H. Spliid, October 20, 2010

Solution to exercise 47

Question 1
The design is an incomplete balanced block design. The applicability of the design is based on additivity between the influence from the block variable and the factor variable.

s = noise scheme effect : $\sum_{i=1}^{5} s_i = 0$
L = litter = random effect: $L_j \in NID(0, \sigma_L^2)$

It is natural to model L_j as a random effect, but if in a particular case it is wanted to model L_j as deterministic, the computations are the same when additivity is assumed (but the interpretation is of course quite different).

Den mathematical model is:

$$Y_{ij} = \mu + s_i + L_j + E_{ij}$$

where $E_{ij} \in NID(0, \sigma^2)$ is the random experimental error. Because of the balance the s_i-effect can be 'cleaned' for L_j-block variation. The variance of the experimental error can also be estimated.

Question 2
From the data we can compute 'raw' sums of squares:

$$\begin{aligned} \text{SSQ}_{tot} &= 10740 - \frac{382^2}{30} = 5875.88; \\ \text{SSQ}_{scheme} &= \frac{38222}{6} - \frac{382^2}{30} = 1506.20 \\ \text{SSQ}_{litter} &= \frac{24138}{3} - \frac{382^2}{30} = 3181.87; \end{aligned}$$

In order to find the adjusted SSQ for the noise schemes we compute Q's as follows:

b =	number of blocks	= 10
t =	number of treatments (sometimes called 'a')	= 5
k =	number of treatments per block	= 3
r =	number of times each treatment appears in the design	= 6
N =	total number of measurements	= 30 (= b·k =t·r)
λ =	number of times each pair of treatments appear in a block in the design	= 3 (=r(k-1)/(t-1)) =(6·2/4 = 3)

$$
\begin{array}{llcr}
Q_i = & T_{i.} - \sum_j n_{ij} \cdot T_{.j}/k & & \\
Q_1 = & 13 - (40 - 23 - 22 - 83 - (-15) - 41)/3 & = & -155/3 \\
Q_2 = & 58 - (40 - 23 - 22 - 2 - 90 - 51)/3 & = & -54/3 \\
Q_3 = & 76 - (40 - 83 - (-15) - 2 - 90 - 45)/3 & = & -17/3 \\
Q_4 = & 92 - (23 - 83 - 41 - 2 - 51 - 45)/3 & = & 31/3 \\
Q_5 = & 143 - (22 - (-15) - 41 - 90 - 51 - 45)/3 & = & 195/3 \\
\hline
 & \text{Control sum} & = & 0
\end{array}
$$

$$
SSQ_{scheme,adjust} = \frac{k \cdot \sum_i Q_i^2}{\lambda \cdot t} = \frac{3 \cdot [(155^2 + 54^2 + 17^2 + 31^2 + 195^2)/3^2]}{3 \cdot 5} = 1471.47
$$

Source of variation	SSQ	*df*	s^2	F-value
Scheme of noise, adjusted	1471.47	4	367.87	4.81
Litters not adust.	3181.87	9	353.54	no test
Residual	1222.53	16	76.41	–
Total	5875.88	29		

Since $F(4,16)_{0.05} = 3.01$ the variation between noise schemes is significant at the 5% level of significance.

Question 3a

We use the Q's to estimate the noise scheme effects:

$$
\widehat{s_i} = \frac{k \cdot Q_i}{\lambda t} = \frac{3 \cdot Q_i}{3 \cdot 5} = \{-10.3, -3.6, -1.1, 2.1, 13.0\}
$$

The residual error variance is estimated from

$$
\widehat{\sigma_E^2} = 76.41 = 8.7^2
$$

Question 3b

$$
\begin{aligned}
C_{high-low} = &\quad Q_A + Q_B - Q_C - Q_D &= -223/3 \\
C_{const-var} = &\quad Q_A - Q_B + Q_C - Q_D &= -149/3
\end{aligned}
$$

We could also find:

$$
\begin{aligned}
C_{interaction} &= Q_A - Q_B - Q_C + Q_D &= -53/3 \\
C_{natural-others} &= -Q_A - Q_B - Q_C - Q_D + 4Q_E &= 975/3
\end{aligned}
$$

$$
\begin{array}{llcl}
SSQ_{high-low} & = \frac{k \cdot C^2_{high-low}}{[1^2+1^2+(-1)^2+(-1)^2](\lambda t)} & & \\
 & = \frac{k \cdot (223/3)^2}{(4)(\lambda t)} & = & 276.27 \\
SSQ_{cons-var} & = \frac{k \cdot (-149/3)^2}{(4)(\lambda t)} & = & 123.34 \\
SSQ_{interaction} & = \frac{k \cdot (-53/3)^2}{(4)(\lambda t)} & = & 15.61 \\
SSQ_{natural-others} & = \frac{k \cdot (975/3)^2}{(20)(\lambda t)} & = & 1056.25 \\
\hline
\text{Control sum} & & = & \underline{1471.47}
\end{array}
$$

$$
\begin{array}{lll}
F_{high-low} & = \frac{276.27}{76.412} & = 3.62 \sim F(1,16)_{0.08} \\
F_{const-var} & = \frac{123.34}{76.41} & = 1.61 \sim F(1,16)_{0.30}
\end{array}
$$

The high-low effect is significant only at the 8% level of significance, while the degree of variation (constant versus varying scheme) probably is not important. Finally it is the difference between the natural and the artificial noise schemes (scheme E versus the others) which is of greatest importance.