H. Spliid, October 20, 2010

Solution to exercise 39

Question 1

The design is an incomplete balanced block design. The blocks are the 5 experimental rounds and the design is balanced. Each plastic type appears the same number of times in the design and the same number of times in the blocks. All pairs of plastic types appear the same number of times in one block namely $\lambda = 3$ times. The blocks are incomplete because all plastic types do not appear in the blocks (the block size is smaller than the number of plastic types).

With this design it is possible to eliminate the variation between blocks from the estimates of the factor effects (the 5 types A, B, C, D and E).

Mathematical model:

$$Y_{ij} = \mu + \tau_i + B_j + E_{ij},$$

where τ_i is the factor effect (A, B, C, D, E) and B_j is the block effect.

Question 2

$$b = 5,\ t = 5,\ k = 4,\ r = 4,\ \lambda = 3$$

$$\begin{aligned}
\text{SSQ}_{tot} &= 218588 - \frac{2030^2}{20} \\
&= 218588 - 206045 = 12513 \\
\text{SSQ}_{block} &= (382^2 + 458^2 + 406^2 + 377^2 + 407^2)/4 - 206045 \\
&= 207075.5 - 206045 = 1030.5
\end{aligned}$$

Adjusted treatment averages are computed:

$$\begin{aligned}
Q_A &= 336 - (382 + 458 + 406 + 377)/4 = -69.75 \\
Q_B &= 544 - (382 + 458 + 406 + 407)/4 = 130.75 \\
Q_C &= 334 - (382 + 458 + 377 + 407)/4 = -72.00 \\
Q_D &= 359 - (382 + 406 + 377 + 407)/4 = -34.00 \\
Q_E &= 457 - (458 + 406 + 377 + 407)/4 = 45.00 \\
\hline
& \hspace{6cm} 0
\end{aligned}$$

$$\begin{aligned}
\text{SSQ}_{plastic,corrected} &= \frac{4(-69.75^2 + 130.75^2 + 72.00^2 + 34.00^2 + 45.00^2)}{3 \cdot 5} = 8086.8 \\
\text{SSQ}_{residual} &= 12513 - 1030.5 - 8086.8 = 3395.7
\end{aligned}$$

ANOVA

Source of variation	SSQ	f	s^2	F
Plastic types	8086.8	4	2021.7	6.55
Blocks	1030.5	4		
Residual	3395.7	11	308.7	
Total	12513.0	19		

Since $F(4,11)_{0.01} = 5.66$ it is concluded that there is a significant effect from the plastic types. The parameters of the model are therefore estimated.

$\widehat{\mu} = \overline{Y}_{..} = 2030/20 = 101.5$

The effects, the τ_i's in the model, are estimated by $\widehat{\tau}_i = k \cdot Q_i/\lambda t$:

$$
\begin{aligned}
\widehat{A} = \widehat{\tau}_1 = k \cdot Q_A/\lambda t &= -18.6 \\
\widehat{B} = \widehat{\tau}_2 = k \cdot Q_B/\lambda t &= 34.9 \\
\widehat{C} = \widehat{\tau}_3 = k \cdot Q_C/\lambda t &= -19.2 \\
\widehat{D} = \widehat{\tau}_4 = k \cdot Q_D/\lambda t &= -9.1 \\
\widehat{E} = \widehat{\tau}_5 = k \cdot Q_E/\lambda t &= 12.0
\end{aligned}
$$

And $\widehat{\sigma}_E^2 = 3395.7/11 = 308.7 \simeq 17.6^2$

Since the design in fact is symmetrically balanced it is also possible to extract estimates for the individual blocks or (which perhaps would be even more relevant) the variance between blocks. The corrected block means can be computed and their theoretical means and variances are known (just as for the corrected treatment means).

In order to assess whether there is a grouping of the treatment effects a Newman-Keul-test could suitably be tried for the Q_i' (no specific characteristica are given which could justify testing certain contrasts).

