IMM - DTU

Statistical Design and Analysis of Experiments
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Solution to exercise 30

Question 1

The design is a partially confounded factorial design.

In the first replication the distribution on extraction chambers was as shown to the left below and in the second replication it was as shown to the right.

Chamber 1	Chamber 2		Chamber 1	Chamber 2
(1)=124	a=171		(1)=123	c=110
c=98	b=153		ac=154	a=185
ab=192	ac=145		b=165	bc=158
abc=165	bc=136		abc=172	ab=201

Replicate 1: I=AB def. contr.   Replicate 2: I=AC def. contr.

When analyzing the factorial model contrasts for confounded effects are not used in the replicate where the confounding is (AB in the first replicate and AC in the second replicate). Contrasts for the remaining effects are found by combining the contrast for the two replicates.

Question 2

The total sum of squares for blocks including confounded factorial effects can be found as the variation between the four blocks in the design as follows:

Chamber	Replicate	Data				Sum
1	1	124	98	192	165	579
2	1	171	153	145	136	605
1	2	123	154	165	172	614
2	2	110	185	158	201	654

Then the variation between the four blocks (including the confounded factorial effects) is

$$\text{SSQ}_{blocks} = \frac{579^2 + 605^2 + 614^2 + 654^2}{4} - \frac{2452^2}{16} = 725.50$$

Factorial effect (not confounded with blocks) sums of squares (see text to exercise):

$$\begin{array}{lcll}
A & : & \frac{(162+156)^2}{2\times 8} & = 6320.25 \\
B & : & \frac{(108+124)^2}{2\times 8} & = 3364.00 \\
AB_{(2)} & : & \frac{(-56)^2}{8} & = 392.00 \\
C & : & \frac{(-96-80)^2}{2\times 8} & = 1936.00 \\
AC_{(1)} & : & \frac{(-10)^2}{8} & = 12.50 \\
BC & : & \frac{(8+8)^2}{2\times 8} & = 16.00 \\
ABC & : & \frac{(-10-4)^2}{2\times 8} & = 12.25
\end{array}$$

Uncertainty (from factorial effects which can be estimated independently in different replications):

$$\begin{array}{lcll}
A & : & \frac{162^2+156^2}{8} - \frac{(162+156)^2}{16} & = 2.25 \\
B & : & \frac{108^2+124^2}{8} - \frac{(108+124)^2}{16} & = 16.00 \\
C & : & \frac{(-96)^2+(-80)^2}{8} - \frac{(-96-80)^2}{16} & = 16.00 \\
BC & : & \frac{8^2+8^2}{8} - \frac{(8+8)^2}{16} & = 0.00 \\
ABC & : & \frac{10^2+4^2}{8} - \frac{(-14)^2}{16} & = 2.25 \\
\hline
 & & & 36.50 \quad ; f = 5
\end{array}$$

ANOVA	SSQ	$d.f.$	s^2	sign.
Blocks	725.50	3		
A (full precision)	6320.25	1	6320.25	+
B (full precision)	3364.00	1	3364.00	+
AB (half precision)	392.00	1	392.00	+
C (full precision)	1936.00	1	1936.00	+
AC (half precision)	12.50	1	12.50	-
BC (full precision)	16.00	1	16.00	-
ABC (full precision)	12.25	1	12.25	-
Uncertainty	36.50	5	7.30	
Total	12815	15		

Conclusion: $Y_{ijkl} = \mu + \mathrm{A}_i + \mathrm{B}_j + \mathrm{AB}_{ij} + \mathrm{C}_k + \mathrm{E}_{ijkl}$
where E denotes the random error (uncertainty) in the experiment, that is the variation *within* chambers .

Question 3

$$
\begin{aligned}
\widehat{\sigma}_E^2 &= \tfrac{12.50+16.00+12.25+36.50}{8} &&= 9.66 = 3.11^2 \\
\widehat{A}_1 &= -\widehat{E}_0 = \tfrac{162+156}{2\times 8} &&= 19.88 \\
\widehat{B}_1 &= -\widehat{F}_0 = \tfrac{108+124}{2\times 8} &&= 14.50 \\
\widehat{AB}_{11} &= \widehat{AB}_{00} = -\widehat{AB}_{01} = -\widehat{AB}_{10} = \tfrac{-56}{8} &&= -7.00 \\
\widehat{C}_1 &= -\widehat{C}_0 = \tfrac{-96-80}{2\times 8} &&= -11.00
\end{aligned}
$$