1 Solutions to exercises 1 – 8

Exercise 1

Mean	=	1.025
Standard deviation	=	0.0001414
Standard deviation of the mean	=	0.0000707

Exercise 2

The two samples can be compared using a conventional t-test or a one-way analysis of variance (unbalanced).

The result is

$$t = \frac{85 - 64}{16.34\sqrt{1/5 + 1/4}} = 1.915$$

From the t-tables $t(4+3)_{0.025} = 2.365$, such that the difference is not significant at the 5% level of significance.
A one-way ANOVA gives

	SS	df	MS	F
Treatment	980	1	980	3.67
Error	1870	7	267.1	
Total	2850	8		

$F_{0.05}(1,7) = 5.59$, which means that the difference between the two types of beams is not significant. Note, that $1.91^2 = 3.67$ and $t(f)^2_{\alpha/2} = F(1,f)_\alpha$.

Exercise 3

A t-test could be carried out under the assumption that the ratings can be approximated by normal distributions with the same variance.

The t-test value becomes t(7+6)=0.53, which is far from significant at any reasonable level. An alternative would be to use a non-parametric test like the two sample Wilcoxon (Mann-Whitney) test. This test can also only be used if the variances within the two groups are identical (questionable here!).

Data ordered value(group)(rank):

1(A)(1) 2(A)(2) 2(A)(3) 2(A)(4) 2(A)(5) 3(B)(6) 3(B)(7) 4(A)(8.5) 4(B)(8.5) 5(B)(10) 7(B)(11) 7(B)(12) 8(B)(13) 9(A)(14) 9(A)(15)

Note that there is one tie.

The rank-sums are
A: 1+2+3+4+5+8.5+14+15 = 52.5 and
B: 6+7+8.5+10+11+12+13=67.5

Critical value for $\alpha = 0.10$ for the B-group is 68, so there is hardly significance for $\alpha = 0.10$.

Exercise 4

The correct method is a paired t-test. For the **differences** the mean, standard deviation and 95% confidence interval for the mean are:

$$\begin{aligned} \widehat{\mu} &= 0.10667 \\ \widehat{\sigma}_E^2 &= 0.0266^2 \\ I(\mu)_{0.95} &= [0.0788; 0.1346] \end{aligned}$$

The assumptions made are that the response variables are (statistically) independent. The distribution of then differences is assumed to be normal with the same mean and variance. It could preferably, for example, be the same person carrying out the two measurements in one sample. It is, for example, not necessary that the same person carries out all 12 experiments.

Exercise 5

Some reasonable questions would be if the test persons were the same gender, age and size. How bad is the asthma before they take the spray? Do they receive the same dosage or is the dosage in accordance to the degree of asthma? If each person can take both A and B a block design could be carried out and a paired t-test (or two-way ANOVA) used for anlyzing the differences.
The results do seem to be paired, as high values in A also have high values in B, so under that assumption a paired t-test leads to a test quantity, Z =

4.79, which is highly significant.
If A and B are not tested on the same person, the persons response to the drug is an extra factor and a paired t-test is not the correct analysis method. Instead a conventional t test for the two means can be used (or a one-way ANOVA). In this case you must check that the variances within the two groups can be assumed to be identical (if in doubt use a method which can handle unequal variances).

Exercise 6

The 95% confidence interval is: L = [420.60;421.04]

Exercise 7

The 10 + 10 measurements are not paired, so a conventional confidence interval for the difference between the two means is asked for. It must again be assumed that the variances within the two groups are identical (if in doubt use a method which does not rely on this assumption). Use $t(9+9)_{0.025} = 2.101$.

The 95% confidence interval of the mean difference in height is: $I(\mu_A - \mu_B)_{0.95} = [-0.2885, 0.8085]$

Exercise 8

There is significant difference between cake-mixture A and B based on a paired t-test. The 95% confidence interval for the difference between the two cake-mixtures is $I(\mu_{A-B})_{0.95} = [-18.38; -4.82]$.