

Exercise 8.1

An inequality is valid if it does not cut away any feasible solutions. The inequality we find should cut away all solutions in the LP-relaxation of X but not in $\text{conv}(X)$. Or stated another way, the LP-relaxation of the modified X should equal $\text{conv}(X)$. The following inequalities will accomplish this.

- (i) $x_1 - x_2 \leq 0$
- (ii) $x - 7y \leq 0$
- (iii) $x - 4y \leq 4$

Exercise 8.2

(i) y is either 0 or 1, $x_1 + x_2 \leq 2y$ and $x_1 \leq 1$ implies $x_1 \leq y$. This inequality cuts off points for which $x_1 > y$, including $(1, 0, 0.5)$.

(ii) See example 8.3 page 115. $b = 9$, $C = 4$, $K = \lceil \frac{b}{C} \rceil = 3$, $\gamma = b - (\lceil \frac{b}{C} \rceil - 1)C = 9 - (3 - 1)4 = 1$ and the inequality is $x \leq b - \gamma(K - y) = 9 - 1(3 - y) = 6 + y$.

(iv) Divide the inequality by 12.

$$\begin{aligned}\frac{9}{12}x_1 + x_2 + \frac{8}{12}x_3 + \frac{17}{12}x_4 + \frac{13}{12}x_5 &\geq \frac{50}{12} \\ \Rightarrow x_1 + x_2 + x_3 + 2x_4 + 2x_5 &\geq \frac{50}{12} \\ \Rightarrow x_1 + x_2 + x_3 + 2x_4 + 2x_5 &\geq 5\end{aligned}$$

The first $\Rightarrow$ follows from the fact that increasing the left hand side of a $\geq$ will not invalidate the inequality. The second $\Rightarrow$ follows from the fact that all coefficients and variables are integer.

Exercise 8.3

An “automatic” approach is to use the Chvátal-Gomory procedure page 119. There is only one constraint, so we need to find a single non-negative weight, u which gives the inequality $y_2 + y_3 + 2y_4 \leq 6$. If we set $u = 1/5$ and multiply u on the original constraint we get:

$$\begin{aligned}\frac{4}{5}y_1 + y_2 + \frac{9}{5}y_3 + \frac{12}{5}y_4 &\leq \frac{34}{5} \\ \Rightarrow 0y_1 + y_2 + y_3 + 2y_4 &\leq \frac{34}{5} \\ \Rightarrow y_2 + y_3 + 2y_4 &\leq 6\end{aligned}$$

The first $\Rightarrow$ follows from the fact that decreasing the left hand side of a $\leq$ will not invalidate the inequality. The second $\Rightarrow$ follows from the fact that all coefficients and variables are integer.