is a limited training budget, which is not comparable with the benefits of the jobs.

The problem is now modelled in the following way: Two matrices, a cost matrix C and a training matrix T is given below. In both matrices, the rows correspond to persons and the columns to jobs. The training budget has an upper limit of 18.

$$C = \begin{pmatrix} 6 & 0 & 4 & 5 & 6 \\ 10 & 2 & 8 & 2 & 9 \\ 7 & 0 & 5 & 4 & 4 \\ 8 & 1 & 6 & 2 & 6 \\ 10 & 2 & 6 & 1 & 7 \end{pmatrix} \quad T = \begin{pmatrix} 4 & 5 & 6 & 5 & 3 \\ 5 & 8 & 4 & 3 & 6 \\ 4 & 10 & 8 & 3 & 5 \\ 2 & 9 & 2 & 10 & 8 \\ 2 & 5 & 3 & 1 & 4 \end{pmatrix}$$

Your assignment from the company:

1. Formulate the problem as a standard LP problem (on paper).
2. Solve the problem as a LP problem using either cplex or GAMS.
3. Solve the problem using Dantzig-Wolfe decomposition, where the budget constraint is the only constraint in the master problem and the sub-problems are assignment problems with cost matrices defined by the dual variables in the master problem. Is the solution integer and if not, what should be done ?