

Exercise 1.7

Define x_i^t as a positive integer decision variable indicating the number of working people on job i in week t .

Define z_i^t as a binary decision variable indicating whether job i is started in week t .

(i) To find a feasible solution:

$$\begin{aligned} \sum_i x_i^t &\leq L_t && \forall t \\ \sum_t z_i^t &= 1 && \forall i \\ z_i^t &= 0 && \forall i, t, t \geq 11 - p_i \\ z_i^t l_{iu} &\leq x_i^{t+(u-1)} && \forall i, u, t, 1 \leq u \leq p_i, 1 \leq t \leq 10 - p_i \end{aligned}$$

(ii) To minimize the maximum number of workers used during any of the nine weeks:

$$\begin{aligned} \min \quad & z \\ & z \geq \sum_i x_i^t \quad \forall t \end{aligned}$$

(iii) Job 1 must start at least two weeks before job 3:

$$\sum_{t_1: t_1 \leq t-2} z_1^{t_1} \geq z_3^t \quad \forall t, t \geq 3, \quad z_3^1 = 0, \quad z_3^2 = 0$$

(iv) Job 4 must start not later than one week after job 5:

$$\sum_{t_4: t_4 \leq t+1} z_4^{t_4} \geq z_5^t \quad \forall t, t \leq 8$$

(v) Jobs 1 and 2 cannot be carried out simultaneously:

$$\begin{aligned} z_1^t + \sum_{t_2: 0 \leq t_2 \leq p_1-1} z_2^{t_2+t} &\leq 1 \quad \forall t \\ z_2^t + \sum_{t_1: 0 \leq t_1 \leq p_2-1} z_1^{t_1+t} &\leq 1 \quad \forall t \end{aligned}$$

Exercise 1.8

For the MIP, y_j is 1 if depot j is built otherwise 0. In the COP this is represented by the set S where a depot $j \in S$ if it is built, that is $y_j = 1 \Leftrightarrow j \in S$. Also for the objective this means that $\sum_{j \in N} f_j y_j = \sum_{j \in S} f_j$.

Now we only need to establish that $\sum_{i \in M} \sum_{j \in N} c_{ij} x_{ij} = c(S) = \sum_{i=1}^m \min_{j \in S} c_{ij}$.

x_{ij} is only larger than 0 if $y_j = 1$ also if $x_{ij} = 0$ then $c_{ij} x_{ij} = 0$. Thus in order for $c_{ij} x_{ij} > 0$ it has to hold that $y_j = 1$.

Since $\forall i : \sum_{j=1}^n x_{ij} = 1$, let's assume that exactly one depot is chosen for each customer (i.e. for given customer i , exactly one of the x_{ij} 's is 1).

The MIP is a minimization problem hence for a given customer i , $x_{ij} = 1$ for the $j \in S$ that has the least cost c_{ij} , i.e. $\min_{j \in S} c_{ij}$. Summing over i we obtain $c(S)$.

The only case when a customer may be served from more than one depot at the same time, is when the distance from the customer to the facilities are the same. In this case a solution with the same value can be obtained by picking arbitrarily one of the facilities and the same argument holds.

Exercise 1.14

$x_{ij}, 1 \leq i \leq N, 1 \leq j \leq N$ is 1 if row i column j contains a queen, 0 otherwise.

We need to express the following things:

1. Each row contains at most (or exactly) 1 queen.
2. Each column contains at most (or exactly) 1 queen
3. Each diagonal contains at most 1 queen.

For the 2 first points, the constraints are:

$$\forall 1 \leq i \leq N : \sum_{1 \leq j \leq N} x_{ij} = 1 \quad (1)$$

$$\forall 1 \leq j \leq N : \sum_{1 \leq i \leq N} x_{ij} = 1 \quad (2)$$

The third point concerning the diagonals is split into 4 cases - see figure 1.

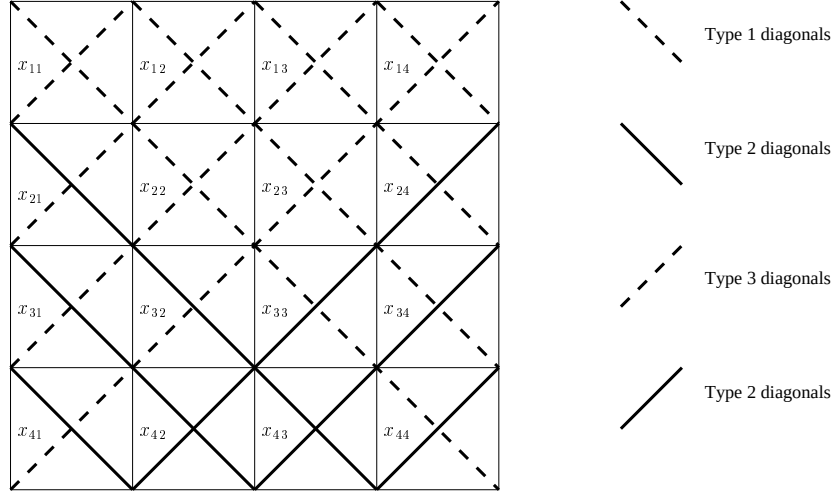


Figure 1: *Specification of the Diagonals for the N-Queen problem.*

The corresponding constraints are specified in the following. Type 1 diagonals are treated first then type 2 and so on.

$$\forall 0 \leq k \leq N - 1 : \sum_{1 \leq i \leq N-k} x_{i(i+k)} \leq 1 \quad (3)$$

$$\forall 1 \leq k \leq N - 1 : \sum_{1 \leq i \leq N-k} x_{(i+k)i} \leq 1 \quad (4)$$

$$\forall 0 \leq k \leq N - 1 : \sum_{1 \leq i \leq N-k} x_{i(N-k-i+1)} \leq 1 \quad (5)$$

$$\forall 1 \leq k \leq N - 1 : \sum_{1 \leq i \leq N-k} x_{N+1-i,k+i} \leq 1 \quad (6)$$

Use cplex to solve it with $N=4$ - how many different solutions exists? Use cplex to find out.

Try solving it with $N=3$ - what happens?