One can also perform the test directly on $\widehat{\tau}_i = k \cdot Q_i/(\lambda t)$. The result is of course the same. The following illustrates both procedures.

$$
\begin{aligned}
s_{res} &= \widehat{\sigma_E} = 17.6 \\
LSR(Q) &= s_{res} \cdot \sqrt{\lambda t/k} \cdot R = 17.6 \cdot \sqrt{3 \cdot 5/4} \cdot R \\
LSR(f) &= s_{res} \cdot \sqrt{k/\lambda t} \cdot R = 17.6 \cdot \sqrt{4/(3 \cdot 5)} \cdot R
\end{aligned}
$$

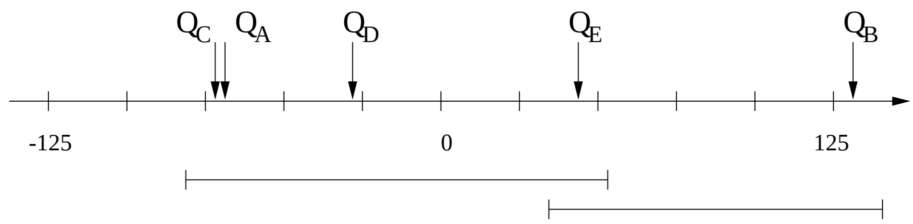
With $f_{res} = 11$, $s_{res} = 17.6$, $k = 4$, $\lambda = 3$ og $t = 5$, the 5% significance table of “Studentized Range” ($q_{.05}(p,f)$) gives:

p	R	$s_{res}\sqrt{\lambda t/k}$	$LSR(Q)$	$s_{res}\sqrt{k/(\lambda t)}$	$LSR(\hat{\tau})$
5	4.58	34.08	156.1	9.089	41.63
4	4.26	34.08	145.2	9.089	38.72
3	3.82	34.08	130.2	9.089	34.72
2	3.11	34.08	106.0	9.089	28.27

Newman-Keuls test for the Q's:

$Q_B - Q_C$	$= 202.75 > 156.1$	Significant
$Q_B - Q_A$	$= 200.50 > 145.2$	Significant
$Q_B - Q_D$	$= 164.75 > 130.2$	Significant
$Q_B - Q_E$	$= 85.75 < 106.0$	Not significant
$Q_E - Q_C$	$= 117.0 < 145.2$	Not significant

Conclusion:



It cannot be determined whether E belongs to the one group or the other, while (B) and (A, C, D) probably define two different groups.

Question 3a

Yet another source of variation is introduced being the 4 mounting plates 'a', 'b', 'c' og 'd'. The problem is that the mounts are not balanced in relation to the factor levels (but of course in relation to the block variabel 'Rounds'), so that differences between different factor levels also contain variation from mounting plates.

The plastic type 'A' will exhibit a higher wear resistance if mounting plate 'd' does not press as hard on the test item as do the other mounting paltes.

Question 3b

If the experiment should be repeated a design which takes into account the mounting plates could be:

	A	B	C	D	E
Round 1	a	b	c	d	
2	b	c	d		a
3	c	d		a	b
4	d		a	b	c
5		a	b	c	d

In this desing balance between mounting plates and plastic types is obtained. The variation from both block variables (rounds and mounts) can be eliminated from the analysis of the factor (plastic type). The design is an incomplete Latin Square which is also called a **Youden Square**.

Suppose the design had been carried out using the suggested design. In that case we would have got the following computation of the variation between mounting plates:

	A		B		C		D		E	
1	a	79	b	139	c	90	d	74		
2	b	93	c	144	d	96			a	125
3	c	58	d	136			a	76	b	136
4	c	106			a	75	b	95	c	101
5			a	125	b	73	c	114	d	95

Compute sums for the 4 mounting plates:

$T_{\cdot a} = 480$
$T_{\cdot b} = 536$
$T_{\cdot c} = 507$
$T_{\cdot d} = 507$

which then gives $\text{SSQ}_{mounts} = (480^2 + 536^2 + 507^2 + 507^2)/5 - 2030^2/20 = 313.8$ with $f = 3$ degrees of freedom.

The new residual SSQ is: $\text{SSQ}_{resid,new} = 3395.7 - 313.8 = 3081.9$ with $f = 11 - 3 = 8$ degrees of freedom.

Since the mounts are balanced in relation to both plastic types and rounds it can be tested whether the influence from the mounts is important or not.

We compute $F = \frac{313.8/3}{3081.9/8} = 0.27 < 1$ and we conclude that there probably is no great effect from the mounts on the particular testing machine used.

The main point is that the balancing of the rounds and of the mounting plates makes the analyses of the interesting factor, the plastic type, independent of whether the rounds and or the mounts are identical or not.

Had there been a marked difference between mounting plates the first design would have been less usefull, or even worse, useless - and the experimenter would not know!