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Solution to exercise 12-2 in Montgomery (5.ed.)

Batches					
1	2	3	4	5	Coded data:
6	19	11	-12	-11	$(Y - 23.40) \times 100$
8	6	24	0	6	
16	2	6	-3	-3	
-1	9	12	6	-8	
0	10	9	-1	-2	
29	46	62	-10	-18	Total = 109

The coding of the data does not make any difference in the analysis of variance. It is a technique that was uses previously when calculations by hand were common and before the invention of the pocket calculator or the PC.

$\text{SSQ}_{Total} = 2321 - 109^2/25 = 1845.76;\ f = 25 - 1$
$\text{SSQ}_{Batches} = (29^2 + 46^2 + 62^2 + 10^2 + 18^2)/5 - 109^2/25 = 969.76;\ f = 5 - 1$

ANOVA	SSQ	f	s^2	F	$E\{s^2\}$
Variation between batches	969.76	5-1	242.44	5.54	$5\sigma^2_{Batch} + \sigma^2_E$
Variation within batches	876.00	25-5	43.80		σ^2_E
Total variation	1845.76	25-1			

a)

$$F(4, 20)_{0.99} = 4.43 < 5.54 \Rightarrow \text{significant difference between batches.}$$

b)

$$\hat{\sigma}^2_E = \frac{43.80}{100^2} = \frac{6.62^2}{100^2};\ \hat{\sigma}^2_{Batch} = \frac{1}{5 \cdot 100^2}(242.44 - 43.80) = \frac{6.30^2}{100^2}$$

c)

$$L = \frac{1}{5}(\frac{242.44}{43.80} \cdot \frac{1}{F(4, 20)_{0.975}} - 1) = \frac{1}{5}(\frac{242.44}{43.80} \cdot \frac{1}{3.51} - 1) = 0.12$$

$$U = \frac{1}{5}(\frac{242.44}{43.80} \cdot F(20, 4)_{0.975} - 1) = \frac{1}{5}(\frac{242.44}{43.80} \cdot 8.56 - 1) = 9.28$$

$$\frac{L}{1+L} < \frac{\sigma^2_{Batch}}{\sigma^2_{Batch} + \sigma^2_E} < \frac{U}{1+U} \Rightarrow$$

$$0.11 < \frac{\sigma^2_{Batch}}{\sigma^2_{Batch} + \sigma^2_E} < 0.90. \text{ (95\% confidence interval)}$$

Note that for the F-distribution tails it applies that: $F(a, b)_\alpha = 1/F(b, a)_{1-\alpha}$