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Solution to exercise 29

Question 1
Mathematical model:
Cut (s) is a deterministic factor.
Plants (P) and twigs (K) are random factors.
Twigs (K) are nested within plants (P) and crossed with the factor cut (s).

$$Y_{ijkl} = \mu + \mathrm{s}_i + P_j + SP_{ij} + K(P)_{k(j)} + SK(P)_{ik(j)} + G_{l(ijk)}$$

EMS-table

Model term	2	4	3	2	ϕ_s	σ^2_P	σ^2_{SP}	$\sigma^2_{K(P)}$	$\sigma^2_{SK(P)}$	σ^2_G
	i	j	k	l						
s_i	0	4	3	2	24	–	6	–	2	1
P_j	2	1	3	2	–	12	6	4	2	1
SP_{ij}	1	1	3	2	–	–	6	–	2	1
$K(P)_{k(j)}$	2	1	1	2	–	–	–	4	2	1
$SK(P)_{ik(j)}$	1	1	1	2	–	–	–	–	2	1
$G_{l(ijk)}$	1	1	1	1	–	–	–	–	–	1

The analysis of variance testing is started with the high order terms in the model (that is 'from below'). Clearly non-significant terms in the model can be removed if adequate.

Computation of sums of squares for the individual terms in the model is done using the sums of squares resulting from a 'complete crossed' computation by adding the proper contributions:

$$\begin{aligned}
\mathrm{SSQ}_s &= 4294.08; \quad f = 1 \\
\mathrm{SSQ}_P &= 23618.16; \quad f = 3 \\
\mathrm{SSQ}_{SP} &= 35.42; \quad f = 3 \\
\mathrm{SSQ}_{K(P)} &= \mathrm{SSQ}_K + \mathrm{SSQ}_{KP} \\
&= 72.54 + 936.96 = 1009.50; \quad f = 8 \\
\mathrm{SSQ}_{SK(P)} &= \mathrm{SSQ}_{SK} + \mathrm{SSQ}_{SKP} \\
&= 373.79 + 717.71 = 1091.50; \quad f = 8 \\
\mathrm{SSQ}_G &= 12.00 + \cdots + 106.21 = 718.00; \quad f = 24
\end{aligned}$$

Question 2

ANOVA

Source of variation		SSQ	f	s^2	F	sign.	
s	cut	4294.08	1	4294.08	32.70	+	
P	plants	23618.16	3	7872.72	59.96	+	
SP		35.42	3	11.81	< 1	-	conclusion 2
K(P)	twigs	1009.50	8	126.19	0.92	-	conclusion 1
SK(P)		1091.50	8	136.44	4.56	+	
G	residual	718.00	24	29.92			

$F(8, 24)_{0.95}$

Conclusion 1: $\sigma^2_{K(P)} \sim 0$.

$\text{SSQ}_{K(P)} + \text{SSQ}_{SK(P)} = 2101.0$; $f = 8 + 8 = 16$, which now is used to test the remaining effects. $s^2 = 2101/16 = 131.31$. One can also proceed with $\text{SSQ}_{SK(P)}$ as it is. The result will be practically the same.

Testing of the term SP ($F(3, 16) = 11.81/131.31 < 1$) shows (conclusion 2), that $\sigma^2_{SP} \sim 0$.

Next a new pooled s^2 value for $SK(P)$ is computed:

$$s^2_{SK(P)} = \frac{1009.50 + 1091.50 + 35.42}{8 + 8 + 3} = 112.44$$

Final conclusion:

$$Y_{ijkl} = \mu + \text{s}_i + P_j + SK(P)_{ik(j)} + G_{l(ijk)}$$

Question 3

Estimates:

$$\begin{aligned}
\hat{\mu} &= 7021/48 = 146.27 \\
\hat{\text{s}}_0 &= 3281/24 - 7021/48 = -9.56;\ \hat{\text{s}}_1 = 3740/24 - 7021/48 = 9.56 \\
\hat{\sigma}^2_G &= 29.92 = 5.47^2 \\
\hat{\sigma}^2_{SK(P)} &= (112.44 - 29.92)/2 = 41.26 = 6.42^2 \\
\hat{\sigma}^2_P &= (7872.72 - 112.44)/12 = 646.69 = 25.4^2
\end{aligned}$$

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