02433 – Written exercise 2

This is the second out of three written exercises. The written report must be handed in at the beginning of course week 11 via the assignment facility on campusnet. The written report is mandatory and will be part of the final oral examination.

Description

In this exercise we consider the theta logistic model for population growth. It is similar to logistic growth, however with the additional parameter θ, which influences how the growth rate depends on the current population size.

Assuming that the population is observed with error we can write the theta logistic model as a state-space model for the log-population size (C_t)

$$C_t = C_{t-1} + r_0 \left(1 - \left[\frac{\exp(C_{t-1})}{K}\right]^{\theta}\right) + e_t \quad (1)$$

$$X_t = C_t + u_t \quad (2)$$

where $e_t \sim N(0, Q)$ are iid., $u_t \sim N(0, R)$ are iid., and e_t and u_t are mutually independent, K is the carrying capacity (the population level when stationarity is reached), r_0 is the intrinsic growth rate, and θ is a shape parameter. So the model has a total of five parameters $\boldsymbol{\lambda} = (\theta, r_0, K, Q, R)$.

Questions

Please include any computer code in an appendix of the written report.

a)

Download the two datasets (population 1: parest_thet05.txt, and population 2: parest_thet15.txt) from the homepage. These files contain noisy observations (X_t) of the log-population size of two different populations. Make a plot of the two time series.

b)

Suppose that the state-space is restricted, ie that $C_t \in [2.1, 8.4]$. Suppose further that the state-space is partitioned into $m = 250$ intervals (b_{i-1}, b_i) ($i \in \{1, \ldots, m\}$) in a way similar to the discretization technique described in Section 13.3.1 in Zucchini09. With the discrete grid, write up the expression for the state dependent densities $p(x_t|C_t = i)$ and for the transition probabilities $\text{Prob}(C_t = j|C_{t-1} = i)$ based on the state-space model above.
Hint: To solve this part use the techniques presented in the lecture notes for week 7 and Section 13.3.1 in Zucchini09.

c)

Write a piece of computer code (e.g. R or Matlab) which evaluates the likelihood function for the parameters of the state-space model: $\boldsymbol{\lambda} = (\theta, r_0, K, Q, R)$. That is, create a script which calculates the forward probabilities of the HMM given by the discretization of the state-space model specified in question b (the process cannot be assumed to be stationary). Then use the forward probabilities to evaluate the likelihood function.
Hint: It may be advantageous to use the R functions in Zucchini09 as a starting point for calculating the forward probabilities.

d)

Use your function for evaluating the parameter likelihood to estimate the parameters of the theta logistic growth model for both of the provided datasets. Also provide confidence intervals of the parameters. Are some of the parameter estimates correlated? if they are can you explain why by inspecting (1)?

e)

To the fitted HMM apply at least one of the following three types of decoding for all $t \in \{1, \ldots, T\}$:

1. Local decoding as specified by equation (5.7) in Zucchini09.
2. Global decoding, as specified by the algorithm in Section 5.3.2 in Zucchini09.
3. Local expectation, as explained on page 7 of the summary slides related to chapter 5.

Plot the decoding(s) and the data in the same plot.

End of exercise