

Exercise 9.4

(i) Use “procedure to lift cover inequalities” page 149. We use the following ordering of $N \setminus C = \{1, 2, 4\}$: $1_1, 2_2, 4_3$. First find α_{1_1} by solving the following knapsack problem and setting $\alpha_{1_1} = |C| - 1 - \zeta_1$.

$$\begin{aligned}\zeta_1 &= \max x_3 + x_5 + x_6 \\ 7x_3 + 5x_5 + 3x_6 &\leq 14 - 12 = 2 \\ x &\in \{0, 1\}^3\end{aligned}$$

The solution value is $\zeta_1 = 0$ so $\alpha_{1_1} = |C| - 1 - \zeta_1 = 3 - 1 - 0 = 2$.

Finding α_{2_2} is done similarly - solve the following knapsack problem and set $\alpha_{2_2} = |C| - 1 - \zeta_2$.

$$\begin{aligned}\zeta_2 &= \max 2x_1 + x_3 + x_5 + x_6 \\ 12x_1 + 7x_3 + 5x_5 + 3x_6 &\leq 14 - 9 = 5 \\ x &\in \{0, 1\}^4\end{aligned}$$

The solution value is $\zeta_2 = 1$ so $\alpha_{2_2} = |C| - 1 - \zeta_2 = 3 - 1 - 1 = 1$.

Finding α_{4_3} - solve the following knapsack problem and set $\alpha_{4_3} = |C| - 1 - \zeta_3$.

$$\begin{aligned}\zeta_3 &= \max 2x_1 + x_2 + x_3 + x_5 + x_6 \\ 12x_1 + 9x_2 + 7x_3 + 5x_5 + 3x_6 &\leq 14 - 5 = 9 \\ x &\in \{0, 1\}^5\end{aligned}$$

The solution value is $\zeta_3 = 2$ so $\alpha_{4_3} = |C| - 1 - \zeta_3 = 3 - 1 - 2 = 0$.

The obtained inequality is then:

$$2x_1 + x_2 + x_3 + 0x_4 + x_5 + x_6 \leq 2$$

Exercise 9.5

(i)

$$36 \leq x_1 + x_2 + x_3 + x_4 \leq 20y_1 + 10y_2 + 10y_3 + 8y_4$$

The last inequality-sign follows from the fact that $x_1 \leq 20y_1$, $x_2 \leq 10y_2$, $x_3 \leq 10y_3$ and $x_4 \leq 8y_4$. Comparing the term to the far right with the term to the far left, we can see that this is indeed a 0-1 knapsack constraint. (Note that “ $\leq$ ” is replaced with “ $\geq$ ”, so it is a weird 0-1 knapsack constraint.)

$$20y_1 + 10y_2 + 10y_3 + 8y_4 \geq 36$$

(ii) Cover constraints can be used to strengthen knapsack constraints, thus we should be looking for a cover constraint. (Note, again, that “ $\leq$ ” should be replaced with “ $\geq$ ” in the cover constraint, since this is the case in the knapsack constraint.)

Finding a cover constraint that cuts of the point $x^* = (20, 10, 0, 6)$ and $y^* = (1, 1, 0, 3/4)$ can be done by using the separation routine on page 150-151, except that this works for knapsack constraints with “ $\leq$ ” only. Modifying the separation routine to take care of this, the following problem is obtained:

$$\begin{aligned} \max & 0z_1 + 0z_2 + 1z_3 + \frac{1}{4}z_4 \\ & 20z_1 + 10z_2 + 10z_3 + 8z_4 < 36 \\ & z \in B^4 \end{aligned}$$

An optimal solution to this problem is $z_1 = z_2 = 0$ and $z_3 = z_4 = 1$. Thus the following inequality is obtained, and since the objective value is $1\frac{1}{4} > 1$, it cuts of y^* .

$$y_3 + y_4 \geq 1$$

Exercise 9.8

In order to prove that the two formulations are equivalent, prove that if a solution is in X then it is in X' and vice versa. Proving that a solution is in a set is proved by considering each constraint in turn.

Assume (x_1, y_2, y_3, y_4) is in X . Let $x_2 = 1000y_2$, $x_3 = 1500y_3$ and $x_4 = 2200y_4$. Since $x_1 \leq 1000y_2 + 1500y_3 + 2200y_4$, then $x_1 \leq x_2 + x_3 + x_4$. Also Let $y_1 = 1$, then $x_1 \leq 2000y_1$ since $x_1 \leq 2000$. The remaining inequalities follows since we have chosen $x_2 = 1000y_2$, $x_3 = 1500y_3$, $x_4 = 2200y_4$ and $y_1 = 1$.

Assume $(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4)$ is feasible for X' . Then $x_1 \leq 1000y_2 + 1500y_3 + 2200y_4$ since $x_1 \leq x_2 + x_3 + x_4$, $x_2 \leq 1000y_2$, $x_3 \leq 1500y_3$ and $x_4 \leq 2200y_4$. Also $x_1 \leq 2000$ since $x_1 \leq 2000y_1$ and $y_1 \leq 1$.

It is now possible to use the separation heuristic for flow cover inequalities for mixed 0-1 inequalities (page 153). The point to cut of is, $(x, y) = (2000, 1000, 1500, 2200, 1, 0, 1, 5/22)$. The separation problem to solve (a knapsack problem) is:

$$\begin{aligned} \min & 0z_1 - 0z_2 - 1z_3 - \frac{5}{22}z_4 \\ & 2000z_1 - 1000z_2 - 1500z_3 - 2200z_4 > 0 \\ & \forall i : z_i \in \{0, 1\} \end{aligned}$$

The solution is $z_1 = z_3 = 1$ and $z_2 = z_4 = 0$ thus $C_1 = \{1\}$, $C_2 = \{3\}$ and $L_2 = 4$ (for $\lambda > 0$). Also $N_2 \setminus (C_2 \cup L_2) = \{2\}$. This gives the following inequality:

$$\begin{aligned} & x_1 + (a_1 - \lambda)^+(1 - y_1) \leq 0 + a_3 + \lambda y_4 + x_2 \\ \Leftrightarrow & x_1 + (2000 - \lambda)^+(1 - 1) \leq 1500 + \lambda y_4 + 1000y_2 \\ \Leftrightarrow & x_1 \leq 1500 + \lambda y_4 + 1000y_2 \Leftrightarrow x_1 - \lambda y_4 - 1000y_2 \leq 1500 \end{aligned}$$

Inserting the point gives:

$$2000 - \lambda \frac{5}{22} - 1000 \cdot 0 \leq 1500 \Leftrightarrow 500 \leq \lambda \frac{5}{22} \Leftrightarrow 2200 \leq \lambda$$

So for $\lambda < 2200$ the inequality cuts of the point.