

Solution to exercise 59

Question 1:

The normal plot shows that the contrasts corresponding to A, B, AB, G and 'blocks' are clearly different from the remaining contrasts. A reasonable conclusion is therefore:

$$Y = \mu + A + B + AB + G + block + \epsilon$$

Question 2

$$\hat{\mu} = 55.95/16 = 3.50,$$

$$\hat{A} = -5.03/8 = -0.6288,$$

$$\hat{B} = 3.29/8 = 0.4113,$$

$$\widehat{AB} = -2.97/8 = -0.3713,$$

$$\hat{G} = -3.85/8 = -0.4813,$$

block effect = $4.35/8 = 0.54$ (difference between the two particular blocks, not a general mean effect or the like).

Question 3

Use the sums of squares for the non-significant effects and compute:

$$\hat{\sigma}_\epsilon^2 = (0.022 + 0.002 + 0.028 + 0.010 + 0.018 + 0.002 + 0.005 + 0.050 + 0.003 + 0.001)/10 = 0.1410/10 = 0.0141 = 0.12^2$$

Question 4

Parameter estimates

$$\widehat{A}_1 = -5.03/16 \simeq -0.31,$$

$$\widehat{B}_1 = +3.29/16 \simeq +0.21,$$

$$\widehat{AB}_{11} = -2.97/16 \simeq -0.19,$$

$$\widehat{G}_1 = -3.85/16 \simeq -0.24,$$

Total A plus B influence				
$\widehat{B}_1 = 0.21$	$\widehat{AB}_{01} = 0.19$	$\widehat{AB}_{11} = -0.19$	\implies	A+B+AB
$\widehat{B}_0 = -0.21$	$\widehat{AB}_{00} = -0.19$	$\widehat{AB}_{10} = 0.19$		0.71 -0.29
	$\widehat{A}_0 = 0.31$	$\widehat{A}_1 = -0.31$		-0.09 -0.33

The optimal expected response is then $\hat{\mu} + \widehat{A}_1 + \widehat{B}_0 + \widehat{AB}_{10} + \widehat{G}_1 = 2.93$, which is attained for A at its high level, B at its low level and G at its high level. The remaining factors are not important (or perhaps determined with a too high uncertainty!).

The block effect is not accounted for, since it is impossible to know how a particular new block will behave (increase or decrease the response). Essentially the block is a randomly varying factor.