Exercise DO.3: Dynamic programming

sct/nkp

Static and Dynamic Optimization

Background

A solution to this exercise is available on the course home page.

1 Optimal tank level control

In this exercise we will use dynamic programming for controlling the level of a tank. A sketch of the tank is shown in figure 1.

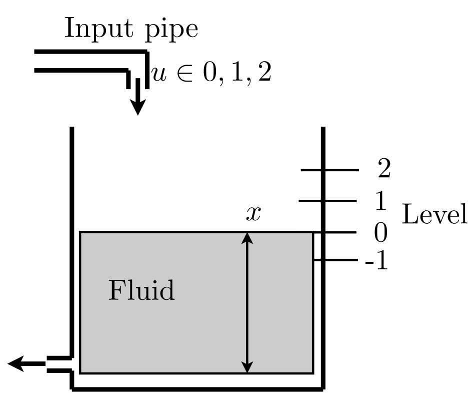

Figure 1. Sketch of tank system

Fluid is exiting the tank with a constant rate (though the hole in the bottom) and fluid can be introduced to the tank through the input pipe in the top. The rate of flow from the input pipe u_i can be controlled.

The level of the tank x_i is described by the simplified dynamics:

$$x_{i+1} = x_i + u_i - 1$$

where we restrict the level of the tank to the finite set

$$x_i \in \{-1, 0, 1, 2\}$$

and the input u_i is restricted to

$$u_i \in \{0, 1, 2\}$$

Question 1

Over the horizon $i = 0, 1, 2, \ldots, 4$ we want the level in the tank to follow a specific reference:

$$r_i = \begin{cases} 0 & , i = 0, 1 \\ 2 & , i = 2, 3, 4 \end{cases}$$

We want to do this in an way such that the following cost is minimized

$$J = (x_4 - r_4)^2 + \sum_{i=0}^{3} (x_i - r_i)^2 + u_i^2$$

Find the optimal trajectory of u_i and x_i for the initial level $x_0 = 0$. Plot the optimal trajectory as function of time (i).

Question 2

Inspect the sensitivity on the optimal cost-to-go for different initial values x_0. Try e.g. $x_0 = \pm 1$ and find the optimal cost and compare the results with the result for $x_0 = 0$.

Question 3

Now introduce the following endpoint constraint:

$$x_4 = r_4 = 2$$

and find the optimal trajectories for $x_0 = -1,\ 0,\ 1$.

Question 4

Change the cost function into

$$J = (x_4 - r_4)^2 + \sum_{i=0}^{3} (x_i - r_i)^2 + \rho u_i^2$$

where $\rho = 0.1$. Find the optimal trajectories for this case (situation as in question 1). Study (see the change in trajectories) the effect of introducing ρ.