

LC7.1

Figure 1 gives node identifier and the suggested y -value for each node. Cost, capacity and flow value are given in the exercise. Below we will describe how y is obtained.

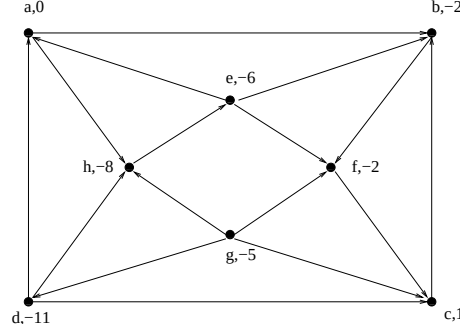


Figure 1: Names and y -value on nodes.

Arbitrarily pick node A to have $y_a = 0$. For all arcs not at their upper or lower bound, the reduced cost (i.e. $\bar{w}$) has to equal 0. Thus assume arc (i, j) is such an arc and assume y_i is known, then y_j can be calculated from $\bar{w}_{ij} = w_{ij} + y_i - y_j = 0 \Leftrightarrow y_j = y_i + c_{ij}$. Like wise if y_j is known, $y_i = y_j - c_{ij}$.

Using this, $y_b = y_a + c_{ab} = 0 - 2 = -2$, $y_e = y_a - c_{ea} = 0 - 6 = -6$, $y_f = y_e + c_{ef} = -6 + 4 = -2$, $y_h = y_e - c_{he} = -6 - 2 = -8$, $y_g = y_f - c_{gf} = -2 - 3 = -5$ and $y_c = y_f + c_{fc} = -2 + 3 = 1$. The only undetermined variable is y_d . There is no arc incident to d , which are not at its upper or lower bound.

In order for the y -vector to certify optimality, the reduced cost of an arc with zero flow (i.e. at its lower bound) must be > 0 and the reduced cost of an arc with flow at its upper bound must be < 0 . Thus instead we write up the reduced cost for each of the arcs incident to d and depending on which bound the flow is at, we require it to be > 0 or < 0 all values except y_d is known, so we get the following:

$$\begin{aligned}\bar{w}_{da} &= w_{da} + y_d - y_a \leq 0 \Rightarrow y_d \leq -6 \\ \bar{w}_{dh} &= w_{dh} + y_d - y_h \leq 0 \Rightarrow y_d \leq -11 \\ \bar{w}_{gd} &= w_{gd} + y_g - y_d \geq 0 \Rightarrow y_d \leq -5 \\ \bar{w}_{dc} &= w_{dc} + y_d - y_c \leq 0 \Rightarrow y_d \leq -4\end{aligned}$$

Thus any $y_d \leq -11$ will be okay, so let us arbitrarily set $y_d = -11$.

Does the suggested y certify optimality? For this to be the case, we have to check that for all arcs e , either $\bar{w}_e = 0$ or if $\bar{w}_e > 0$ then $x_e = 0$ or if $\bar{w}_e < 0$ then $x_e = u_e$. y has been constructed taking account for this, so it only remains to check that this holds for the remaining arcs, i.e. ah , eb , bf , cb , gc and gh . (Do it!)