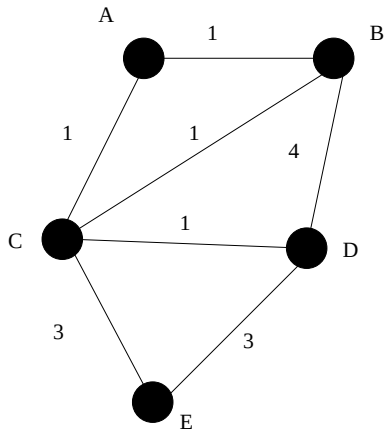
Decomposition Exercise 6

The Multi Commodity Flow Problem: This is a classic routing problem which has traditionally been solved using Column Generation. The specific version of the problem we consider can be stated as follows: **Given a network and a set of node-pairs between which flow should be possible, maximize the summed flow between the node-pairs**.

Below in the figure, a network with 5 nodes and 7 links is shown. Each link has a capacity associated. This means that the flow on that link (in both directions !) may at most sum to the capacity.

Given a node pair: B→E, a path between the two nodes can be given as a binary vector: $\{AB:0, AC:0, BC:1, BD:0, CD:0, CE:1, DE:0\}$. This path $p^{BE}_{\{ij\}}$ has the value 1 if that link is used and 0 otherwise. The flow on this path can be represented with a variable x and the *profit* (we are maximizing) of that variable is 1. Maximizing the total flow between several different node-pairs, is then simply summing the flow on the variables. But the link capacity limitations should also be obeyed, so there has to be a constraint for each link, which ensures that the capacity is not exceeded. Assume now that you want to calculate the greatest *sum* of flow which may be transported for the demands A→E and B→E ?



Demand1: A −> E

Demand2: B −> E

1. Give an initial guess on the maximal summed throughput.
2. Formulate the column generation master problem (i.e. **assume** that you have a set of all the paths from A→E and from B→E)
3. What is the initial columns ? (If we are not allowed to add actual paths ?)
4. What is the column generation subproblem ?
5. What is the stopping criteria ?
6. Perform the colum generation algorithm, solving the master problem using CPLEX and solving the subproblem by hand (you are **not** required to perform more than a few iterations).