

Project Assignment One

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The project period formally starts on Tuesday 28 September and the report must be handed in at the latest Friday 29 October at 15:00 to David Pisinger. The report is expected to be appr. 5 pages long excluding tables, figures and appendices. The report must be written in Danish or in English. The project may be solved in groups – permissible group sizes are 1, 2 and 3.

Introduction

After having finished your university degree you have been hired to work at the consultancy Optimal Solutions. The company is present in many European countries and you soon realize that the challenges presented in front of you are also diverse and enjoy the many different tasks.

Question 1

One of your first tasks is a planning problem from the planning department of a company is to manage the rental contracts for the fleet of vans used by your company. For the next six months period the forecast for vans look like shown in Table 1.

October	November	December	January	February	March
430	410	450	390	425	450

Table 1: Requirements for vans for six months

At the 1 October the company has 180 vans from the former period. The rental period for these vans terminates at the end of November. In order to rent vans there is a choice of three contracts that may start at the first day of every month:

1. 3-month contracts for a total cost of 1800 Euros per van.
2. 4-month contracts for a total cost of 2200 Euros per van.
3. 5-month contracts for a total cost of 2400 Euros per van.

Now the question is how many contracts of the different types need to be started every month in order to satisfy the company's needs at the least cost and to have *no remaining vans* rented after the end of March.

Question 1.1

Initially, you just want to have an idea of how expensive a solution might be.

- *Devise a **greedy heuristic** and use it to solve the problem. For the given data presented describe the choices made by the greedy heuristic in the order they are made. What is the result?*

Question 1.2

Let x_{cm} be the variable stating the number of vans hired on a c -month contract starting in month m .

- *Describe the integer linear program to solve the optimization problem minimizing the cost and having no remaining vans rented at the end of the planning period.*

As you remember a general integer linear program can be quite cumbersome to solve for even the best solver. To get an idea of the solution value you start by solving the LP relaxation.

- *What is the solution and the optimal value of the LP relaxation? What does that tell you about the optimal solution to the integer linear program?*

Question 1.3

You discover some interesting structure in the problem. In order to understand the structure you just focus on the variables for $m = 1$ and $m = 2$ and the three different contract types. This corresponds to six variables.

What we really want is to describe the problem in *equality* form. So write up the constraint matrix, add a “surplus” variable y_i for each row i . Finally, add a redundant row $0 \cdot x + 0 \cdot y = 0$. As our focus is on the matrix just skip the right hand. Note that what has been done does not change the problem and therefore not the feasibility and optimality of solutions. The problem is the same, it has just been re-written.

Now perform the following elementary row operations for each $i = m, m - 1, \dots, 1$, in the stated order: subtract i th constraint from the $(i + 1)$ th constraint.

- *How does the resulting constraint matrix look like? What structure does it have? Can you use that to solve the problem efficiently? If yes, how?*

Question 1.4

Assume now that the van rental company introduces a new contract, flex-rent, instead of the contracts already described. A *flex-rent contract* makes it possible to rent a van for up to 4 months at a fixed cost of 2300 euro. The 4 months do not need to be consecutive, and they may start the first of any month. However, due to a high demand before Christmas, November and December both count as 1.5 months.

Let M be an upper bound on the number of vans needed, and let x_{cm} be a binary variable indicating whether van contract c ($c \leq M$) covers month m .

- a *Formulate an integer programming model for the problem, using the just introduced notation.*
- b *It is also possible to write up all possible flex-rent periods, as in Question 1.2. Formulate the problem using the rental periods.*
- c *Now (a) and (b) besides defining two modelles also defines the corresponding formulations. Prove that the formulation (b) is stronger than the formulation (a). (Hint: Show that all feasible LP-solutions to formulation (b) are also feasible LP-solutions to formulation (a), and that there exists LP-feasible solutions to (a) which are not feasible LP-solutions to (b). It is allowed to use smaller demands for the six months when constructing an instance to prove the latter.)*

Question 2

Your success on the first assignment quickly gets you involved in another case. A medium sized company with branches geographically distributed all over Denmark has approached Optimal Solutions. The company has decided to build an intra-net based on communication capacity leased from a large telecommunication vendor, Yellow. This operator has a back-bone network connecting a number of locations, and you can lease communication capacity between individual pairs of points in the back-bone network and in that way build your own network. Fortunately, all branches are located in one of the locations of the back-bone network, however, not all locations house a branch of the company.

