

Solutions for exercises in shortest path algorithms 2003, 13.3

Question 2

Figure 1 shows a transshipment problem including the tree-solution found in question 1 as initial solution. Numbers on nodes are the capacity/demand (capacity is negative, demand is positive). Numbers on arcs are the flow. Only arcs in the tree-solution are used and so the flow can be uniquely determined.

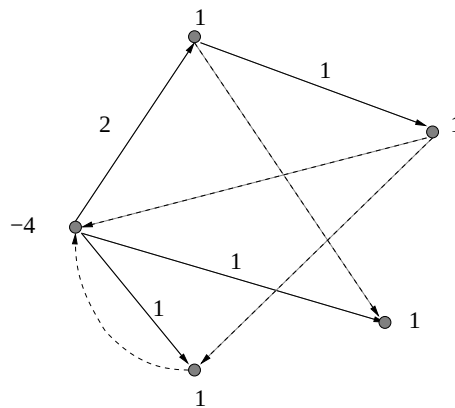


Figure 1: *Transshipment problem with the initial solution, including flow.*

Figure 2 shows the same transshipment problem. Numbers on arcs are the cost of sending one unit flow through the arc. Numbers on nodes are node-potentials (y_i). The node potential of node i is the distance (total cost) from the root node to i in the *tree-solution*.

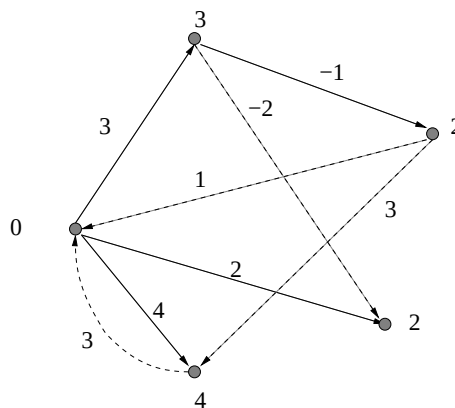


Figure 2: *Transshipment problem with node potentials.*

The solution is not optimal since $y_3 = 2 > y_1 + c_{13} = 3 - 2 = 1$ (see node numbers in the assignment).

Using a) Ford-Bellmann, the correct distances and shortest path tree is found by setting $y_3 = 2$ and replacing arc $s3$ with arc 13 . We check that the new solution is in fact optimal by checking that for all arcs ij : $y_i \leq y_j + c_{ji}$.

Using b) the transshipment algorithm, we note that since $y_3 = 2 > y_1 + c_{13} = 3 - 2 = 1$, arc 13 identifies a negative cycle. The cycle is shown in figure 3 and consist of the arcs $s1$, 13 and $s3$. The value of the cycle is calculated by adding forward arcs and subtracting backward arcs. So the value of the cycle is $c_{s1} + c_{13} - c_{s3} = 3 + (-2) - 2 = -1$ which is exactly the difference between y_1 and y_3 .

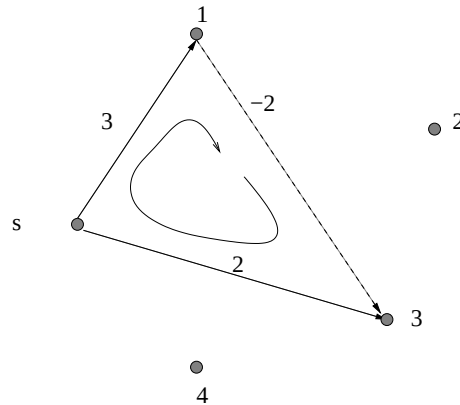


Figure 3: *Transshipment problem - a negative cycle.*

There is only one backward arc in the cycle, namely arc $s3$, so the minimum amount of flow in a backward arc of the cycle is 1 (see figure 1). Hence increase all forward arcs with one and reduce all backward arcs with 1, such that the flow in arc $s3$ is 0 and arc 13 can replace it in the solution. The final solution is shown in figure 4.

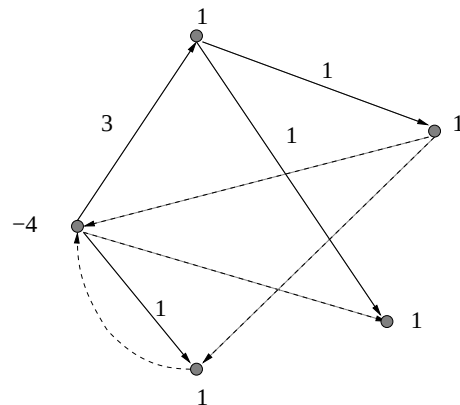


Figure 4: *Transshipment problem with the final solution, including flow.*