Assignment on Canonical Discriminant Functions

Consider 3 normal populations corresponding to mean values

$$\boldsymbol{\mu}_1 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \quad \boldsymbol{\mu}_2 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \quad \boldsymbol{\mu}_3 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

and the common dispersion matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

1. Verify that the matrix corresponding to the variation between group means is

$$\boldsymbol{B} = \sum_{i=1}^{3} (\boldsymbol{\mu_i} - \overline{\boldsymbol{\mu}})(\boldsymbol{\mu_i} - \overline{\boldsymbol{\mu}})' = \begin{bmatrix} \mathbf{14} & \mathbf{4} \\ \mathbf{4} & \mathbf{2} \end{bmatrix}$$

Here $\overline{\boldsymbol{\mu}}$ is the average of the three mean values

Consider a random variable $\boldsymbol{X}$ following one of the three distributions and introduce the linear combination

$$Y = \boldsymbol{d}'\boldsymbol{X} = (d_1\ d_2) \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = d_1 X_1 + d_2 X_2.$$

For i=1, 2, 3 the mean and variance of Y are equal to

$$\nu_i = E(Y|i) = \boldsymbol{d}'\boldsymbol{\mu}_i$$

$$\sigma_Y^2 = V(Y|i) = \boldsymbol{d}'\boldsymbol{\Sigma}\boldsymbol{d}$$

2. Verify for $\bar{\nu}$ equal to the average of the ν_i-values that (3-1) × (the empirical variance of the ν_i-values) is

$$\sum_{i=1}^{3} (\nu_i - \bar{\nu})^2 = \boldsymbol{d}'\boldsymbol{B}\boldsymbol{d}$$

Consider the vectors $\boldsymbol{d}_1$and $\boldsymbol{d}_2$ and ((3-1)×) the ratio between the empirical variance of the ν_i-values and the variance of Y

$$\frac{\sum_{i=1}^{3} (\nu_i - \bar{\nu})^2}{\sigma_Y^2} = \frac{\boldsymbol{d}_k'\boldsymbol{B}\boldsymbol{d}_k}{\boldsymbol{d}'\boldsymbol{\Sigma}\boldsymbol{d}}, \quad k = 1, 2.$$

3. Determine the vectors $\boldsymbol{d_k}$ so that the ratio is maximized for *k=1* and maximized under the constraint $\boldsymbol{d}_1'\boldsymbol{\Sigma}\boldsymbol{d_2} = \mathbf{0}$ for *k=2.*

4. Same question as above, but now for the case

$$\boldsymbol{\Sigma} = \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$$

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