

Decomposition Exercise 2

1. Consider the system (P):

$$\begin{aligned} x_1 - 2x_2 &\geq -4 & (E1) \\ -2x_1 - x_2 &\geq -5 & (E2) \\ 5x_2 &\geq -1 & (E3) \end{aligned}$$

Check the feasibility of the polyhedron defined by (E1)-(E3). Project out first x_1 , then also x_2 from the system.

- (a) Is the system feasible and if so, what are the resulting multipliers ?
- (b) Replace (E3) with (E3'): $-x_2 \geq -1$. Is the system feasible and if so, what are the resulting multipliers ?
- (c) Replace (E3) with (E3''): $x_2 \geq 1000$. Is the system feasible and if so, what are the resulting multipliers ?
2. Consider Example 2.15. Give an educated guess (that is, with a good explanation) of the optimal dual solution to the LP
3. Solve, using projection, the LP problem:

$$(P1) : \min x_1 - 2x_2 \quad \text{S.t. } (E1) - (E3)$$

What is the dual solution ?

4. What happens when solving an infeasible LP by projection ? Test your answer by solving (P1) with (E3'') instead of (E3).
5. What happens when solving an unbounded LP by projection ? Test your answer by solving:

$$(P3) : \min x_1 - x_2 \quad \text{S.t. } (E1) - (E3')$$

6. Understand Lemma 2.16 and its proof. Prove Corollary 2.18 and 2.19.