Solution to exercise 17

We have a confounded 3^2 factorial design with two repetitions per factor combination. It is assumed that randomization within each day is used. In the experiment the part of the interaction corresponding to the TL^2 term is confounded with days.

Mathematical model:

$$Y_{ijk} = \mu + T_i + L_j + TL_{ij} + \varepsilon_{k(ij)} \quad i = 0,1,2;\ j = 0,1,2;\ k = 0,1$$

or in Kempthorne's formulation (in that $TL_{i,j} = TL_{i+j} + TL^2_{i+2j}$):

$$Y_{ijk} = \mu + T_i + L_j + TL_{i+j} + TL^2_{i+2j} + \varepsilon_{k(ij)}$$

where all indices are computed modulo 3.

When distribution on days using TL^2 we use $Block = (i + 2j)_3$ (i.e. 'modulo 3'), which gives the following placement:

		T = 0	T = 1	T = 2
L	0	0	1	2
	1	2	0	1
	2	1	2	0

, where I $\sim$ 0, II $\sim$ 1, III $\sim$ 2

Computation af sums of squares

$$\text{ssq}_{resid} = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}\left(Y_{ijk} - \overline{Y}_{ij\cdot}\right)^2 = 58.5$$

Cell totals:

	T		
L	47	33	32
	53	35	24
	50	42	35

$$Tot_{\ldots} = \textstyle\sum_{ijk} Y_{ijk} = 47 + 33 + \cdots + 35 = 351$$

$$\text{ssq}_L \quad \boxed{\begin{matrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{matrix}} \rightarrow \frac{(47+33+32)^2}{2\cdot 3} + \frac{(53+35+24)^2}{2\cdot 3} + \frac{(50+42+35)^2}{2\cdot 3} - \frac{Tot_{...}^2}{18}$$

$$= 25.00, \; df = 2$$

$$\text{ssq}_T \quad \boxed{\begin{matrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{matrix}} \rightarrow \frac{(47+53+50)^2}{2\cdot 3} + \frac{(33+35+42)^2}{2\cdot 3} + \frac{(32+24+35)^2}{2\cdot 3} - \frac{Tot_{...}^2}{18}$$

$$= 302.33, \; df = 2$$

$$\text{ssq}_{TL} \quad \boxed{\begin{matrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{matrix}} \rightarrow \frac{(47+24+42)^2}{2\cdot 3} + \frac{(33+53+35)^2}{2\cdot 3} + \frac{(32+35+50)^2}{2\cdot 3} - \frac{Tot_{...}^2}{18}$$

$$= 5.33, \; df = 2$$

$$\text{ssq}_{TL^2} \quad \boxed{\begin{matrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{matrix}} \rightarrow \frac{(47+35+35)^2}{2\cdot 3} + \frac{(33+24+50)^2}{2\cdot 3} + \frac{(32+53+42)^2}{2\cdot 3} - \frac{Tot_{...}^2}{18}$$

$$= 33.33, \; df = 2$$

Which gives:

ANOVA

Variation	Name	SSQ	*df*	s^2	F
Temperature	T	302.33	2	151.17	23.26
Alloy	L	25.00	2	12.50	1.92
Interaction	TL	5.33	2	2.67	0.41
Interaction+Days	TL^2	33.33	2	16.67	(2.56)
Residual		58.50	9	6.50	
Total			17		

It is seen that only the temperature main effect is significant. The alloy has probably no important influence (F-value $\approx F(2,9)_{0.80}$), and with 9 degrees of freedom for the residual it is safe to conclude the following model:

$$Y_{ijk} = \mu + T_i + \varepsilon_{ijk}$$

with

$$\begin{aligned} \widehat{T}_1 &= \overline{Y}_{1..} - \overline{Y}_{...} = 5.50 \\ \widehat{T}_2 &= \overline{Y}_{2..} - \overline{Y}_{...} = -1.17 \\ \widehat{T}_3 &= \overline{Y}_{3..} - \overline{Y}_{...} = -4.33 \end{aligned}$$

$$\begin{aligned}
\widehat{\mu} &= \overline{Y}_{\dots} = 19.50 \\
\widehat{\sigma}^2 &= \frac{58.50 + 5.33 + 25.00}{9 + 2 + 2} = 6.83 = 2.61^2
\end{aligned}$$

The indicated test-quantity for TL^2, it exhibits a weak significance. That could indicate that some variation between days is present.

It can also be noted that the test for the TL_{i+j} interaction is a test for an unconfounded part of the TL interaction. Since the term is not significant it indicates that there is no TL interaction and that the weak significance for the TL^2_{i+2j} part probably is caused by the blocks (days).

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