

Canonical Correlations Analysis (CCA)

1 Handwritten exercise

Consider the 4-dimensional random variable

$$\begin{bmatrix} y_1 \\ y_2 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y} \\ \mathbf{x} \end{bmatrix}$$

with mean 0 and dispersion matrix

$$\begin{bmatrix} 1 & \rho & 0 & 0 \\ -\rho & 1 & \rho & 0 \\ 0 & \rho & 1 & \rho \\ 0 & 0 & \rho & 1 \end{bmatrix} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

1. Find the canonical correlations and the canonical variables between the \mathbf{y} and the \mathbf{x} variables.
2. Write a SAS-program and verify your results for $\rho = 0.25$.
3. What happens if $\rho = 0.75$?

2 SAS exercise

Run the following SAS program to do canonical correlations analysis on the cement data set. Interpret the results.

```

Title '51 experiments with measurement of cement strength';
data cement (type=corr);
infile cards missover;
input __type_ $ __Name_ $ C3S C3A BLAINE Strgth3 Strgth28;
cards;
N . 51 51 51 51
corr C3S 1
corr C3A -0.309 1
corr BLAINE 0.091 0.192 1
corr Strgth3 0.158 0.120 0.745 1
corr Strgth28 0.344 -0.166 0.320 0.464 1
;
Title 'Correlations matrix for 51 trials on measuring cement strength';
proc print data=cement;
run;
Title 'Eigenvalues of correlations matrix for 51 trials on measuring cement strength';
proc cancorr data=cement;
var Strgth3 Strgth28;
with C3S C3A BLAINE;
run;
Title 'Analysis where there is conditioned on the fineness BLAINE';
proc cancorr data=cement;
var Strgth3 Strgth28;
with C3S C3A;
partial BLAINE;
run;

```