Solution to exercise 42

1): The design construction has obviously been :

A	B	C	D=-BC	E=-AC	Code
-1	-1	-1	-1	-1	(1)
1	-1	-1	-1	1	ae
-1	1	-1	1	-1	bd
1	1	-1	1	1	abde
-1	-1	1	1	1	cde
1	-1	1	1	-1	acd
-1	1	1	-1	1	bce
1	1	1	-1	-1	abc

Defining relations $I_1 = -BCD$ and $I_2 = -ACE$ giving $I_1 I_2 = +ABDE$ and alias relations:

I	=	-BCD	=	-ACE	=	+ABDE
A	=	-ABCD	=	-CE	=	+BDE
B	=	-CD	=	-ABCE	=	+ADE
AB	=	-ACD	=	-BCE	=	+DE
C	=	-BD	=	-AE	=	+ABCDE
AC	=	-ABD	=	-E	=	+BCDE
BC	=	-D	=	-ABE	=	+ACDE
ABC	=	-AD	=	-BE	=	+CDE

The design is a resolution III design, and main effects are confounded with two-factor interactions which is the best that can be achieved with 5 factors in 8 measurements.

Often these relations are reduced by taking away higher order interactions. We could remove all terms with 3 or more letters:

I	=	-BCD	=	-ACE	=	+ABDE
A	=		=	-CE	=	
B	=	-CD	=		=	
AB	=		=		=	+DE
C	=	-BD	=	-AE	=	
AC	=		=	-E	=	
BC	=	-D	=		=	
(ABC)	=	-AD	=	-BE	=	

2): We find the parameter estimates, the sums-of-squares and the relevant effect estimates using Yates' algorithm:

Code	Response	I	II	Contrast	Parameter	SSQ	Effect
(1)	36	81	182	388	48.5	-	
a e	45	101	206	-4	-0.5	2	$\widehat{A} = -1.0$
b d	45	69	20	88	11.0	968	$\widehat{B} = 22.0$
ab de	56	137	-24	-4	-0.5	2	
c de	39	9	20	24	3.0	72	$\widehat{C} = 6.0$
ac d	30	11	68	-44	-5.5	242	$-\widehat{E} = -11.0$
bc e	76	-9	2	48	6.0	288	$-\widehat{D} = 12.0$
abc	61	-15	-6	-8	-1.0	8	

One way of writing the model (as a regression model) is:

$$Y = \mu + \alpha_A \cdot \Delta_A + \alpha_B \cdot \Delta_B + \alpha_C \cdot \Delta_C + \alpha_D \cdot \Delta_D + \alpha_E \cdot \Delta_E + \epsilon$$

where , for example, $\Delta_A = A - A_0 = A - 75^oC$ is the deviation of A from the center point of the design and $\widehat{\alpha}_A = -1/(80^oC - 70^oC) = -0.1/^oC$.

Factor	Variable	Unit	Center point	$\widehat{\alpha}$	Unit
μ	Level of response			48.5 ($\hat{\mu}$)	%
A	Temperature	^{o}C	75	-0.1	% / ^{o}C
B	Concentration	%	50	1.1	% / %
C	Reac.Time	min	35	0.60	% / min
D	Catal.Conc.	%	0.15	-120	% / %
E	pH	pH	6.0	11	% / pH

The direction of the steepest ascent (increase) is the direction in which the response Y increases most rapidly. It is given by the α's: $\widehat{\Delta}_{optimal} = \{\widehat{\alpha}_A, \widehat{\alpha}_B, \widehat{\alpha}_C, \widehat{\alpha}_D, \widehat{\alpha}_E\}$

3): At the given point (65, 70, 40, .25, 5.0) the deviation from the center point is

$$\Delta = [(65 - 75), (70 - 50), (40 - 35), (0.25 - 0.15), (5 - 6)] = [-10, 20, 5, 0.1, -1]$$

and the estimated mean response is

$$\widehat{Y} = 48.5 - 0.1 \cdot (-10) + 1.1 \cdot 20 + 0.6 \cdot 5 - 120 \cdot 0.1 + 11 \cdot (-1) = 51.5\%$$

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