Decomposition Exercise 4

Benders algorithm cook-book: When you want to solve a problem with Benders, do the following:

1. Write up the original problem.
2. Assume that the y variables are **constant**. Use this to reformulate the original problem:
   (a) Move the y constants to the right hand side of the constraints.
   (b) Move the y variables in the objective out (they are constant).
   (c) Change the constraints to sensible constraints ($\geq$ for minimization problems and $\leq$ for maximization problems).
3. Dualize the model (we need to find the u values, keeping the y constants in the in the objective. This is the Benders Primal Sub Problem (BPSP)
4. Assume you get an arbitrary y value (inside the definition domain).
5. Set in the fixed y into the BPSP and solve to get u and (and set z_{UP}).
6. Generate new constraint to the Benders Master Problem (BMP).
7. Add the generated constraint (to the other constraints) in BMP.
8. Solve BMP to get new y values (and set z_{LO}).
9. Terminate if $z_{UP} - z_{LO} \leq \epsilon$.
10. Go to 5

1. RKM Exercises 10.1
2. RKM Exercises 10.2.
3. Solve, using Bender's Decomposition, the problem in 1 x-variable (continuous) and 1 y-variable (integer):

   Min:

   $$z_0 = 5x - 3y$$

   S.T.:

   $$\begin{aligned} x + 2y &\geq 4 \\ 2x - y &\geq 0 \\ x - 3y &\geq -13 \\ x &\geq 0 \\ y \in \{0, 1, \ldots, 10\} \end{aligned}$$

   Use the initial trial solution $(\overline{y}, \overline{z_0}) = (0, 0)$. Solve the sub-problems by hand, including finding the optimal dual solutions. For each Master Problem, draw the problem in the (y, z_0)-space (with y on the horizontal axis).
4. Show that the optimal Master Problem objective value $\overline{z_0}$ always is a lower bound on the optimal objective value to the original problem, and that the sub-problem objective $\hat{z} + f^T \overline{y}$ always is an upper bound on the optimal objective value.