The pricing policy of Yellow is as follows: Each customer individually buys in on a set connections chosen by the customer. The price is calculated as the sum of the prices for the individual connections. Each customer is offered a "customized" price for each connections, and these prices (although usually positive) may be negative to reflect a discount given by Yellow.

Figure 1 shows a small example, where 3 branches have to be connected in a network with 6 locations.

Your task is to analyze the situation of the company and determine which connections to lease in order to establish the cheapest intranet. Yellow guarantees that the connections will always be operative. You do therefore not have to consider failure protection.

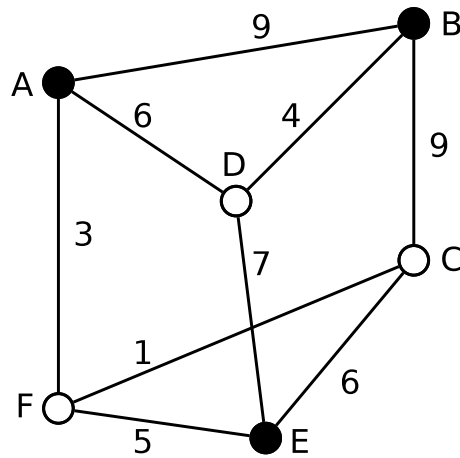


Figure 1: An example of a telecommunications network - the branches to be connected are the black vertices.

Question 2.1

- *Formulate a general mathematical programming model for the optimization problem of establishing the cheapest intra-net, given that all costs are non-negative.*

The model should have a decision variable x_{ij} for each connection (i, j) , and a set of constraints expressing that for each partition of the set of locations into two non-empty subsets with a branch in each, at least one connection joining the two subset must be chosen. In your model let p_{ij} be the price for establishing a direct connection between i and j , and a_i is equal to 1 if i is branch and 0 otherwise.

- *Can the model be used also when prices may be negative? Would you recommend the use of the model for large companies with hundreds of branches?*

Question 2.2

- *Use the model to solve the example problem of Figure 1 using CPLEX with 0-1 variables. Solve the model also when the integrality constraints on the variables are removed. Comment on the two solutions obtained and their objective function values.*

Question 2.3

An alternative to solving the problem to optimality is to use a method, which gives a good (but not necessarily optimal) solution fast. Such a method is called a *heuristic*.

As you realize the proximity of the problem to the minimum spanning tree you hit upon the following idea inspired by Prim's algorithm: Start by choosing (randomly) a branch. Let us call it b_0 . Now for each other branch compute the shortest distance to b_0 . The branch with the shortest distance to b_0 will be denoted b_1 . Then add the connections

on the path from b_1 to b_0 to the set of selected connections. In each of the subsequent iterations we continue this approach. Determine the unconnected branch b_i with the shortest distance to the tree already generated, that is, find an unconnected branch the shortest distance to any location that has been selected.

The process is terminated when all branches are added to the tree, that is, all branches are connected.

Obviously, the “killer” part of the described algorithm is the shortest path calculations. However, there are two challenges: 1) the network is not directed, and 2) there may be edges of negative length.

- *How would you cope with the non-directedness of the network, and which shortest path algorithm would you suggest to use given that negative edge lengths are present?*

However, you realize that it may be easier to solve the problem regarding negative edge lengths by changing the problem to reflect that it is always beneficial to choose an edge of negative cost, even though you don’t need the connection.

- *Describe the corresponding problem transformation, whereby you end up with a network with only non-negative edges lengths.*

You now are in the position that you may choose any of the shortest path algorithms touched upon in the course. In Figure 2, the network of your clients company is shown.

- *Which connections should be leased, if these are determined with the method just described with b_0 chosen to be A? Show the sequence in which the locations are considered, and the connections added in each iteration.*
- *What is the time complexity of the combined heuristic using a the shortest path method iteratively?*

