

Extra exercise on Gomory's fractional cutting plane algorithm

Removing the integrality constraints we can solve the LP-relaxed problem in three simplex iterations as follows:

	x_1	x_2	s_1	s_2	
Z	6	6	0	0	0
s_1	1	2	1	0	5
s_2	2	1	0	1	5
Z	0	3	0	-3	-15
s_1	0	3/2	1	-1/2	5/2
x_1	1	1/2	0	1/2	5/2
Z	0	0	-2	-2	-20
x_2	0	1	2/3	-1/3	5/3
x_1	1	0	-1/3	2/3	5/3

The optimal solution to the LP-relaxed problem is $x_1 = 5/3, x_2 = 5/3, z^* = 20$. Using the multipliers data in any of the last two rows of the third simplex tableau and using the Chvátal-Gomory cut (inequality 8.11 on page 125 of the book) we get the following inequality:

$$2/3s_1 + 2/3s_2 \geq 2/3$$

We can now add this inequality to the original LP-relaxed problem and resolve the problem from the scratch. Using dual simplex method, however, we can add the inequality to the final simplex tableau and carry on doing only the necessary dual simplex iterations. But first we need to change the inequality into a $\leq$ form and add positive slack s_3 to get an equation:

$$\begin{aligned} -2/3s_1 - 2/3s_2 &\leq -2/3 \\ -2/3s_1 - 2/3s_2 + s_3 &= -2/3 \end{aligned}$$

Picking up from the third simplex tableau we now have:

	x_1	x_2	s_1	s_2	s_3	
Z	0	0	-2	-2	0	-20
x_2	0	1	2/3	-1/3	0	5/3
x_1	1	0	-1/3	2/3	0	5/3
s_3	0	0	-2/3	-2/3	1	-2/3
Z	0	0	0	0	-3	-18
x_2	0	1	0	-1	1	1
x_1	1	0	0	1	-1/2	2
s_3	0	0	1	1	-3/2	1

Now the optimal solution to the LP-relaxed problem with addition of one Chvátal–Gomory cut becomes $x_1 = 2, x_2 = 1, z^* = 18$. Since this is an integer solution we have found the optimal solution to the original integer problem.

Reminder note on dual simplex method: In dual simplex method we start with an infeasible initial tableau. The pivot element is chosen by considering the most negative right hand side (-2/3 in our case) and the minimum ratio test is performed by dividing the values in the objective function row by the values in the row with most negative right hand side. The chosen column is the one for which the absolute value of the ratio is smallest.