IMM - DTU Statistical Design and Analysis of Experiments
H. Spliid, October 20, 2010

Solution to exercise 23

Question 1

The experiment is a 2^3-factorial in two blocks (litters) with two repetitions of each factorial combination. It is assumed that complete randomization within blocks has been used.

We call age A, treatment B and gender C, and it is seen that the confounding is done using ABC as defining contrast and the blocking is:

Litter I	Litter II	
(1) ab ac bc	a b c abc	$I = ABC$

Mathematical model:

$$
\begin{aligned}
Y_{ijkl} &= \mu + A_i + B_j + AB_{ij} + C_k + AC_{ik} + BC_{jk} + ABC_{ijk} + blocks + E_{l(ijk)} \\
&\sum_i A_i = \cdots = \sum_k ABC_{ijk} = 0 \\
&E_{l(ijk)} \in \text{NID}(0, \sigma^2) \\
&ABC \text{ is confounded with blocks (litters).}
\end{aligned}
$$

Question 2

To illustrate we use Yates' analysis:

Treatm.	Sum	(1)	(2)	(3)	SSQ	Parameter estimate			F
(1)	-48	-44	7	-108	729.00	-6.75	=	$\widehat{\mu}$	
a	4	51	-115	148	1369.00	9.25	=	$\widehat{A}_1$	84.25
b	15	-94	73	168	1764.00	10.50	=	$\widehat{B}_1$	108.55
ab	36	-21	75	4	1.00	0.25	=	$\widehat{AB}_{11}$	0.06
c	-57	52	95	-122	930.25	-7.63	=	$\widehat{C}_1$	57.25
ac	-37	21	73	2	0.25	0.13	=	$\widehat{AC}_{11}$	0.02
bc	-38	20	-31	-22	30.25	-1.38	=	$\widehat{BC}_{11}$	1.86
abc	17	55	35	66	272.25				ABC and/or litters
					5096.00				

$\text{SSQ}_{residual} = 8.0 + 12.5 + 4.5 + 32.0 + 18.0 + 18.0 + 12.5 + 24.5 = 130,\ f = 8$
$\sum y_{ijkl}^2 = 5226, \quad \text{SSQ}_{resid.} = 5226 - 5096 = 130.0$ (control!)
All terms in the model are tested against $\text{SSQ}_{resid.} = 130$ with 8 degrees of freedom. The term ABC cannot be tested because of the confounding with blocks (litters).

Testing with level of significance $\alpha = 5\%$ is based on the critical F-value $F(1, 8)_{0.95} = 5.32$.

Thus the critical region for a one degree of freedom SSQ is : $\text{SSQ} > 5.32 \cdot 130/8 = 86.45$.

Estimates for significant parameters:

$$
\begin{aligned}
\widehat{\mu} &= -6.75 \\
\widehat{A}_1 &= -\widehat{A}_0 = 9.25 \quad \text{Age} \\
\widehat{B}_1 &= -\widehat{B}_0 = 10.50 \quad \text{Treatment} \\
\widehat{C}_1 &= -\widehat{C}_0 = -7.63 \quad \text{Gender}
\end{aligned}
$$

The effect estimates are $\widehat{A} = 2\widehat{A_1}$, $\widehat{B} = 2\widehat{B_1}$ and $\widehat{C} = 2\widehat{C_1}$, respectively.

Residual variation variance is estimated by

$$\widehat{\sigma}_E^2 = \frac{130 + 1.0 + 0.25 + 30.25}{8 + 3} = 14.68 \simeq 3.8^2$$

Question 3

Using the generator equation $D = ABC$, where $D =$ litter, we obtain the defining relation $I = ABCD$. The complete 2^4 factorial design is divided in two (complementary) $\frac{1}{2} \times 2^4$ factorial designs:

(1)	ab	ac	bc
ad	bd	cd	abcd

and

a	b	c	abc
d	abd	acd	bcd

It is seen that the first of these fractions is the experiment considered, as given in question 1.

In order to find the sign of the alias relation we may consider one single experiment. Take fx (1) where the indices $(i, j, k, l) = (0, 0, 0, 0)$. Therefore D_0 and ABC_{000} are confounded, such that $D_0 = ABC_{000} \iff D_1 = ABC_{111}$. Thus $ABC = +D$.

Litter I $D = 0$		Litter II $D = 1$
(1) ab ac bc	and	ad bd cd abcd

Alias relations:

$$\begin{aligned}
I &= +ABCD \\
A &= +BCD \\
B &= +ACD \\
AB &= +CD \\
C &= +ABD \\
AC &= +BD \\
BC &= +AD \\
ABC &= +D
\end{aligned}$$

Question 4

From the above it is seen that if all D interactions are zero or very small and $ABC = 0$ we get the same analysis as obtained in question 1 and 2, with the only change that the sum of squares for ABC is interpreted as the result of the D-effect alone.

Estimates of significant effects are as in questions 1 and 2 and now further

$$\widehat{D}_1 = -\widehat{D}_0 = 4.13$$

in that we now find that D is strongly significant, because $SSQ_{ABC=D} = 272.25$ $\Longrightarrow F_{ABC=D} = 272.25/(130/8) = 16.75 > F(1, 8)_{0.95} = 5.32$.

. o O o .